Detecting and measuring crucial differences between cognitive maps

Timo J. Septer  
University of Groningen, The Netherlands

Jacob Dijkstra  
University of Groningen, The Netherlands

Frans N. Stokman  
University of Groningen, The Netherlands

Abstract

The reasoning underlying the attitude of stakeholders towards the implementation of a means can be captured in causal cognitive maps about the effects of the means on relevant goals. For acyclic cognitive maps with weighted-directed signed links we propose quantitative measures for the weight of the paths between means and goals and a measure for the total result of the means on all goals. In data concerning the cognitive maps of 94 employees about their perceived consequences of a merger, the latter measure correlates strongly with their attitude towards the merger. Finally we propose a method to detect for which links in the map the cognitive differences between individuals contribute most to their differences in attitudes towards the means, in the sense that agreement on these links would decrease the variance in the attitudes most.

Keywords

Cognitive mapping, conflict analysis, networks and graphs, organizations, rationality

Corresponding author:
Timo J. Septer, Department of Sociology, Interuniversity Center for Social Science Theory and Methodology (ICS), University of Groningen, Grote Rozenstraat 31, 9712 TG Groningen, The Netherlands. 
Email: t.j.septer@rug.nl
Introduction

In January 2006, two Dutch inspection agencies merged into one national authority concerning the inspection of food and commodities. The contents of the inspections by each of the former organizations differed, but they visited to a large extent the same companies. The merger was intended to increase productivity, by combining the inspections of both former organizations into one visit, and by combining their best practices. This merger fits into the ambition of the Dutch government to reduce the size of the civil service. Among employees were both supporters and opponents of the merger. Employees had constructed their own theories about the cause–effect relations concerning the merger (cf. Sims and Gioia, 1986; Weick, 1979). For instance, not all employees shared the view of management that productivity and quality levels would increase due to the merger.

Some conflicts concerning policy changes are rooted in differences in the cause–effect reasoning of individuals, that is, in different cognitions about the extent to which a policy change will have a positive or negative effect on the realization of the goals (Hammond et al., 1966; McGrath, 1984; Tjosvold, 1985, 2008). In order to change people’s overall attitudes towards policy changes, these cognitions should be changed (Bartunek, 1984). In our study in the food inspection agency, we have collected data concerning the attitudes of employees towards the merger, as well as their underlying causal theories. We have captured these causal theories concerning the consequences of the merger in so-called cognitive maps. A cognitive map is a directed network with nodes representing concepts and links between those nodes representing whether a change in one concept (the cause) will result in a change in another concept (the effect), according to a certain individual. Since we are interested in the cognition concerning the causal consequences of a policy change on a set of end goals, the maps we consider have a means-ends structure (Montibeller and Belton, 2006). In our study the means is the merger. The other nodes in the maps are sub goals and end goals. The extent to which these goals are reached cannot be changed directly, but only via changes in the means. An example of a cognitive map of an employee concerning the consequences of the merger is given in Figure 1.

If according to the employee two of these variables are causally related, an arrow is drawn between the corresponding nodes. For instance, the arrow from responsibility to productivity in Figure 1 indicates that, according to the employee, a change in the level of his responsibility will result in a change in productivity. More information about the links will be added later by assigning signs and strengths to the arrows.
We will present a mathematical measure to calculate an individual’s attitude towards the means based on their cognitive maps. With attitude towards a means we mean the extent to which individuals support or oppose the implementation of the means. In our data, our measure indeed correlates highly with the attitude of employees towards the merger. Subsequently, we introduce an algorithm to detect those cognitive differences between individuals that contribute most to the variance in their attitudes towards the means. Thus, the central question in this paper is how to compare causal cognitive maps, such that those cognitive differences in a group can be detected that add greatly to divergent attitudes, that is, the level of support or opposition towards implementation of the means.

Several measures have been proposed to compare maps between individuals. Eden et al. (1992) argue that there is no general approach to analyse maps. The relevance of using certain measures to compare maps depends on the research question and on the type of data in the cognitive map. They propose several measures to compare maps, such as a measure for cognitive

![Cognitive map](image-url)
complexity and measures to compare the structures of maps. Complexity measures are based on the number of nodes and links in the maps; individuals that perceive more links have a more complex cognition. The measures for structure focus on the level of linkage, so whether the nodes in the maps are highly interconnected or whether there are clusters in the maps that are disconnected from other clusters. Another type of analysis for cognitive maps is to focus on the nodes that are central in a map, namely those nodes with a large number of incoming or outgoing links (Eden, 2004; Eden et al., 1992). Other scholars compare the maps of individuals by using a distance ratio (Langfield-Smith and Wirth, 1992; Markóczy and Goldberg, 1995). Maps of two individuals are then compared by summing up the differences between the maps. Although all these measures give insight in the extent to which the maps of several individuals are aligned, they do not relate the maps of individuals to their attitudes.

We argue that the attitude of individuals towards a means will be related to the consequences they expect from that means. Since cognitive maps capture the reasoning concerning these consequences of the means, these maps must be a good indicator for the attitude of the individual towards the means. However, individuals that differ to a large extent in cognitive complexity can still have the same attitude towards the means. An individual with a more complex cognition will perceive more pros and cons than an individual with less cognitive complexity, but after balancing their pros and cons they might end up with the same attitude. However, these individuals would be very distinct, based on the distance ratio (Langfield-Smith and Wirth, 1992; Markóczy and Goldberg, 1995) and the measures of complexity and structure (Eden, 2004; Eden et al., 1992). On the other hand, if individuals only differ in cognition on the strength and the sign of one certain link and that link is very central in their maps, their attitudes towards the means might be very different. This is so, because a central link is part of several reasonings concerning the effect from the means on the goals. Yet, in the latter case the individuals’ cognitions would be rather similar according to the distance ratio (Langfield-Smith and Wirth, 1992; Markóczy and Goldberg, 1995) and the measures of complexity and structure (Eden, 2004; Eden et al., 1992).

Consequently, these measures cannot detect which inter-individual cognitive differences should be aligned in order to reach consensus concerning the implementation of the means. However, such information is extremely relevant in situations in which implementation of the means will have an impact on a large group of individuals. Even if the affected individuals lack formal power in the decision-making process, their support of the means is
frequently necessary, since they could obstruct successful implementation. The method we propose will detect those cognitive differences that contribute most to divergent attitudes towards the means under consideration. Some scholars (Anthony et al., 1994) have analysed strategic manipulation of information in political debate, using cognitive maps based on what was said and emphasized in debates. We use maps that capture the actual cognition of stakeholders and deduce what to bring up in a debate, in order to reduce conflict.

In the next section we will introduce a measure to infer the attitudes of the employees from their cognitive maps. In our data, this measure correlates highly with the actual attitudes towards the merger. Subsequently, we propose an algorithm to detect those links in the cognitive maps on which divergent views of the employees contribute most to the differences in attitudes towards the merger. Then we will sketch the background of the merger of the food inspection services and describe how the cognitive map data were collected. Our measures and the method to detect links on which reaching consensus is most relevant in order to decrease variance in attitudes are illustrated on the data. We end with a discussion.

Modelling the cognitive maps and predicting attitude

Technically, a cognitive map is a directed weighted graph \( G = (N, W) \), where \( N \) is the set of nodes \( v_i \) \( (i = 1, \ldots, n) \) and \( W \) the set of weights \( w_{ij} \) assigned to the link from node \( v_i \) to node \( v_j \). In graph theory one often uses the term vertex instead of node and the terms arrow, arc or directed edge instead of link. In line with the literature on cognitive maps, we will use the terms links and nodes. The links have a weight and a sign and have maximum strength \( \beta \) \((w_{ij} \in [-\beta, \beta])\). A weight of zero means that there is no perceived causal relation between the nodes. In general we denote nodes by \( v_i \). Since the means and goals are nodes of special interest, we sometimes denote them by, respectively, \( m \) and \( g_i \) \((i = 1, \ldots, n_g; \) with \( n_g \) being the number of goals).

A cognitive map can be represented by an \( n \times n \) valency matrix \( W \), with entries \( W_{ij} = w_{ij} \) (Axelrod, 1976; Harary et al., 1965). In Figure 2 a weighted cognitive map of one employee is given, together with the corresponding valency matrix.

A path is a sequence of nodes such that from each of its nodes there is a link to the next node in the sequence. By raising the matrix \( W \) to the power \( l \), paths of length \( l \) can be detected. If \( W_{ij}^l \neq 0 \), a path from \( v_i \) to \( v_j \) exists of length \( l \) (Axelrod, 1976; Harary et al., 1965). The number of paths...
from node \( v_i \) to \( v_j \) is \( n_{P_{ij}} \). These paths are denoted by \( P_{ij}^{[q]} = \{ v_i v_{i1}, v_{i1} v_{i2}, \ldots, v_{i_{l_q-2}} v_{i_{l_q-1}}, v_{i_{l_q-1}} v_{i_j} \} \), with \( v_k \in \mathbb{N}, 0 \leq q \leq n_{P_{ij}} \) and \( l_q \) being the length of the specific path, that is, the number of links in the path.

In our analysis the determination of the weight of a path, based on the weights of the links in that path, is crucial. If we are able to calculate the weight of a path from the weights of the links in the path, we can calculate what a change in the weight of one single link in the path would do to the weight of that whole path. Montibeller and Belton (2006) proposed measures to calculate the weight of a path from the weights of the links in that

\[
W = \begin{bmatrix}
1. Merger & 0 & 2 & 2 & -2 & -1 & -2 & -1 & -2 \\
2. IT systems & 0 & 0 & 0 & 0 & -2 & 0 & -1 & -1 \\
3. New tasks & 0 & 0 & 0 & 0 & -2 & -3 & -1 & -3 \\
4. Responsibility & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 2 \\
5. Job satisfaction & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6. Collaboration & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 \\
7. Productivity & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8. Quality of output & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix}
\]

**Figure 2.** Weighted cognitive map of sample employee with corresponding valency matrix, based on the questionnaire. Dotted arcs represent links with negative weights.
path. We adopt their terminology of partial effects $PE$ and total effects $TE$, but we will propose different ways to calculate them.

The weight of the path $P_{ij}^{[q]}$ is referred to as the partial effect $PE(P_{ij}^{[q]})$ from $v_i$ on $v_j$ (Montibeller and Belton, 2006). $TE(P_{ij})$ denotes the weight of the total effect from $v_i$ to $v_j$ of these $nP_{ij}$ paths taken together. We will introduce measures for $PE(P_{ij}^{[q]})$ and $TE(P_{ij})$ that fulfil some desirable criteria.

Criteria for measures of path strength

We expect that the attitude of individuals towards a means, hence the extent to which individuals support or oppose the implementation of it, will be correlated with the total effect they expect from the means. Therefore, we want a mathematical measure that assigns a single number to a cognitive map, capturing the extent to which the individual expects positive or negative effects of the means. As a first step we need a measure that assigns a weight to a path consisting of several links, that is, a measure for the partial effect $PE(P_{ij}^{[q]})$. We formulate three desirable criteria for this measure.

Criterion 1, sign of $PE$: concerning the sign of a $PE$ there is consensus that paths with an odd number of negative links have a negative sign and that paths with an even number of negative links have a positive sign (Axelrod, 1976; Montibeller and Belton, 2006):

$$\text{sgn}(PE(P_{ij}^{[q]})) = \text{sgn} \left( \prod_{kl \in P_{ij}^{[q]}} w_{kl} \right)$$

Criterion 2, range of strengths of $PE$: the strength of the causal effect between two nodes should not depend on the number of sub steps in the causal reasoning. A causal reasoning consisting of one link could be equally as strong as a more complex causal reasoning consisting of several links. In other words, the range of possible partial effects should be independent of path length:

$$PE(P_{ij}^{[q]}) \in [-\beta, \beta] \forall l_q$$
**Criterion 3, strength of PE:** the weight of all links in a path should contribute to the strength of the partial effect. Two paths that differ only in the weight assigned to one particular link should differ in strength. Some scholars (Kosko, 1986; Montibeller and Belton, 2006) consider the weighted links in cognitive maps as ordinal and proposed that PE is the minimum of the weights in a path. A disadvantage of this measure is that the most extreme values are crucial. Both the paths $v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_4$ and $v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_4$ would have a partial effect of weak (1). We consider the weights in the cognitive maps not as just ordinal, but as measured on a ratio level (cf. Langfield-Smith and Wirth, 1992; Markóczy and Goldberg, 1995; Roberts, 1976). In our measure for the strength of the partial effect of the two paths given above, the first path should have a stronger PE than the second.

There is no unique measure for the partial effect that fulfils criteria 1–3. Both measures, based on the addition of weights of links and measures based on multiplication, could fulfil these criteria. Measures for partial effect that are based on addition are suitable if the effect from node $i$ to $j$ represents some distance between $i$ and $j$. Since in the causal cognitive maps the weights of a path indicate some kind of intensity, rather than a distance, a multiplicative measure seems more natural. A measure for the partial effect that fulfils criteria 1–3 is the multiplication of the weights in a path, normalizing for the path length:

$$PE(P_{ij}^{[q]}) = \frac{1}{\beta^{l_{ij} - 1}} \prod_{kl \in P_{ij}^{[q]}} w_{kl}$$

(3)

Such a multiplicative measure has also been proposed by Roberts (1976), without normalizing for path length, however. Since the maximum weight of a path of length $l$ is $\beta^l$, we divide the measure through $\beta^{l-1}$ in order to fulfil criterion 2. As a result, the strength of longer paths is in the same range of values as the strength of shorter paths. Furthermore, the measure fulfils criterion 1, simply because the multiplication of an odd number of negative numbers results in a negative number. In addition, if the weight of only one link in the path is changed, the partial effect will also change through multiplication of the weights, except if one link has weight zero, then no path exists. Hence, the measure fulfils criterion 3 as well. For example, the path $v_1 \rightarrow^3 v_2 \rightarrow^1 v_3 \rightarrow^2 v_4$ would have a partial effect of $PE = \frac{1}{3^3}(-3 \cdot 1 \cdot 2) = \frac{2}{3}$ and
$v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_4$ a partial effect of $PE = \frac{1}{3!}(1 \cdot 1 \cdot 1) = \frac{1}{9}$, according to Equation (3).

Based on the partial effects, we want a measure for the strength of the total effect $TE$ of the means on a goal. We will formulate two desirable criteria for the strength of the total effect.

**Criterion 4, range of $TE$:** the strength of the total effect of the means on a goal should not depend on the number of partial effects from the means on the goals. Therefore, $TE(P_{ij}) \in [-\beta, \beta]$.

**Criterion 5, strength of $TE$:** the partial effects $PE$ of each path from a means to a goal should contribute to the strength of the total effect $TE$. If one individual perceives one partial effect on a goal as stronger than another individual, while their cognitive maps are otherwise identical, the former individual should perceive a stronger total effect $TE$ than the latter. This criterion excludes the measure for the total effect of Montibeller and Belton (2006), who proposed to distinguish between the total negative and the total positive effect, the total negative effect being the largest negative partial effect and the total positive effect being the largest positive partial effect. A disadvantage of that measure is that the most extreme values are crucial. If there are several paths with a moderate positive partial effect and only one path with a strong negative partial effect, the $TE$ would be ($-\text{strong, moderate}$), suggesting the negative paths are stronger. However, if the number of positive paths were taken into account, this might be unrealistic.

A measure for the total effect $TE$ meeting criteria 4 and 5 is the average over all possible partial effects. Let $n_{P_{ij}}$ denote the number of all possible partial effects from $v_i$ to $v_j$. Then the total effect is

$$TE(P_{ij}) = \frac{1}{n_{P_{ij}}} \sum_{q=1}^{n_{P_{ij}}} PE(P_{ij}^q)$$

We emphasize that we analyse maps for which the set of links is the same for all individuals. Let $A$ be the samples’ adjacency matrix with $A_{ij} = 1$ if the individuals are asked to assign a weight to the link from $v_i$ to $v_j$ and $A_{ij} = 0$ if the link is not evaluated. For example, in the data of the merger we have asked each employee to ascribe a weight to each of the 22 links drawn in Figure 1. Therefore, the adjacency matrix contained 22 1-entries. The reachability matrix $R = \sum_{i=2}^{n-1} A^i$ is the matrix indicating the total number of possible
indirect paths between each pair of nodes, that is, the total number of indirect paths from \( v_i \) to \( v_j \) is \( n_{P_{ij}} = R_{ij} \). Note that we sum up the powers of the adjacency matrices starting with \( A^2 \). Since we are interested in the number of indirect effects between two nodes, we have excluded the direct effects given in matrix \( A \).

It might be that according to a certain individual a path does not exist (i.e. it is assigned a weight of 0). However, in the calculation of the total effect of this individual the average is taken over all possible indirect paths. The total effects in Equation (4) are elements from the matrix in which the partial effects are summed up, divided by the elements from the reachability matrix. Hence Equation (4) can also be written as

\[
TE(P_{ij}) = \frac{\left[ \sum_{t=2}^{n-1} \beta \beta^t \right]}{R_{ij}} = \frac{\left[ \sum_{t=2}^{n-1} \beta \beta^t \right]}{\left[ \sum_{t=2}^{n-1} A^t \right]} \quad (5)
\]

To illustrate our measures, we will calculate some partial effects \( PE \) and total effects \( TE \) in the map from Figure 2. In this map there are six indirect paths from the merger to the goal productivity. Four of these paths consist of two links, namely the paths via IT systems, responsibility, new tasks and collaboration. Since several paths from merger to productivity are possible, we introduced \( q \) to denote which path we refer to. It is arbitrary which path number we assign to these paths and therefore we are free to refer to these paths as, respectively, path 1 to path 4. This means that \( (P_{Merger,Productivity}^{[1]}) \) refers to the path merger \( \rightarrow \) IT systems \( \rightarrow \) productivity. In Figure 2 there are two paths consisting of three links, namely the path merger \( \rightarrow \) new tasks \( \rightarrow \) collaboration \( \rightarrow \) productivity \( (P_{Merger,Productivity}^{[5]}) \) and the path merger \( \rightarrow \) responsibility \( \rightarrow \) collaboration \( \rightarrow \) productivity \( (P_{Merger,Productivity}^{[6]}) \).

We will calculate the partial effect for those six paths using Equation (3), with the maximum weight that can be assigned to a link being \( \beta = 3 \). The partial effect from merger \( \rightarrow \) IT systems \( \rightarrow \) productivity of the employee in Figure 2 is then \( PE(P_{Merger,Productivity}^{[1]}) = \frac{2-1}{3} = \frac{2}{3} \) and further we find \( PE(P_{Merger,Productivity}^{[2]}) = \frac{-2}{3} = \frac{2}{3} \), \( PE(P_{Merger,Productivity}^{[3]}) = \frac{2-1}{3} = \frac{2}{3} \) and \( PE(P_{Merger,Productivity}^{[4]}) = \frac{-2}{3} = \frac{2}{3} \). For the partial effects of the three paths we divide by \( \beta^2 = 9 \). Thus, the partial effect of merger \( \rightarrow \) responsibility \( \rightarrow \) collaboration \( \rightarrow \) productivity is \( PE(P_{Merger,Productivity}^{[5]}) = \frac{-2}{3} = \frac{2}{3} \) and for
merger → new tasks → collaboration → productivity it is
\[ PE(P_{\text{Merger, Productivity}}^6) = \frac{2}{3} \times \frac{3}{2} = \frac{4}{3}. \]

As can be seen in Figure 1, the possible number of paths from merger to productivity is 10, but since the individual whose map is depicted in Figure 2 assigned a weight of zero from job satisfaction to productivity, four of these 10 possible paths do not exist in his perception. Hence the paths \((P_{\text{Merger, Productivity}}^7)\) to \((P_{\text{Merger, Productivity}}^{10})\) are the paths via job satisfaction and all have a partial effect of zero.

The total effect from the merger to productivity can now be calculated by Equations (4) or (5) and is
\[ TE(P_{\text{Merger, Productivity}}) = \frac{2}{3} + \frac{2}{3} + \frac{4}{3} + \frac{8}{3} + \frac{4}{3} + 0 + 0 + 0 + 0 + 0 = \frac{8}{3}. \]

For the direct effect from merger to productivity, the employee in Figure 2 could choose an integer between 2 and 3 and he chose 1, which is in line with the total effect that we have calculated. Using Equations (3) and (4), the total effects from the merger to the three other goals in the map from Figure 2 are
\[ TE(P_{\text{Merger, Jobsatisfaction}}) = \frac{4}{3}, \quad TE(P_{\text{Merger, Collaboration}}) = \frac{5}{3} \] and \( TE(P_{\text{Merger, Quality of output}}) = \frac{34}{45} \).

**Measure for the total result TR perceived by individuals**

In order to relate the maps of the employees to their attitudes towards the means, we want to combine the total effects on the goals into one measure, which we will call the total result \( TR \). The total result is a measure for how satisfied the employee is with all the consequences (s)he expects from the merger, taken together. We define the total result \( TR(m) \) that an employee assigns to the means \( m \) as a weighted average of the total effects (s)he expects from \( m \) on the goals. The weights \( s_{g_i} \) indicate the relative importance of goal \( g_i \) to the employee. The total result \( TR \) of means \( m \) is

\[ TR(m) = \frac{\sum_{i=1}^{n_{s}} s_{g_i} \cdot TE(P_{m, g_i})}{\sum_{i=1}^{n_{s}} s_{g_i}} \] (6)

The total result of the employee in Figure 2 can be calculated with this formula. Since in these data we have no measure for goal salience, we assume that all goals are equally important to each employee and set all
saliences equal to 1. The total result is then \( TR(\text{Merger}) = \frac{-5}{9} + \frac{4}{3} + \frac{-5}{3} + \frac{-34}{45} = -1.70 \approx -1.08. \)

**Method for detecting links on which agreement is crucial**

We expect that the attitudes towards the means, that is, the extent to which individuals support or oppose it, are grounded in the causal consequences they expect. As we will show later in the Results section, in the data concerning the merger our measure for the total result \( TR \) indeed correlates highly with attitude. Thus, divergent attitudes towards the means in a group are the result of divergent cognitions. Cognitions could be more aligned through debate and of course if consensus is reached in the group concerning the weights of each link, that is, if the maps become identical, the attitudes towards the means will be similar. However, discussing each link is time consuming. It is relevant to detect one link, or a small set of links, on which reaching agreement contributes largely to more similar attitudes towards the means.

As a next step we want to identify those causal links on which divergent causal estimates among the employees contribute most to the diversity in attitudes. If the group wants to align their attitudes, these are the links about which consensus is crucial. Intuitively, one could argue that the link with the highest variance is the most crucial link. However, since a single link can be part of multiple paths, it could be that consensus on another link would reduce the divergence in attitudes more.

As a measure for the divergence in attitudes we use the variance in total result, thus the sum of the squared deviations of the mean, from all individuals in our sample. Consider a sample of \( n \) individuals in which the total result of individual \( \alpha \) is denoted by \( TR^{[\alpha]} \). Then this variance is

\[
\sigma^2 (TR) = \frac{1}{n} \sum_{\alpha = 1}^{n} \left( TR^{[\alpha]} - \frac{1}{n} \sum_{k = 1}^{n} TR^{[k]} \right)^2
\]

(7)

The weights assigned from \( v_i \) to \( v_j \) in a map of individual \( \alpha \) are denoted by \( w_{ij}^{[\alpha]} \). We assume that if agreement would be reached within the sample concerning the weight of the link from \( v_i \) to \( v_j \), this agreement will be the mean weight of the sample on that link. We denote the sample average of the weights of the link from \( v_i \) to \( v_j \) by \( \bar{w}_{ij} \):
\[
\overline{w}_{ij} = \frac{1}{n} \sum_{\alpha=1}^{n} w_{ij}^{[\alpha]}
\]

(8)

We denote the variance in total result \(TR\), given that all individuals agree that the weight from node \(v_i\) to \(v_j\) is equal to the sample mean \(\overline{w}_{ij}\), by \(\sigma^2\left( TR | w_{ij}^{[\alpha]} = \overline{w}_{ij} \right)\). The latter value indicates what the variance would be if the sample would agree on the link from \(v_i\) to \(v_j\), while on all other links the individuals keep their initial link weight. The link on which agreement would result in the smallest variance in \(TR\) is called the most crucial link on which agreement should be reached. Consensus on the causal effect of this link maximally decreases the level of conflict concerning the total result \(TR\) of the means.

However, consensus on several links simultaneously might be considered, in order to align attitudes. Let \(\overline{S} \subset W\) denote a subset of links on which we define the sample to agree, then \(w_{ij}^{[\alpha]} = \overline{w}_{ij}\) if \(\{ij\} \in \overline{S}\). Then \(\sigma^2\left( TR | \overline{S} \right)\) is the variance in the total result if everyone agrees on the links within \(\overline{S}\). Let \(\overline{S}_x\) denote a set with exactly \(x\) links. For each \(x\) we can compute the set of links on which the variance in \(TR\) will reduce most by:

\[
\min_{\overline{S}_x \subset W} \left( \sigma^2\left( TR | \overline{S}_x \right) \right)
\]

(9)

For example, if the individuals want to align their attitudes by discussing three causal links, they could solve the minimization problem (9) for \(x = 3\).

**Collecting cognitive map data**

We have collected cognitive map data about the consequences of the merger between two inspection agencies among employees working in one region of the Netherlands. The data were collected in 2007, a year after the merger had taken place. We have collected the data among all employees by means of a questionnaire (Hart, 1976). In this questionnaire we have asked respondents to indicate the weight of links in a nomothetic map. This means that we had already selected nodes and links concerning the consequences of the merger, and respondents were restricted to give their cognition on these links. The selection of these nodes and links was based on interviews with 10 employees, held about a month prior to the mailing of the questionnaire. Before the questionnaire was sent to all employees as a web survey, it was piloted among seven employees.
We have interviewed 10 employees from different departments and hierarchical functions. We asked what, according to them, the main consequences of the merger were, both intended and unintended. We also asked to identify consequences that they thought others might expect to arise from the merger. The interviews were audio-recorded and took one and a half to two hours each. In six of the ten interviews we let the interviewees draw a map explicitly on a large piece of paper. First we let them write down the concepts, that is, the nodes, and let them draw links between the concepts. We asked them to speak out loudly which concepts were linked and why. When they were finished we asked them whether the links were equally strong and, if not, if they could draw thicker arcs for the links that were stronger. The map of one interviewee is given in Figure 3. In the other four interviews there was no time to let the interviewees draw the maps, but we did ask them what the goals of the merger were and we asked them to argue whether the merger positively or negatively would contribute to realizing those goals.

Some concepts that were phrased differently in different interviews seemed to be similar. In certain interviews it was indicated that, as a consequence of the merger, the level of responsibility was decreased, while others stated that the level of control was increased. We have checked whether these arguments were two sides of the same coin, and they were. Similarly, some interviewees used the word quantity of output, which appeared to be the same as productivity. There were seven consequences of the merger that were mentioned in at least five of the ten interviews, namely new IT systems, new tasks, responsibility, job satisfaction, quality of output, collaboration (between employees of the former organizations) and productivity. All other concepts were mentioned by at most two interviewees, and were not included in the questionnaire.

The causal links that were strong according to some interviewees were perceived to exist by all or most of the other interviewees. Those links were selected for the questionnaire. The other links, which were excluded, were mentioned in only one interview and were indicated as weak. We ended up with the eight nodes and 22 links drawn in Figure 1. Of these eight nodes, the merger is the means. Four concepts were indicated as goals by at least five interviewees, namely productivity, quality of output, collaboration and job satisfaction. No other concept was mentioned as a goal. Although the merger was not primarily implemented in order to change job satisfaction, interviewees expected the merger to have an impact on it and mentioned that management used increasing job satisfaction as an argument to defend the merger.
Figure 1 shows that the maps contain three sub goals, namely new tasks, IT systems and responsibility. From both former organizations some IT systems were selected, which meant that employees had to work with at least some new IT facilities. Since the responsibility of employees differed between the two former organizations, they expected this responsibility would change.

The 22 links that we selected contain no cycles. This means that there are no feedback loops in the causal reasoning of the individuals, such as ‘an increase in concept A increases concept B and increasing B increases A’. The measures we propose to derive an individual’s attitude from his cognitive map only work in acyclic maps, since the ultimate strength of a cycle is undetermined. In practice, individuals hardly indicate cycles in their cognitive maps (Axelrod, 1976; Weick, 1979), and our 10 interviewees also hardly mentioned any cycles. The only cycle that appeared during the drawing of a map was that job satisfaction increased quality of output, while a higher quality of output also increased job satisfaction. When we asked the interviewee whether this worked as a downward or upward spiral, he indicated this was not the case and the effect was primarily from job satisfaction to quality of output.

The weight of each of the 22 links is determined in the questionnaire by sentences, such as ‘If my responsibility would increase, then my job satisfaction would [strongly increase, increase, slightly increase, remain unchanged, slightly decrease, decrease, strongly decrease]’. These seven-point Likert items were coded from $-3$ (strongly decreased) to $+3$ (strongly increased). Respondents could also choose ‘I don’t know’, which was coded as a
missing value. Two exceptions are the links from merger to IT systems and from merger to new tasks, since these links are not bipolar; they only measure the extent to which new tasks and IT systems changed as a consequence of the merger. The question was ‘How much have the IT systems (respectively, new tasks) changed after the merger?’ with four answer categories (not at all, a little, moderately, much), which were coded from 0 to 3.

In order to improve clarity of instruction and to check whether relevant variables or relationships were missing, the questionnaire was piloted among seven employees of the organization, different from those interviewed earlier. The employees completed the questionnaire in the presence of a researcher, and were encouraged to comment. Besides some small remarks, the questionnaire appeared to be clear and, according to respondents, no crucial concepts or links were missing.

An email with a link to the final version of the questionnaire was sent out to those 247 employees who already worked in one of the ancestor organizations prior to the merger. Anonymity was assured. After a week a reminder was sent and after three weeks the questionnaire was closed. By then 151 respondents (61.1%) had completed the questionnaire.

The use of a questionnaire to collect cognitive map data has advantages and disadvantages. Some scholars (e.g. Eden and Ackermann, 1998) prefer idiosyncratic maps, in which individuals have the freedom to add their own concepts and links, to nomothetic maps as we have collected through our questionnaire. We agree that by limiting respondents to give their view on only a selected set of nodes and links, some information might be lost. On the other hand, in idiosyncratic maps it is sometimes unclear whether respondents using different wording actually refer to different concepts and, if they do not indicate a certain link, whether they actually believe it is not present. Therefore, it is necessary to return to the respondents to validate the meaning of certain words and explicate whether they forgot a link or that the link actually does not exist. Consequently, the collection of idiosyncratic cognitive map data among a large set of individuals is very time consuming. The advantage of collecting nomothetic maps using a questionnaire is that the cognitive maps of a large group of employees can be collected. All employees indicate their cognitions concerning the same set of links, rendering maps comparable between employees.

Results

A first test of the validity of our total effects measure $TR$ in Equation (5) is its correlation with the overall effects that individuals report. In our data,
respondents were asked to indicate the overall effect they expected the merger to have on each of the four goals.

Our measure for the total effects $TE(P_{mg})$ correlated highly with the overall direct effects $w_{mg}$, namely 0.755 for job satisfaction, 0.578 for productivity, 0.676 for quality of output and 0.638 for collaboration ($N = 94$, $p < 0.001$). This means that the computation of our measure for the total effect $TE$ is meaningful.

We expect that the total result that individuals expect from the means will determine their attitudes towards the means. In the questionnaire we have measured the attitude towards the merger with three items. These items were measured on a seven-point scale from ‘strongly disagree’ to ‘strongly agree’ and were worded; ‘I find the merger has resulted in a better organization’, ‘It would be better if the merger had not taken place’ and ‘I find that I personally have benefited from the merger’ (comparable with items from the cognitive resistance scale of Oreg, 2006). The scale was reliable (Cronbach’s $\alpha = 0.64$) and, on average, employees believed that the merger was not beneficial ($M = -0.53$; $SD = 1.43$). The correlation between attitude and $TR(m)$ is 0.57 ($p < 0.001$), indicating that our total result measure $TR$ is a good indicator for the attitudes towards the merger.

**Results: detection of crucial conflict links**

We want to detect the links on which the variance in the causal weights between individuals contributes most to the variance in the total result. Table 1 gives the variance for the links $w_{ij}$, ordered by size. The variance is the squared deviation of the mean weight on the specific link, among our 94 respondents. The largest variances are on the links merger $\rightarrow$ responsibility and new tasks $\rightarrow$ job satisfaction. So the respondents differ most in their perceived weight of the effect from the merger on responsibility and the effect from the new tasks on job satisfaction. On the other hand, their cognition concerning the strength of the effects from the merger on the IT systems and from collaboration on quality of output, are most in line with one another. This can be concluded from the relatively small variances of these two links in Table 1.

Note that the variances of the links merger $\rightarrow$ productivity and merger $\rightarrow$ quality of output are not given in Table 1. The reason for this is that the weights on these links do not contribute to the calculation of the total result $TR$, since they are not part of an indirect path from means to ends. The other 20 links are part of a path from means to ends.
We will show that the links on which the variance is relatively large are not necessarily the links on which reaching agreement contributes most to alignment of the attitudes towards the merger, that is, to reduction of the variance in the total result $\sigma^2(\mathbf{w}_{ij})$ using Equations (6)–(8).

The actual variance in $\mathbf{TR}$, that is, the variance before agreement is reached on any link, is $\sigma^2(\mathbf{TR}) = 0.2765$. We will calculate the variance in $\mathbf{TR}$ that we expect if consensus is reached on one of the links in the cognitive map, assuming that this consensus will be the average weight $\mathbf{w}_{ij}$ assigned to that link by the respondents in our sample. Firstly, we overwrite the actual weight a respondent assigned to a specific link by that average weight, using Equation (8), while we leave the rest of the weights unchanged. Then we use Equations (3)–(6) to calculate the total result of each of the respondents, based on these maps in which we have overwritten the weight of this one link. From these total results we calculate the variance $\sigma^2(\mathbf{TR}|\mathbf{w}_{ij}^{[\alpha]} = \mathbf{w}_{ij})$. Table 2 gives an overview of the variance in $\mathbf{TR}$ when the weight of one link would be equal to the average sample weight assigned to that link. The

<table>
<thead>
<tr>
<th>Link $v_i \rightarrow v_j$</th>
<th>$\sigma^2(\mathbf{w}_{ij})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merger $\rightarrow$ responsibility</td>
<td>2.575</td>
</tr>
<tr>
<td>New tasks $\rightarrow$ job satisfaction</td>
<td>2.192</td>
</tr>
<tr>
<td>Merger $\rightarrow$ job satisfaction</td>
<td>2.133</td>
</tr>
<tr>
<td>Merger $\rightarrow$ collaboration</td>
<td>1.782</td>
</tr>
<tr>
<td>IT systems $\rightarrow$ job satisfaction</td>
<td>1.520</td>
</tr>
<tr>
<td>New tasks $\rightarrow$ productivity</td>
<td>1.239</td>
</tr>
<tr>
<td>New tasks $\rightarrow$ collaboration</td>
<td>1.180</td>
</tr>
<tr>
<td>IT systems $\rightarrow$ productivity</td>
<td>1.180</td>
</tr>
<tr>
<td>New tasks $\rightarrow$ quality of output</td>
<td>1.132</td>
</tr>
<tr>
<td>IT systems $\rightarrow$ quality of output</td>
<td>0.929</td>
</tr>
<tr>
<td>Responsibility $\rightarrow$ job satisfaction</td>
<td>0.900</td>
</tr>
<tr>
<td>Merger $\rightarrow$ new tasks</td>
<td>0.852</td>
</tr>
<tr>
<td>Responsibility $\rightarrow$ collaboration</td>
<td>0.808</td>
</tr>
<tr>
<td>Responsibility $\rightarrow$ productivity</td>
<td>0.746</td>
</tr>
<tr>
<td>Job satisfaction $\rightarrow$ productivity</td>
<td>0.735</td>
</tr>
<tr>
<td>Responsibility $\rightarrow$ quality of output</td>
<td>0.688</td>
</tr>
<tr>
<td>Collaboration $\rightarrow$ productivity</td>
<td>0.687</td>
</tr>
<tr>
<td>Job satisfaction $\rightarrow$ quality of output</td>
<td>0.644</td>
</tr>
<tr>
<td>Collaboration $\rightarrow$ quality of output</td>
<td>0.580</td>
</tr>
<tr>
<td>Merger $\rightarrow$ IT systems</td>
<td>0.528</td>
</tr>
</tbody>
</table>
variance in the total result expected from the merger will be reduced most if there would be consensus on the effect of the merger on the responsibility, followed by the link between new tasks and collaboration and the link from new tasks to job satisfaction.

In particular, the effect of consensus on the link from new tasks to collaboration is interesting. The variance on six other links was larger than the variance on this link (see Table 1). However, if we regard the level of reduction in the variance of TR, agreement on this link ranks second. In contrast, the variance on the link from merger to job satisfaction was quite large (Table 1), while consensus on this link would hardly reduce the level of variance in the total result TR (Table 2).

In Table 3, the reduction in variance in the total result TR is calculated if consensus would be reached on two links. This is the result of the minimization problem (9) for two links, thus for $\mathbb{S}_2$. We only report the four pairs of links on which consensus reduces the variance in TR most. Table 3 shows that the variance of TR reduces most if consensus is reached on the link merger → responsibility, supplemented with respectively new tasks → collaboration ($\sigma^2(TR|\mathbb{S}) = 0.067$) or new tasks → job satisfaction ($\sigma^2(TR|\mathbb{S})$)

| $v_i \rightarrow v_j$: link on which agreement is reached | $\sigma^2(tr|w_{ij}^{[x]} = \bar{w}_{ij})^a$ |
|--------------------------------------------------------|----------------------------------|
| Merger → responsibility                                 | 0.117                            |
| New tasks → collaboration                               | 0.190                            |
| New tasks → job satisfaction                            | 0.205                            |
| New tasks → quality of output                           | 0.221                            |
| Merger → new tasks                                      | 0.221                            |
| IT systems → job satisfaction                           | 0.223                            |
| Responsibility → collaboration                          | 0.231                            |
| Responsibility → job satisfaction                       | 0.241                            |
| Merger → IT systems                                     | 0.253                            |
| Merger → collaboration                                  | 0.253                            |
| Merger → job satisfaction                               | 0.255                            |
| Collaboration → quality of output                       | 0.262                            |
| New tasks → productivity                                | 0.265                            |
| Collaboration → Productivity                            | 0.266                            |
| Job satisfaction → productivity                         | 0.266                            |
| Job satisfaction → quality of output                    | 0.267                            |
| IT systems → productivity                               | 0.267                            |
| IT systems → quality of output                          | 0.268                            |
| Responsibility → productivity                           | 0.269                            |
| Responsibility → quality of output                      | 0.269                            |

$^a$The variance in TR before agreement is reached on any link is $\sigma^2(TR) = 0.277$. 
However, consensus on the links merger → job satisfaction and merger → collaboration will also largely decrease the variance in the total result ($\sigma^2(\text{TR}|S) = 0.102$).

It might seem obvious that consensus on the pair of links merger → responsibility and new tasks → collaboration results in the largest reduction of variance, since these were the two links that separately reduced variance in TR most (Table 2). However, it is possible that consensus on each member of a set of links separately hardly reduces variance in TR, while consensus on the entire set would reduce TR variance to a large extent. An example is shown in Table 3. If consensus is reached on the links merger → job satisfaction and merger → collaboration, this ranks fourth in the ordering of how much TR variance is reduced, while these links separately hardly reduce the variance in TR (see Table 2).

These results indicate that dissensus on the weight of the link merger → responsibility contributes most to the differences in attitudes, followed by dissensus on the link new tasks → collaboration. In particular, the effect of consensus on this latter link would not have been found if only the variances on each of the links were considered in isolation of the other links in the maps and the paths it is part of (see Table 1). Thus, a naive person would argue that on links on which the variance is large, there is still ‘much consensus to win’. However, consensus on these links does not necessarily contribute much to agreement among the employees concerning their attitude towards the merger. This leads to the counterintuitive conclusion that it might be more relevant to promote consensus on a link on which there already is little variance, if this link is part of many paths from means to goals.

### Discussion

Attitudes of individuals towards a means are rooted in their cognition concerning the causal effect the means will have on relevant goals. We have proposed a measure to compute the total results $\text{TR}$ individuals expect from

| | $\sigma^2(\text{TR}|S)^a$ |
|---|---|
| Merger → responsibility and new tasks → collaboration | 0.067 |
| Merger → responsibility and new tasks → job satisfaction | 0.073 |
| Merger → responsibility and IT systems → job satisfaction | 0.081 |
| Merger → job satisfaction and merger → collaboration | 0.102 |

$a$ The variance in TR before agreement is reached on any link is $\sigma^2(\text{TR}) = 0.277$. 

Table 3. Variance in the total result TR if consensus is reached on two links
the means, based on their weighted cognitive map. We have found that this measure correlates with their attitudes towards the means, suggesting that the attitudes of individuals can be derived from their cognitive maps. As a next step we have proposed and applied a method to detect links in the cognitive maps of a group of individuals on which the different cognitive views contribute most to the variance in the expected total result of the means.

Our measures and methods are practical tools to search for differences in the cognitive maps of individuals that result in different attitudes towards the means. In order to reduce the variance in attitudes towards a means, the cognition of employees should be aligned, for instance through debate. Aiming for perfect alignment of the causal cognition of employees is often unrealistic and will be time consuming. Therefore, it is interesting to detect which causal differences contribute most to the variance in attitudes. As our data show, the links on which the variance is large are not necessarily the links on which reaching consensus will decrease the variance in attitude most. Disagreement on the strength of a link that is part of several paths can result in more diverse attitudes towards a means than a larger disagreement on a link that is only part of one path from means to ends. Put differently, putting much effort in increasing consensus on a link on which individuals have divergent cognitions is hardly useful if it concerns a rather irrelevant link, that is, a link that will be part of hardly any arguments supporting or rejecting the means.

Our approach also has implications for the debate about whether cognitive diversity in a work team is beneficial or detrimental for team performance. Some scholars have argued that in teams whose members have a shared cognition, widely supported decisions are made easily, thereby enhancing team performance (Cannon-Bowers and Salas, 2001; Cooke et al., 2000; Klimoski and Mohammed, 1994; Mohammed and Dumville, 2001). Others argue that such teams lack the disagreement and debate necessary to identify optimal solutions, and that cognitively diverse teams will perform better (De Dreu, 2006; De Dreu and Van de Vliert, 1997; Tjosvold, 1998, 2008). Our measures and methods complement this debate, since they express the extent to which cognitive differences result in different attitudes towards the means. This makes explicit the existing trade-off between a fuller exploration of alternative means on the one hand, and conflicting attitudes towards the chosen means on the other.

Collection and interpretation of weighted cognitive map data

There is debate over the way to collect data capturing the cognitive maps of individuals, and over the quality of the data resulting from either method.
Eden and Ackermann (1998) prefer idiosyncratic cognitive maps to nomothetic ones, giving individuals freedom to add their own concepts and links. We agree that by limiting respondents to give their view on only a selected set of nodes and links, some information might be lost. However, in order to compare maps, we have chosen to use questionnaires and collect data on the same set of links and nodes for all individuals, and to develop measures and methods for such data. This set of links and nodes was selected after interviews in which idiosyncratic data was collected. If the preliminary work of selecting the crucial links and nodes is done properly, we believe the missing information is not crucial for the attitude of an employee. An advantage of using questionnaires is that all individuals give their vision on the same set of relationships, also explicitly indicating when a certain relationship does not exist, according to them.

Through the selection of the set of links no cycles could appear in the maps of the employees. The measures we propose can only be used in acyclic maps, since a path containing a cycle would be infinitely long. According to some scholars (Bougon et al., 1990; Weick, 1979), loops occur often in real life and maps without cycles should raise questions. However, in the interviews we conducted, the only cycle that was mentioned appeared not to be a continuous loop and the interviewee could indicate which direction was stronger. Axelrod (1976) also found that cycles hardly pop up in maps. Eden et al. (1992) state that loops deserve special attention for two reasons. One reason is that the loop can be the result of a coding error that needs correcting and the cycle can be coded into a hierarchy. Another reason is that the loop actually exists, so the individual recognizes growth, decline or feedback control.

According to some scholars (Kosko, 1986; Larichev, 1992), in layman theories the causal assertions individuals make are at best ordinal, for example weak, moderate and strong. We have asked respondents to indicate on a seven-point Likert scale the extent to which an increase in one concept in the map would make another concept decrease, increase or remain unchanged. We have interpreted these answers as being on a ratio level. Although we agree that it is a difficult cognitive task to assign such weights and that it is not defined what, for instance, ‘strongly increase’ would mean in reality, respondents seemed to be able to answer the questions. Furthermore, the measure $TR$ we propose for the total result of the means, based on these maps, did correlate high with attitude, as expected. So we believe that individuals are able to intuitively assign weights to the links in the maps that can be interpreted at a ratio level.
**Future research**

In the data used in this paper, we have no information concerning the priorities that employees assign to each of the four goals, and we assumed the salience to be equal over the goals. Therefore, employees with the same cognitive maps are assumed to have the same attitude towards the change. However, individuals might also attach conflicting saliences to each of those goals. This might result in conflict about the means, even if they agree about the effects from means to goals. Suppose they agree that a means will have a positive effect on one goal, for instance productivity, and a negative effect on another goal, for instance quality. If for one individual the goal productivity is relatively more salient than quality of output, while for the other individual the goal priorities are vice versa, the latter might oppose the means while the former supports it. Then the conflict is not rooted in cognitive differences, but in different goal saliences. While in the current paper we assume that employees with exactly the same cognitive map will have the same attitude towards the means, the addition of goal salience will give a more diverse image of the roots of conflict.

In our example, the merger was the only means. Individuals could either support the merger or oppose it. Cognitive maps could also be collected about the consequences of several alternatives. Then the individuals might oppose a means of which they do see mainly positive consequences, if in their view another alternative is even better. The presented methodology is able to investigate this phenomenon.

We have assumed that after a causal link is discussed in a group, each employee will agree on the average sample weight on that link. To us this seems a natural reference point, but other reference points could be chosen and incorporated in our measures. Other possible reference points are, for instance, the mode of the sample or the weight that a certain group of experts expects, or the weight in a map of a person who wants to convince a grassroots organization. If, for instance, a politician has insight in the cognitive maps of a group of voters concerning the consequences of a certain policy change, (s)he could compute on which links to convince these voters in order to gain more support for the policy change.

In order to test whether our method to detect the crucial cognitive differences actually helps to reach agreement more easily through discussing certain links in the group, experiments should be conducted in which one part of the group just discusses the matter, while in the other part of the group those links are discussed that are most crucial according to our algorithm. If in the latter group the level of conflict is reduced more than in the former group, our method would proof useful.
Funding

This research received no specific grant from any funding agency in the public, commercial or not-for-profit sectors.

References


