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Poverty and natural resource management in the Central Highlands of Eritrea

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Publication date:
2005

Chapter 6

Bio-economic Model of the Farming Systems in the Highlands of Eritrea

6.1 Introduction

In Chapter four, we have presented the major biophysical and socio-economic components of the mathematical model that will be used to analyse farmers' decisions in the Central Highlands of Eritrea. We have described current and potential economic activities and technologies as well as interactions among the various economic activities and biophysical components of the farming system. The socio-economic activities include crop production, livestock production, and tree planting and off-farm employment. The major resources of the farm households include labour and land. The biophysical processes included in our model are soil erosion and nutrient balance (particularly nitrogen). The use of organic and chemical fertilizers, the application of soil conservation structures (stone bund), and tree planting (planting eucalyptus and allowing natural regeneration of woodlands) are the major technologies under consideration.

6.2 Mathematical modelling of the farming systems

Farmers make a large number of decisions. A distinction can be made between decisions on production, consumption and trade and decisions on the application of soil conservation techniques. Production decisions and decisions aimed at improving the natural resource base are often interdependent. The most part of what farmers consume is often produced at the farm and therefore farmers' production decisions are influenced by their consumption habit or vice versa in that activities related to one can have a positive or negative impact on the other. For example, a decision to apply manure on cropland to increase crop production will also improve the soil quality. On the other hand a decision to cultivate steep-slope to increase production, leads to higher erosion.

As farmers make a large number of decisions, we will concentrate on the main decisions about the allocation of their limited resources (mainly land and labour) on crop production, raising livestock and tree planting. Towards this end we will

identify the relevant production, consumption and trade decisions. A decision variable refers to a variable whose optimal level must be determined. A parameter refers to an exogenous factor. A state variable is determined by the values of already defined decision variable(s) and parameter(s).

Once the relevant decisions are identified, we will discuss which exogenous factors affect the decision and the parameters involved. We will adopt a certain rule of notation: all decision variables and state variables will be written in capital letters, and the exogenous factors or parameters by a small letter. The relevant production, consumption and trade decisions, the factors that affect the decisions as well as the relationship among them will be described and modelled in the following sections.

6.3   The planning year and the planning period

As the mathematical model we develop incorporates activities and investments with long-term returns such as tree planting and the construction of soil conservation structures, we will develop a dynamic (multi-annual) model. Thus we will define a planning period as the period (number of years) during which the benefits of all activities and investments will be fully utilized. The number of years in the planning period is called n. We number the years of the planning period as 1, 2, …n and define the set T:

$$T = \{1, 2, …, n\} \tag{1}$$

Farming activity in general and crop production in particular is a seasonal activity. As the farming system in the highlands of Eritrea is generally characterized by rain-fed crop production, both rural income and labour demand for agricultural activities have seasonal characteristics. Most of the jobs have to be done between June and November, which is a peak season in agricultural activities and crops are harvested between October and December. In addition, the availability of labour varies accordingly as many farmers who have off-farm jobs may return to their farming activities and young family members are available full time for farming activities as schools are closed.

We, therefore, define a planning year, which consists of the twelve months following the beginning of the first of January. As the availability of labour as well as labour requirement for farm activities vary considerably from time to time we will divide each planning year into 18 different periods of two weeks or one month. We will divide the six months starting the beginning of June through the end of November (peak period for agricultural activities) into periods of two weeks and the rest 6 months in periods of one month. Thus we will have a set P:

$$P = \{p_1, p_2, \ldots p_{18}) \qquad (2)$$

Where:

p_1: the month of January p_{10}: the first half of August
p_2: the month of February p_{11}: the second half of August
p_3: the month of March p_{12}: the first half of September
p_4: the month of April p_{13}: the second half of September
p_5: the month of May p_{14}: the first half of October
p_6: the first half of June p_{15}: the second half of October   (3)
p_7: the second half of June p_{16}: the first half of November
p_8: the first half of July p_{17}: the second half of November
p_9: the second half of July p_{18}: the month of December

A schematic presentation of the planning period and planning year is given in figure 6.1

Figure 6.1   Planning period and planning year
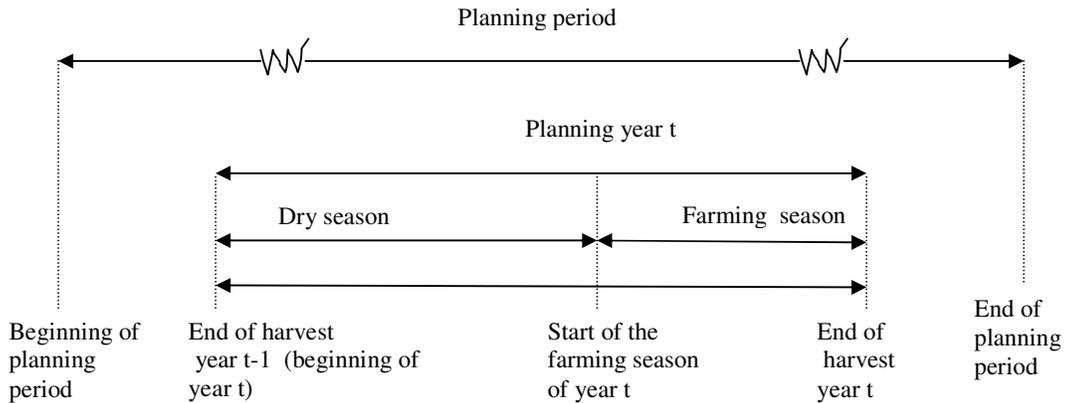
6.4   Land use decisions and land constraints

Each village in the highlands of Eritrea has a limited land area with different slope and soil quality categories. Soil depth, nutrient contents and soil erosion vary across fields of different soil types. As a result, while some soil types are cultivated continuously, others have to be left fallow after cultivating two or three years. A detailed classification of land with respect to slope, soil depth and organic matter content is not available for the study area. As the rate of soil erosion is very much related with slope of land, we will use only the later as a basis for classifying various categories of land. We distinguish four different soil types called s_1, s_2, s_3, and s_4 with:

$s_{1:}$ a slope of 0-8 percent
$s_{2:}$ a slope of 8-16 percent
$s_{3:}$ a slope of 16-30 percent $\hspace{4cm}$ (4)
$s_{4:}$ a slope of 30 percent or more

We will refer to these as "categories of land", "soil types" or "type of land". The set of these four types of land will be defined as:

$$S = \{s_1, s_2, s_3, s_4\} \hspace{4cm} (5)$$

The decisions faced by farmers with respect to land use include:

- Fields to be cultivated and fields to be used for grass and tree planting
- The types of crops to be cultivated on each soil type
- Types and doses of fertilizer to be applied on the different soil types
- The type of soil conservation structure to be built on the different soil
- types

In connection with these decisions, we define four sets relating to types of crops, types of fertilizer, types of soil conservation structures and types of trees. The major types of crops that grow in the study villages include barley, wheat, maize, sorghum, beans, millet, and taff. For reasons of convenience grass and fallow are defined as crops as well. Let us first define a set C as the set that consists of nine types of crops:
$$C = \{c_1, c_2, c_3, c_4, c_5, c_{6,} c_7\}, \hspace{3cm} (6)$$

where

c_1 = barley $\hspace{2cm}$ c_6 = fallow $\hspace{3cm}$ (7)
c_2 = millet $\hspace{2.3cm}$ c_7 = grass
c_3 = beans
c_4 = sorghum
c_5 = wheat (taff)

Manure, crop residues and chemical fertilizers (Urea and DAP[22]) can be applied as fertilizers. Crop residues may also be left on the farm as mulch to improve soil structure. Each fertilizer can be applied in different rates (kg/ha) on its own or in combination with another fertilizer (see also Section 6.7 in this chapter). We will define a set F that consists of seven types of fertilizers.

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[22] Urea contains 46% of Nitrogen by weight and DAP (Diammonium Phosphate) contains 18% of Nitrogen and 46% of phosphorus by weight.

$$F = \{f_0, f_1, f_2, f_3, f_4, f_5, f_6\}, \tag{8}$$

where

f_0 = no fertilizer
f_1 = 1800 kg of manure $\qquad\qquad$ (9)
f_2 = 3600 kg of manure
f_3 = mulch and manure (500kg of crop residues + 500 kg of Manure)
f_4 = mulch and chemical fertilizer (500kg of crop residues + 50 kg of Urea + 50 kg DAP)
f_5 = 50 kg of Urea + 50 kg DAP
f_6 = 100 kg of Urea + 50 kg DAP

We also define the set W as a set that consists of two types of soil conservation structures.

$$W = \{w_0, w_1\}, \tag{10}$$

where

w_0 = no conservation structure $\qquad\qquad$ (11)
w_1 = stone bund

We finally define a set Y as a set that consists of two types of woodlands (see Chapter four)

$$Y = \{y_1, y_2\}, \tag{12}$$

where

y_1 = natural woodlands dominated by acacia trees $\qquad\qquad$ (13)
y_2 = plantations dominated by eucalyptus trees

In the definitions to be introduced below, land of soil type s with soil conservation structure w, is simply called 'land of type (s,w)'. Similarly, crop c is often used in combination with fertilizer f. We will, therefore, simply talk about 'crop (c,f)'. If in a certain year t on a plot of land of soil type s stone bunds are constructed, then the plot is of type (s,w_0) at the beginning of the year and of type (s,w_1) at the end of the year. By definition, 'land type (s,w)' in year t refers to the situation at the end of the year.

As villages in the highlands of Eritrea have clear boundaries, the total area of land available for the various activities (cropland, grassland and woodland) as

well as the current allocation of land among those activities is given. For $s \in S$, $w \in W$, $y \in Y$, we introduce the following parameters:

cland0(s,w)   area of cropland of soil type (s,w) at the beginning
                       of the planning period, in ha   (14)

tland0(s,w,y) area of treeland of soil type (s,w) under tree y at the
                       beginning of the planning period, in ha

At the village level decisions will be made on the allocation of the available land for different uses. Land may be used for crop production, tree planting or grazing. In our base model we assume that fertilizer can be applied only on croplands but soil conservation structures can be built on croplands, woodlands, as well as grasslands. We introduce two types of time indices t and tt, which will help us to remember the age of the trees and to calculate the age dependent wood production. For all $s \in S$, $w \in W$, $c \in C$, $f \in F$, $y \in Y$, $t \in T$ and tt = 0, 1, 2 .. t, we define the following decision variables:

CLAND(s,w,c,f,t)   area of land of soil type (s,w) where in year t   (15)
                          crop (c,f) is cultivated, in ha.

TLAND(s,w,y,tt,t) area of land of soil type (s,w) where in year t trees   (16)
                          of species y grow which were planted in year tt,
                          in ha.

Special attention deserves TLAND(s,w,y,0,t) which refers to the area of land of soil type (s,w) where in year t trees of species y grow, which were planted before the planning period. From definition (16) also follows that, for all tt = 1, 2, … t :

TLAND(s,w,y,tt,tt) area of land of soil type (s,w) where in year tt trees
                          of species y are planted, in ha.   (17)

The establishment of soil conservation structures reduces the area of land that can be used for crop production or tree planting. The length of conservation structures required to control soil erosion from different land categories is different because the extent of erosion varies with the slope of the land. The area that will be occupied by the conservation structures also varies proportionally. Thus we define the following parameter:

pstone(s)   proportion of land type (s) occupied by stone bunds   (18)

If stone bunds are constructed on a piece of land of size a, the area of (1-pstone(s)) × a can be used for crop or tree planting. We will assume that stone bunds can be constructed on woodlands only if the trees are first cleared. The

land may then be replanted or used for crop production. We introduce the following decision variables for all $s \in S$, $p \in P$, $t \in T$:

TSTONE(s,p,t)	area of land of soil type s, where in year t-1 trees grow, which are cut in year t, and where stone bunds are constructed in period p of year t, in ha	(19)
CSTONE(s,p,t)	area of land of soil type s, where in year t-1 crops are grown and in period p of year t stone bunds are constructed, in ha.	

Farmers may also decide to cut trees and use the land either to cultivate crops or replant it with trees. We define for all $s \in S$, $w \in W$, $y \in Y$, $t \in T$ and $tt = 0,1,2$ ..t-1:

TCLAND(s,w,y,tt,p,t) area of land of soil type (s,w), where in period p of year t trees of species y, which were planted in year tt, are cut, in ha	(20)

Stone bunds can be constructed only on land that had no stone bunds previously. Thus we have the following conditions for $s \in S$, $t \in T$, $t \neq 1$, and $tt = 1, 2, \ldots t$:

$$\sum_p CSTONE(s, p, t) \leq \sum_{c, f} CLAND(s, w_0, c, f, t-1) \tag{21}$$

$$TSTONE(s, p, t) \leq \sum_{y, tt} TCLAND(s, w_0, y, tt, p, t) \tag{22}$$

Equations (21) and (22) also apply if the right hand sides are replaced by

$cland0(s, w_0)$ and $\sum_y TCLAND(s, w_0, y, 0, p, 1)$ respectively. $\qquad$ (23)

Note that for reasons of convenience we adopt the notation $\sum_{c, f}$ to refer to summation over all elements of $c \in C$ and $f \in F$. Similarly, $\sum_y$ and $\sum_p$ refer to summations over all elements of $y \in Y$ and $p \in P$ respectively.

6.4.1 Tree growth and the land constraints

The repartition of the available land into crop and woodland, with and without stone bunds, in year t can be different from the repartition in year t-1, because of the following possible activities in year t:
- construction of stone bunds

- cutting of trees
- planting of trees

If trees are cut, woodland is converted into croplands or trees are replanted. If trees are planted, cropland is converted into woodland or woodland is replanted with trees. These possible activities show up in the following expressions, which represent the interrelations between planting, growth and cutting of trees in various years of the planning period. We call these expressions the *tree-balance equations*. For all $s \in S$, $w \in W$, $y \in Y$, $t \in T$ but $t \neq 1$, and $tt = 0,1,2\ldots t-1$, it may be written:

$$TLAND(s,w,y,tt,t) = TLAND(s,w,y,tt,t-1) - \sum_p TCLAND(s,w,y,tt,p,t) \quad (24)$$

For $t = 1$ we have:

$$TLAND(s,w,y,0,1) = tland0(s,w,y) - \sum_p TCLAND(s,w,y,0,p,t) \quad (25)$$

All variables introduced so far are supposed to be non-negative. It follows from (24) therefore, that for all $s \in S$, $w \in W$, $y \in Y$, $t \in T$, and $tt = 0,1,2\ldots t-1$, it has to be satisfied:

$$0 \leq \sum_p TCLAND(s,w,y,tt,p,t) \leq TLAND(s,w,y,tt,t-1)$$

Of course no more trees can be cut in year t than there are available.

The variables defined in (15) and (17) are decision variables determining the planting of crops and new trees in year t. For all $s \in S$, $t \in T$, the expression

$$\sum_{y,p} TLAND(s,w_1,y,t,t) + \sum_{c,f} CLAND(s,w_1,c,f,t)$$

represents the total area of land type (s,w_1) where in year t crops are cultivated and new trees are planted. The available land for these crops and for new trees consists of the following parts:

a) the crop land of type (s,w_1) in year t-1:

$$\sum_{c,f} CLAND(s,w_1,c,f,t-1)$$

b) the crop land of type (s,w_0) in year t-1, where in year t stone bunds are constructed:

$$(1 - pstone(s)) \times \sum_p CSTONE(s,p,t)$$

c) the woodland of type (s,w_0) in year t-1, where in year t trees are cut and stone bunds are constructed

$$(1 - pstone(s)) \times \sum_p TSTONE(s, p, t)$$

d) the woodland type (s,w_1) in year t-1, where in year t trees are cut:

$$\sum_{y,p} \sum_{tt=0}^{t-1} TCLAND(s, w_1, y, tt, p, t)$$

Thus for all $s \in S$, $t \in T$, $t \neq 1$, and for $w = w_1$, it may be written:

$$\sum_y TLAND(s, w_1, y, t, t) + \sum_{c,f} CLAND(s, w_1, c, f, t) = \sum_{c,f} CLAND(s, w_1, c, f, t-1)$$

$$+ (1 - pstone(s)) \times \sum_p \{CSTONE(s, p, t) + TSTONE(s, p, t)\}$$

$$+ \sum_{y,p} \sum_{tt=0}^{t-1} TCLAND(s, w_1, y, tt, p, t)$$

$$(26)$$

Equation (26) holds also for t = 1, if $\sum_{c,f} CLAND(s, w_1, c, f, t-1)$ is replaced

by cland0(s,w_1). $\qquad (27)$

Similarly for $w = w_0$ and $t \in T$, and $t \neq 1$:

$$\sum_y TLAND(s, w_0, y, t, t) + \sum_{c,f} CLAND(s, w_0, c, f, t) = \sum_{c,f} CLAND(s, w_0, c, f, t-1)$$

$$- \sum_p \{CSTONE(s, p, t) + TSTONE(s, p, t)\}$$

$$+ \sum_{y,p} \sum_{tt=0}^{t-1} TCLAND(s, w_0, y, tt, p, t)$$

$$(28)$$

Equation (28) holds also for t=1 if $\sum_{c,f} CLAND(s, w_0, c, f, t-1)$ is replaced

by *cland0(s,w_0)*. $\qquad (29)$

Note that in (26) and (27) the losses of available land due to the construction of stone bunds are taken into account by including the parameter *(1-pstone(s))*. Understandably, the parameter is not included in (28) and (29).

The expressions (24) – (29) represent the *land constraints*, which postulate that in each year and for each land type (s,w), the land in use for crop and tree cultivation equals the available land. The equality says that all the village land will be used for one or the other type of economic activity at any given time.

This is because grass and fallow land, which require little labour input, are included. The total land area of type s available at the village remains the same throughout the planning period. The area where stone bunds have been constructed, however, changes over time.

Moreover, additional constraints will be imposed that reflect the suitability of different soil types for crop production. For example soil type s_4 is too steep to be used for crop production. Thus we have the following constraints:

CLAND(s,w,c,f,t) = 0 where s = s_4 for all c $\in$ C, c $\neq c_7$; f $\in$ F, w $\in$ W
and t $\in$ T (30)

6.5   Crop production and consumption modelling

 Crop production at any given time is a function of yields and crop area. Yield, in turn, is a function of various factors including soil type, the type and quantity of fertilizer applied, as well as the application of soil conservation methods (see Chapter seven). For all s $\in$ S, w $\in$ W, c $\in$ C, f $\in$ F, t $\in$ T, we define the following parameter.

yld(s,w,c,f)   yield of crop (c,f) from soil type (s,w) in year t (31)

Fallowing is a common practice in the Central Highlands of Eritrea (see Chapter five). We will assume that if a certain plot is cultivated at any time, some other parcel of land will remain fallow at the same time. We call this parcel the fallow supplement of the cultivated crop. We will assume that the length and frequency of fallowing (and hence the size of the fallow supplement) does not depend on the type of soil, or the application of fertilizer. Thus we define

fal   the ratio of area of land left fallow in year t to the area of land
        cultivated in year t (32)

If fallow land corresponds to crop (c_6, f_0), the total fallow land type (s,w) in year t is given as follows:

$$CLAND(s,w,c_6,f_0,t) = \sum_{c,f} fal \times CLAND(s,w,c,f,t)$$ (33)

No fertilizer is applied on fallow and grassland. Thus:

CLAND(s,w,c,f,t) = 0, for c = c_6, c_7; f $\in$ F, f $\neq$ f_0 (34)

The total production of each crop, in any given year will be the yield per hectare of that crop multiplied by the area of land occupied by that crop under the various land management practices. We define for all c ∈ C, t ∈ T:

TPROD(c,t) total amount of crop c produced in year t, in kg　　　　　　(35)

The crop production of crop c ∈ C in year t is given by:

$$TPROD(c,t) = \sum_{s,w,f} yld(s,w,c,f,t) \times CLAND(s,w,c,f,t) \qquad (36)$$

6.5.1　Crop residues

Crop production results into the production of an important by-product – crop residues. Crop residue is an important supplement as animal feed in the Central Highlands where shortage of animal feed is the major constraint to livestock production. Crop residues, particularly residues of maize and sorghum are also used as fuel. Another possible use of crop residues which is not currently practised in the study area is as fertilizer (see chapter five). We will assume that the amount of crop residues produced is proportional to crop production, i.e., the ratio between crop yield and a crop residue does not depend on the type of fertilizer applied or the construction of stone bunds. However the ratios of crop residue to crop yield vary from one crop to the other. Thus we define the following parameter and state variable for all c ∈ C, t ∈ T:

CROPRES(c,t)　total amount of crop residues of crop c produced in
　　　　　　year t, in kg
resid(c)　　　　the ratio of crop residues to yield of crop c　　　　　(37)

The total amount of crop residue from each crop is given as follows:

CROPRES(c,t) = resid(c) × TPROD(c,t)　　　　　　　　　　(38)

Crop residues produced in a given year may be used in the same year or stored for use in the following years. Thus the sum of crop residues used for the various uses in a given year should not exceed the crop residues of each crop produced in that year plus residues carried over from the previous year, which implies the conditions in (40) and (41). We first define the following variables and a parameter for c ∈ C and t ∈ T:

CRESFUEL(c,t)	the portion of residues of crop c used as fuel in year t, in kg	(39)
CRESFEED(c,t)	the portion of residues of crop c used as animal feed in year t, in kg	
CRESFERT(c,t)	the portion of residues of crop c used as fertilizer in year t, in kg	
CRESTOCK(c,t)	the amount of residues from crop c that remain in stock at the end of year t, in kg	
crestock0(c)	the amount of residues from crop c available in stock at the beginning of the planning period, in kg	

The *stock equations* for the residues for c ∈ C and t > 1 and t = 1 are given in (40) and (41) respectively.

$$CRESTOCK(c,t) = CRESTOCK(c,t\text{-}1) + CROPRES(c,t) - CRESFUEL(c,t) \\ - CRESFEED(c,t) - CRESFERT(c,t) \tag{40}$$

Equation (40) also applies for t=1 if *CRESTOCK(c,t-1)* is replaced by *crestock0(c)* (41)

Figure 6.2   Periods when crop residues are produced and applied for mulching
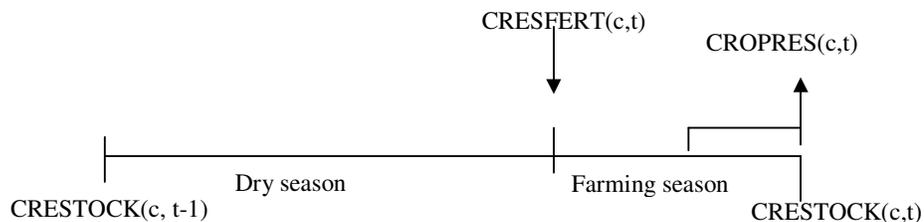
As shown in the above diagram, if crop residue is applied as fertilizer (mulching) in year t, it has to be applied at the beginning of the farming season, in which case crop residue to be used for mulching in a given year has to come from crop residues produced in previous year. Thus for all c ∈ C and t ∈ T and t ≠ 1, the following condition must be satisfied.

$$CRESFERT(c,t) \leq CRESTOCK(c,t-1) \tag{42}$$

Condition (42) also applies to t=1 if *CRESTOCK(c,t-1)* is replaced by *crestock0(c)* (43)

6.5.2   Consumption, buying and selling of crops

The household can do a number of things with its production. It may consume part of its produce, keep part of the produce to be used as seed, sell it in the market and/or store it for use in the coming year. The household may also buy crops from the market or use a stock of crops from previous years. Price variations in different periods of the year and farmers' buying and selling strategies in different periods within one year are not taken into consideration. However, due to the fact that farmers usually sell crops right after the harvest when prices are lower and buy in later periods when price are higher, the buying and selling prices of each crop will be different. For c ∈ C and t ∈ T we define the following variables and parameters.

BUYCROP(c,t) the amount of crop c bought during year t, in kg   (44)
SELLCROP(c,t)   the amount of crop c sold during year t, in kg
FOOD(c,t)   amount of crop c consumed by the village
                 members during year t, in kg
SEED(c,t)   amount of crop c used as seed in year t, in kg
STOCK(c,t)   stock of crop c at the end of year t, in kg
popl(t)   total number of people in the village in year t
calcont(c)   amount of calorie in one kilogram of crop c, kilocalories
calreq   annual amount of calorie requirement per person, in kilocalories
sdreq(c)   seed requirement of crop c, in kg/ha
popl0   total number of persons in the village at the beginning of
                 the planning period
stock0(c)   amount of crop c available in stock at the beginning of
                 the planning period

The minimum calories required for all residents of the village have to be met from the crops consumed by the members of the village. Thus for all t ∈ T:

$$\sum_{c=1}^{5} calcont(c) \times FOOD(c,t) \geq calreq \times popl(t) \tag{45}$$

The amount of seed of crop c required in each year is proportional to the area of land cultivated with that crop in the same year. For all c ∈ C and t ∈ T we write:

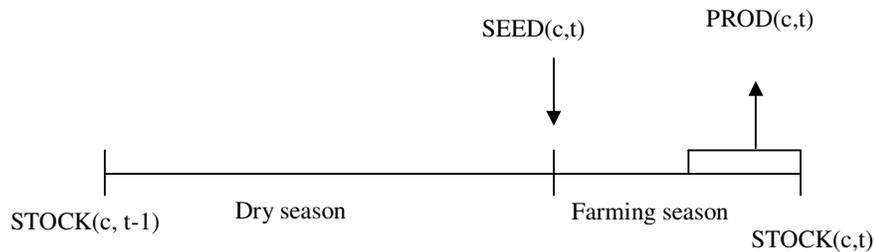$$SEED(c,t) = \sum_{s,f,w} sdreq(c) \times CLAND(c,s,f,w,t) \tag{46}$$

We also write the *crop balance equations* for c ∈ C and t∈ T, t ≠ 1 as follows:

$$STOCK(c,t) = STOCK(c,t-1) + PROD(c,t) + BUYCROP(c,t)$$
$$- SELLCROP(c,t) - FOOD(c,t) - SEED(c,t) \tag{47}$$

To write the crop balance equation for t = 1, the first term in (47) will be replaced by stock0(c), i.e., the stock of crop c at the beginning of the planning year. (48)

Figure 6.3   Crop production and seed requirement

The diagram in Figure 6.3 shows that while the production of crops is harvested at the end of the farming season, seed is required at the beginning of the farming season. Equation (49) ensures that the seed required in each year must be met from what remains in stock from previous year and/or from crop bought in the year in question.

$$SEED(c,t) \leq STOCK(c,t-1) + BUYCROP(c,t) \tag{49}$$

Condition (49) also applies to t=1 if *STOCK(c,t-1)* is replaced
by *stock0(c)* (50)

We also postulate *STOCK(c,t)* ≥ 0 for all t $\in$ T (51)

This implies that there is enough crop to cover food demand and seed requirement in each year. We will later make modifications to reflect the situation where rural households face shortages.

6.6   Wood and grass production

As discussed in Chapter three rural households in the highlands of Eritrea widely use tree products mainly for fuel wood and construction. These resources may be obtained from natural woodlands or from individual or community plantations. Thus farmers will have to decide on the area of land they want to keep under natural woodlands and under plantations. The decision will be influenced by the opportunity cost of the land (forgone crop production or grazing) and the costs involved in establishing and maintaining plantations on the one hand and the benefits in terms of output of tree products on the other. The production of wood for various uses in year t will be expressed in the

variables already introduced. We define for all s ∈ S, w ∈ W, y ∈ y, t ∈ T, tt = 0,1,2…t:

WDHARV(s,w,y,t) harvested wood in year t from trees of species y   (52)
                              growing on land type (s,w), in kg
vwtland(s,w,y,tt,t) volume of wood in year t of all trees of species y,
                              which were planted in year tt and have grown
                              all years tt, tt+1,…t on the land of type (s,w), in kg/ha
wdyld(s,w,y)   annual increase in the volume of wood from tree
                              species y planted on land type (s,w), in (kg/ha/year)
vwtland0(s,w,y)   initial volume of wood on woodlands of tree species y,
                              in kg/ha
VWDWDL(y,t)   total amount of wood type y in woodlands in year t

Initially there is a stock of wood in the woodlands. The stock of wood will decrease when rural people collect wood for fuel and other uses and when woodlands are converted into crop or grasslands. On the other hand, the stock of wood will increase over time by the establishment of new woodlands and the natural growth on the woodlands. For the purpose of simplicity we will express the volume of wood on a per hectare basis. We will also assume that if farmers collect wood, they cut all the trees on a piece of land[23]. Moreover, although the rate of growth of trees is of a non-linear nature where yield (the mean annual increment) depends on the age of the trees, due to lack of data we will assume a linear growth. The yield of wood varies by the type of trees planted as well as by the type of land. For s ∈ S, w ∈ W, y ∈ y, t ∈ T, tt = 1,2…,t, the parameters in (52) can approximately be written as:

$$vwtland(s,w,y,tt,t) = wdyld(s,w,y) \times (t - tt) \qquad (53)$$

For tt = 0 we have:

$$vwtland(s,w,y,tt,t) = vwtland0(s,w,y) + wdyld(s,w,y) \times (t - tt) \qquad (54)$$

Note that the parameter *vwtland(s,w,y,tt,t)* is expressed on a per ha basis and changes over time only due to natural growth. The initial values *vwtland0(s,w,y)* are estimated on the basis of the age of the trees. Also note the wording "have grown all years tt, tt+1,…,t on the land type (s,w)" in the definition of *vwtland(s,w,y,tt,t)* in (52). If trees grow in year t on land without stone bunds, then they have grown on land without stone bunds throughout all years tt, tt+1,…, t. Thus for s ∈ S, w ∈ W, y ∈ y, t ∈ T, tt = 0,1,2…,t, the amount of wood harvested may be written as:

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[23] In practice farmers may collect dry wood or cut only branches of a tree as well.

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$$WDHARV(s,w,y,t) = \sum_{tt=0}^{t}\{vwtland(s,w,y,tt,t) \times \sum_{p}TCLAND(s,w,y,tt,p,t)\ \}(55)$$

The total amount of wood that is available on the woodlands, for y ∈ y, t ∈ T, will then be written as follows:

$$VWDWDL(y,t) = \sum_{s,w,tt} vwtland(s,w,y,tt,t) \times TLAND(s,w,y,tt,t) \tag{56}$$

The wood farmers harvest can be used as fuel, but there is also the possibility of selling it either for fuel wood or for construction purposes in the case of eucalyptus. Only some portion of the eucalyptus can be used for construction purposes (say 20%). Rural households may also buy fuel wood. Thus we introduce the following three non-negative decision variables:

WDFUEL(y,t)   amount of wood type y that is used as fuel in year t, in kg
SELLWOOD(y,t) amount of wood type y sold in year t, in year t in kg   (57)
BUYWOOD(y,t) amount of wood type y bought in year t, in kg
WDSTSOCK(y,t) stock of wood from tree species y at the end of year t, in kg
wdstock0(y)   initial stock of wood type y available for use, in kg

This leads, for all s ∈ S, w ∈ W, y ∈ y, t ∈ T, t ≠ 1, to the following constraints:

$$WDSTOCK(y,t) = WDSTOCK(y,t-1) + \sum_{s,w}WDHARV(s,w,y,t)$$
$$+ BUYWOOD(y,t) - SELLWOOD(y,t) - WDFUEL(y,t) \tag{58}$$

Equation (58) also applies to t = 1 if the first term in the right hand side of the equation is replaced by *wdstock0(y)*. (59)

Equations (58) and (59) show that households can use wood harvested or bought in previous years if it is not used or sold in that year.

6.6.1   Grass production
Grass is produced from grasslands, fallowlands and woodlands. We assume that the yield of grass from grasslands and fallowlands are the same. The yield of grass from woodlands, however, differs due to differences in tree density which,

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in turn, differs by types of trees[24]. The total amount of animal feed available to the farmer is the sum of grass from all the above sources. So we introduce the following parameters and state variables.

gryld(s,w)   yield of grass from grassland of land type (s,w)
gryldw(s,w,y)   yield of grass from woodland of land type (s,w)   (60)
                     where tree type y is planted, kg/ha
GRASS(t)   total amount of grass produced in year t, in kg

The total grass production in year t, for all $s \in S$, $w \in W$, $y \in y$, $t \in T$, $tt = 0,1,2\ldots,t$, is written as follows:

$$GRASS(t) = \sum_{s,w} gryld(s,w) \times \{CLAND(s,w,c_6,f_0,t) + CLAND(s,w,c_7,f_0,t)\}$$

$$ + \sum_{s,w,y,tt} gryldw(s,w,y) \times TLAND(s,w,y,tt,t))$$

(61)

6.7 Livestock modelling

The growth of the livestock is determined by weight gain and birth and mortality rates. Households decide the number and composition of livestock they keep in each period. They also buy and sell livestock if it is economically attractive. The common types of animals they keep include oxen, cows, donkeys, sheep and goat. Different animals are kept for different purposes. Thus we will distinguish between four types of livestock where

     v_1: oxen
     v_2: cows   (62)
     v_3: donkeys
     v_4: sheep and goats

and define the set V which consists of four types of livestock.
$V = \{v_1,v_2,v_3,v_4\}$ (63)

In deciding on the number of livestock they keep, farmers compare the benefits and costs of keeping additional livestock. Benefits include milk from cattle, sheep and goats; cash from selling livestock; and animal power for traction and transport by oxen and donkeys respectively. The costs, on the other hand, include cash outlays for veterinary services and reduced income from other

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[24] Leaves from natural woodlands are important source of animal feed in the Central Highlands. See chapter seven on estimations of yield of grass from natural woodlands and eucalyptus plantations.

activities (such as crop production) as raising livestock competes for limited resources such as labour and land.

The number of livestock in any given year is determined by the number of livestock at the beginning of the year, the natural rate of growth, as well as the buying and selling decisions of the farmers (see Section 7.6 how the natural rate of growth of livestock is determined). We introduce the following variables and parameters:

LVSTK(v,t)	the number of livestock units v in the village at the end of year t	(64)
SELVSTK(v,t)	the number of livestock units of type v sold during year t	
BUYLVSTK(v,t)	the number of livestock units of type v bought during year t	
lvstck0(v)	the number of livestock units of type v available in the village at the beginning of the planning period	
grlvstk(v)	annual natural rate of increase in the number of livestock v	

Thus for all v ∈ V and t ∈ T, t ≠ 1, the number of livestock v in year t will be:

$$LVSTK(v,t) = (1 + grlvstk(v)) \times [LVSTK(v,t-1) + BUYLVSTK(v,t) - SELLVSTK(v,t)]$$

$$(65)$$

Equation (65) also applies to t = 1 if $LVSTCK(v,t-1)$ is replaced by $lvstck0(v)$ (66)

6.7.1 Feed availability and livestock

We assume that some minimum number (proportional to the cropland cultivated) of oxen and donkeys will be required for ploughing and transport purposes respectively. The maximum number of livestock farmers can keep is also determined by the availability of fodder i.e. the upper limit of this choice will be determined when the village decides to allocate land among crop production, grazing and tree-planting activities. This is because such a decision determines the amount of forage available for livestock.

We have already defined *CRESFEED(c,t)*, (39) which indicates the proportion of crop residue to be used as animal feed. We have also defined *GRASS(t)* the amount of animal feed available from grasslands, woodlands and croplands (60).

We introduce the parameters in (67) which indicate feed required for each type of livestock, dry organic matter content of grass and crop residues respectively.

feedreq(v)	amount of feed required per unit of livestock type v, in kg DOM/year
domcong	Dry Organic Matter content of grass, in kg/kg of grass
domconcr	Dry Organic Matter content of crop residue, in kg/kg of crop residue

(67)

For t ∈ T, we impose the following constraint:

$$\sum_v feedreq(v) \times LVSTK(v,t) \leq \sum_c domconcr \times CRESFEED(c,t) \\ + domcong \times GRASS(t)$$

(68)

i. e the total animal feed requirement must be satisfied by all the resources used as animal feed. We will later include the possibility of purchasing animal feed.

6.7.2 Animal power requirement and livestock

The minimum livestock units required for ploughing, transporting crops, crop residues, manure etc. to and from the farm, home and market, determines the lower limit of the number of livestock kept by farmers. Almost all ploughing is done by animal traction using a pair of oxen. Therefore, the amount of area cultivated for crop production is limited by the availability of oxen (v_1). The availability of iron plough and other farm implements is considered less a constraint and therefore not considered here. The number of days an ox is available for work is limited by religious reasons and physical capacity of the ox. The amount of area cultivated in any given year will also be limited by the amount of adult male labour that has to accompany the pair of oxen. As some agricultural activities have to be done in a given period (sometimes of very short duration) that it is necessary to impose constraints regarding animal labour requirement on a period by period basis (see the cropping calendar in fig 5.1). We define the following parameters for all c ∈ C, p ∈ P:

oxcult(p,c)	number of oxen days required in period p per ha of land category s, used to cultivate crop c.
oxdays(p)	number of days an ox can work in period p

(69)

For t ∈ T and p ∈ P we write the constraint:

$$\sum_{c,s,f,w} oxcult(p,c) \times CLAND(c,s,f,w,t) \le oxdays(p) \times LIVESTOCK(v_1,t) \qquad (70)$$

Donkeys (v_4) serve a number of activities including transporting crops and crop residues from the farm to the house, transporting manure to the field, fuel collection, fetching water and transporting goods to and from the market. However, donkeys are more intensively used in periods when agricultural activities are at peak. We will, therefore, impose a constraint relating only to those periods. The transportation capacity is determined by the number of donkeys, the number of days a donkey can be used in a given period and the average ton/km capacity of a donkey. For $p \in P$ we introduce the following parameters:

donkday(p) number of days a donkey can work in period p (71)

wdonkey weight potentially transported by a donkey, in ton km/day
distf average distance of cropland from the village (settlement), in km

As discussed in Chapter five, livestock are allowed to graze on croplands after the harvest. Thus farmers have to collect their crops and crop residues as soon as possible. Thus the transportation of crops and crop residues has to be carried out in a specific period. Similarly, the application of Manure has to be done before the onset of the farming season so that the manure would be mixed with the soil during land preparation. Figure 6.4 shows the periods during which the application of manure and transportation of crops and crop residues should be made.

Figure 6.4   Periods where animal power is required for transport of manure, crop and crop residues
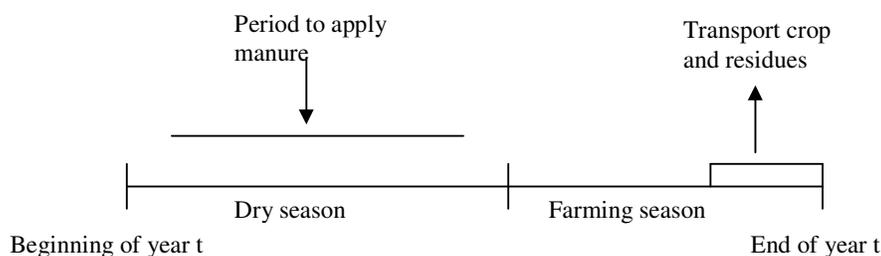
Thus we include additional constraints as in (72) and (73). The constraints show that the total number of animal days required to transport crops, crop residues and fertilizer between the farm and the house in each period should be less or equal to the transportation capacity of the livestock. For $t \in T$ we write the constraints:

$$\sum_{p=12}^{18} LIVSTCK(v_3,t) \times donkday(p) \times wdonkey \geq \sum_{c} CROPRES(c,t) \times distf$$

$$(72)$$

$$+ \sum_{c} PROD(c,t) \times wdonkey$$

$$\sum_{p=2}^{6} LIVSTCK(v_3,t) \times donkday(p) \times wdonkey \geq MANFERT(v,t) \times distf \qquad (73)$$

6.7.3 Milk production

The average milk production multiplied by the total number of livestock gives the total volume of milk production. For all $v \in V$ and $t \in T$ we define the following variable and parameter (74) and write an equation for milk production as shown in (75):

MILK(t)   Total milk production   in year t in litres
myld(v)   the average yield of milk per head livestock v in litres/year   (74)

$$\sum_{v} myld(v) \times LVSTK(v,t) = MILK(t) \qquad\qquad (75)$$

6.7.4 Manure production and use

Manure is another important resource produced by livestock. In the Central Highlands of Eritrea where farmers are too poor to afford chemical fertilizer, manure is the major type of fertilizer applied to maintain soil fertility. Manure, particularly from cattle, is also an important source of fuel for domestic energy. Both the quantity of manure produced from each type of livestock and the nutrient content of the manure depend on the quantity and type of animal feed. We, nevertheless, assume that livestock are fed a certain fixed quantity (according to feed requirement for each type of animal) and produce the same quantity and quality of manure. We also assume that the nutrient content of manure is the same. We introduce the following variables, parameters and an equation for manure production for $v \in V$ and $t \in T$.

MANURE(v,t) total amount of manure produced by livestock type v
          in year t, in kg   (76)
manyld(v)   average amount of manure per animal produced
          by livestock type v, in kg/year

$$MANURE(v,t) = manyld(v) \times LVSTK(v,t) \qquad (77)$$

The total amount of manure will be used either for fuel or fertilizer. Manure produced in a given year may be used in the same year or stored for use in the following years. We thus define two non-negative decision variables, a state variable and a parameter for $v \in V$ and $t \in T$ as follows: and write the manure constraint as follows:

MANFERT(v,t)   quantity of manure from livestock v used as fertilizer, in kg

MANFUEL(v,t)   quantity of manure from livestock v used as fuel, in kg   (78)

MANSTOCK(v,t) amount of manure from livestock v available in year t, in kg

manstock0(v)   mount of manure available in the village at the beginning of the planning period, in kg

Thus the *manure balance* for $v \in V$, t =1 and t > 1 respectively are written as:

$$MANSTOCK(v,t) = MANSTOCK(v,t-1) + MANURE(v,t)$$
$$- MANFERT(v,t) - MANFUEL(v,t) \qquad (79)$$

Equation (79) also applies to t=1 if $MANSTOCK(v,t-1)$ is replaced by *manstock0(v)*   (80)

Livestock produces manure throughout the year. However manure produced during the farming season is not available for use as fertilizer in the same year. Assuming that only half of the manure produced in a given year could be applied as fertilizer in that same year, we need the following additional constraints. For $v \in V$, t > 1 and t =1 respectively:

$$MANFERT(v,t) = MANSTOCK(v,t-1) + 0.5 * MANURE(v,t) \qquad (81)$$

Equation (81) also applies to t=1 if $MANSTOCK(v,t-1)$ is replaced by *manstock0(v)*   (82)

6.8   Fertilizer balance

Farmers can use fertilizer produced on the farm (manure and crop residues) and/or buy chemical fertilizer. The amount of crop residues and manure available for use as fertilizer is influenced by decisions of the farmers relating to

crop and livestock production as well as his decision on how to allocate these resources among different uses such as fertilizer, fuel and animal feed. The amount of input used as fertilizer should not exceed the amount of that input available for use as fertilizer. We assume that farmers will use each type of fertilizer either at a prescribed rate or do not use that input at all. For f ∈ F, we define the following parameters:

manurate(f) amount of manure applied when fertilizer type f is applied, in kg/ha   (83)

residrate(f) amount of crop residue applied when fertilizer type f is applied, in kg/ha

urearate(f) amount of Urea applied when fertilizer type f is applied, in kg/ha

daprate(f) amount of DAP applied when fertilizer type f is applied, in kg/ha

Note that for all f ∈ F the values of these parameters are given due to the definitions in (9).

The available fertilizer is composed of four components, two of which, *CRESFERT(c,t)* and *MANFERT(v,t)*, already described as the portion of crop residues and manure respectively used as fertilizer in (39) and (76). The remaining two, Urea and DAP, refer to chemical fertilizers and we introduce two related decision variables indicating the amount of each type of chemical fertilizer bought and used in each year. We first define the decision variables for t ∈ T:

BUYUREA(t) amount of Urea purchased in year t, kg   (84)

BUYDAP(t) amount of DAP purchased in year t, kg

To ensure that the demand for each type of fertilizer does not exceed the supply, for t ∈ T, we include the following constraints:

$$\sum_{v} MANFERT(v,t) = \sum_{s,w,c,f} manurate(f) \times CLAND(s,w,c,f,t)$$

$$\sum_{c} CRESFERT(c,t) = \sum_{s,w,c,f} residrate(f) \times CLAND(s,w,c,f,t)$$
(85)

$$BUYUREA(t) = \sum_{s,w,c,f} urearate(f) \times CLAND(s,w,c,f,t)$$

$$BUYDAP(t) = \sum_{s,w,c,f} daprate(f) \times CLAND(s,w,c,f,t)$$

6.9 Energy modelling

Rural households use fuel wood, dung, crop residues and kerosene for cooking, heating and lightening purposes. Using kerosene and purchasing wood from the market involve cash outlays. Alternatively, farmers may obtain biomass resources freely from natural sources but this involves opportunity costs in terms of labour for their collection as well as forgone output from alternative use of the resources. Thus the costs and benefits associated with each source of energy determine the composition of fuels for households use. We define a decision variable and some parameters as follows:

KEROSENE(t)	amount of kerosene bought in year t, in litres	(86)
crencont(c)	the amount of useful energy per unit of residues from crop c, in MJ/kg	
mnencont(v)	the amount of useful energy per unit of manure from livestock v, in MJ/kg	
wdencont(y)	the amount of useful energy per unit of wood from tree of species y, in MJ/kg	
krencont	the amount of useful energy per unit of kerosene, in MJ/litre	
enreq	average energy requirement per person per year, in MJ	

The total amount of energy households' use from all those resources should at least be equal to the minimum energy requirements of the population. For $v \in V$, $c \in C$, $y \in Y$, and $t \in T$, we have the following constraint:

$$enreq * popl(t) \leq \sum_c crencont(c) \times CRESFUEL(c,t)$$
$$+ \sum_v mnencont \times MANFUEL(v,t)$$
$$+ \sum_y wdencont(y) \times WDFUEL(y,t) \quad (87)$$
$$+ krencont \times KEROSENE(t)$$

We will later consider other options related to rural energy such as new energy saving stoves and the use of liquefied petroleum gas (LPG).

6.10 Population and labour

As discussed in 6.3 both the demand for and the supply of labour in the rural areas vary considerably from one period to the other. Thus it is important that we put the labour constraint for each period. In addition, we will also distinguish between adult male labour and total labour available for agricultural activities.

This distinction is necessary because while most farming activities can be done by any member of the household, ploughing (land preparation and sowing) is traditionally done by men. Due to the 30 years war of independence and the recent war with Ethiopia, shortage of adult male labour is a serious constraint in agricultural production in Eritrea (see Chapter five).

The number of days each labour category is available for various economic activities is an important constraint for the farming household. In this study we will assume that this is a parameter. We will calculate the number of days members of a household (adult male, adult female and children) will be available for agricultural activities by taking into account all relevant variables. For example children will be available only for part of the days when schools are open; women's time will be adjusted for the time required to undertake household responsibilities and both adult male and adult female time will be adjusted for other social obligations. Finally the number of days rural household can undertake agricultural activities in each period is limited by religious holidays. However such constraints do not apply to non-farm activities. Thus we distinguish between number of days in which any activity can be done and number of days in which only some activities can be done in each period. Population size in any given year is the population in the previous year adjusted for the natural rate of growth. Migration into and out of the village is not considered in this study. For p ∈ P, and t ∈ T, we define some parameters.

avmlbag(p,t)	the quantity of male labour (days) available for agricultural activities in period p of year t	(88)
avmlbal(p,t)	the quantity of male labour (days) available for all activities in period p of year t	
avlbag(p,t)	the total amount of labour (days) available for agricultural activities in period p of year t	
avlbal(p,t)	the total amount of labour (days) available for agricultural activities in period p of year t	

Along with land, labour, is the most important input in the rural areas of Eritrea for all economic activities. Crop production, livestock and tree planting all require labour. The amount of labour required to cultivate a hectare of cropland depends on the land management practices of the farmer. The application of fertilizer and soil conservation activity influences the amount of labour input required in crop production. Application of manure and crop residues as a fertilizer requires labour (and/or animal power) to transport them to the fields. The constructions of soil conservation structures also require considerable amount of labour. The amount of labour required for transporting fertilizer varies with distance to the farm. For simplicity we will consider the average distance of the farms from the village. The amount of labour required for

undertaking conservation activity and the area occupied by the conservation structures, on the other hand, vary with land categories. Farms on steep slopes require longer structures because conservation structures have to be built close to each other on steeper farms than on farms with gentle slope. Thus the amount of labour required for undertaking conservation activities varies with the slope of the land.

In addition, labour is required to keep livestock. The amount of labour required to take care of livestock is very difficult to model precisely, mainly because of the various types of arrangements made amongst households in the villages of the Central Highlands (see Chapter five and Chapter seven).

Labour is also required to plant trees. The amount of labour required for this activity is available from the records of the Ministry of Agriculture (see Chapter seven). Finally labour is required for the collection of fuel wood and dung. However, discussions with farmers show that households collect fuel wood and dung either in periods when they are free from agricultural activities or on their way home after accomplishing their agricultural task. Therefore, we do not include labour required for fuel collection in our labour constraint in (90). The benefits from tree planting will be included in the model not from the time saved in fuel collection but from the manure and crop residue that can eventually be used as fertilizer and animal feed respectively (rather than using them for fuel) and the potential to sell fuel wood and/or construction materials.

Farmers in the Highlands of Eritrea, particularly those in villages close to the capital city and major towns, are also involved in off-farm employments. The availability of off-farm employment opportunity was difficult to determine from the fieldwork as most of the young people are mobilized to join the army (see Chapter five). We assume only adult males will have access to off-farm jobs. We define a decision variable and parameters as follows:

OFFARM(p,t)   the number of days farmers in the village engage in
                off-farm jobs, in mandays/year
labcult(c,p)   amount of labour required in period p to cultivate one ha
                of land under crop c excluding labour required for
                the construction and maintenance of stone bunds and
                application of fertilizer, in mandays/ha
labcutre (y)   amount of labour required to cut trees of species y,
                in mandays/ha
  labcons(s)   amount of labour required to build stone bunds on land
                category w, in mandays/ha   (89)
labtree(p,y)   amount of labour required for planting trees of type y in
                period p, in mandays/ha

lablivs(p,v)   amount of labour required to tend one head of livestock v
                        in period p

mlab(p,c)   the amount of male labour needed in period p to plough one
                        ha of land under crop c, in days

Therefore the *labour constraints* for p ∈ P, and t ∈ T, will be as follows:

$$
\begin{aligned}
&\sum_{c,s,f,w} labcult(c,p) \times CLAND(c,s,f,w,t) \\
&+ \sum_{s} \left(labcons(s) \times \left[CSTONE(s,p,t) + TSTONE(s,p,t) \right] \right) \\
&+ \sum_{tt=0}^{t-1} \sum_{s,w,y} labcutr(y) \times TCLAND(s,w,y,tt,p,t) \\
&+ \sum_{s,w,c,f} (labfert(p,f) \times CLAND(s,w,c,f,t)) \\
&+ \sum_{v} labliv(v,p) \times LIVSTK(v,t) + \sum_{s,w,y} labtree(y,p) \times TLAND(s,w,y,t,t) \\
&\leq avlbal(p,t) - OFFARM(p,t)
\end{aligned}
\tag{90}
$$

$$
\begin{aligned}
&\sum_{c,s,f,w} mlab(p,c) \times CLAND(c,s,f,w,t) \\
&+ \sum_{tt=1}^{t-1} \sum_{s,w,y} labcutr(y) \times TCLAND(s,w,y,tt,p,t) \times TCLAND(s,w,y,tt,p,t) \\
&+ \sum_{s} (labcons(s) \times \left[CSTONE(s,p,t) + TSTONE(s,p,t) \right] + OFFARM(p,t) \\
&\leq avmlbag(p,t)
\end{aligned}
\tag{91}
$$

i.e. the total amount of labour required in period p for the cultivation of the total cropland, keeping livestock, constructing stone bunds and planting trees should not exceed the total labour available in the village less the labour spent on off-farm jobs. In addition, the amount of male labour required to cultivate the cropland should be less or equal to the availability of male labour in each period minus labour spent on off-farm jobs.

6.11   Cash constraint

The sources of cash to the typical farmer in the highlands of Eritrea include, selling livestock and livestock products, selling crops, off-farm employment, and remittances. Although due to lack of collateral rural households have very limited access to credit, we also include the possibility of credit. Cash not spent

in a given year is carried over to the following years. The major expenditures in rural areas include consumer goods and services as well as some farm inputs. The average per capita expenditure on non-cereal items is estimated based on our findings during the field work. Households earn cash from the sale of crops, livestock and from non-farm activities. The selling and buying prices are different mainly due to transport, storage and other marketing costs (see 7. 10). The amount of money rural households spend in any given year should not exceed their earnings during the year and saving from previous years. We assume that prices as well as the wage rate remain the same throughout the planning period. We first define the following decision variables and parameters for $c \in C$, $v \in V$, and $t \in T$:

CASHBAL(t) cash remaining at the end of year t, after all expenses are paid, in Nakfa

CREDIT(t) amount of cash the village borrows in year t, in Nakfa

PAYCREDIT(t) amount of money paid in year t in settlement of loans plus interest in year t

INTEREST(t) amount of money paid as interest in year t

cash0 total amount of cash available in the village at the beginning of the planning period

bpricec(c,t) buying price of crop c in year t, in Nakfa/kg

spricec(c,t) selling price of crop c   in year t, in Nakfa/kg

priceu(t) price of Urea in year t, in Nakfa/kg

priced(t) price of DAP in year t, in Nakfa/kg

bpricev(v,t) buying price of livestock v in year t, in Nakfa/kg   (92)

spricev(v,t) selling price of livestock v in year t, in Nakfa/kg

pricem(t) price of milk in year t, in Nakfa/litre

bpricew(t) buying price of fuel wood in year t, in Nakfa/kg

spricew(t) selling price of fuelwood in year t, in Nakfa/kg

pricek(t) price of kerosene in year t, in Nakfa/litre

wage(t) wage rate in year t, in Nakfa/person/day (from off-farm jobs)

remit(t) average amount of money households receive from relatives in one year in year t, in Nakfa

hhexp(t) amount of money required to buy basic non cereal items such as oil, salt, sugar, etc in year t, in Nakfa/year/person

r the rate of interest

The cash balance for the village at the end year t for $c \in C$, $v \in V$, $y \in Y$, and $t \in T$, $t \neq 1$ is given in (93).

120

$$CASHBAL(t) = CASHBAL(t-1) + \sum_c spricec(c,t) \times SELLCROP(c,t)$$

$$+ \sum_v spricev(v,t) \times SELLLIVSTC(v,t) +$$

$$+ \sum_y pricew(y,t) \times SELLWOOD(y,t)$$

$$+ \sum_p wage(t) \times OFFARM(p,t) + CREDIT(t)$$

$$+ pricem(t) \times milk(t) + remit(t)$$

$$- \sum_c bpirce(c,t) \times BUYCROP(c,t)$$

$$- \sum bpricev(v,t) \times BUYLIVSTC(v,t)$$

$$- \sum_y bpricew(y,t) \times BUYWOOD(y,t)$$

$$- pricek(t) \times KEROSENE(t) - priceu(t) \times BUYUREA(t)$$

$$- priced(t) \times BUYDAP(t)$$

$$- PAYCREDIT(t) - hh\exp(t) \times POPL(t) \tag{93}$$

Equation (93) will also hold for t = 1, if the first term in the right hand of (93) is replaced by *cash0*. (94)

We assume that credit obtained in any given year plus interest will be paid in the following three years in equal instalments. Thus for all t ∈ T:

$$\sum_t PAYCREDIT(t) = \sum_{\tau=t-3}^{\tau=t-1} 1/3 \times (1+3r) \times Credit(\tau) \tag{95}$$

We will also postulate that credit cannot be obtained in the last two years of the planning period because there will not be sufficient years to settle the credit. Thus,

$$CREDIT(t) = 0 \text{ for } t = T\text{-}1, T\text{-}2 \tag{96}$$

The amount of interest paid in year t for all t ∈ T is given in (97)

$$INTEREST(t) = \sum_{\tau=t-3}^{\tau=t-1} r \times CREDIT(\tau) \tag{97}$$

6.12   Land management, crop yield, soil and nutrient loss

Crop yield is one of the most important parameters in our model. Crop yield is influenced by a number of factors such as the amount and distribution of rainfall, soil type, and application of fertilizer. While rainfall is an exogenous factor, soil depth, the type and quantity of fertilizer applied, as well as the intensity and frequency of farm activities (such as ploughing and weeding) are influenced by the decisions of farming households. The estimation of crop yield for different crops where all the above factors are variable is very difficult. Thus we obtain crop yield under average conditions of rainfall and labour input (for land preparation, weeding and harvesting) and concentrate on the effect of soil type, construction of stone bunds, and the application of different types of fertilizer, which are of prime importance to our study. The relevant parameters are derived in Chapter seven.

Soil type and the type of crops cultivated as well as the construction of stone bunds and application of various types of fertilizers also influence the amount of soil and nutrient loss, which are used as indicators of the sustainability of production systems. See Chapter seven for discussion on the relationships between soil and nutrient loss on the one hand, and the type of land, crops cultivated and various management practices on the other. For $s \in S$, $w \in W$, $c \in C$, $f \in F$, $y \in Y$, $t \in T$, we define:

erosc(s,w,c,f)   rate of soil loss from land of soil type (s,w), cultivated   (98)
with crop (c,f), in tons/ha/year
erost(s,w,y)   rate of soil loss from land of soil type (s,w), planted
with tree species y, in kg/ha/year
TSLOSS(t)   total amount of soil loss in year t, in tons

For $s \in S$, $w \in W$, $c \in C$, $f \in F$, $y \in Y$, $t \in T$, the total amount of soil lost in year t can be written as follows:

$$TSLOSS(t) = \sum_{s,w,c,f} erosc(s,w,c,f) \times CLAND(c,s,f,w,t)$$
$$+ \sum_{s,w,y,tt} erost(s,w,y) \times TLAND(s,w,y,tt,t)$$
(99)

Similarly the rate of nutrient loss is influenced by type of land, land use, and land management techniques. Nutrient balance refers to the difference between nutrient inflows and nutrient outflows from a given land. The major sources of nutrient inflow are the application of mineral and organic fertilizers, nutrient deposition by rainfall, inputs of nutrients due to soil sedimentation and nitrogen inputs due to N-fixation. Outflow of nutrients on the other hand include removal

of nutrients due to harvest of crops and residues, leaching of nutrients, nitrogen gaseous losses and nutrient losses due to soil erosion. In this study we focus only on the nitrogen balance. The processes involved in nitrogen inflows and outflows and the parameters associated with each process are described in Chapter seven.

We define the following variables and parameters for all t ∈ T

NBAL(t)   Average nitrogen balance in year t, in kg/ha
ncontf(f)   amount of nitrogen in fertilizer type f, in kg
ncontc(c)   amount of nitrogen in crop and crop residue in kg/kg   (100)
nrain   amount of nitrogen supplied by rainfall in year t, in kg/ha
nfix   amount of nitrogen supplied due to nitrogen fixation, in kg/year
nfal   amount of nitrogen supplied by fallow land, in kg/year
neros   the amount of nitrogen lost through erosion in kg/tons

Nitrogen balance on crop lands for all t ∈ T, is given as follows:

$$
\begin{aligned}
NBAL(t) = &\ (\sum_{s,w,c,f} CLAND(s,w,c,f,t) \times ncontf(f)) / \sum_{s,w,c,f} CLAND(s,w,c,f,t) \\
&+ nrain + nfix + nfal \\
&- (\sum_{s,w,c,f} ncontc(c) \times CLAND(s,w,c,f,t)) / \sum_{s,w,c,f} CLAND(s,w,c,f,t) \\
&- (\sum_{s,w,c,f} neros \times erosc(s,w,c,f,t) \times CLAND(s,w,c,f,t)) / \sum_{s,w,c,f} CLAND(s,w,c,f,t)
\end{aligned}
$$

(101)

6.13 Objective function

As stated in Chapter four, farmers have multiple objectives and they maximize net discounted income only when other objectives are met. Thus we have included the objectives of securing sufficient food for the family and sufficient energy for cooking as constraints in the model. Now we write the farmer's objective of maximizing net benefits from his farming and other activities. The Net benefit of the farmer is defined as the difference between his total earnings from sale of crops, livestock, milk and wood (adjusted for changes in stock of livestock and wood) as well as income from off-farm employment and expenditures on purchased inputs and kerosene.

For all c ∈ C, v ∈ V, y ∈ Y, and t ∈ T, we define net benefits as:

$$NETBENEFIT(t) = \sum_{c} spricec(c,t) \times SELLCROP(c,t)$$
$$- \sum_{c} pricec(c,t) \times BUYCROP(c,t)$$
$$+ \sum_{v} spricev(v,t) \times SELLIVSTCK(v,t)$$
$$- \sum_{v} bpricev(v,t) \times BUYLIVSTCK(v,t)$$
$$+ \sum_{v} spricev(v,t) \times [LVSTK(v,t) - LVSTK(v,t-1)]$$
$$+ \sum_{y} spricew(y,t) \times SELLWOOD(y,t) \qquad (102)$$
$$- \sum_{y} bpricew(y,t) \times BUYWOOD(y,t)$$
$$+ \sum_{y} spicew(y,t) \times [VWDWDL(y,t) - VWDWDL(y,t-1)]$$
$$+ pricem(t) \times MILK + wage(t) \times OFFARM(t)$$
$$- priceu(t) \times BUYUREA(t) - priced(t) \times BUYDAP(t)$$
$$- pricek(t) \times KEROSENE(t)$$

The farmer will maximize the discounted net benefits. Thus we write the objective function, for t ∈ T, as follows: c ∈ C, v ∈ V, w ∈ W, f ∈ F and t ∈ T:

    c ∈ C, v ∈ V, w ∈ W, f ∈ F and t ∈ T:

$$\text{Max} \sum_{t} (1/(1+r))^t \times NETBENEFIT(t) \qquad (103)$$

A summary of the linear programming model, which includes as well references to the definitions of variables and parameters and to values of parameters, is presented in appendix 1 (Table A1). All the equations of the model are also presented.