# Essays on Customization Applications in Marketing

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# Chapter 4
# Promotion Customization across Multiple Categories

## 4.1 Introduction

In the previous chapter, split questionnaire designs have been proposed as a means of efficiently collecting data and a methodology was developed to design optimal split questionnaires based on prior information, which is obtained using full questionnaires from pilot or past studies. In this chapter, we apply the Bayesian framework to customize promotions across multiple categories. The purpose is to identify the optimal subset of categories to promote to each individual, based on associations in purchase behavior across categories.

As a consequence of technological developments, especially with the explosion in the usage of the World Wide Web, it is now possible to acquire immediate information about consumers and to meet their needs by customizing products and product-related information in real time. Nowadays, more and more companies use such customized and targeted online promotion programs, including customization of ads, email-messages, sales-promotions to loyalty card users, electronic coupons etc. Recently, Bucklin et al. (2002) proposed customization and automation as key areas of research in e-commerce. Targeting and customization issues have long been of interest in marketing. For example, Rossi et al. (1996) have applied targeting to a coupon delivery problem. Researchers such as Ansari and Mela (2003), and Bertsimas and Mersereau (2003) have examined the customization of information by means of e-mail marketing content on the internet; Gooley and Lattin (2000) have studied customization of advertisement content; Zhang and Krishnamurthi (2004) have shown how to customize online price promotions and Ansari, Essagaier and Kohli (2000) have analyzed customization of product offers.

Online retailing has increased its revenues by almost $72 million. This demonstrates that online retailing has become a major force in the consumer industry. Repeat buyers account for more than half of online revenues, and stimulating repeat-buying renders online retailing more efficient and therefore more profitable. Because of the information available on repeat customers, online retailers can customize their offers to them. This improves conversion rates and may justify higher margins (www.bcg.com/publications). Retailers therefore are continuing to invest in tools to facilitate repeat-buying, promotional tools being chief among these. The dynamic character of the Internet makes relatively easy for retailers to offer promotions to individual customers "on the fly" to guide their current decisions, by using information from their previous choices. Hence, customized delivery of promotions via email or on the web is becoming vital in online retailers strategies.

Online grocery stores such as Peapod (www.peapod.com) and NetGrocer (www.netgrocer.com) use various customization services for the grocery shopping process on the Internet. Such services involve creating personal lists for products (that frequently purchased, for weekend parties, or for special occasions), creating lists of the items available in a customer's pantry and refrigerator, and then suggesting recipes in which those items can be used. Another example of an advanced customized promotion program is that employed by CVS Pharmacy (www.cvs.com). CVS uses loyalty cards to offer different sales-promotions for low-tier, middle-tier, and top-tier customers. It also uses target mailings with segment-level content and customized offers. Moreover, CVS customizes promotions at the cash register using previous category purchase histories. Price promotions and coupons are an important part of any customization process.

According to the Association of Coupon Professionals, Internet-delivered coupons, although still a controversial topic in the industry, saw a five-fold increase in distribution as entrepreneurial marketers sought better ways to target and deliver effective incentives (www.couponpros.org). Merchants use information about their shoppers to target e-coupons from these three possible types of data. The first one is shoppers' socio-demographic profiles, which can be obtained from external sources or directly obtained from shoppers in the form of answers to questionnaires and surveys. The second is the shopper's clickstream, which is a raw log of the web pages requested by the shopper in the merchant's store. The last one is shoppers' transactions, such as items purchased, the recipient of the purchases, items added to a shopping cart, and e-coupons offered, accepted and used. Customized coupon offers can be delivered through e-mail or on the web. The e-coupon is a short piece of text that can carry a short message. Promotional web pages show a certain number of coupon offers for different products and categories. Electronic distribution of coupons has become more widespread under programs such as Catalina Marketing Incorporated’s (CMI; www.catalinamarketing.com) checkout coupon and frequent shopper programs, in which households receive in store coupons and/or volume discounts through the Internet (www.valupage.com/Entry.pst). In addition, specialized web-based promotion companies offer promotions for a range of products and categories for different manufacturers and retailers (www.dealcatcher.com, www.coolsavings.com, www.allonlinecoupons.com, www.findsavings.com).

A limitation of the customized promotion programs in practice as well as those described in the academic literature to date is that electronic coupons are issued by companies based on customer information in only one category at a time. In this chapter, we consider the development of a customization method by focusing on the selection of what product categories to promote, across multiple product categories, taking into account the dependencies in consumers’ purchase behavior across those categories. Our method can be applied to, for example, checkout coupons printed on cash register receipts or e-coupon promotions delivered on web pages or through e-mail. In practice, these receipts, web pages and e-mails can show only a limited number of coupon offers, mostly due to space limitations, and our purpose is to select the most suitable categories to promote to individuals to minimize the customer effort, as well as maximize the retailer revenue.

Our approach obtains cross-category purchase incidence and expenditure information and analyzes this as input to develop customized coupon programs. Our promotions design problem is to determine which category from many possible categories to assign to a customer for promotion. The approach has two stages. First of all, we obtain prior information about consumer choices, price and the promotional environment in online retail stores, at the category level. We obtain current and past purchase history information from online transaction data. We construct a consumer response model using total expenditure and multicategory purchase incidence information, while dealing with consumer heterogeneity. Our flexible model of heterogeneity accommodates observable and unobservable heterogeneity and produces household level inferences for targeting purposes. We consider the interdependence in consumer purchases among multiple categories (see e.g., Manchanda, Ansari, and Gupta, 1999). We model the incidence decisions and expenditures jointly. We use the expected expenditures on a shopping trip if we promote a category or not as a criterion to find the optimal promotion design, i.e. our design maximizes consumer expenditure, and thus the firm's revenue. The promotion design problem, however, increases exponentially with the number of categories. At the second stage, therefore, using estimates of this model, we search all possible category-promotions designs with the modified Federov algorithm and determine the optimal design, which gives maximum expenditures on a shopping basket to maximize revenue of online retailers.

Our objectives in this chapter are summarized as follows:

- Developing a joint heterogeneous model of category purchase incidence and expenditure decisions across multiple categories,
- Using this category buying behavior information, we develop a method for designing an optimal customized promotion plan, specifying what categories to promote to which consumers.

## 4.2 Literature Review

We model consumer responses to promotions considering purchase incidence and expenditure together to obtain knowledge about correlations across categories. Several previous studies in the marketing literature have considered promotion effects at the multicategory level. These studies have been developed and estimated in three different ways: 1) for one product category and for a specific purchase variable at a time, 2) for three purchase variables (purchase incidence, brand choice and purchase quantity) simultaneously within a single product category, 3) for multiple purchase outcome variables simultaneously across multiple product categories. For example, Bolton (1989) finds that the effects of category display and feature activity are much larger than the effects of brand prices, display, and feature activity. While Gupta (1988) models customers' brand choice, inter-purchase time and purchase quantity decisions separately, he uses category level marketing-mix variables in the inter-purchase time and purchase quantity models. Seetharaman, Ainslie and Chintagunta (1999) study category level household state dependence considering brand choice behavior across five product categories. In this paper, they investigated whether households exhibit similar sensititivities to the marketing mix variables in different product categories. Fader and Lodish (1990) use IRI Marketing Factbook data from 331 product categories to explore the relationship between category structure (e.g. purchase cycle, penetration, etc.) and promotional movement (e.g. volume sold on price cuts, display and feature, etc.). They report systematic relationships between category characteristics and the effect of promotional policies. Narasimhan et al. (1996) study the relationship between product category characteristics and promotional elasticity using data from 108 product categories. They consider three types of promotions (regular price cuts, features, and displays) and seven category characteristics. They report that promotions obtain the highest response for brands in easily stockpiled, high penetration categories with short purchase cycles. Whereas the latter two studies ignore interdependence in consumer's purchases across multiple categories, Mulhern and Leone (1991), Chintagunta and Haldar (1998), and Manchanda and Gupta (1997) explicitly allow for dependency across multi-category items. Ainslie and Rossi (1998) measure the covariance of both observed (linked to measured characteristics of households) and unobserved heterogeneity in marketing sensitivity across two categories. Their focus is on the measurement of cross-category correlations in conditional choice behavior. Chib et al. (2002) modeled and estimated the purchase incidence model for twelve categories. In this paper, they illustrated disregarding cross-correlations across multiple categories in shopping basket models causes underestimation of the magnitude of cross-category correlations and overestimation of the effectiveness of the marketing mix, and additionally ignoring unobserved individual heterogeneity results in overestimation of cross-category correlations and underestimation of the effectiveness of the marketing mix. Our model in this chapter is in the third class i.e. multiple purchase outcome variables are estimated simultaneously across multiple product categories. As far as we know, there is no previous study on joint modelling of purchase incidence and expenditure across multiple categories in the marketing literature and our study differs from the previous studies due to the consideration of correlations between purchase incidence and expenditure in the model estimation.

## 4.3 Methodology

Our approach consists of two connected stages. At the first stage, we construct a consumer response model to obtain information about cross-category promotion effects, considering heterogeneity across households. At the second stage jointly executed with the first, we choose for every individual a limited set of promotions from all possible promotions with the modified Federov algorithm, using total shopping expenditure as a criterion.

### 4.3.1 The Model

Consumer purchase of multiple categories in a shopping trip can be characterized in terms of two related decisions: which categories to choose, and how much to spend. The role of the price and promotion variables in the purchase process at the category level makes the joint modeling of the incidence decision and the expenditure decision on a shopping trip necessary. Modeling these two decisions separately, that is, using for example a multivariate probit model for category incidence and a regression model for the expenditures, is incorrect and yields biased estimates if expenditure decisions are not independent of the category incidence decisions. For this reason, we use a censored regression (tobit) model. A similar approach was previously used by Krishnamurthi and Raj (1988) to jointly model purchase quantity and brand-choices. We will use a hierarchical multivariate type-2 tobit to jointly model category choice incidence and expenditure (previous applications of the tobit models in marketing include those by DeSarbo and Choi (1999), for modeling consumer search behavior; by DeSarbo and Jedidi (1995) in a consideration set application; and by Bucklin and Sismeiro (2003) to model web site browsing behavior). The details of the model are explained below.

### 4.3.2 Consumer Response Model

We investigate the predictive relationship of covariates with category-incidences and expenditures through a censored regression model describing category incidence and expenditures simultaneously. Let us assume H households, represented by index h=1,...,H, making purchase incidence decisions across a set of J product categories, j=1,..,J, on a total of T shopping trips, indexed by t=1,…,T. $Y_{2ht} = [Y_{2h1t}, Y_{2h2t}, ..., Y_{2hjt}]$ are binary dependent variables with consumer's product-category incidence decision outcomes. $Y_{1ht} = [Y_{1h1t}, Y_{1h2t}, ..., Y_{1hjt}]$ is our expenditure vector. We denote the predictor variables (marketing policy variables comprise price and promotion), as $X_{ht} = [X_{h1t}, X_{h2t}, ..., X_{hjt}]$. The observed choice vector (incidence of category-choice), $Y_2$ is

$$Y_{2hjt} = \begin{cases} 1, & \text{if } Y^*_{2hjt} \geq 0 \\ 0, & \text{if } Y^*_{2hjt} < 0 \end{cases} \tag{4.1}$$

Expenditures are observed only when the indicator variable for category choice, $Y_{2hjt}$, takes on the value 1:

$$Y_{1hjt} = \begin{cases} Y^*_{1hjt} & \text{if } \quad Y_{2hjt} = 1 \\ 0 & \text{otherwise} \end{cases} \tag{4.2}$$

We can write the models for the latent utilities, $Y^*_{2j}$, and the logarithm of the latent expenditure variable, $Y^*_{1j}$, for the J categories as:

$$\begin{bmatrix} Y^*_{1h1} \\ Y^*_{1h2} \\ ... \\ ... \\ Y^*_{1hJ} \end{bmatrix} = \begin{bmatrix} X_{1h1} & 0 & .... & 0 \\ ... & X_{1h2} & .... & 0 \\ ... & ... & .... & ... \\ 0 & 0 & .... & X_{1hJ} \end{bmatrix} \begin{bmatrix} \beta_{1h1} \\ \beta_{1h2} \\ .. \\ .. \\ \beta_{1hJ} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1h1} \\ \varepsilon_{1h2} \\ ... \\ ... \\ \varepsilon_{1hJ} \end{bmatrix} \tag{4.3a}$$

$$\begin{bmatrix} Y^*_{2h1} \\ Y^*_{2h2} \\ ... \\ ... \\ Y^*_{2hJ} \end{bmatrix} = \begin{bmatrix} X_{2h1} & 0 & .... & 0 \\ ... & X_{2h2} & .... & 0 \\ ... & ... & .... & ... \\ 0 & 0 & .... & X_{2hJ} \end{bmatrix} \begin{bmatrix} \beta_{2h1} \\ \beta_{2h2} \\ .. \\ .. \\ \beta_{h2J} \end{bmatrix} + \begin{bmatrix} \varepsilon_{2h1} \\ \varepsilon_{h22} \\ ... \\ ... \\ \varepsilon_{2hJ} \end{bmatrix} \tag{4.3b}$$

Each of the $Y^{*}\cdot_{h1}$, $Y^{*}\cdot_{h2},\ldots,Y^{*}\cdot_{hj}$ and $\varepsilon\cdot_{h1}$, $\varepsilon\cdot_{h2},\ldots,\varepsilon\cdot_{hj}$ are $T\times1$ vectors. The matrices $X_{\cdot hj}$ are of order $T\times k$ and the vectors $\beta\cdot_{hj}$ are of order $k\times1$, with k the number of predictor variables. X is the matrix of category price, category promotion and intercept variables and has dimensions $T\times kJ$. Thus, $\varepsilon_{1j}$, j=1,…,J, is an HT vector of error terms such that $E(\varepsilon_{1i})=0$ and $E(\varepsilon_{1i}\varepsilon'_{1j})=\sigma_{1ij}I_{HT}$ *i,j*=1,2,…..J and $\varepsilon_{2j}$, j=1,…,J, is an HT vector of error terms such that $E(\varepsilon_{2i})=0$ and $E(\varepsilon_{2i}\varepsilon'_{2j})=\sigma_{2ij}I_{HT}$ *i,j*=1,2,…..J. Further, we have $E(\varepsilon_{1i}\varepsilon'_{2j})=\sigma_{12ij}I_{HT}$.

We can re-write our equations as:

$$Y^{*}_{1ht}=X'_{ht}\beta_{1h}+\varepsilon_{1ht} \quad (4.4a)$$

$$Y^{*}_{2ht}=X'_{ht}\beta_{2h}+\varepsilon_{2ht} \quad (4.4b)$$

where the j-th row of the matrix $X_{ht}$ contains all explanatory variables including price and promotion, and the intercepts for each product category, influencing the utility and expenditure of the j-th category. The error terms are $\varepsilon_{ht}=[\varepsilon_{h1t},\varepsilon_{h2t},\ldots,\varepsilon_{hjt}]$. As unobserved factors informing the utilities may be common across categories, we assume that $\varepsilon_{2ht}\sim MVN[0,\Sigma_{2(J\times J)}]$, where $\Sigma$ is a $J\times J$ covariance matrix. Coincidence captured by the correlated error structure of the purchase utilities, i.e. choice in one category alters the utility of choices in other categories. Thus, if the covariance of the errors is positive, then an increase in the purchase utility of category i will lead to an increase in the purchase utility of category j. In other words, the error correlations capture the linkages between the uncontrollable factors that drive joint purchases (Manchanda, Ansari, and Gupta, 1999). The unobserved factors that affect total expenditures of shopping trips are $\varepsilon_{1ht} \sim MVN[0, \Sigma_{1(J\times J)}]$. The unobserved incidence and expenditure errors are correlated, i.e. $\mathrm{cov}[\varepsilon_{1ht}, \varepsilon_{2ht}] = \Sigma_{12(J\times J)}$, as will be explained in more detail below.

As explanatory variables, we only consider the own effects. Own effects show the impact of explanatory variables on the same category purchase. We do not consider the cross effects which reflect the change in purchase utility of category j due to the marketing actions of other categories, since these effects are likely to be small for the categories that we study. In addition; including those cross-effects would render our model very highly parameterized, where as our main interest is in representing the covariance of incidence and expenditure of multiple categories (Note that Manchanda, Ansari, and Gupta (1999) did not find any significant cross-effects of the incidence of four categories, except for detergent-softener and mix-cake category pairs, which are obviously related).

### 4.3.3 Individual Level Heterogeneity

We include individual level heterogeneity into the model by considering household specific coefficients.

$$\beta_{1h} = Z_h \Theta_1 + \zeta_{1h}, \qquad h = 1,.....H \qquad (4.5a)$$

$$\beta_{2h} = Z_h \Theta_2 + \zeta_{2h}, \qquad h = 1,.....H \qquad (4.5b)$$

We can write individual level parameters $\beta_{1h}$ and $\beta_{2h}$ as
$\beta_{1h} \sim MVN(Z_h \Theta_{1(j^*m^*k)}, \Lambda_{1(j^*m^*k \times j^*m^*k)})$, and $\beta_{2h} \sim MVN(Z_h \Theta_{2(j^*m^*k)}, \Lambda_{2(j^*m^*k \times j^*m^*k)})$

and m is the number of household-level explanatory variables. These variables are measured at the individual level and characterize a household's shopping behavior (see Rossi and Ainslie, 1998). While $\Theta_1$ and $\Theta_2$ indicate the impact of household level explanatory variables, $\Lambda_1$ and $\Lambda_2$ represent the unobserved sources of heterogeneity across households. In our application, we do not have any individual level explanatory variables, so $Z_h$ contains an intercept term only. Since $Z_h$ is equal to 1 for all h, $\Theta$ is the mean vector for the explanatory variables. The error terms are distributed as $\zeta_{1h} \sim MVN(0, \Lambda_{1(j^*m^*k \times j^*m^*k)})$, $\zeta_{2h} \sim MVN(0, \Lambda_{2(j^*m^*k \times j^*m^*k)})$.

### 4.3.4 Joint Model (Hierarchical Multivariate Type-2 Tobit Model)

The full model can now be represented as

$$Y_{ht}^* = \begin{bmatrix} Y_{1ht}^* \\ Y_{2ht}^* \end{bmatrix} = \begin{bmatrix} X_{1ht} & 0 \\ 0 & X_{2ht} \end{bmatrix} \begin{bmatrix} \beta_{1h} \\ \beta_{2h} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1ht} \\ \varepsilon_{2ht} \end{bmatrix} \tag{4.6}$$

$$\varepsilon_{ht} = \begin{bmatrix} \varepsilon_{1ht} \\ \varepsilon_{2ht} \end{bmatrix} \sim MVN\left(0, \begin{bmatrix} \Sigma_1 & \Sigma_{12} \\ \Sigma_{21} & \Sigma_2 \end{bmatrix}\right) \tag{4.7}$$

$$\beta_h = \begin{pmatrix} \beta_{1h} \\ \beta_{2h} \end{pmatrix} \sim MVN(V_h \Theta, \Lambda) \tag{4.8}$$

$$\Lambda = \begin{bmatrix} \Lambda_1 & Cov(\Lambda_1, \Lambda_2) \\ Cov(\Lambda_2, \Lambda_1) & \Lambda_2 \end{bmatrix}, \ \Theta = \begin{bmatrix} \Theta_1 & 0 \\ 0 & \Theta_2 \end{bmatrix}, \ V_h = I_{(2\times 2)} \otimes Z_h \tag{4.9}$$

## 4.4 Estimation with MCMC

We use Markov Chain Monte Carlo Methods (MCMC) to estimate our model. Chib (1992) describes MCMC methods for the standard tobit censored regression model, and proposes data-augmentation algorithms to simulate the unobserved variables $Y_{ht}^{*}$ in every step of the Gibbs sampler. We apply those procedures to estimate our model. Details of an MCMC algorithm for a hierarchical multivariate type-2 tobit model are provided by Fox, Montgomery and Lodish (2004). For reasons of identifiability, in the multivariate probit model diagonal elements of the covariance matrix are set to ones, i.e. $\sigma_{2ii}=1$, for all i, so that the matrix $\Sigma_2$ is a correlation matrix. The full conditional distribution of the correlation matrix is not tractable form and therefore not easy to draw. Liechty, Ramaswamy and Cohen (2000) draw R using griddy Gibbs sampler methods, and Barnard, McCulloch and Meng (2000) generate R using variants of the Metropolis-hasting algorithm. However these methods are computationally intensive. We estimate covariance matrix $\Sigma_0$ ($\Sigma_{01}$, $\Sigma_{02}$ and $\Sigma_{012}$) and coefficients $\beta_h$ and post-process the draws of $\Sigma$, $\beta_h$ and $\Theta$ with the use of a diagonal matrix C ($C = \mathrm{diag}(\Sigma_{02})^{-1/2}$) and obtained the correlation matrix $\Sigma = C\Sigma_0 C'$ (Edwards and Allenby, 2003). In the MCMC algorithm we use 50000 iterations and burn-in 20000 i.e. saved every fifth draws after that for real data set, and monitor convergence of the chain through plots of the hyper-parameters.

### 4.4.1 Gibbs Sampling

Technical details of MCMC estimation of the hierarchical multivariate type-2 tobit model are similar to Fox et al. (2004), with the exception that the expenditures and purchase incidence of categories are dependent in our model. Moreover, individual level heterogeneity is considered not only through an intercept term ($\beta_0$) as in Fox et al., but the effects of marketing actions (price and promotion coefficients, $\beta_{1hJ}$ and $\beta_{2hJ}$ respectively) are also allowed to be heterogeneous. $Y_{h(j)t}$ illustrates expenditures or utilities of category i, and $Y_{h(-j)t}$ is all the others. We need to specify conditional distributions of the relevant variables for Gibbs sampling. Natural conjugate priors are chosen for estimation. The stages in the Gibbs sampler are represented by s below, and the conditional draws are shown:

$$\Sigma^{(s+1)} \mid Y^{*(s)}_{hjt}, \beta^{(s)}_{h}, \Theta^{(s)}, \Lambda^{(s)}$$

$$Y^{*(s+1)}_{1h(j)t} \mid Y^{*(s)}_{1h(-j)t)}, \beta^{(s)}_{h}, \Theta^{(s)}, \Lambda^{(s)}, \Sigma^{(s+1)}$$

$$Y^{*(s+1)}_{2h(j)t} \mid Y^{*(s)}_{2h(-j)t}, \beta^{(s)}_{h}, \Theta^{(s)}, \Lambda^{(s)}, \Sigma^{(s+1)}$$

$$\beta_h^{(s+1)} \mid Y^{*(s+1)}_{ht}, \Sigma^{(s+1)}, \Theta^{(s)}, \Lambda^{(s)}$$

$$\Theta^{(s+1)} \mid Y^{*(s+1)}_{ht}, \beta^{(s+1)}_{h}, \Sigma^{(s+1)}, \Lambda^{(s)}$$

$$\Lambda^{(s+1)} \mid Y^{*(s+1)}_{ht}, \beta^{(s+1)}_{h}, \Sigma^{(s+1)}, \Theta^{(s+1)}$$

*4.4.1.1 Prior Distributions:*

In Gibbs sampling, we need to specify prior distributions for the parameters of interest. We used noninformative priors.

1. The prior distribution of $\Sigma^{-1}$ is Wishart $W[\rho_1, R_0]$, where $\rho_1 = J * 2 + 2$, J is the number of categories, and $R_0 = I$.
2. The prior distribution of $\Lambda^{-1}$ is $W_p[\rho_2, R_1]$, where $\rho_2 = p + 2$, $R_1 = I$, and p is the rank of $\Theta$.
3. The prior distribution of $\Theta$ is a multivariate normal $MVN[\Theta_0, V_\Theta]$ where $\Theta_0 = 0$ and $V_\Theta = diag(10^{-3})$.

*4.4.1.2 Full Conditional Distributions:*

1. The full conditional distribution of the residual covariance matrix, $\Sigma^{(s+1)}$ is inverse wishart, $\Sigma^{-1(s+1)} \sim W[N + \rho_1, S]$, where N is the total number of observations ($N = H * T$) and $S = (Y_h^* - X_h\beta_h^{(s)})'(Y_h^* - X\beta_h^{(s)}) + R_0$.

2. The full conditional distribution of latent utilities ($Y_2$) for the probit part of the model is a truncated multivariate normal, $Y_{2hjt}^{*(s+1)} \sim TMVN(\mu_{20}, \Sigma_{20})$, with mean $\mu_{20}$ and variance $\Sigma_{20}$, which are shown below. If the indicator variable $Y_2 = 1$, then $Y_2^*$ is drawn from a normal distribution, truncated below at 0. Otherwise, $Y_2^*$ is drawn from a normal distribution, truncated above at 0. We have:

$$Y^*_{ht} = \begin{bmatrix} Y^*_{hjt} \\ -- \\ Y^*_{h,(-j),t} \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \sigma_{jj} & | & \sigma_{j(-j)} \\ -- & + & -- \\ \sigma_{(-j)j} & | & \Sigma_{(-j)(-j)} \end{bmatrix}, \text{ and}$$

$$\mu_{20} = X_{2hjt}\beta^{*(s)}_{2h} + (Y^{*(s)}_{2h,(-j),t} - E(Y^{*(s)}_{2h,(-j),t}))\Sigma^{-1(s)}_{2(-j)(-j)}\sigma^{(s)}_{2j(-j)}$$

$$\Sigma_{20}{}^{(s+1)} = \sigma^{(s+1)}_{2jj} - \sigma^{(s+1)}_{2j(-j)}\Sigma^{-1(s+1)}_{2(-j)(-j)}\sigma^{(s+1)}_{2(-j)j}$$

3. $Y^*_1$ is $Y_1$ if $y_1 > 0$, otherwise $Y^*_1$ is drawn from a normal distribution, truncated above at 0. The full conditional distribution of expenditures is a truncated multivariate normal distribution, $Y^{*(s+1)}_{1hjt} \sim \text{TMVN}(\mu_{10}, \Sigma_{10})$, with mean $\mu_{10}$ and variance $\Sigma_{10}$, where

$$\mu^*_{10} = X_{1hjt}\beta^{*(s)}_{1h} + (Y^{*(s)}_{1h,(-j),t} - E(Y^{*(s)}_{1h,(-j),t}))\Sigma^{-1(s)}_{1(-j)(-j)}\sigma^{(s)}_{1j(-j)}$$

$$\Sigma_{10}{}^{(s+1)} = \sigma^{(s+1)}_{1jj} - \sigma^{(s+1)}_{1j(-j)}\Sigma^{-1(s+1)}_{1(-j)(-j)}\sigma^{(s+1)}_{1(-j)j}$$

The inverse cdf method is used to draw truncated normal values for $Y_1 > 0$. It is explained below (see also Fox et al., 2004):

- Compute the upper limit for the uniform interval,
  $L = \Phi\left[\left(0 - E[Y^*_{1hjt} \mid Y^*_{1h(-j)t}, \beta_h, \Theta, \Sigma, \Lambda]\right) / (\sigma_{jj} - \sigma_{j(-j)}\Sigma^{-1}_{(-j)(-j)}\sigma_{j(-j)})\right]$, where $\Phi[.]$ represents the Normal cumulative distribution function (cdf) and
  $E[Y^*_{1hjt} \mid Y^*_{1h(-j)t}, \beta, \Theta, \Sigma, \Lambda] = X_{hjt}\beta^{*(s)}_h + (Y^{*(s)}_{h,(-j),t} - E(Y^{*(s)}_{h,(-j),t}))\Sigma^{-1(s)}_{(-j)(-j)}\sigma^{(s)}_{j(-j)}$
- Draw a uniform variate, U ~Uniform(0,L)
- Compute the value of the uniform draw:
  $Y^*_{1i} = \Phi^{-1}(U)(\sigma_{jj} - \sigma_{j(-j)}\Sigma^{-1}_{(-j)(-j)}\sigma_{j(-j)}) + E[Y^*_{1i} \mid Y^*_{1(-i)}, \beta, \Theta, \Sigma, \Lambda]$

4. The full conditional distribution of individual level coefficients $\beta_h$ is multivariate normal, $\beta_h^{(s+1)} \sim MVN[M_c, V_c]$, where

$M_c = V_c vec((X_h' Y_h) \times S + \Sigma^{-1(s+1)}(Z\Theta^{(s)})')$, and $V_c = (S^{-1} \otimes (X_h' X_h) + \Sigma^{-1(s+1)})^{-1}$.

5. The full conditional distribution of $\Theta$ is a multivariate normal, $\Theta^{(s+1)} \sim MVN[M_c, V_c]$ with mean $M_c = vec(Z_h' \beta_h^{(s+1)} \times \Lambda^{-1(s)}) \times V_c$ and variance $V_c = (\Lambda^{-1(s)} \otimes (Z_h' Z_h) + V_\Theta)^{-1}$.

6. The full conditional distribution of error of individual coefficients, $\Lambda$ is inverse wishart, $\Lambda^{-1(s+1)} \sim W(H + \rho_2, S)$, where H is the number of subjects and $S = (\beta_h^{(s+1)} - Z\Theta^{(s+1)})'(\beta_h^{(s+1)} - Z\Theta^{(s+1)}) + R_1$.

## 4.5 Customized Promotions Design

The typical retailer's decision problem is to choose the right categories to promote, since not all of them can be promoted, or necessarily should be promoted, at the same time. This is in contrast with the manufacturers' planning problem, in which each product line and brand has its own promotional plan. The retailer needs certain selection criteria (common ones are store traffic generation, profitability of the item, revenue it generates or the image it creates). The components of customized promotion decisions are when, where, what, and to whom to promote. In this section, we focus on the optimization of promotion decisions regarding which product categories from many possible to promote to whom, in the online shopping venue. We maximize the retailer's revenue, based on the model of the two related consumer decisions –in which categories to purchase and how much to spend in each of them. We will use combinatorial optimization routines to customize the promotion plan to individual customers.

We assume that the categories that we choose offer a price promotion (the proposed approach can also be applied at the point of purchase for 'in store' targeted coupon distribution). Since we have many categories and the unit measure of size for these categories differs between categories, we consider consumer expenditure as a criterion function to optimize. Cost and margin data are seldom available, and are not available to us in this study. In marketing, revenue has been used as a criterion for establishing pricing policies (see Anjos et al., 2005), and has been quite popular, for example, in the airline industry. As Rossi et al. (1996) state, "Any successful customization approach must deal directly with the problem of partial information and take parameter uncertainty into account in the decision problem." Therefore, we use a Bayesian decision theoretic approach (Dorfman, 1997) to our promotion allocation problem. That is, we estimate our objective function --retailer revenue-- in every draw of the Gibbs sampler, which enables us to minimize expected loss or, equivalently, maximize expected revenue, across the draws integrating out parameter uncertainty. To compute the optimization criterion, we estimate the a-posteriori expected difference in category expenditure for each consumer if we promote, respectively if we do not promote, the category in question. After we estimate the expected expenditures in these two cases and compute the difference for each category at every MCMC draw, we generate promotion designs by choosing that allocation of promotions that maximizes, for each individual, the revenue difference arising from promoting across categories. In doing so, we reflect restrictions on the number of categories to promote, say P, that operate in most applied situations. The question then becomes which P categories to choose for each consumer. For each consumer, we estimate the expected difference in expenditure if we promote a category and do not promote it, in order to generate all possible optimal promotion designs for each consumer, and then accept the design that has the maximum total expenditure. This procedure will give us the information of which categories to promote to whom.

In generating promotion designs, we set restrictions on the total number of categories to promote: at most P categories will be promoted at the same time, since obviously the online retailer cannot promote all categories at the same time. This approach is needed because of the space limitations on web pages, cash register receipts, and e-mail messages for displaying electronic coupons to customers. Additionally, these restrictions reduce the number of possible promotion plans (contained in the candidate promotion-plan matrix, C), which reduces computing time to find an optimal design[12]. In our optimization problem, instead of a candidate set of a size $2^J$, we will

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[12] For example, the candidate set (C) for J=4 categories (columns) with P=3 being chosen to promote consists of 4 candidate promotion plans (rows): $C = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

use a candidate set (C), with size $\binom{J}{P}$. For example, in our application, there are 4368 possible promotion plans, choosing five categories to promote out of sixteen. We define the criterion function as a function of expected consumer expenditures, which are a function of the price and promotion variables. Assume that all parameters of the model are collected in $\psi$. Our optimization problem for each consumer h is defined as:

$$\underset{\{d_{hj}\}}{\arg\max}\, E[\pi_i] = E_{\psi}\left[\sum_{j=1}^{J}\left(E\left[(Y_{1hj}^{\ 1})\right] - E\left[(Y_{1hj}^{0})\right]\right)^{d_{hj}}\right] \qquad (4.10)$$

$$\text{s.t.} \quad \sum_{j=1}^{J} d_{hj} = K \ ; \text{j=1,…,J} \quad \text{h=1,…,H, with,}$$

$$d_{hj} = \begin{Bmatrix} 1 & \textit{if category } j \textit{ is chosen} \\ 0 & \textit{if category } j \textit{ is not chosen} \end{Bmatrix}$$

$d_{hj}$ is determined by a search over all possible designs in the candidate matrix C, using the modified Federov algorithm as illustrated below. $E\left[Y_{1hj}^{1}\right]$ is the expected expenditure of a customer h given that we promote *j,* and $E\left[Y_{1hj}^{0}\right]$ is the expected expenditure of a customer h given that we do not promote *j.* We estimate these quantities for each consumer and in each Gibbs iterations. So at draw s of the Gibbs sampler, we obtain $\psi^{(s)}$, which enables us to compute $E^{(s)}\left[Y_{1hj}^{0} \mid X'_{hj}, \psi^{(s)}\right]$ and $E^{(s)}\left[Y_{1hj}^{1} \mid X''_{hj}, \psi^{(s)}\right]$. Here $X'_{hj}$ and $X''_{hj}$ are our design matrices, in which the promotion variables are set to 0 and 1, respectively, for category j and consumer h. We estimate the expected expenditure of category j if we promote it, through the expectation of $Y_{1hj}$ conditional on $X'_{hj}$ :

$$E\left[Y^0_{1hj} \mid X'_{hj}\right] = P(Y^*_{2hj} > 0) \cdot E(Y_{1hj} \mid X'_{hj}, Y^*_{2hj} > 0) \qquad (4.11a)$$

$$= \left(\frac{1}{\sigma_{2jj}}\right)\Phi\left(\frac{Y_{2hj} - X'_{hj}\beta_{2h}}{\sigma_{2jj}}\right) \times \left(X'_{hj}\beta_{1h} + \frac{\Sigma_{2j(-j)}}{\sqrt{\Sigma_{2jj}\Sigma_{2(-j)(-j)}}}\sigma_{2jj}\frac{\phi(-X'_{hj}\beta_{2h})}{1-\Phi(-X'_{hj}\beta_{2h})}\right) \qquad (4.11b)$$

Note that we use a correlation matrix in the MCMC estimation for identification reasons and the diagonals elements are equal to 1 ($\sigma_{1jj}$=1, $\sigma_{2jj}$=1,..). A similar expression is obtained for $E^{(s)}\left[Y^1_{1hj} \mid X''_{hj}, \psi^{(s)}\right]$. Here we have used:

$$\Sigma_1 = \begin{bmatrix} \sigma_{1jj} & | & \sigma_{1j(-j)} \\ -- & | & -- \\ \sigma_{1(-j)j} & | & \Sigma_{1(-j)(-j)} \end{bmatrix}, \ \Sigma_2 = \begin{bmatrix} \sigma_{2jj} & | & \sigma_{2j(-j)} \\ -- & | & -- \\ \sigma_{2(-j)j} & | & \Sigma_{2(-j)(-j)} \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} \Sigma_{jj} & | & \Sigma_{j(-j)} \\ -- & | & -- \\ \Sigma_{(-j)j} & | & \Sigma_{(-j)(-j)} \end{bmatrix}.$$

Now we need to take the expectation of (4.11b) over the distribution of the parameters. The expectation for, for example, $Y^0_{1hj}$ equals

$$E_\psi\left[E\left[Y^0_{1hj} \mid X'_{hj}\right]\right] = \int E\left[Y^0_{1hj} \mid X'_{hj}, \psi\right] f(\psi)d\psi \, .$$

This enables us to compute the two components of the criterion function across the iterates of the Gibbs sampler as:

$$E_\psi\left[E\left[Y^1_{1hj} \mid X''_{hj}\right]\right] \approx \sum_s \frac{E\left[Y^1_{1hj} \mid X''_{hj}, \psi^{(r)}\right]}{S}$$

$$E_{\psi}\left[E\left[Y_{1hj}^{0} \mid X_{hj}''\right]\right] \approx \sum_{s} \frac{E\left[Y_{1hj}^{0} \mid X_{hj}', \psi^{(r)}\right]}{S} \qquad (4.12)$$

### 4.5.1 Modified Fedorov Algorithm

Exchange algorithms are mainly used for finding plans in experimental designs where all the decision variables can be set at specified values in combinations determined by the plan. In our decision, our variable is 1/0 variable of whether or not to promote a category out of J categories. There are $2^{16}$-1 (=65535) different promotion plans for sixteen categories. Since our promotion plans have the restrictions that only five categories be promoted, the candidate promotion plan matrix is reduced to 4368 different promotion plans. Now we want to choose the best plan for each household from this candidate set. The Federov (1972) algorithm is used for this purpose, and is described below in the context of the specific design criterion chosen:

1. A candidate list of all feasible combinations of the promotion plans is constructed.

2. One combination is randomly selected from the candidate list for each household as a starting plan. We calculate the value of design criterion ($E(\pi_i)$) for this plan.

3. We exchange each combination of promotion plans for each household with the remaining promotion plans in the candidate list, and calculate the value of the design criterion for the new plan. This process is repeated, and the exchange that leads to the largest reduction in the design criterion is accepted. In the modified Federov algorithm (Cook and

Nachtsheim, 1980), any exchange that reduces the value of design criterion is made as soon as it is found, which speeds up the algorithm.

4. Until the improvement in design criterion is smaller than some specified tolerance, we repeat step 3.

5. We repeat steps 2 to 4 four times to try to avoid local optima, and the best plan found is used as an optimal promotion design.

## 4.6 Synthetic Data Results for Model Estimation

In this section, we discuss the results of three simulation studies investigating the performance of the models and estimation algorithms. In these studies, we checked the performance of models and algorithms in three cases: the simulation results for the simple multivariate type-2 tobit model when there is no covariance between incidence and expenditure, and without individual heterogeneity; with covariance and no individual heterogeneity; with covariance and individual heterogeneity. The model is estimated with the Gibbs sampling for two categories, and 100 households in three cases. Synthetic data are generated with known parameters (true values), which were compared to the estimated parameter values. That is, the design of the simulation study mimics the structure of the empirical data, so that good recovery in the simulation gives us confidence for the performance of the algorithm with the real data.

### 4.6.1 No Covariance between Incidence and Expenditure, No Individual Heterogeneity

In the first case, we assume that all subjects have the same preference ($\beta$) coefficients, i.e. no individual heterogeneity, and that there is no interdependence between purchase incidence and expenditure between categories. We assume that there are two explanatory variables. The error matrix ($\Sigma$) is in correlation form, and for this reason, incidence diagonal $\sigma$ values are equal to 1. As we can see, parameter estimates of correlation (Table 4.1), $\beta$'s (Table 4.2) and expenditures' $\sigma$ (Table 4.3) are close to true ones, and results reveal a satisfactory performance of the Gibbs sampling algorithm.

Table 4.1: Simulation results for the correlation ($\Sigma$) matrix

| | **True $\Sigma$** | | | | **Posterior Mean $\Sigma$** | | | | **Posterior SE $\Sigma$** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | expenditure(I) | | incidence(II) | | expenditure(I) | | incidence(II) | | expenditure(I) | | incidence(II) | |
| **category** | ***1*** | ***2*** | ***1*** | ***2*** | ***1*** | ***2*** | ***1*** | ***2*** | ***1*** | ***2*** | ***1*** | ***2*** |
| ***(I) 1*** | 1.00 | -0.29 | 0.00 | 0.00 | 1.00 | -0.10 | -0.02 | 0.04 | 0.00 | 0.03 | 0.05 | 0.04 |
| ***(I) 2*** | -0.29 | 1.00 | 0.00 | 0.00 | -0.10 | 1.00 | -0.04 | 0.02 | 0.03 | 0.00 | 0.05 | 0.05 |
| ***(II) 1*** | 0.00 | 0.00 | 1.00 | 0.30 | -0.02 | -0.04 | 1.00 | 0.34 | 0.05 | 0.05 | 0.00 | 0.04 |
| ***(II) 2*** | 0.00 | 0.00 | 0.30 | 1.00 | 0.04 | 0.02 | 0.34 | 1.00 | 0.04 | 0.05 | 0.04 | 0.00 |

Table 4.2: Simulation results for $\beta_i$'s

| ***Expenditure*** | | | | ***Incidence*** | | | |
|---|---|---|---|---|---|---|---|
| True $\beta_{1i}$ | | | | True $\beta_{2i}$ | | | |
| -0.508 | 1.949 | 0.259 | -0.679 | 0.049 | -0.148 | -0.183 | 0.042 |
| Posterior $\beta_{1i}$ | | | | Posterior $\beta_{2i}$ | | | |
| -0.700 | 2.138 | 0.220 | -0.644 | -0.024 | -0.138 | -0.139 | 0.055 |
| Posterior SE of $\beta_{1i}$ | | | | Posterior SE of $\beta_{2i}$ | | | |
| 0.212 | 0.200 | 0.126 | 0.127 | 0.192 | 0.192 | 0.203 | 0.203 |

Table 4.3: Simulation results for expenditure's standard deviation ($\sigma_{1ii}$)

| | True | Posterior Mean | Posterior SE |
|---|---|---|---|
| ***expenditure*** | 5.766 | 5.721 | 0.144 |
| | 3.873 | 3.667 | 0.092 |

### 4.6.2 With Covariance between Incidence and Expenditure, No Individual Heterogeneity

In this case, we still assume that all subjects have the same preference ($\beta$) coefficients, i.e. no individual heterogeneity, and we have two explanatory variables, but there is interdependence between purchase incidence and expenditure between categories (see correlation between expenditure and incidence in the correlation matrix, Table 4.4). We obtain satisfactory results for parameter estimates ($\Sigma$ in Table 4.4, $\beta$'s in Table 4.5 and expenditures' $\sigma$ in Table 4.3), i.e. the posterior means are close to true parameter values.

Table 4.4: Simulation results for the correlation ($\Sigma$) matrix

| | **True Σ** | | | | **Posterior Mean Σ** | | | | **Posterior SE Σ** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | expenditure(I) | | incidence(II) | | expenditure(I) | | incidence(II) | | expenditure(I) | | incidence(II) | |
| **category** | ***1*** | ***2*** | ***1*** | ***2*** | ***1*** | ***2*** | ***1*** | ***2*** | ***1*** | ***2*** | ***1*** | ***2*** |
| ***(I) 1*** | 1.00 | -0.29 | 0.22 | 0.99 | 1.00 | -0.31 | 0.26 | 0.99 | 0.00 | 0.03 | 0.04 | 0.00 |
| ***(I) 2*** | -0.29 | 1.00 | 0.31 | -0.32 | -0.31 | 1.00 | 0.29 | -0.29 | 0.03 | 0.00 | 0.04 | 0.03 |
| ***(II) 1*** | 0.22 | 0.31 | 1.00 | 0.30 | 0.26 | 0.29 | 1.00 | 0.31 | 0.04 | 0.04 | 0.00 | 0.04 |
| ***(II) 2*** | 0.99 | -0.32 | 0.30 | 1.00 | 0.99 | -0.29 | 0.31 | 1.00 | 0.00 | 0.03 | 0.04 | 0.00 |

Table 4.5: Simulation results for $\beta_i$'s

| ***Expenditure*** | | | | ***Incidence*** | | | |
|---|---|---|---|---|---|---|---|
| True $\beta_{1i}$ | | | | True $\beta_{2i}$ | | | |
| -1.228 | 0.497 | 0.437 | 1.830 | 0.243 | 0.453 | 0.064 | 0.270 |
| Posterior $\beta_{1i}$ | | | | Posterior $\beta_{2i}$ | | | |
| -1.052 | 0.638 | 0.373 | 1.820 | 0.240 | 0.494 | 0.069 | 0.263 |
| Posterior SE of $\beta_{1i}$ | | | | Posterior SE of $\beta_{2i}$ | | | |
| 0.156 | 0.141 | 0.128 | 0.116 | 0.173 | 0.198 | 0.177 | 0.157 |

Table 4.6: Simulation results for expenditure's standard deviation ($\sigma_{1ii}$)

| | **True** | **Posterior Mean** | **Posterior SE** |
|---|---|---|---|
| ***expenditure*** | 5.766 | 5.908 | 0.123 |
| | 3.873 | 3.885 | 0.093 |

### 4.6.3 With Covariance between Incidence and Expenditure, and Individual Heterogeneity

In the final case, we use two explanatory variables, but there is interdependence between purchase incidence and expenditure between categories, and all subjects have unique preference ($\beta_h$) coefficients, i.e. the model now accommodates individual heterogeneity. We do not report individual $\beta$'s for convenience, since we have 100 subjects. We report the correlation matrix of the errors for expenditure and incidence ($\Sigma$), the mean level preference coefficients ($\Theta$), the standard deviation of expenditure errors ($\sigma$), and the covariance ($\Lambda$) of the marketing mix variables. We again obtain satisfactory results for all parameter estimates. Estimates for $\Sigma$ are presented in Table 4.7, for $\Theta$ in Table 4.8, for $\sigma$ in Table 4.9, and for $\Lambda$ in Table 4.10. Note that $\Lambda$ values for the incidence part are not as good as $\Lambda$ values for the expenditure part in Table 4.10. The simulation results are satisfactory, and now we can apply this method to the real data set with sixteen product categories.

Table 4.7: Simulation results for the correlation ($\Sigma$) matrix

| | **True $\Sigma$** | | | | **Posterior Mean $\Sigma$** | | | | **Posterior SE $\Sigma$** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | expenditure(I) | | incidence(II) | | expenditure(I) | | incidence(II) | | expenditure(I) | | incidence(II) | |
| **category** | ***1*** | ***2*** | ***1*** | ***2*** | ***1*** | ***2*** | ***1*** | ***2*** | ***1*** | ***2*** | ***1*** | ***2*** |
| ***(I) 1*** | 1.00 | -0.29 | 0.22 | 0.99 | 1.00 | -0.31 | 0.22 | 0.98 | 0.00 | 0.04 | 0.04 | 0.00 |
| ***(I) 2*** | -0.29 | 1.00 | 0.31 | -0.32 | -0.31 | 1.00 | 0.34 | -0.26 | 0.04 | 0.00 | 0.04 | 0.04 |
| ***(II) 1*** | 0.22 | 0.31 | 1.00 | 0.30 | 0.22 | 0.34 | 1.00 | 0.34 | 0.04 | 0.04 | 0.00 | 0.04 |
| ***(II) 2*** | 0.99 | -0.32 | 0.30 | 1.00 | 0.98 | -0.26 | 0.34 | 1.00 | 0.00 | 0.04 | 0.04 | 0.00 |

Table 4.8: Simulation results for $\Theta$

| ***Expenditure*** | | | | ***Incidence*** | | | |
|---|---|---|---|---|---|---|---|
| True $\beta_{1i}$ | | | | True $\beta_{2i}$ | | | |
| 0.182 | -1.801 | -1.086 | 1.434 | -0.227 | -0.104 | 0.308 | -0.389 |
| Posterior $\beta_{1i}$ | | | | Posterior $\beta_{2i}$ | | | |
| 0.134 | -1.612 | -1.381 | 1.414 | -0.237 | -0.159 | 0.267 | -0.406 |
| Posterior SE of $\beta_{1i}$ | | | | Posterior SE of $\beta_{2i}$ | | | |
| 0.212 | 0.196 | 0.179 | 0.195 | 0.377 | 0.383 | 0.367 | 0.400 |

Table 4.9: Simulation results for expenditure's standard deviation ($\sigma_{1ii}$)

| | True | Posterior Mean | Posterior SE |
|---|---|---|---|
| ***expenditure*** | 5.766 | 5.829 | 0.136 |
| | 3.873 | 3.836 | 0.094 |

Table 4.10: Simulation results for $\Lambda$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | **True $\Lambda$** | | | | |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| | | | **Posterior $\Lambda$** | | | | |
| 0.827 | -0.226 | 0.132 | 0.120 | 0.192 | 0.036 | 0.744 | -0.312 |
| -0.226 | 0.968 | -0.052 | -0.253 | -0.492 | 0.024 | -0.204 | 0.960 |
| 0.132 | -0.052 | 1.082 | -0.076 | -0.168 | -0.048 | 0.084 | -0.228 |
| 0.120 | -0.253 | -0.076 | 1.278 | -0.012 | 0.432 | -0.060 | -0.504 |
| 0.192 | -0.492 | -0.168 | -0.012 | 1.342 | 0.024 | 0.180 | -0.132 |
| 0.036 | 0.024 | -0.048 | 0.432 | 0.024 | 1.262 | -0.012 | 0.132 |
| 0.744 | -0.204 | 0.084 | -0.060 | 0.180 | -0.012 | 1.108 | -0.085 |
| -0.312 | 0.960 | -0.228 | -0.504 | -0.132 | 0.132 | -0.085 | 1.180 |
| | | | **SE of Posterior $\Lambda$** | | | | |
| 0.417 | 0.263 | 0.241 | 0.241 | 0.344 | 0.337 | 0.339 | 0.344 |
| 0.263 | 0.339 | 0.216 | 0.192 | 0.325 | 0.317 | 0.324 | 0.323 |
| 0.241 | 0.216 | 0.275 | 0.169 | 0.275 | 0.277 | 0.295 | 0.282 |
| 0.241 | 0.192 | 0.169 | 0.219 | 0.258 | 0.261 | 0.268 | 0.266 |
| 0.344 | 0.325 | 0.275 | 0.258 | 0.770 | 0.554 | 0.593 | 0.575 |
| 0.337 | 0.317 | 0.277 | 0.261 | 0.554 | 0.789 | 0.579 | 0.570 |
| 0.339 | 0.324 | 0.295 | 0.268 | 0.593 | 0.579 | 0.909 | 0.621 |
| 0.344 | 0.323 | 0.282 | 0.266 | 0.575 | 0.570 | 0.621 | 0.897 |

## 4.7 Data Description

We have purchase data from a random sample of customers of a leading online grocery retailer. The data are from May 1996 to July 1997, from a total of 279 households. We have the purchase history data of 16 product-categories over time, and the incidence frequencies are shown in Table 4.11. We know that 133 households shopped regularly from one of these product-categories from 1996-1997. We have a total of 4281 shopping trips, and at least one category was purchased on 3632 occasions during these trips. We use 62 shopping weeks. The numbers of pair-wise purchases for all 16 product-category pairs are shown in Table 4.12. Detergents, paper towels and toilet paper are the most frequently purchased categories and also the most promoted categories, as is evident from Table 4.11. Squeeze margarine, butter, allergy medicines and coffee instant decaf are the most infrequently purchased product categories (Table 4.11). The online purchase data contains the number of units of SKUs bought from each of the 16 categories, size, price paid per unit, and whether or not the price reflected a deal purchase. If the same SKU was bought more than once for the same price in one shopping trip, it was treated as a single purchase by aggregating the quantities. However, if different SKUs in the same product category were chosen on the same shopping trip, we randomly selected one of them. The number of shopping trips ranged from 9 to 69, with the average being 32 per household. The average total expenditure per shopping trip is $125 per household. This calculation included the total expenditure of those consumers for whom an itemization of expenditure per product category was not available, i.e. purchases of “unknown categories.”

From the correlation matrix of purchase incidence of product categories (Table 4.13), paper towel tissue and toilet paper tissue has the highest correlation (0.413). We observe that paper towel tissue and paper toilet tissue (0.413), paper towel tissue and laundry detergent (0.262), toilet paper tissue and laundry detergent (0.243), spaghetti sauce and toilet paper tissue (0.226), toilet paper tissue and soft margarine (0.219), spaghetti sauce and soft margarine (0.205) and finally toilet paper tissue and soap (0.203) are purchased the most frequently together compared to other category pairs. Again in this table, there are quite a few negative but low correlation values. The noticeable negatively correlated category pairs are: Squeeze margarine and stick margarine are negatively correlated with butter (-0.010 and -0.022, they are 3 and 17 times purchased together, respectively). Note that if we estimate the correlation matrix with only purchase weeks or including nonpurchase weeks, we obtain almost the same results.

The expenditure correlation matrix is presented in Table 4.14. Again toilet paper tissue and paper towel tissue have the highest correlation (0.295), but this correlation is not as high as their incidence correlation (0.413). The other highly correlated category pairs are toilet paper tissue and laundry detergent (0.256), paper towel tissue and laundry detergent (0.251), soft margarine and crackers (0.224), paper towel tissue and butter (0.208), toilet paper tissue and stick margarine (0.157), soap and toilet paper tissue (0.149), and, finally, spaghetti sauce and toilet paper tissue (0.148).

Table 4.15 illustrates the correlation of purchase incidence and expenditures of categories. The correlation of paper toilet tissue incidence and expenditure is equal to 0.787 and paper towel tissue is 0.750, which are the lowest. This means consumers purchase these two categories frequently but do not spend much. There may be two explanations for that: these categories are promoted a lot (which is actually confirmed by the descriptive statistics in Table 4.12), or consumers purchase these categories in high quantities.

In Table 4.16, we illustrate some descriptive statistics for purchase and nonpurchase weeks. We can easily see that except for allergy tablets and squeeze margarine (average the same prices in two cases), the prices of remaining categories are higher for nonpurchase weeks than the purchase weeks.

Table 4.11: Descriptive statistics of the online purchase

| Category | # of SKU in Category | Purchased # of SKU | # of Purchase on Promotion | # of Category Purchase | % of Purchase on Promotion |
|---|---|---|---|---|---|
| Allergy medicine | 79 | 8 | 1 | 17 | 0.059 |
| Butter | 32 | 9 | 227 | 686 | 0.331 |
| Coffee Gr. Decaf | 85 | 24 | 11 | 159 | 0.069 |
| Coffee Gr. Regular | 187 | 40 | 27 | 224 | 0.121 |
| Coffee Ins. Decaf | 24 | 8 | 0 | 30 | 0.000 |
| Coffee Ins. Regular | 68 | 20 | 3 | 87 | 0.034 |
| Cold medicine | 222 | 27 | 6 | 48 | 0.125 |
| Crackers | 32 | 14 | 60 | 225 | 0.267 |
| Laundry | 179 | 48 | 195 | 833 | 0.234 |
| Margarine Soft | 62 | 24 | 124 | 638 | 0.194 |
| Margarine Squeeze | 4 | 2 | 36 | 60 | 0.600 |
| Margarine Stick | 42 | 15 | 97 | 358 | 0.271 |
| Paper Toilet | 114 | 38 | 240 | 1954 | 0.123 |
| Paper Towel | 74 | 28 | 246 | 1863 | 0.132 |
| Soap | 137 | 45 | 72 | 411 | 0.175 |
| Spaghetti Sauce | 249 | 76 | 193 | 744 | 0.259 |

Table 4.12: Descriptive statistics: joint purchase frequencies

| ***Categories*** | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** | ***6*** | ***7*** | ***8*** | ***9*** | ***10*** | ***11*** | ***12*** | ***13*** | ***14*** | ***15*** | ***16*** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***1*** Allergy | **17** | | | | | | | | | | | | | | | |
| ***2*** Butter | 0 | **686** | | | | | | | | | | | | | | |
| ***3*** Coffee GD. | 0 | 24 | **159** | | | | | | | | | | | | | |
| ***4*** Coffee GR. | 1 | 32 | 26 | **224** | | | | | | | | | | | | |
| ***5*** Coffee ID. | 0 | 4 | 1 | 4 | **30** | | | | | | | | | | | |
| ***6*** Coffee IR. | 0 | 8 | 2 | 5 | 8 | **87** | | | | | | | | | | |
| ***7*** Cold | 4 | 3 | 3 | 1 | 0 | 2 | **48** | | | | | | | | | |
| ***8*** Cracker | 0 | 49 | 15 | 15 | 0 | 6 | 1 | **225** | | | | | | | | |
| ***9*** Laundry | 4 | 145 | 20 | 52 | 10 | 21 | 13 | 32 | **833** | | | | | | | |
| ***10*** Marg. Soft | 3 | 80 | 27 | 53 | 7 | 8 | 9 | 44 | 152 | **638** | | | | | | |
| ***11*** Marg.Sqz. | 2 | 3 | 1 | 1 | 0 | 0 | 0 | 2 | 8 | 9 | **60** | | | | | |
| ***12*** Marg. Stick | 1 | 17 | 21 | 27 | 2 | 5 | 3 | 32 | 72 | 80 | 4 | **358** | | | | |
| ***13*** Paper Toilet | 11 | 365 | 78 | 102 | 16 | 39 | 27 | 109 | 444 | 348 | 27 | 171 | **1954** | | | |
| ***14*** Paper Towel | 7 | 385 | 72 | 107 | 11 | 40 | 15 | 127 | 458 | 277 | 18 | 177 | 1035 | **1863** | | |
| ***15*** Soap | 4 | 69 | 16 | 30 | 1 | 10 | 5 | 36 | 98 | 72 | 6 | 67 | 245 | 213 | **411** | |
| ***16*** Spag. Sauce | 2 | 127 | 41 | 58 | 4 | 13 | 7 | 61 | 143 | 186 | 11 | 79 | 394 | 331 | 103 | **744** |

Table 4.13: Descriptive statistics: bivariate correlations of purchase incidence

| ***Category*** | **1** | **2** | ***3*** | ***4*** | **5** | **6** | ***7*** | ***8*** | ***9*** | ***10*** | ***11*** | ***12*** | ***13*** | ***14*** | ***15*** | ***16*** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***1*** Allergy | **1** | | | | | | | | | | | | | | | |
| ***2*** Butter | -0.014 | **1** | | | | | | | | | | | | | | |
| ***3*** Coffee GD. | -0.006 | **0.039** | **1** | | | | | | | | | | | | | |
| ***4*** Coffee GR. | 0.009 | **0.048** | **0.118** | **1** | | | | | | | | | | | | |
| ***5*** Coffee ID. | -0.003 | 0.011 | 0.006 | **0.039** | **1** | | | | | | | | | | | |
| ***6*** Coffee IR. | -0.005 | 0.004 | 0.003 | 0.019 | **0.151** | **1** | | | | | | | | | | |
| ***7*** Cold | **0.137** | -0.005 | ***0.024*** | -0.003 | -0.005 | ***0.023*** | **1** | | | | | | | | | |
| ***8*** Cracker | -0.008 | **0.096** | **0.063** | **0.045** | -0.010 | ***0.026*** | -0.003 | **1** | | | | | | | | |
| ***9*** Laundry | 0.020 | **0.119** | 0.012 | **0.073** | **0.047** | **0.049** | **0.043** | ***0.028*** | **1** | | | | | | | |
| ***10*** Marg. Soft | 0.017 | **0.051** | **0.049** | **0.100** | **0.035** | 0.006 | **0.032** | **0.074** | **0.135** | **1** | | | | | | |
| ***11*** Marg.Sqz. | **0.059** | -0.010 | -0.002 | -0.006 | -0.005 | -0.009 | -0.007 | 0.003 | 0.009 | ***0.023*** | **1** | | | | | |
| ***12*** Marg. Stick | 0.003 | ***-0.022*** | **0.062** | **0.063** | 0.007 | 0.007 | 0.007 | **0.081** | **0.076** | **0.117** | 0.010 | **1** | | | | |
| ***13*** Paper Toilet | **0.044** | **0.219** | **0.086** | **0.094** | **0.043** | **0.052** | **0.059** | **0.111** | **0.243** | **0.219** | **0.044** | **0.129** | **1** | | | |
| ***14*** Paper Towel | 0.021 | **0.250** | **0.079** | **0.105** | 0.021 | **0.059** | 0.017 | **0.142** | **0.262** | **0.152** | 0.016 | **0.143** | **0.413** | **1** | | |
| ***15*** Soap | **0.039** | **0.080** | **0.033** | **0.072** | -0.005 | **0.031** | 0.019 | **0.092** | **0.109** | **0.087** | 0.020 | **0.138** | **0.203** | **0.164** | **1** | |
| ***16*** Spag. Sauce | 0.004 | **0.107** | **0.086** | **0.106** | 0.009 | 0.021 | 0.015 | **0.111** | **0.100** | **0.205** | ***0.028*** | **0.100** | **0.226** | **0.173** | 0.129 | **1** |

Bold categories are significantly correlated with 0.01. Bold and italic categories are significantly correlated with 0.05.

Table 4.14:Descriptive statistics: bivariate correlations of expenditure

| ***Category*** | **1** | **2** | ***3*** | ***4*** | ***5*** | ***6*** | ***7*** | ***8*** | ***9*** | ***10*** | ***11*** | ***12*** | ***13*** | ***14*** | ***15*** | ***16*** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***1*** Allergy | **1** | | | | | | | | | | | | | | | |
| ***2*** Butter | -0.011 | **1** | | | | | | | | | | | | | | |
| ***3*** Coffee GD. | -0.005 | ***0.027*** | **1** | | | | | | | | | | | | | |
| ***4*** Coffee GR. | 0.005 | **0.039** | **0.118** | **1** | | | | | | | | | | | | |
| ***5*** Coffee ID. | -0.002 | -0.002 | 0.004 | **0.037** | **1** | | | | | | | | | | | |
| ***6*** Coffee IR. | -0.004 | -0.004 | 0.001 | 0.010 | **0.139** | **1** | | | | | | | | | | |
| ***7*** Cold | **0.077** | -0.004 | 0.008 | -0.003 | -0.004 | ***0.023*** | **1** | | | | | | | | | |
| ***8*** Cracker | -0.005 | **0.057** | **0.043** | **0.031** | -0.008 | ***0.025*** | -0.004 | **1** | | | | | | | | |
| ***9*** Laundry | ***0.026*** | **0.118** | ***0.027*** | **0.053** | **0.029** | **0.058** | **0.030** | 0.017 | **1** | | | | | | | |
| ***10*** Marg. Soft | 0.002 | **0.040** | **0.050** | **0.086** | **0.046** | 0.004 | **0.046** | **0.224** | **0.094** | **1** | | | | | | |
| ***11*** Marg.Sqz. | **0.056** | -0.003 | -0.006 | -0.006 | -0.005 | -0.007 | -0.006 | 0.000 | 0.017 | 0.014 | **1** | | | | | |
| ***12*** Marg. Stick | **0.082** | ***-0.024*** | **0.050** | **0.046** | 0.001 | -0.001 | 0.000 | **0.049** | **0.129** | **0.064** | 0.001 | **1** | | | | |
| ***13*** Paper Toilet | 0.010 | **0.142** | **0.050** | **0.090** | **0.046** | **0.094** | **0.040** | **0.085** | **0.256** | **0.208** | **0.034** | **0.143** | **1** | | | |
| ***14*** Paper Towel | 0.011 | **0.208** | **0.079** | **0.063** | 0.014 | **0.070** | 0.000 | **0.100** | **0.251** | **0.100** | 0.005 | **0.157** | **0.295** | **1** | | |
| ***15*** Soap | 0.016 | **0.059** | ***0.028*** | **0.043** | 0.003 | **0.052** | 0.000 | **0.065** | **0.099** | **0.070** | 0.017 | **0.141** | **0.149** | **0.089** | **1** | |
| ***16*** Spag. Sauce | -0.001 | **0.070** | **0.087** | **0.068** | -0.001 | **0.048** | 0.008 | **0.103** | **0.062** | **0.159** | 0.017 | **0.052** | **0.148** | **0.104** | **0.092** | **1** |

Bold categories are significantly correlated with 0.01. Bold and italic categories are significantly correlated with 0.05.

4.15: Descriptive statistics: bivariate correlations of expenditure&purchase incidence

| ***Expenditure*** | ***Purchase Incidence*** 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***1*** Allergy | **0.869** | -0.012 | -0.006 | 0.003 | -0.002 | -0.004 | **0.085** | -0.007 | 0.009 | 0.004 | **0.065** | ***0.024*** | **0.040** | 0.021 | ***0.025*** | 0.001 |
| ***2*** Butter | -0.013 | **0.924** | ***0.027*** | **0.039** | 0.005 | 0.003 | -0.007 | **0.077** | **0.117** | **0.046** | -0.005 | -0.014 | **0.200** | **0.229** | **0.065** | **0.093** |
| ***3*** Coffee GD. | -0.006 | **0.041** | **0.956** | **0.123** | 0.008 | 0.000 | 0.013 | **0.058** | 0.015 | **0.049** | -0.005 | **0.047** | **0.078** | **0.078** | **0.036** | **0.089** |
| ***4*** Coffee GR. | 0.012 | **0.049** | **0.105** | **0.941** | **0.046** | 0.013 | 0.001 | **0.040** | **0.070** | **0.100** | -0.006 | **0.068** | **0.093** | **0.095** | **0.070** | **0.109** |
| ***5*** Coffee ID. | -0.003 | 0.002 | 0.003 | **0.032** | **0.960** | **0.168** | -0.004 | -0.010 | **0.044** | **0.035** | -0.005 | 0.000 | **0.042** | 0.016 | -0.007 | 0.005 |
| ***6*** Coffee IR. | -0.004 | 0.000 | 0.004 | 0.012 | **0.118** | **0.880** | **0.032** | **0.032** | **0.051** | 0.007 | -0.008 | -0.003 | **0.053** | **0.053** | **0.039** | **0.036** |
| ***7*** Cold | **0.122** | -0.001 | 0.017 | -0.005 | -0.004 | 0.016 | **0.880** | -0.004 | **0.032** | **0.049** | -0.006 | 0.002 | **0.055** | 0.015 | 0.007 | 0.020 |
| ***8*** Cracker | -0.006 | **0.073** | **0.049** | **0.035** | -0.008 | 0.021 | -0.003 | **0.818** | 0.018 | **0.075** | 0.002 | **0.061** | **0.080** | **0.112** | **0.066** | **0.092** |
| ***9*** Laundry | **0.035** | **0.111** | ***0.023*** | **0.060** | **0.032** | **0.048** | **0.043** | ***0.024*** | **0.855** | **0.098** | 0.019 | **0.120** | **0.211** | **0.248** | **0.095** | **0.080** |
| ***10*** Marg. Soft | 0.012 | **0.046** | **0.052** | **0.089** | **0.048** | 0.004 | ***0.027*** | **0.086** | **0.129** | **0.872** | 0.019 | **0.111** | **0.200** | **0.142** | **0.074** | **0.184** |
| ***11*** Marg.Sqz. | **0.051** | -0.007 | -0.004 | -0.006 | -0.005 | -0.008 | -0.006 | 0.001 | 0.009 | 0.017 | **0.955** | 0.006 | **0.043** | 0.011 | 0.013 | ***0.025*** |
| ***12*** Marg. Stick | ***0.028*** | **-0.032** | **0.064** | **0.043** | 0.008 | 0.010 | 0.004 | **0.075** | **0.070** | **0.060** | 0.003 | **0.858** | **0.104** | **0.137** | **0.138** | **0.060** |
| ***13*** Paper Toilet | 0.010 | **0.145** | **0.061** | **0.084** | **0.048** | **0.096** | **0.042** | **0.116** | **0.245** | **0.214** | **0.032** | **0.165** | **0.787** | **0.340** | **0.153** | **0.160** |
| ***14*** Paper Towel | 0.006 | **0.218** | **0.080** | **0.080** | 0.018 | **0.094** | 0.002 | **0.123** | **0.240** | **0.090** | 0.007 | **0.160** | **0.264** | **0.750** | **0.095** | **0.118** |
| ***15*** Soap | ***0.025*** | **0.066** | ***0.026*** | **0.048** | 0.011 | **0.048** | 0.006 | **0.083** | **0.097** | **0.086** | ***0.023*** | **0.141** | **0.173** | **0.148** | **0.838** | **0.113** |
| ***16*** Spag. Sauce | 0.000 | **0.081** | **0.096** | **0.065** | 0.003 | **0.036** | 0.006 | **0.122** | **0.079** | **0.175** | 0.019 | **0.091** | **0.196** | **0.131** | **0.096** | **0.863** |

Bold categories are significantly correlated with 0.01. Bold and italic categories are significantly correlated with 0.05.

Table 4.16: Descriptive statistics for purchase and non-purchase weeks

A. PURCHASE WEEKS

| Product | N | Price | Price per Volume | Promotion | # of Weeks on Sale | % of Prom |
|---|---|---|---|---|---|---|
| Allergy medicine | 3557 | 6.649 | 0.271 | 0.240 | 852 | 0.240 |
| Butter | 3557 | 2.129 | 0.135 | 0.798 | 2840 | 0.798 |
| Coffee Ground Decaf | 3557 | 6.240 | 0.399 | 0.540 | 1920 | 0.540 |
| Coffee Ground Regular | 3557 | 5.678 | 0.275 | 0.755 | 2684 | 0.755 |
| Coffee Instant Decaf | 3557 | 5.747 | 0.864 | 0.418 | 1486 | 0.418 |
| Coffee Instant Regular | 3557 | 4.942 | 0.873 | 0.482 | 1715 | 0.482 |
| Cold medicine | 3557 | 6.901 | 0.274 | 0.464 | 1651 | 0.464 |
| Crackers | 3557 | 2.103 | 0.131 | 0.801 | 2849 | 0.801 |
| Laundry | 3557 | 6.366 | 0.072 | 0.967 | 3438 | 0.967 |
| Margarine Soft | 3557 | 1.525 | 0.155 | 0.941 | 3347 | 0.941 |
| Margarine Squeeze | 3557 | 1.567 | 0.134 | 0.681 | 2424 | 0.681 |
| Margarine Stick | 3557 | 1.355 | 0.083 | 0.851 | 3026 | 0.851 |
| Paper Toilet | 3557 | 2.144 | 0.550 | 0.985 | 3505 | 0.985 |
| Paper Towel | 3557 | 1.864 | 1.050 | 1.000 | 3557 | 1.000 |
| Soap | 3557 | 2.775 | 0.539 | 0.862 | 3067 | 0.862 |
| Spaghetti Sauce | 3557 | 2.351 | 0.087 | 1.000 | 3557 | 1.000 |

B. NON-PURCHASE WEEKS

| Product | N | Price | Price per Volume | Promotion | # of Weeks on Sale | % of Prom |
|---|---|---|---|---|---|---|
| Allergy medicine | 4689 | 6.639 | 0.271 | 0.244 | 1143 | 0.244 |
| Butter | 4689 | 2.131 | 0.137 | 0.813 | 3810 | 0.813 |
| Coffee Ground Decaf | 4689 | 6.292 | 0.401 | 0.555 | 2602 | 0.555 |
| Coffee Ground Regular | 4689 | 5.730 | 0.277 | 0.761 | 3567 | 0.761 |
| Coffee Instant Decaf | 4689 | 5.773 | 0.868 | 0.421 | 1972 | 0.421 |
| Coffee Instant Regular | 4689 | 4.972 | 0.880 | 0.485 | 2275 | 0.485 |
| Cold medicine | 4689 | 6.921 | 0.275 | 0.414 | 1940 | 0.414 |
| Crackers | 4689 | 2.119 | 0.132 | 0.754 | 3535 | 0.754 |
| Laundry | 4689 | 6.463 | 0.074 | 0.969 | 4542 | 0.969 |
| Margarine Soft | 4689 | 1.522 | 0.169 | 0.931 | 4367 | 0.931 |
| Margarine Squeeze | 4689 | 1.569 | 0.134 | 0.646 | 3029 | 0.646 |
| Margarine Stick | 4689 | 1.360 | 0.085 | 0.830 | 3890 | 0.830 |
| Paper Toilet | 4689 | 2.150 | 0.472 | 0.983 | 4608 | 0.983 |
| Paper Towel | 4689 | 1.878 | 1.050 | 1.000 | 4689 | 1.000 |
| Soap | 4689 | 2.785 | 0.587 | 0.849 | 3982 | 0.849 |
| Spaghetti Sauce | 4689 | 2.377 | 0.090 | 1.000 | 4689 | 1.000 |

### 4.7.1 Model Specification and Variable Definition

Since our model considers category incidence, we need to construct category level price and promotion variables. Category price is computed as the share-weighted average price of brands. We have only online price-promotion information. We have information on two kinds of price cut promotions: a) the product is on a "special promotion" available to all customers, b) the product is available at a special price to preferred customers who have a store card. As a promotion variable, we consider a dummy variable representing whether or not the category is on promotion. We will use category-specific intercepts to capture category preference. We denote the explanatory variables for category incidence as follows:

$$X_1 = X_1\{\mathit{Category\ specific\ intercepts, Category\ prices, Category\ promotions}\}$$

On each purchase occasion, we have price and promotion information on the product category that was chosen. If the purchased SKU is on promotion, then the category promotion variable takes the value 1 in the incidence part of the model. We also include an intercept term. The promotion variable in the expenditure part is constructed to be equal to 1 if the purchase on promotion, 0 if not. The category price variable is obtained by calculating the weighted average price per volume of SKUs in the category and weights are market share for each category. The explanatory variables for expenditure are:

$$X_2 = X_2\{\mathit{Intercepts, Category\ prices, Purchase\ is\ on\ promotion\ or\ not}\}$$

For the hierarchical regression on the price and promotion coefficients, we have only an intercept term:

$$Z_h = Z_h\left\{ Intercept \right\}$$

We have 16 categories. Our Y matrix consists of two stacked matrices: $Y_2$ represents the observed choices of 16 categories, and $Y_1$ is the spending for these categories across the shopping trips.

## 4.8 Results and Discussion

We report the estimated cross-category correlation matrix for expenditure and purchase incidence across 16 categories in Table 4.17, Table 4.18 and Table 4.19. Highly correlated category pairs for expenditure are allergy and paper toilet tissue (0.772), allergy and coffee instant decaf (0.619), allergy and paper towel tissue (0.594), allergy and soft margarine (0.588), allergy and soap (0.588), allergy and stick margarine (0. 568), allergy and spaghetti sauce (0. 506), allergy and laundry (0.500), soft margarine and coffee instant decaf (0.523), stick margarine and soft margarine (0.501), paper toilet tissue and squeeze margarine (0.511), and finally paper towel and toilet paper (0.503). Among the margarine categories (stick, soft and squeeze), stick and soft margarine have the highest and significant correlation (0.501) for expenditure. Among the coffee categories, though the value of correlation is not very high (0.274), only the coffee instant regular and coffee instant decaf categories are significantly correlated. Expenditures of allergy and cold tablets are not significantly correlated with each other. Although expenditures of allergy tablets seem to be highly correlated with many categories' expenditures, cold tablets do not show that pattern, since this category is not correlated with any of others. We can explain this situation by considering that allergy tablets are purchased regularly, whereas customers purchase cold tablets only when necessary. Allergy tablets may thus be a store traffic driver. The paper tissue categories (towel and toilet) are significantly correlated (0.503).

Estimated purchase incidence correlations are illustrated in Table 4.18. In the paper tissue categories, purchase incidence of paper towel tissue and paper toilet tissue has one of the highest correlations, 0.615. Coffee ground decaf and coffee ground regular have the highest correlation (0.663), which is also higher compared to the correlations among the other coffee categories. The other highly correlated categories are coffee ground regular and allergy (0.515), butter and stick margarine (0.521), butter and toilet paper (0.523), butter and paper towel (0.530), toilet paper and laundry (0.515), paper towel and laundry (0.510), toilet paper and soft margarine (0.526), toilet paper and stick margarine (0.539). Chib et al. (2002) also found high correlation between toilet tissue and laundry detergents. Among the margarine categories, stick margarine and soft margarine have the highest correlation (0.559). While the expenditure of allergy and cold tablets are not correlated, purchase incidences are significantly correlated. Coincidence of coffee ground regular and coffee ground decaf, paper toilet tissue and paper towel tissue, and the butter and margarine categories can be explained from the fact that probably these category-pairs are shown on the same web page in the online shopping environment (this effect similar to the shelf effect in brick and mortar shopping). The "shelf effect" is also observed by Chib et al. (2002) in their purchase incidence model for twelve categories. Online retailers may improve profitability by displaying high-margin products and frequently purchased products on the same web page or by presenting pop-up ads. Moreover, some high correlations between four categories in coffee and three categories in the margarine group (butter can also be included in this group) may cause consumers to purchase these categories for variety's sake rather than viewing these categories as substitutes. Although there are some negative correlations for expenditures, the purchase incidence correlations are all positive, similar to the results obtained by Chib et al. (2002).

The correlation between expenditure and purchase incidence are shown in Table 4.19. Categories that do not show a significant correlation between expenditure and purchase incidence are coffee ground decaf (0.516), coffee ground regular (0.189), cold medication (0.192), and squeeze margarine (0.369). Normally we would expect higher correlation values between purchase incidence and expenditure within a category. Expenditures on allergy medications are related to many categories' purchase incidences. Expenditure of allergy is highly related with purchase incidence of paper toilet tissue (0.812), paper towel tissue (0.649), coffee ground regular (0.637), soft margarine (0.620), spaghetti sauce (0.561), laundry detergent (0.559), coffee ground decaf (0.553) and butter (0.507). Among the coffee categories, coffee instant decaf expenditure is related with purchase incidence of coffee ground regular (0.596), coffee instant regular (0.448) and coffee ground decaf (0.459). Margarine expenditures are related to purchase incidence of butter (which is correlated with soft margarine 0.448, stick margarine 0.500 and squeeze margarine 0.416). Among margarine categories, stick margarine and soft margarine affect each other's expenditures and purchase incidence more than squeeze margarine. In the paper tissue categories, paper towel tissue and toilet tissue significantly affect each other's expenditures and purchase incidences. Expenditure of allergy tablets is significantly correlated with purchase incidence of cold tablets (0.338), however, interestingly, not the other way around (0.007). From this table, we can conclude that expenditures of coffee ground decaf, coffee ground regular, and cold tablets are mostly not significantly correlated with purchase incidence of other categories. Expenditures of cold tablets are negatively affected by the purchase incidence of the most of the categories (though these correlations are very low and therefore not significant). Expenditures of cold tablets have very low correlation values for almost all purchase incidences, which means that consumers purchase cold medicines without considering other category purchases. However, expenditures of allergy are highly related to almost all other category purchase decisions, indicating that this may be a traffic driver.

We summarize the estimated covariate effects (i.e. marketing mix, price and promotion) for the sixteen categories in Table 4.20. Modeling purchase incidence and expenditures together provides useful insights into the differing nature of price and promotion sensitivities in the choice and expenditure stages of the purchase decision. All price coefficients in the incidence part of the model are negative as expected, except for insignificant laundry price coefficients. Crackers (-0.111), squeeze margarine (-0.156), cold (-0.130), allergy (-0.129) and spaghetti sauce (-0.147) have the highest price sensitivity of purchase incidence; allergy (-0.689), coffee instant decaf (-0.622), and cold (-0.482) have the highest price sensitivity of expenditures. We obtain a few significant positive price coefficients, which is unexpected, in the expenditure part: butter (0.078), coffee ground regular (0.267), laundry detergent (0.869). However, most of the price coefficients for both purchase incidence and expenditure are negative as expected.

According to the results in Table 4.20, promotion is not a very significant factor in the decision process of purchases of many categories in online grocery shopping except for the spaghetti sauce and toilet paper categories. The promotion coefficient of purchase incidence of spaghetti sauce is significant, equal to 1.096. In the decision process of spaghetti sauce, promotion is an important factor for purchase. Sales promotions are also an important factor in the purchase decision for paper toilet tissue (0.147). The highest promotion effects on expenditures are 18.344 of soap, 19.754 of allergy, 9.937 of squeeze margarine, 6.451 of crackers, and finally 7.592 of coffee instant regular. Spending for all margarine categories is affected by sales promotions. If consumers choose to purchase that category, sales promotion is important in influencing the decision of how much to spend (consumers may buy more than they used to, and stockpile). We can state that people’s decisions regarding how much to spend are more strongly affected by sales promotions than is the decision of what category to buy. We see that promotion effects on expenditures may be negative as well, i.e. if the price cut is large, people spend less on the product because it is cheaper, if they buy the same amount on average.

## 4.9 Optimization Results

We report the optimization results in this section. We present the estimated objective function for each customer in Table 4.22. The objective function ($E[\pi_i]$) illustrates the difference between the expected expenditure of selected categories depending on whether it is promoted or not. We estimate the expected expenditure of each category for each consumer with and without promotion, so, as shown in equation 4.11. Selecting category allocations from many different possible allocations is a combinatorial optimization problem. A similar combinatorial optimization approach to optimize the design and content of electronic communications were first applied by Ansari and Mela (2003). We generate many designs with the modified Federov design generating algorithm, which we also used to select allocations of blocks of questions to splits in the split questionnaire design problem. The promotion design with maximum promotional lift in spending for each category is accepted as the optimal promotion design. Though some revenues ($E[\pi_i]$) for some customers are negative in Table 4.22, most of them are positive, as expected (expected expenditures can be negative, since if there is a high price cut, spending will decrease if the customer purchases the regular amount, as if it were not on promotion). That is, promoting a category increases consumers spending on average, but for some customers the predicted effect on spending is negative. In these cases, it may be optimal to promote fewer than the five categories that are fixed in promotional plans. We note that in our optimal promotion design, negative revenues generally belong to customers with small shopping baskets (i.e. they spend more) or very large baskets (i.e. they spend much). In other words, managers do not need to offer promotions for customers who are already ready to spend a large amount, or for customers who spend little for their necessities during the online shopping process. In such a situation, determining the optimal number of categories to promote (i.e. number of coupons), considering the most profitable categories from many, is another important topic which must be considered.

We report the pairwise frequencies of selected categories in the final promotion design for all consumers in Table 4.23. According to this table, the promotion design offers sales promotions the most frequently for squeeze margarine (72 customers out of 133), coffee ground regular (68), cold (59), cracker (54), and finally butter (51). The least offered categories are allergy tablets (13), coffee instant decaf (17) and soap (26). Some categories are offered together more often than the others. For example, squeeze margarine and coffee ground regular are offered together 34 times, cold and squeeze margarine 32 times, cold and coffee ground regular 30 times, and toilet paper and squeeze margarine are offered together to customers (i.e. appeared together in a customized promotion plan) 26 times. On the other hand, allergy medication and coffee instant regular, stick margarine, toilet paper, and soap and coffee instant decaf, and stick margarine are offered together only once. As we notice, toilet paper and paper towel tissue are the most frequently promoted categories in the data, however our optimal customized promotion design does not suggest them to be promoted the most frequently; they appear only 13 times together.

Table 4.17: Estimated bivariate correlations of expenditure across categories

| | Category | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Allergy | 1 | 0.100 | 0.199 | 0.184 | 0.084 | 0.118 | 0.205 | 0.094 | 0.057 | 0.058 | 0.163 | 0.070 | 0.063 | 0.064 | 0.056 | 0.073 |
| 2 | Butter | **0.463** | 1 | 0.138 | 0.148 | 0.089 | 0.134 | 0.189 | 0.073 | 0.044 | 0.041 | 0.131 | 0.053 | 0.032 | 0.037 | 0.051 | 0.040 |
| 3 | Coffee GD. | 0.300 | 0.188 | 1 | 0.166 | 0.179 | 0.144 | 0.168 | 0.161 | 0.143 | 0.143 | 0.155 | 0.164 | 0.136 | 0.164 | 0.172 | 0.155 |
| 4 | Coffee GR. | 0.247 | 0.089 | 0.052 | 1 | 0.180 | 0.150 | 0.132 | 0.159 | 0.120 | 0.146 | 0.117 | 0.147 | 0.112 | 0.161 | 0.148 | 0.155 |
| 5 | Coffee ID. | **0.619** | **0.380** | 0.268 | 0.235 | 1 | 0.137 | 0.158 | 0.124 | 0.133 | 0.091 | 0.149 | 0.134 | 0.091 | 0.127 | 0.112 | 0.108 |
| 6 | Coffee IR. | **0.353** | **0.376** | 0.142 | 0.055 | **0.274** | 1 | 0.169 | 0.161 | 0.115 | 0.114 | 0.134 | 0.180 | 0.105 | 0.087 | 0.131 | 0.136 |
| 7 | Cold | -0.095 | 0.075 | -0.021 | 0.005 | 0.049 | -0.059 | 1 | 0.188 | 0.196 | 0.170 | 0.165 | 0.190 | 0.193 | 0.200 | 0.163 | 0.219 |
| 8 | Cracker | **0.298** | **0.188** | 0.011 | -0.105 | 0.105 | 0.129 | -0.087 | 1 | 0.088 | 0.084 | 0.135 | 0.085 | 0.072 | 0.075 | 0.076 | 0.067 |
| 9 | Laundry | **0.500** | **0.325** | 0.254 | 0.122 | **0.458** | **0.295** | -0.179 | 0.138 | 1 | 0.043 | 0.137 | 0.047 | 0.027 | 0.027 | 0.042 | 0.036 |
| 10 | Marg. Soft | **0.588** | **0.415** | 0.144 | 0.122 | **0.523** | **0.256** | 0.135 | **0.339** | **0.367** | 1 | 0.169 | 0.041 | 0.029 | 0.031 | 0.048 | 0.036 |
| 11 | Marg.Sqz. | **0.369** | **0.402** | 0.164 | 0.109 | 0.260 | **0.305** | -0.029 | 0.085 | **0.409** | 0.161 | 1 | 0.140 | 0.113 | 0.130 | 0.167 | 0.124 |
| 12 | Marg. Stick | **0.568** | **0.478** | 0.173 | 0.151 | **0.388** | 0.195 | -0.055 | **0.362** | **0.401** | **0.501** | 0.208 | 1 | 0.032 | 0.031 | 0.046 | 0.045 |
| 13 | Paper Toilet | **0.772** | **0.434** | **0.285** | **0.237** | **0.477** | **0.319** | -0.081 | **0.261** | **0.444** | **0.451** | **0.511** | **0.463** | 1 | 0.020 | 0.039 | 0.027 |
| 14 | Paper Towel | **0.594** | **0.467** | 0.222 | 0.112 | **0.492** | **0.393** | -0.189 | **0.237** | **0.408** | **0.423** | **0.374** | **0.431** | **0.503** | 1 | 0.031 | 0.030 |
| 15 | Soap | **0.588** | **0.409** | 0.137 | 0.146 | **0.296** | **0.349** | **-0.364** | **0.236** | **0.320** | **0.368** | 0.160 | **0.450** | **0.445** | **0.420** | 1 | 0.044 |
| 16 | Spag. Sauce | **0.506** | **0.365** | 0.081 | 0.063 | **0.381** | **0.302** | 0.096 | **0.320** | **0.327** | **0.383** | **0.305** | **0.375** | **0.419** | **0.367** | **0.365** | 1 |

Lower triangle values are correlations and upper triangle (with italics) values are standard errors. Bold categories are significantly correlated with significance level 0.05.

Table 4.18: Estimated bivariate correlations of purchase incidence across categories

| Category | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** Allergy | **1** | *0.145* | *0.136* | *0.113* | *0.136* | *0.131* | *0.129* | *0.145* | *0.094* | *0.112* | *0.122* | *0.114* | *0.102* | *0.103* | *0.111* | *0.120* |
| **2** Butter | 0.113 | **1** | *0.063* | *0.056* | *0.098* | *0.088* | *0.108* | *0.048* | *0.031* | *0.039* | *0.107* | *0.047* | *0.022* | *0.024* | *0.040* | *0.031* |
| **3** Coffee GD. | **0.387** | **0.395** | **1** | *0.072* | *0.108* | *0.104* | *0.119* | *0.069* | *0.059* | *0.071* | *0.122* | *0.065* | *0.050* | *0.054* | *0.060* | *0.055* |
| **4** Coffee GR. | **0.515** | **0.402** | **0.663** | **1** | *0.108* | *0.097* | *0.131* | *0.080* | *0.052* | *0.051* | *0.119* | *0.055* | *0.039* | *0.036* | *0.055* | *0.051* |
| **5** Coffee ID. | **0.275** | **0.259** | **0.307** | **0.424** | **1** | *0.110* | *0.166* | *0.132* | *0.089* | *0.084* | *0.162* | *0.122* | *0.080* | *0.079* | *0.109* | *0.117* |
| **6** Coffee IR. | **0.270** | **0.366** | **0.360** | **0.421** | **0.424** | **1** | *0.131* | *0.119* | *0.059* | *0.087* | *0.142* | *0.109* | *0.054* | *0.053* | *0.090* | *0.078* |
| **7** Cold | **0.410** | 0.125 | **0.259** | **0.358** | 0.225 | 0.220 | **1** | *0.144* | *0.081* | *0.090* | *0.153* | *0.123* | *0.072* | *0.076* | *0.090* | *0.106* |
| **8** Cracker | 0.154 | **0.354** | **0.317** | **0.363** | 0.067 | **0.272** | 0.089 | **1** | *0.059* | *0.045* | *0.138* | *0.055* | *0.036* | *0.038* | *0.055* | *0.040* |
| **9** Laundry | **0.368** | **0.376** | **0.313** | **0.370** | **0.244** | **0.300** | **0.238** | **0.252** | **1** | *0.031* | *0.076* | *0.036* | *0.020* | *0.022* | *0.037* | *0.028* |
| **10** Marg. Soft | **0.312** | **0.479** | **0.331** | **0.453** | **0.353** | **0.293** | **0.251** | **0.424** | **0.451** | **1** | *0.088* | *0.033* | *0.024* | *0.026* | *0.039* | *0.029* |
| **11** Marg.Sqz. | 0.136 | **0.323** | 0.166 | 0.133 | 0.101 | 0.204 | 0.170 | 0.234 | **0.474** | **0.367** | **1** | *0.093* | *0.075* | *0.067* | *0.116* | *0.091* |
| **12** Marg. Stick | **0.296** | **0.521** | **0.450** | **0.426** | **0.245** | **0.259** | 0.195 | **0.464** | **0.460** | **0.559** | **0.285** | **1** | *0.024* | *0.028* | *0.040* | *0.036* |
| **13** Paper Toilet | **0.268** | **0.523** | **0.457** | **0.422** | **0.267** | **0.400** | **0.260** | **0.424** | **0.515** | **0.526** | **0.347** | **0.539** | **1** | *0.015* | *0.027* | *0.021* |
| **14** Paper Towel | **0.276** | **0.530** | **0.397** | **0.482** | **0.299** | **0.384** | **0.238** | **0.429** | **0.510** | **0.490** | **0.307** | **0.490** | **0.615** | **1** | *0.027* | *0.024* |
| **15** Soap | **0.263** | **0.431** | **0.361** | **0.427** | 0.182 | **0.326** | 0.086 | **0.359** | **0.385** | **0.403** | **0.330** | **0.490** | **0.485** | **0.459** | **1** | *0.034* |
| **16** Spag. Sauce | **0.272** | **0.439** | **0.345** | **0.430** | 0.211 | **0.298** | **0.320** | **0.426** | **0.397** | **0.468** | **0.323** | **0.466** | **0.504** | **0.457** | **0.426** | **1** |

Lower triangle values are correlations and upper triangle (with italics) values are standard errors. Bold categories are significantly correlated with significance level 0.05.

Table 4.19: Estimated bivariate correlations of purchase incidence-expenditure across categories.

| | Purchase Incidence | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***Expenditure*** | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** | ***6*** | ***7*** | ***8*** | ***9*** | ***10*** | ***11*** | ***12*** | ***13*** | ***14*** | ***15*** | ***16*** |
| ***1*** Allergy | ***0.528*** | ***0.507*** | ***0.553*** | ***0.637*** | **0.348** | **0.438** | **0.338** | **0.436** | ***0.559*** | ***0.620*** | **0.337** | ***0.595*** | ***0.812*** | ***0.649*** | ***0.600*** | ***0.561*** |
| ***2*** Butter | 0.081 | ***0.966*** | **0.355** | **0.360** | **0.236** | **0.338** | 0.093 | **0.332** | **0.348** | **0.443** | **0.308** | **0.497** | **0.485** | ***0.510*** | **0.397** | **0.406** |
| ***3*** Coffee GD. | 0.182 | 0.211 | 0.516 | 0.315 | 0.158 | 0.241 | 0.123 | 0.078 | 0.275 | 0.159 | 0.113 | 0.172 | **0.300** | 0.243 | 0.136 | 0.107 |
| ***4*** Coffee GR. | 0.184 | 0.101 | 0.021 | 0.189 | 0.211 | 0.123 | 0.257 | -0.092 | 0.152 | 0.127 | 0.094 | 0.165 | 0.245 | 0.128 | 0.151 | 0.074 |
| ***5*** Coffee ID. | **0.412** | **0.415** | **0.459** | ***0.596*** | ***0.672*** | **0.448** | **0.347** | 0.208 | ***0.519*** | ***0.549*** | 0.248 | **0.406** | ***0.519*** | ***0.528*** | **0.302** | **0.426** |
| ***6*** Coffee IR. | 0.134 | **0.392** | 0.187 | 0.194 | 0.171 | **0.491** | 0.172 | 0.226 | **0.320** | **0.273** | 0.332 | 0.212 | **0.346** | **0.420** | **0.351** | **0.328** |
| ***7*** Cold | 0.007 | 0.077 | -0.025 | -0.031 | 0.066 | -0.043 | 0.192 | -0.142 | -0.193 | 0.132 | -0.056 | -0.054 | -0.080 | -0.188 | **-0.377** | 0.086 |
| ***8*** Cracker | 0.095 | **0.204** | **0.198** | **0.258** | 0.001 | 0.162 | 0.048 | ***0.833*** | 0.145 | **0.350** | 0.161 | **0.374** | **0.279** | **0.256** | **0.242** | **0.346** |
| ***9*** Laundry | **0.326** | **0.349** | **0.279** | **0.308** | **0.193** | **0.262** | **0.191** | **0.233** | ***0.960*** | **0.399** | **0.464** | **0.421** | **0.467** | **0.445** | **0.345** | **0.377** |
| ***10*** Marg. Soft | **0.288** | **0.448** | **0.297** | **0.419** | **0.337** | **0.268** | **0.237** | **0.404** | **0.416** | ***0.970*** | **0.351** | ***0.522*** | ***0.500*** | **0.467** | **0.375** | **0.435** |
| ***11*** Marg.Sqz. | 0.051 | **0.416** | 0.189 | 0.124 | 0.093 | 0.221 | 0.149 | 0.177 | **0.425** | 0.176 | 0.369 | 0.225 | ***0.521*** | **0.403** | 0.153 | **0.331** |
| ***12*** Marg. Stick | **0.284** | ***0.500*** | **0.436** | **0.409** | 0.232 | **0.239** | 0.170 | **0.448** | **0.438** | ***0.535*** | **0.255** | ***0.946*** | ***0.510*** | **0.471** | **0.460** | **0.417** |
| ***13*** Paper Toilet | **0.235** | **0.470** | **0.421** | **0.372** | **0.233** | **0.373** | **0.240** | **0.398** | **0.486** | **0.474** | **0.319** | **0.489** | ***0.984*** | ***0.561*** | **0.444** | **0.467** |
| ***14*** Paper Towel | **0.251** | **0.483** | **0.360** | **0.446** | **0.278** | **0.351** | **0.221** | **0.404** | **0.472** | **0.443** | **0.273** | **0.447** | ***0.555*** | ***0.982*** | **0.411** | **0.413** |
| ***15*** Soap | **0.243** | **0.440** | **0.355** | **0.413** | 0.183 | **0.325** | 0.070 | **0.351** | **0.358** | **0.394** | **0.323** | **0.477** | **0.486** | **0.467** | ***0.963*** | **0.403** |
| ***16*** Spag. Sauce | 0.243 | **0.395** | **0.307** | **0.383** | 0.183 | **0.268** | **0.307** | **0.388** | **0.342** | **0.412** | **0.295** | **0.421** | **0.453** | **0.408** | **0.387** | ***0.973*** |

Bold categories are significantly correlated with significance level 0.05.

Table 4.20: Posterior Θ (mean across all customers) with full Λ

| | Expenditure | | | Incidence | | |
|---|---|---|---|---|---|---|
| Categories | intercept | price | promotion | intercept | price | promotion |
| Allergy medicine | -3.189 | **-0.689** | **19.754** | -0.261 | **-0.129** | 0.166 |
| Butter | -2.024 | **0.078** | 0.269 | -1.409 | **-0.034** | -0.021 |
| Coffee Gr. Decaf | -9.983 | **-0.156** | -0.923 | -0.403 | **-0.077** | 0.103 |
| Coffee Gr. Regular | -3.148 | **0.267** | **-15.053** | 0.006 | **-0.097** | 0.001 |
| Coffee Ins. Decaf | 4.452 | **-0.622** | **-26.226** | 0.632 | **-0.054** | -0.018 |
| Coffee Ins. Regular | -21.588 | **-0.137** | **7.592** | -0.500 | **-0.039** | 0.012 |
| Cold medicine | -9.441 | **-0.482** | 0.718 | 0.159 | **-0.130** | -0.031 |
| Crackers | -1.477 | **-0.203** | **6.541** | -1.274 | **-0.111** | -0.031 |
| Laundry | -8.797 | **0.869** | **-10.209** | -1.598 | 0.009 | -0.029 |
| Margarine Soft | -0.916 | **-0.268** | -0.254 | -0.639 | **-0.085** | -0.100 |
| Margarine Squeeze | 2.020 | **-0.144** | **9.937** | -1.545 | **-0.156** | 0.058 |
| Margarine Stick | -1.929 | **-0.140** | **-16.147** | -1.611 | **-0.087** | -0.029 |
| Paper Toilet | 3.552 | **-0.076** | 0.038 | 0.177 | **-0.026** | **0.147** |
| Paper Towel | 1.873 | **-0.072** | 0.023 | -0.138 | -0.004 | -0.536 |
| Soap | 1.631 | **-0.106** | **18.344** | -0.301 | -0.030 | 0.013 |
| Spaghetti Sauce | 1.216 | **-0.327** | -0.006 | -0.439 | **-0.147** | **1.096** |

Table 4.21: Posterior mean of standard errors of Θ's with full Λ

| | Expenditure | | | Incidence | | |
|---|---|---|---|---|---|---|
| Categories | intercept | price | promotion | intercept | price | promotion |
| Allergy medicine | 0.967 | 0.197 | 0.734 | 0.552 | 0.021 | 0.263 |
| Butter | 0.263 | 0.023 | 0.192 | 0.166 | 0.011 | 0.080 |
| Coffee Gr. Decaf | 1.652 | 0.056 | 0.927 | 0.722 | 0.020 | 0.212 |
| Coffee Gr. Regular | 0.993 | 0.090 | 1.294 | 0.332 | 0.013 | 0.111 |
| Coffee Ins. Decaf | 1.249 | 0.098 | 1.677 | 0.605 | 0.009 | 0.236 |
| Coffee Ins. Regular | 0.444 | 0.047 | 0.615 | 0.598 | 0.007 | 0.280 |
| Cold medicine | 0.589 | 0.041 | 1.111 | 0.844 | 0.029 | 0.149 |
| Crackers | 0.522 | 0.046 | 0.955 | 0.476 | 0.035 | 0.152 |
| Laundry | 0.718 | 0.093 | 1.799 | 0.356 | 0.045 | 0.121 |
| Margarine Soft | 0.614 | 0.046 | 0.384 | 0.489 | 0.031 | 0.140 |
| Margarine Squeeze | 0.889 | 0.045 | 0.999 | 0.421 | 0.036 | 0.217 |
| Margarine Stick | 0.519 | 0.044 | 1.333 | 0.310 | 0.036 | 0.129 |
| Paper Toilet | 1.597 | 0.035 | 0.055 | 0.592 | 0.012 | 0.073 |
| Paper Towel | 2.482 | 0.030 | 0.094 | 0.418 | 0.005 | 0.420 |
| Soap | 0.453 | 0.012 | 0.695 | 1.050 | 0.018 | 0.072 |
| Spaghetti Sauce | 0.636 | 0.072 | 0.151 | 0.879 | 0.047 | 0.546 |

Table 4.22 Optimization results

| **Consumer** | $E[p_i]$ | **Consumer** | $E[p_i]$ | **Consumer** | $E[p_i]$ | **Consumer** | $E[p_i]$ |
|---|---|---|---|---|---|---|---|
| 1 | -2.923 | 35 | 0.878 | 69 | -0.728 | 103 | -0.062 |
| 2 | 3.349 | 36 | -1.589 | 70 | -2.297 | 104 | -1.018 |
| 3 | 0.621 | 37 | -1.133 | 71 | -1.799 | 105 | 2.303 |
| 4 | 1.880 | 38 | -2.813 | 72 | 1.726 | 106 | -2.670 |
| 5 | 0.212 | 39 | -0.675 | 73 | 1.707 | 107 | -3.201 |
| 6 | -0.618 | 40 | 0.354 | 74 | -0.511 | 108 | -0.299 |
| 7 | 0.764 | 41 | 0.958 | 75 | -0.300 | 109 | -0.597 |
| 8 | -3.687 | 42 | 1.714 | 76 | -1.970 | 110 | -0.995 |
| 9 | 1.225 | 43 | -0.737 | 77 | -2.033 | 111 | 0.504 |
| 10 | -2.926 | 44 | 0.574 | 78 | 0.480 | 112 | 1.538 |
| 11 | 1.444 | 45 | -3.032 | 79 | 0.709 | 113 | -1.576 |
| 12 | -0.627 | 46 | -1.159 | 80 | 1.296 | 114 | -0.208 |
| 13 | -1.487 | 47 | -0.379 | 81 | -2.495 | 115 | -1.053 |
| 14 | 2.769 | 48 | -0.340 | 82 | 1.330 | 116 | 0.590 |
| 15 | -1.826 | 49 | 0.778 | 83 | 1.180 | 117 | 0.763 |
| 16 | -0.960 | 50 | 0.832 | 84 | 3.072 | 118 | 0.723 |
| 17 | 0.032 | 51 | -0.016 | 85 | -2.370 | 119 | -3.069 |
| 18 | -0.586 | 52 | -0.607 | 86 | 1.392 | 120 | 1.584 |
| 19 | -0.360 | 53 | 0.976 | 87 | -0.391 | 121 | -3.365 |
| 20 | 0.581 | 54 | 0.067 | 88 | -0.740 | 122 | -0.327 |
| 21 | 0.410 | 55 | 2.538 | 89 | 0.880 | 123 | -0.439 |
| 22 | -0.652 | 56 | -0.258 | 90 | -1.698 | 124 | 0.308 |
| 23 | 1.084 | 57 | -1.917 | 91 | -2.312 | 125 | 0.764 |
| 24 | -0.306 | 58 | -2.967 | 92 | 2.218 | 126 | 2.249 |
| 25 | -1.185 | 59 | -0.832 | 93 | 3.136 | 127 | -0.753 |
| 26 | 1.151 | 60 | -3.235 | 94 | 0.063 | 128 | -1.635 |
| 27 | -3.463 | 61 | 0.099 | 95 | 2.176 | 129 | 1.071 |
| 28 | -0.781 | 62 | -0.490 | 96 | -0.485 | 130 | 0.520 |
| 29 | -0.444 | 63 | -2.274 | 97 | 3.175 | 131 | -0.288 |
| 30 | -3.347 | 64 | 1.707 | 98 | -1.401 | 132 | -0.709 |
| 31 | 0.155 | 65 | 0.821 | 99 | -1.475 | 133 | 4.001 |
| 32 | 2.284 | 66 | 1.108 | 100 | -2.470 | | |
| 33 | 1.305 | 67 | 0.212 | 101 | -0.751 | | |
| 34 | 1.461 | 68 | 0.486 | 102 | -0.231 | | |

Note: $E[\pi_i]$ is the difference between expenditure of a category if promoted or not promoted, calculated using price per volume

Table 4.23: Frequencies of offers across categories

| | Category | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Allergy | 13 | | | | | | | | | | | | | | | |
| 2 | Butter | 3 | 51 | | | | | | | | | | | | | | |
| 3 | Coffee GD. | 4 | 9 | 36 | | | | | | | | | | | | | |
| 4 | Coffee GR. | 6 | 25 | 11 | 68 | | | | | | | | | | | | |
| 5 | Coffee ID. | 2 | 4 | 4 | 9 | 17 | | | | | | | | | | | |
| 6 | Coffee IR. | 1 | 14 | 8 | 12 | 2 | 30 | | | | | | | | | | |
| 7 | Cold | 3 | 18 | 13 | 30 | 4 | 7 | 59 | | | | | | | | | |
| 8 | Cracker | 4 | 15 | 15 | 25 | 5 | 10 | 25 | 54 | | | | | | | | |
| 9 | Laundry | 5 | 14 | 11 | 16 | 4 | 4 | 9 | 11 | 33 | | | | | | | |
| 10 | Marg. Soft | 4 | 17 | 8 | 18 | 4 | 7 | 17 | 5 | 5 | 38 | | | | | | |
| 11 | Marg.Sqz. | 3 | 22 | 18 | 34 | 11 | 19 | 32 | 25 | 17 | 15 | 72 | | | | | |
| 12 | Marg. Stick | 1 | 12 | 4 | 12 | 4 | 4 | 8 | 8 | 6 | 7 | 16 | 28 | | | | |
| 13 | Paper Toilet | 1 | 14 | 14 | 19 | 5 | 11 | 21 | 23 | 11 | 13 | 26 | 9 | 49 | | | |
| 14 | Paper Towel | 7 | 18 | 7 | 18 | 5 | 7 | 18 | 18 | 9 | 12 | 20 | 10 | 13 | 46 | | |
| 15 | Soap | 6 | 8 | 9 | 14 | 1 | 6 | 9 | 10 | 5 | 10 | 8 | 1 | 5 | 4 | 26 | |
| 16 | Spag. Sauce | 2 | 11 | 9 | 23 | 4 | 8 | 22 | 17 | 5 | 10 | 22 | 10 | 11 | 18 | 8 | 45 |

## 4.10 Conclusion

In this chapter, we focus on customization of promotions across multiple categories. We have a model which allows estimating the effects of promotions what categories consumers purchase and how much they spend for each category. We use a hierarchical Bayes multivariate type-2 tobit model for that. We used the Gibbs sampling to estimate model parameters. After estimating the model, we approach the design of customized promotion plan design with the Bayesian decision approach. Our objective function in the optimization problem is revenue, which is a function of the expected spending of a category when we do or do not promote. We estimate the objective function in each MCMC chain, which allows us to integrate out the uncertainty in the parameters. After we estimate the expected promotional lift in expenditure for each customer for each category, we use the Federov design generating algorithm to choose the best among many possible customized promotion designs. The designs with the maximum expected promotional lift in spending for the selected categories are the optimal customized promotion designs. We found an optimal promotion design for each customer, and thus customization was successfully performed with the Bayesian decision framework, although it appeared that for some customers the chosen numbers of promotions (five) results in a decrease in revenue, so that this subsection of customers, less than five promotions may be optimal.

The estimated correlation matrix illustrates that purchase incidence and expenditure should be modeled together. Interdependence between purchase incidence and expenditure between categories should be examined more carefully. Taking these cross-correlations into account helps to create more efficient marketing strategies, including cross-selling strategies, setting optimal prices, creating more efficient shopping websites, creating better shelf designs in stores, creating more successful online or in store coupon strategies, and designing better customization strategies.

Empirical results illustrate that sales promotion is an important factor for the decision of how much to spend. Consumers may buy more than they used to and stockpile under sales-promotions. People’s decisions of how much to spend is more affected by sales promotions compared to the decision of what category to buy. Promotion effects of expenditures may also be negative, i.e. if the price cut is large, people spend less on the product because it is cheaper, if sales volume does not increase. We observe effects of shelf layout in online shopping, similar to in brick and mortar stores. Since some categories are presented on the same web page, they are purchased together more frequently. For example, more frequent coincidence of coffee ground regular and coffee ground decaf, paper toilet tissue and paper towel tissue, and butter and margarine categories can be explained by this. We found sales promotions to be most important for the purchase decision of spaghetti sauce. Price effects are significant in almost all categories for both purchase incidence and expenditure decisions. The expenditures on coffee ground decaf, coffee ground regular, and cold tablets are mostly not significantly correlated with purchase incidence of other categories. Expenditures of allergy tablets are highly related to purchase decisions in almost all other categories, indicating that this may be a traffic driver.

The model used in this chapter can be potentially improved in four ways: 1. Include consumer budget constraints in the estimation and the optimal allocation of promotions. 2. Include dynamics in the price and promotion parameters (state-space approach) so that the optimal allocation of promotion varies over time and is dependent on reactions to the most recently promoted categories. 3. Build a brand-choice model on top of the category expenditure and incidence model, so that we also know what brands to promote in each category. 4. Reformulate the model into a purchase quantity and incidence model, considering budgetary constraints.