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## Perspectives on productivity and business cycles in Europe

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## Chapter 2 The euro area business cycle<sup>14</sup>

### 2.1 Introduction

The European Central Bank (ECB) is charged with maintaining price stability in the euro area and to set monetary policy, it needs information about the state of the euro area economy. A business cycle index (BCI) can provide this information at a glance. However, BCIs have mainly been developed for the U.S., with much less research focusing on the euro area. In case of the euro area as a whole, an added complexity is that it only recently became an economic entity in its own right due to the ECB's common monetary policy. The scarcity of research does not leave the ECB or other policy makers without tools to analyze the euro-economy: there is an area-wide structural model (see Fagan, Henry and Mestre, 2005) as well as more recent work on Bayesian dynamic general equilibrium models (Smets and Wouters, 2004). However, BCIs are a useful complement as they do not rely on a detailed description of the economic structure and do not make assumptions regarding the behaviour of consumers or firms. Rather, BCIs are constructed based on statistical analysis of the economy in question.

This chapter compares different methods of constructing BCIs. The main question it addresses is how the selection of a set of variables affects the performance of different BCIs. In a recent paper Boivin and Ng (2003) also address this issue and, using simulation techniques, they come to the conclusion that indexes, which are based on a limited number of variables, perform at least as well or even better than those based on the full dataset they consider.<sup>15</sup> The analysis here is largely complementary: the setting is applied and uses economic logic rather than statistical algorithms to reduce the data set.

The comparison of BCI construction methods focuses on ability of BCIs to capture relevant historical cyclical facts, not on their performance in real-time forecasting.

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<sup>14</sup> This chapter is largely based on Inklaar, Jacobs and Romp (2003). See acknowledgements for further details.

<sup>15</sup> Bai and Ng (2002) also find that the number of series need not be very large to get precise estimates for factor models, one of the methods used in constructing BCIs.

Furthermore, the turning points of the index are considered to be the most relevant cyclical fact, although others would focus on, for example, the business cycle as a periodic cycle or a serially correlated phenomenon.<sup>16</sup> To evaluate forecasting performance, an analysis would be needed along the lines of e.g. Diebold and Rudebusch (1991) or McGuckin, Ozyildirim and Zarnowitz (2003) and more attention would need to be paid to end-of-sample problems as in Forni, Hallin, Lippi and Reichlin (2003).

The methodology for constructing BCIs was originally developed at the National Bureau of Economic Research (NBER) in the U.S. in the 1930s and described in the seminal book of Burns and Mitchell (1946). It has since then been widely used (see e.g. Zarnowitz, 1992). In recent years these "NBER method" indexes are maintained and regularly published by The Conference Board (TCB), which has also developed similar indexes for other countries. A more recent development in the construction of BCIs is the use of dynamic factor models. Early applications of dynamic factor models are described in Sargent and Sims (1977) and Geweke (1977). Recent examples are Stock and Watson (1989, 2002), Camba-Mendez, Kapetanios, Smith and Weale (2001), and the Generalized Dynamic Factor Model of Forni, Hallin, Lippi and Reichlin (2000) and Forni and Lippi (2001).

After a general introduction to business cycle measurement, Section 2.2 describes the two different methods used for constructing BCIs, namely the generalized dynamic factor model of Forni *et al.* (2000) and the NBER method. Section 2.3 presents three BCIs for the euro area. The first uses a relatively small dataset and is constructed using the NBER method (EuroTCB). The second is estimated using a dynamic factor model and a very large dataset (EuroCOIN from Altissimo *et al.*, 2001). The final index is based on a small dataset and constructed using a dynamic factor model (EuroIJR). This final index can be used to gauge the relative importance of data selection to arrive at a small dataset on the one hand and the method of index construction on the other hand. Section 2.4 compares the three business cycle indexes in terms of how they are correlated and track the euro area cycle. In Section 2.5 the EuroIJR index is used to shed some light on cyclical dynamics in the euro area.

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<sup>16</sup> See Harding and Pagan (2005) for an overview of these approaches.

The main finding is that the business cycle indexes analyzed here are very similar in terms of correlations. The dates of peaks and troughs are also similar. The leads and lags around the turning points of euro area GDP are generally modest and none of the three indexes is consistently more accurate in pinpointing peaks and troughs than the other two. This suggests that a useful BCI can be constructed using only a limited number of variables. Furthermore, it is not necessary to include data from each euro area country to adequately capture the euro area cycle. As Chapter 1 already showed, cycles of European countries tend to move together and the analysis in this chapter confirms that for some purposes, the differences across euro area countries are not crucial and the area can be treated like a single economic entity.

However, the analysis of cyclical dynamics brings some of the heterogeneity of the different countries to the fore. So, for example, German industrial production is one of the most important variables in the EuroIJR index, whereas French and Spanish industrial production make only modest contributions to the index. This means that a shock to German industrial production tends to have a different effect on the euro area cycle than similar shocks in France and Spain. So even for short-run policy making, the euro area cannot be treated as fully homogeneous.

## **2.2 Methodology**

Business cycles are more or less regular patterns in fluctuations in economic activity. In the well-known definition of Burns and Mitchell (1946, p. 3):

A cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own.

In other words, expansions and contractions in economic activity are observed in time series of many variables across different sectors of most (market) economies at roughly the same time. This suggests that an index capturing these movements would be very useful. This idea lies at the core of the NBER approach as originally proposed by

Burns and Mitchell (1946). An alternative method to measure business cycles is to formulate a formal statistical model which identifies underlying “shocks” that drive the business cycle.<sup>17</sup> Dynamic factor models are part of this latter group and provide a more formal way to select and weight relevant cyclical variables. This chapter will compare business cycle indexes constructed according to both approaches.

Burns and Mitchell (1946) define the business cycle in terms of fluctuations in economic activity. However, the choice of a measure of "economic activity" is not straightforward. The usual choice is GDP, but since GDP is only available on a quarterly frequency, additional variables are needed to establish a monthly chronology. Furthermore, the usual publication lag of GDP makes it unsuitable for gaining a timely insight into the state of the economy. Therefore, the NBER Business Cycle Dating Committee has adopted a broader approach in the U.S. by also looking at other (monthly) economic variables such as industrial production or retail sales.

Contractions of economic activity are an essential ingredient of the classical definition of a business cycle. However, some economic theories predict movements around a permanent component or “trend”. This has given rise to the analysis of fluctuations around a trend, a category of cycles usually referred to as deviation cycles or growth cycles. While policy makers are primarily interested in classical cycles, academics tend to focus on deviation cycles (Harding and Pagan, 2000). A third type of cycles looks at turning points in the growth rate of economic activity. These growth rate cycles are related to deviation cycles, since growth rates can be interpreted as a trend filter (cf. Harding and Pagan, 2005).

Given a reference series of economic activity, turning points of business cycles can be determined in levels, deviations from trend, or growth rates. This chapter focuses on the classical cycle and the growth rate cycle concepts, as discussed further below. The deviation cycle is central to the next chapter and the main methods are described there. The standard method to determine turning is to use the algorithm of Bry and Boschan (1971). This algorithm calculates moving averages of different lengths to narrow down

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<sup>17</sup> The term "shock" should not be taken to mean that business cycles are set in motion through economic events such as, for example, stock market crashes or technology shocks. In a purely statistical sense, a shock is a phenomenon that causes a variable to diverge from its long-run average.

the region where the turning points are likely to be located and then pinpoints the exact month where the peak or trough occurred using the original series. The only restrictions are that a full business cycle (peak to peak or trough to trough) should last at least fifteen months, each business cycle phase (peak to trough, trough to peak) should last at least five months and peaks and troughs should alternate. These criteria have been developed to pinpoint turning points in the classical business cycle using monthly data. A generalization to quarterly or even annual data is relatively straightforward, as discussed in Harding and Pagan (2001). Application of the Bry-Boschan algorithm to the dating of growth rate cycles is less well established, mostly because no independent reference chronology exists for a growth rate cycle, as is the case for the classical cycle in the United States. However, Zarnowitz and Ozyildirim (2001) have recently used the Bry-Boschan algorithm to compare various filtering methods and the algorithm is also applied to the dating of EuroCOIN turning points (see Altissimo *et al.*, 2001).<sup>18</sup> In this chapter, the same rules are used for dating growth rate cycles as for dating classical cycles, but further research on the appropriateness of this choice is called for.

### **The NBER method**

In establishing a monthly business cycle chronology, the NBER relies on four monthly variables: employment, personal income, industrial production and manufacturing and trade sales. Together these make up the composite coincident index for the United States. The choice of these variables (in some form) can be traced back to the work of Burns and Mitchell (1946). Since then, the four components of the coincident index have stood up as a good representation of the reference business cycle.

Potentially relevant economic variables are evaluated based on how closely they track the classical cycle of the reference series. This can be done by looking at the correlations with the reference series at various leads and lags and whether variables exhibit peaks and troughs around the same time as the reference series. Consistently leading and lagging variables are then combined into leading and lagging composite indexes. The change in a composite index is calculated as the unweighted average of

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<sup>18</sup> In contrast, Artis, Marcellino and Proietti (2002) identify periods with negative growth rates for a certain number of months as a recession.

changes in the components, after normalisation; the level of the index is computed by cumulating the changes from a specified base year.<sup>19</sup>

The choice of variables depends for a large part on the judgment of the researcher. One has to construct a "good" reference series based on a measure of "economic activity", find a way to determine its peaks and troughs and then evaluate whether other variables have a "close" relationship to the reference series. The degree of subjectivity of the NBER method has been a motivation to develop more statistically oriented methods.

### The Generalized Dynamic Factor Model

Although statistical methods also involve a number of (subjective) choices, generally speaking they do impose more (theoretical) structure on the problem of measuring business cycles. The basic idea of factor models is that a dataset consisting of a number of time series can be decomposed into a common component and an idiosyncratic component, where the common component is driven by only a few common shocks. Although many factor models fit this general description, the remainder of the analysis uses the Generalized Dynamic Factor Model (GDFM) of Forni *et al.* (2000). The model is "generalized" in the sense that, contrary to the earlier dynamic factor models such as those of Sargent and Sims (1977) or Geweke (1977), the idiosyncratic components can be correlated.<sup>20</sup> The factor model is basically a method of rank-reduction, where the information in the large matrix of observations is summarized in the matrix of common components of smaller rank.<sup>21</sup> The GDFM can be written as follows.

$$(2.1) \quad x_{it} = \chi_{it} + \xi_{it} \equiv b_{i1}(L)u_{1t} + b_{i2}(L)u_{2t} + \dots + b_{iQ}(L)u_{Qt} + \xi_{it},$$

where  $x_{it}$  is the  $t$ -th observation on the  $i$ -th time series and  $L$  is the lag operator. The dynamic factor loading  $b_{ij}(L)$  describes the dynamic impact of the  $j$ -th common shock  $u_j$  on the  $i$ -th series. The common shocks and the factor loadings together make up the *common components*  $\chi_{i,t}$ . After the influence of common shocks has been removed, only

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<sup>19</sup> See The Conference Board (2001) for more details

<sup>20</sup> The theoretical criterion is that idiosyncratic shocks can be correlated as long as this correlation can still be distinguished from the common shocks. In practice, it is harder to draw a strict line. See the appendix to this chapter for more details.

<sup>21</sup> A more extensive discussion can be found in Appendix 2.A.

the idiosyncratic components  $\xi_i$  remain. Equation (2.1) shows that the model is explicitly dynamic since a common shock can affect a variable with leads or lags.

As Forni *et al.* (2000) show, the common component in this model is only uniquely identified in a dataset with an infinite number of observations and time series, but they present an estimator that is reasonably precise for datasets of more modest dimensions. The main identifying assumption is that there are only a limited number of common shocks that explain an increasing percentage of the variance of the dataset as the number of time series in the dataset grows, while the importance of the idiosyncratic shocks remains bounded. The common components of Equation (2.1) can then be estimated by employing principal component analysis in the frequency domain.

The common component of a series is the part that is driven by shocks that are common to all series, while the remainder is idiosyncratic noise. The common component of GDP is the logical candidate for a business cycle index. Abstracting from mathematical complexities, the common component of a series is a linear combination of all the variables, where the weights on each of the variables are chosen to maximize the variance explained by the common component. Since the main interest is in cyclical fluctuations we also want to filter out the high-frequency noise of GDP's common component:

$$(2.2) \quad x_{it} = \chi_{it}^C + \chi_{it}^{NC} + \xi_{it} = \sum_{n=1}^N \sum_{m=-M}^M K_{i,n,m}^C x_{n,t+m} + \chi_{it}^{NC} + \xi_{it},$$

where  $K_i^C$  is the matrix of weights used in calculating the cyclical common component  $\chi_{it}^C$ . Weights are available for each variable  $i$  and each lead and lag, denoted by  $m$ . A similar decomposition can also be made for the irregular, high-frequency noise in  $\chi_{it}^{NC}$ .

Instead of the GDFM, other factor models could have been used as well, but there are a number of reasons for using the GDFM. First, an important characteristic of business cycles is that not all variables move exactly in phase: leads and lags are quite common. This makes it essential to use a dynamic factor model, to take these leading and lagging relationships into account. Second, in the model of Forni *et al.* (2000) it is very straightforward to remove high-frequency noise and only focus on longer run



fluctuations. Finally, the GDFM allows for a certain amount of correlation between idiosyncratic components. This can be very important for cyclical indicators. For example, if two industrial production series are included in the index, it is quite possible that measurement errors in the two series are partly correlated, although both of these measurement errors are unrelated to cyclical fluctuations. If idiosyncratic components were required to be strictly uncorrelated, part of this measurement error would show up in the common component. To avoid this, it is desirable to allow for some amount of correlation between idiosyncratic components. A more pragmatic reason for choosing the GDFM is that this way, a more parsimonious specification of the EuroCOIN index can be compared to EuroCOIN.

The GDFM is basically a multivariate filter for euro area GDP. The information from other series allows the model to a) eliminate idiosyncratic measurement errors, b) filter out high-frequency noise, c) estimate economic activity within a quarter and d) use information from leading variables and their relation to GDP to forecast economic activity for recent periods. Although the final point is quite important for the usefulness of a BCI, the model applied here is only equipped to take the first three points into account.<sup>22</sup>

One problem in applying this method is that the selection of the number of common shocks is not straightforward; see also Bai and Ng (2002). Here, one of the criteria suggested by Forni *et al.* (2000) is used, namely that each common shock should explain at least a pre-specified percentage of total variance.<sup>23</sup>

### 2.3 Euro Area Business Cycle Indexes

This section describes three business cycle indexes for the euro area: EuroTCB, a coincident index constructed along the lines of the NBER methodology, EuroCOIN, in which the Generalized Dynamic Factor Model is applied to a large set of data, and the hybrid EuroIJR, in which the GDFM is used on the limited set of variables used in the construction of coincident and leading indexes for European countries by TCB. EuroCOIN is constructed by others and published online monthly, but EuroTCB and

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<sup>22</sup> See Forni *et al.* (2003) for details on end-of-sample estimation.

<sup>23</sup> See Appendix 2.A for further discussion.

EuroIJR were specifically constructed for the analysis here. In all cases, quarterly GDP for the euro area is included in the construction of the index. The information from the other series is used to get the information that is necessary for a monthly business cycle chronology.

### **EuroTCB**

The Conference Board (TCB) publishes business cycle indexes for a number of euro area countries on a monthly basis. At present, coincident and leading indexes are constructed for France, Germany and Spain. The components of the coincident indexes have been selected based on the components of the U.S. coincident index as well as their ability to match the business cycle turning points of GDP in the individual countries. The leading indexes have been constructed so that they generally lead the coincident index at business cycle turning points.

The components of the leading and coincident indexes differ across countries, but they generally contain the same type of time series. The coincident indexes include measures of sales, income, production and employment. The leading indexes usually contain financial variables such as bond yields and share prices, natural leading series like orders for new goods and building permits, and finally surveys of consumer or business confidence.<sup>24</sup> All series have been selected to match classical cycle turning points in each of the individual countries. It is therefore not clear whether they will provide a good representation of the euro area growth rate cycle, but given that these three countries account for about 60 percent of euro area GDP, the representation is assumed to be reasonably good.

In constructing the EuroTCB index the NBER and TCB procedures are followed as well as possible.<sup>25</sup> The procedure starts off with 12 coincident variables, four from each country, and euro area GDP from the OECD Quarterly National Accounts. In the construction of a monthly index, the quarter-on-quarter growth rate of GDP is applied to each month in the quarter. This procedure corresponds to linearly interpolating the level

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<sup>24</sup> Table 2.4 below provides a list of the components of the leading and the coincident indexes for the three countries

<sup>25</sup> For a more extensive discussion of the methods, see The Conference Board (2001).

of GDP for each month.<sup>26</sup> To construct a BCI for a country, The Conference Board weights each series by a standardization factor. This standardization factor is calculated from the inverse standard deviations of each of the series, normalized to sum to one. An analogous procedure is followed here for the 12 variables plus euro area GDP.<sup>27</sup> It would be possible to weight by the relative size of the economies in addition to the relative volatility of each series. However, only using the relative volatility is more in line with the NBER tradition and it is also more in line with the procedures used in calculating the other two BCIs. According to Altissimo *et al.* (2001), EuroCOIN is calculated from a set of series after normalizing and in calculating the EuroIJR index we also do not perform further weighting. Euro area GDP in the index is included mostly to make the index more comparable to the other two. However, the index including and excluding euro area GDP are very similar (correlation of 0.99) and the turning points are never more than a few months apart.

### **EuroCOIN**

The EuroCOIN index is published monthly by the Centre for Economic Policy Research (CEPR) ([www.cepr.org](http://www.cepr.org)). Altissimo *et al.* (2001) describe the index in detail, so only the highlights are covered here. The authors construct their index from a database with monthly observations for 951 series for France, Germany, Spain, and Italy, the Netherlands, Belgium and a number of euro area wide variables. Using a number of criteria, such as timeliness in publication and concordance to the common shocks, they reduce their dataset to 246 series. The series cover a wide range of variables such as industrial production, prices, interest spreads and surveys of business and consumer confidence. The generalized dynamic factor model is applied to this database, after first differencing to render the series stationary. The authors include all common shocks that capture 10 percent or more of total variance, which leads to the choice of four factors. The first four dynamic principal components together explain 55 percent of all variance

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<sup>26</sup> An alternative would be to interpolate the growth rates instead of the level. There is no strong case for either of these options, but at least for the chosen procedure, it is immediately clear that the month-to-month developments in GDP are not known, since the same growth rate for each month is used.

<sup>27</sup> To be precise, the month-on-month changes in each of the variables are multiplied by the standardization factor and summed across variables. For comparability to the other two BCIs, the 3-month average of this summation is used as the index.

in the data. Altissimo *et al.* (2001) then use the cyclical part of the common component of euro area GDP as their business cycle index, i.e. all fluctuations with a periodicity higher than 14 months. Due to the stationarity requirement of the GDFM, GDP is included in growth rates, so their BCI models the growth rate cycle of the euro area.

### **EuroIJR**

For the third euro area business cycle index, the generalized dynamic factor model is applied to the components of the coincident and leading indexes for France, Germany and Spain of TCB. In the construction of this index, which is referred to as EuroIJR, features from both approaches are combined.<sup>28</sup> On the one hand, data is used that analysts consider as informative for the cyclical development in euro area countries. The turning points of these series generally lead or coincide with GDP of the country in question. The fact that only a limited number of series enters into the index makes it easier to relate changes in the index to changes in the components and therefore to interpret these changes. On the other hand, the GDFM is used to weight the series. This way, we can examine whether selecting only a limited number of variables for the index leads to a serious loss of information. Note that TCB has selected the coincident and leading variables to match classical turning points in economic activity, without reference to turning points in the growth rate cycle. It is therefore possible that some other variables would be selected if this latter cycle concept were used. This possibility is not investigated further, but left for further research. It should be noted though, that a selection based on the correspondence to the growth rate cycle may even improve the performance of the index.

In total, 37 indicators enter into the coincident and leading indexes of France, Germany and Spain (see the overview in Table 2.4). For each country, there are four coincident series. For France, the leading index contains ten indicators, while the corresponding indexes for Germany and Spain contain eight and seven series respectively. In addition to these variables, we include quarterly GDP growth for the euro

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<sup>28</sup> The IJR in EuroIJR refers to the initials of the authors of the original paper on which this chapter is based, Inklaar, Jacobs and Romp (2004).

area. All series are analyzed as normalized exponential growth rates (first differences in logs), since stationarity is a prerequisite to the GDFM.

As mentioned above, the criterion of Forni *et al.* (2000) is used with common shocks included as long as they explain at least five percent of total variance. This leads to the selection of six common factors that together capture fifty percent of total variance and sixty percent of cyclical variance in the data. Just as in Altissimo *et al.* (2001), all fluctuations with a frequency lower than 14 months are defined as cyclical.

## 2.4 Comparison

This section compares the three BCIs for the euro area, looking at the period from January 1988 to September 2002. The data from The Conference Board are available for a longer period of time but EuroCOIN only starts in 1988, which limits the time span for comparison. Some other differences between the indexes are also of interest. EuroCOIN covers the widest range of countries, namely Belgium, France, Germany, Italy, Netherlands and Spain; the other two indexes only include data from France, Germany and Spain. As a result, EuroIJR and EuroTCB are based on data from countries representing 60 percent of euro area GDP, while EuroCOIN's countries cover nearly 90 percent. Although all three BCIs include euro area GDP as one of the components, the GDP series used to construct EuroCOIN seems slightly different from the series used for the other two indexes (compare Figure 2.1 and Figure 6 from Altissimo *et al.*, 2001).<sup>29</sup> Finally, the EuroIJR index is based on the first six common factors, while EuroCOIN is based on four factors. Given the differences in the two datasets, these factors are not comparable. However, the resulting index is qualitatively similar whether four or six common factors are selected.

In the remainder of this section the three BCIs are compared in terms of their correlation with GDP growth and in terms of cyclical peaks and troughs, both in the classical cycle and the growth rate cycle. The euro area classical cycle is defined by the peaks and troughs of euro area GDP. Figures 2.1 and 2.2 show monthly series of euro

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<sup>29</sup> The GDP series used by Altissimo *et al.* (2001) could not be acquired, but visual inspection of the two GDP series does not reveal any major discrepancies.

area GDP and the BCIs.<sup>30</sup> Figure 2.1 shows euro area GDP and the three BCIs as indexes with January 1988=100. The euro area growth rate cycle is defined by the growth rate of euro area GDP and consequently all three BCIs are taken as growth rates, (Figure 2.2). In both sets of figures the area between cyclical peaks and troughs is shaded. In Figure 2.1, these correspond to classical business cycle recessions, in Figure 2.2 these correspond to periods of declining growth rates. To determine the turning points the algorithm of Bry and Boschan (1971) is used.<sup>31</sup> Although the figures show differences in short-term fluctuations, overall the similarities between the indexes are striking. Especially the recession of 1993 clearly stands out in all three indexes.

Table 2.1 shows the correlations between the three indexes (in growth rates) as well as the change in euro area GDP. Correlations are based on the monthly series and the quarterly aggregates. The quarterly results are included because for GDP only quarterly data is available, whereas monthly GDP is interpolated. For the correlation coefficients this interpolation has little or no effect. The correlations confirm the conclusions from visual inspection by showing large and positive coefficients (all significant at the 1 percent level). In other words, the three indexes all capture a large amount of the variation in euro area GDP since 1987.

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<sup>30</sup> Appendix 2.B shows the same type of figures for quarterly aggregates of the indexes. The quarterly series are calculated as the average of each of the indexes within the quarter. For EuroTCB and EuroIJR the basic data could also be aggregated to a quarterly frequency and the indexes calculated from these new series, but this changes little.

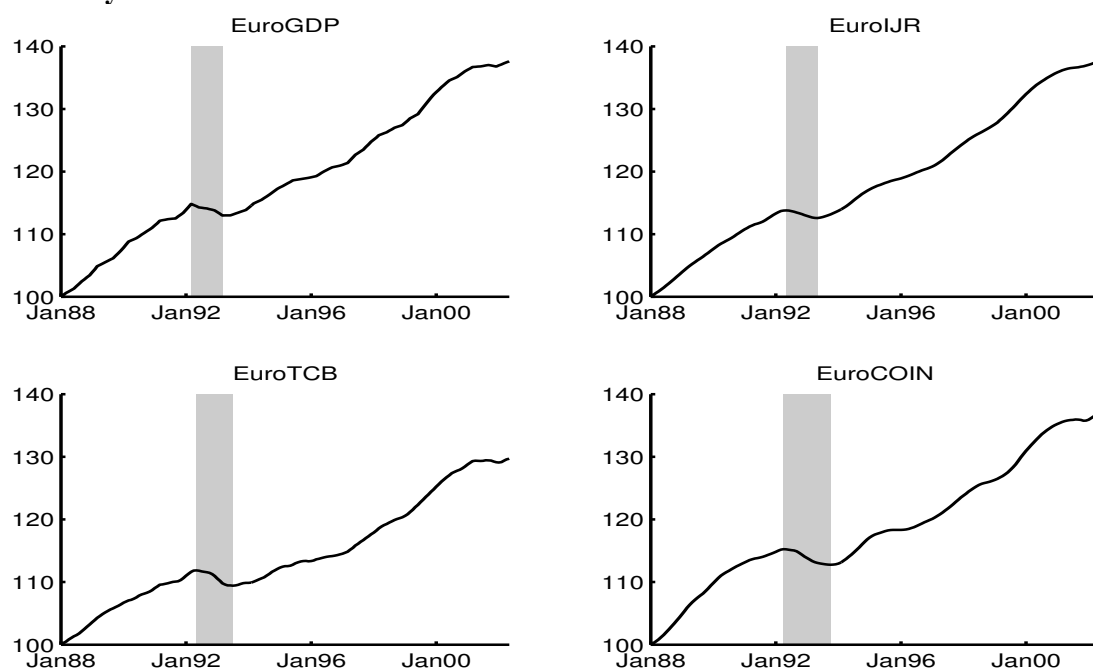
<sup>31</sup> The program implementing the Bry-Boschan algorithm is taken from Mark Watson, converted from Gauss to Matlab and adapted along the lines of Harding and Pagan (2001) to also determine turning points in quarterly series.

**Table 2.1 Correlation between euro area GDP and business cycle indexes**

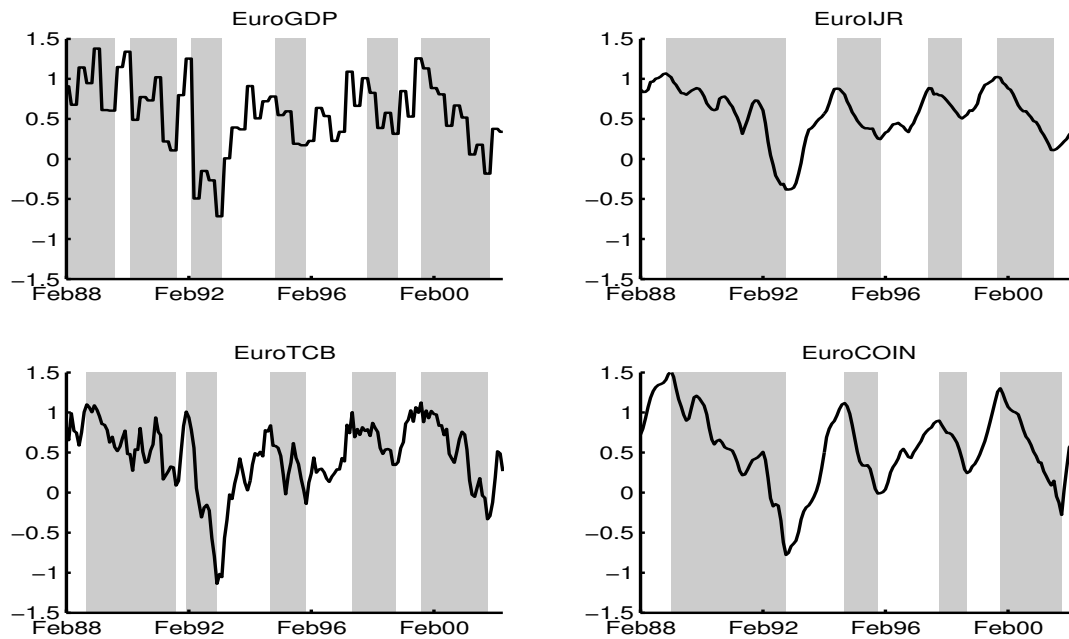
	EuroGDP	EuroIJR	EuroTCB
<i>Monthly index</i>			
EuroIJR	0.84		
EuroTCB	0.80	0.88	
EuroCOIN	0.80	0.92	0.84
<i>Quarterly index</i>			
EuroIJR	0.84		
EuroTCB	0.83	0.91	
EuroCOIN	0.80	0.93	0.86

Note: the quarterly index is calculated as the average over the quarter of the values of the monthly index.

**Figure 2.1 Euro area GDP and monthly business cycle indexes (levels), 1988-2002, January 1988=100**



Note: shaded areas mark business cycle recessions with absolute declines in economic activity.

**Figure 2.2 Euro area GDP and monthly business cycle indexes (growth rates), 1988-2002**

Note: shaded areas mark growth rate cycle recessions with decreasing growth rates of economic activity.

In Figures 2.1 and 2.2 recessionary periods are shaded to facilitate visual inspection, but it is informative to look at the differences in turning points in some more detail. Table 2.2 shows the turning points of the indexes in levels (cf. Figure 2.1). These turning points correspond to the turning points of the classical cycle and signal absolute expansions and contractions in economic activity. Table 2.3 shows the turning points for the growth rates of the indexes (cf. Figure 2.2). These turning points signal slowdowns and accelerations in economic growth and correspond to the growth rate cycle. A turning point of the growth rate cycle will generally lead a turning point of the classical cycle since a slowdown in growth usually occurs before growth turns negative. Furthermore, a series generally has more growth rate cycle turning points than classical cycle turning points since absolute declines in economic activity are rarer than slowdowns in growth. This is confirmed by comparing the turning points of GDP in Table 2.2 and Table 2.3. In the period 1988-2002, GDP showed only one classical cycle, but five growth rate cycles. As discussed before, turning points are shown for both monthly and quarterly series, first of all to ensure the chronology is robust to the interpolation method for GDP, but also to check whether it is robust for the different BCIs.



**Table 2.2 Business cycle turning points of euro area GDP and business cycle indexes**

	<i>Peak and trough dates</i>				<i>Leads/lags versus EuroGDP</i>		
	EuroGDP	EuroIJR	EuroTCB	EuroCOIN	EuroIJR	EuroTCB	EuroCOIN
<i>Monthly indexes</i>							
Peak	Mar-92	May-92	May-92	Apr-92	2	2	1
Trough	Mar-93	May-93	Jul-93	Oct-93	2	4	7
<i>Quarterly indexes</i>							
Peak	1992Q1	1992Q2	1992Q2	1992Q1	1	1	0
Trough	1993Q1	1993Q2	1993Q2	1993Q3	1	1	2

Note: positive figures denote a lag of x months

**Table 2.3 Growth rate cycle turning points of euro area GDP and business cycle indexes**

	<i>Peak and trough dates</i>				<i>Leads/lags versus EuroGDP</i>		
	EuroGDP	EuroIJR	EuroTCB	EuroCOIN	EuroIJR	EuroTCB	EuroCOIN
<i>Monthly indexes</i>							
Peaks	Mar-90	Dec-88	Oct-88	Feb-89	-15	-17	-13
	Mar-92		Jan-92		M	-2	M
	Dec-94	Jul-94	Oct-94	Oct-94	-5	-2	-2
	Dec-97	Jul-97	Jun-97	Nov-97	-5	-6	-1
	Sep-99	Oct-99	Sep-99	Nov-99	1	0	2
Trough	Sep-89				M	M	M
	Sep-91		Sep-91		M	0	M
	Mar-93	Nov-92	Jan-93	Nov-92	-4	-2	-4
	Dec-95	Dec-95	Dec-95	Nov-95	0	0	-1
	Dec-98	Aug-98	Nov-98	Oct-98	-4	-1	-2
	Dec-01	Aug-01	Nov-01	Nov-01	-4	-1	-1
Average lead/lag				-4.5	-3.1	-2.8	
Average absolute lead/lag				4.8	3.1	3.3	
<i>Quarterly indexes</i>							
Peaks	1989Q1	1988Q4	1988Q4	1989Q1	-1	-1	0
	1994Q1	1994Q3	1994Q4	1994Q4	2	3	3
	1997Q2	1997Q3	1997Q2	1997Q4	1	0	2
	1999Q3	1999Q4	1999Q3	1999Q4	1	0	1
Trough	1993Q1	1992Q4	1993Q1	1992Q4	-1	0	-1
	1995Q4	1995Q4	1995Q4	1995Q4	0	0	0
	1998Q4	1998Q3	1998Q4	1998Q4	-1	0	0
Average lead/lag				0.1	0.3	0.7	
Average absolute lead/lag				1.0	0.6	1.0	

Note: positive figures denote a lag of x months, negative figure a lead of x months, M denotes missed turning points.

Tables 2.2 and Table 2.3 show that none of the three indexes perfectly matches the peaks and troughs of the cycles of GDP. However, the similarity between the turning points of the indexes and those of GDP is considerable. Table 2.2 shows that the euro area had one classical cycle between March 1992 and March 1993. The EuroIJR index had both its peak and trough two months later than GDP. The EuroCOIN index lagged one month at the peak and lagged seven months at the trough. EuroTCB lagged one month at the peak and four months at the trough. The other indexes as well as GDP also showed negative growth in 2001, but the period was too short to signal a turning point. The Bry-Boschan algorithm smoothed these dips and thus did not produce recession signals. The quarterly chronology matches the monthly turning points with generally modest lags.

Table 2.3 shows the turning points of the euro area growth rate cycle and the turning points for (the growth rates of) each of the BCIs. As could already be seen from Figure 2.2, it is much harder for the BCIs to match turning points of the GDP growth rate cycle than those of the classical business cycle. This is partly due to the fact that there are simply more growth rate cycles, but also because the Bry-Boschan algorithm was originally designed to pinpoint classical turning points. As discussed in Section 2.2, the algorithm includes a number of decision rules, such as the minimum period between a peak and a trough and between two peaks or two troughs. These criteria are based on the U.S. classical business cycle chronology as maintained by the NBER Dating Committee and unfortunately, it is not known how appropriate these rules are for dating growth rate cycle turning points.

Some of the difficulties in dating peaks and troughs in growth rate cycles show up when comparing the monthly chronology in the top panel of Table 2.3 with the quarterly chronology in the bottom panel. Based on the monthly series, GDP showed a trough in September 1991 and a peak in March 1992. In the quarterly chronology this upswing was too short to show up as a turning point. Focusing on the monthly chronology in the top panel of Table 2.3, the early 1990s is a period for which turning points are hard to determine. Compared to the other BCIs, EuroTCB performs best with only one missed trough, compared to one missed peak and two missed troughs by the other two BCIs. In the second half of the 1990s, the BCI turning points are in better accordance with the

GDP turning points, although leads of up to six months can be seen. The average absolute lead/lag is slightly larger than one quarter for both EuroTCB and EuroCOIN and nearly five months for EuroIJR.

The bottom panel of Table 2.3 shows that the BCIs perform relatively better in identifying growth rate cycle turning points at a quarterly frequency. As mentioned before, the number of peaks and troughs that were picked out are the same and the average absolute lead or lag is no bigger than one quarter. EuroTCB once again has a very low lead/lag, with EuroIJR and EuroCOIN having an average lead/lag of one quarter.

In summary, the three BCIs pick up the two classical turning points in euro area GDP, but have more difficulties in signalling the growth rate cycle turning points. The performance of the three BCIs is roughly comparable, with a slight advantage for EuroIJR in determining classical turning points and a modest advantage for EuroTCB in identifying growth rate cycle turning points. However, differences in growth rate cycle peaks and troughs between monthly and quarterly series are an indication that further research is warranted.

### ***2.5 Driving Forces of the Euro Area Growth Rate Cycle***

One of the main advantages of using only a relatively small dataset to construct a business cycle index is that it makes it easier to analyze the impact of individual series. Although the EuroIJR index includes only variables for a limited number of countries, such an analysis should help to understand some of the dynamics of the euro area growth rate cycle. A first piece of information in this analysis is the weighting matrix, (see equation 2.2). Another useful perspective is given by the average contribution of each variable to the index. The contribution of a variable to the index is given by the weight times the value of the variable in a particular month. Table 2.4 shows these weights and contributions for all variables. The weight and contribution of each variable is calculated by summing across leads and lags. Two columns with average contributions are included, one averaged over all months and another averaged over those months where the value of the index in absolute sense was larger than 0.1 (this was the case in more than 85 percent

of the months). The reason for the latter adjustment is that in months with small values of the index, the relative contributions can be quite large.

Table 2.4 shows that euro area GDP is the most important variable in the index, although by no means the only important one. There are also a number of variables with negative weights, meaning that these variables were negatively correlated with euro area GDP over this period. In other words, these variables are countercyclical. As the final two columns show, these variables do make, on average, a positive contribution to the index. Some variables make negative contributions, but this is mostly due to large contribution shares in months where the index is close to zero, because excluding these months nearly eliminates negative contributions.

It is also interesting to see that the contribution of GDP is considerably larger than its weight (around two times as large). Since all variables have been normalised, this finding cannot reflect differences in the relative size of changes in variables. Instead, the reason might be that some variables move more in line with the index as a whole than others, but no useful measure was found to show this. Other important variables are German industrial production (with an average contribution of 11.5 percent), French imports (6.3 percent) and Spanish household consumption (15.3 percent). Quite significant is also the presence of variables with small contributions, such as for example the Spanish order book survey. It would be very useful if statistical criteria could be used to remove such variables from the index since they are likely to mostly add noise to the system.<sup>32</sup>

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<sup>32</sup> See Boivin and Ng (2003) for criteria to include variables or weight them according to the correlation of the error with other series' errors. This seems a potentially fruitful approach, although the statistical foundations are less clear. For the state of the art on inference for large factor models see Bai (2003).

**Table 2.4 Weights and contributions of the EuroIJR index**

Country	Type	Variable	Weights	Contribution share	
				All months	Excl. outliers
France	LEAD	Bond Yield 10 year	4.3	3.7	3.4
France	LEAD	Yield Spread - 10 year minus Day-Day Loan	-3.1	3.2	0.2
France	LEAD	Stock Price SBF 250 Index	-0.1	1.2	0.2
France	LEAD	Personal Consumption of Manuf. Goods	2.9	-3.5	1.1
France	LEAD	Building Permits - Residential	0.4	0.1	0.2
France	LEAD	New Unemployment Claims	-0.3	0.3	0.7
France	LEAD	Industrial New Orders	0.1	-1.2	0.0
France	LEAD	Consumer Confidence Index	1.6	0.4	0.3
France	LEAD	Change in Stocks	4.2	3.8	1.8
France	LEAD	Ratio Deflator of Manuf. Value Added to Unit Labor Cost	2.3	0.6	0.5
France	COIN	Retail sales	2.0	0.1	0.7
France	COIN	Industrial Production	1.8	0.6	0.7
France	COIN	Real Imports	6.3	7.2	6.3
France	COIN	Paid Employment	5.1	-2.9	7.1
Germany	LEAD	New Orders - Investment Goods	0.4	0.7	0.1
Germany	LEAD	Yield Spread - 10 year minus 3 month	2.4	1.2	1.0
Germany	LEAD	New Orders - Consumer Confidence Index	-0.7	0.5	-0.2
Germany	LEAD	Change in Inventories	-2.4	3.0	1.0
Germany	LEAD	New Orders - Residential Construction	5.5	-3.4	1.0
Germany	LEAD	Stock Prices	0.9	1.6	0.3
Germany	LEAD	Gross Enterprise and Property Income	5.7	-1.5	1.6
Germany	LEAD	Growth Rate for Consumer Price Index for Services	-2.1	1.3	0.2
Germany	COIN	Industrial Production	10.3	17.0	11.5
Germany	COIN	Employment - Number of People Employed	0.7	2.0	0.4
Germany	COIN	Manufacturing Sales	4.7	3.6	2.8
Germany	COIN	Retail sales	7.1	10.9	4.1
Spain	LEAD	Construction Component of Industrial Production (3-m ma)	1.1	-0.1	0.4
Spain	LEAD	Capital Equipment Component of Industrial Production (3-m ma, s.a.)	1.6	-0.7	0.2
Spain	LEAD	Spanish Contribution to Euro M2 (s.a.)	2.5	1.8	1.2
Spain	LEAD	Spanish Equity Price Index	-0.7	0.3	-0.1
Spain	LEAD	Long-term Government Bond Yield (Inverted)	-3.0	2.0	1.9
Spain	LEAD	Order Books Survey (3-m ma, s.a.)	0.9	0.0	0.0
Spain	LEAD	Job Placings (3-m mov. av., s.a.)	2.2	0.5	0.3
Spain	COIN	Final Household Consumption (Q)	9.3	9.5	15.3
Spain	COIN	Industrial Production Excluding Construction (3-m ma)	4.1	1.6	2.0
Spain	COIN	Real Imports (3-m ma)	5.3	1.9	2.9
Spain	COIN	Retail Sales Survey (s.a.)	1.4	-0.2	0.1
EuroArea	GDP	Gross Domestic Product (Q)	15.4	32.9	28.7
Total			100.0	100.0	100.0

Source: Indicators: The Conference Board ([www.globalindicators.org](http://www.globalindicators.org)). Weights and contributions: own calculations.

Notes: COIN: Coincident indicators; LEAD: Leading indicators; Weights: share, summed across leads and lags, calculated from projection matrix. 3-m ma: 3 month moving average. sa: seasonally adjusted. Contribution share: calculated as the share of the changes in each indicator times its weight in the total change of the index. Outliers are defined as month where the EuroIJR index value was less than 0.1 in absolute sense. This cut-off point was chosen because nearly all large contributions (of more than 100% of the index in that month) occurred in such months.

To further illuminate the contributions of variables to the index, Figure 2.3 shows a number of plots, where the shaded areas give the contribution by (a set of) variables. Panel A shows how the variables for the different countries, as well as euro area GDP

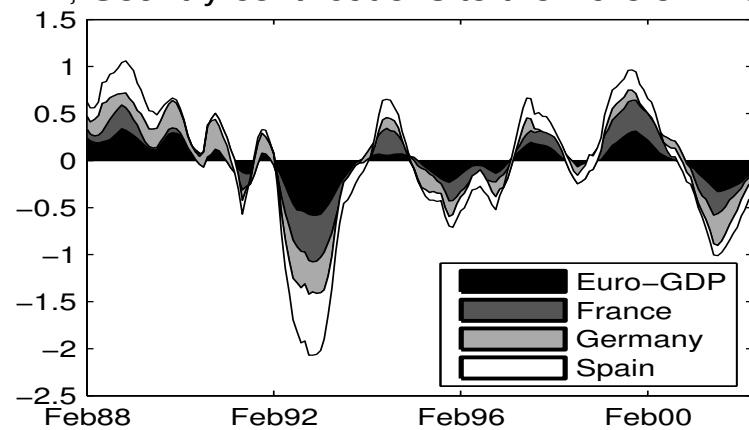
affect the EuroIJR index.<sup>33</sup> This reveals part of the pattern of economic growth in the 1990s. In the early 1990s, most of European growth was driven by Germany with little contribution from France and Spain. The main recession starting around the end of 1991 shows large negative contributions from all three countries, as well as from the overall euro area GDP. Toward the end of the 1990s, France and Spain were making large contributions to positive values of the index, but German variables hardly made any contribution. The relative size of the countries in terms of GDP does not seem to be the most important predictor of their share of the contributions, although this set of countries may simply be too close in relative size to easily distinguish. A more thorough test of this matter would include data for countries such as Finland or Ireland. Finally, the recession starting around 2001 once again features sizable contributions from each of the countries. From this limited set of countries and time span, there seems to be no evidence of idiosyncratic shocks that led to recessions in only one of the countries. This issue is analyzed in more detail in the next chapter.

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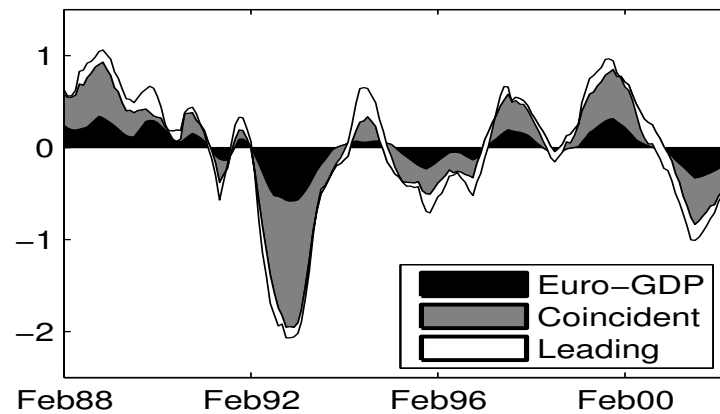
<sup>33</sup> Each of the countries also influences the index indirectly by their contribution to Euro area GDP, but we have not separately distinguished this impact.

**Figure 2.3 Contribution to EuroIJR by its components**

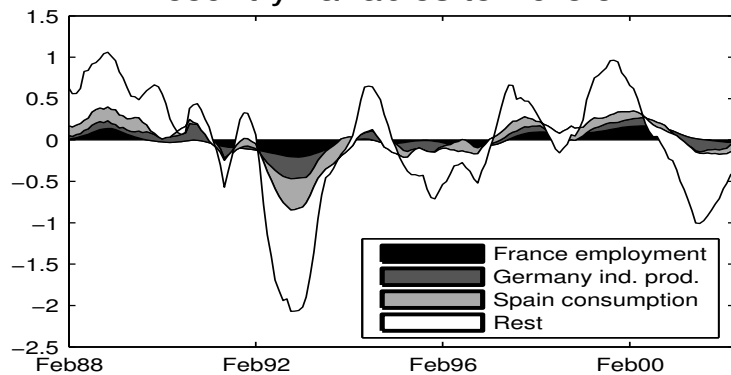
**A, Country contributions to the EuroIJR index**



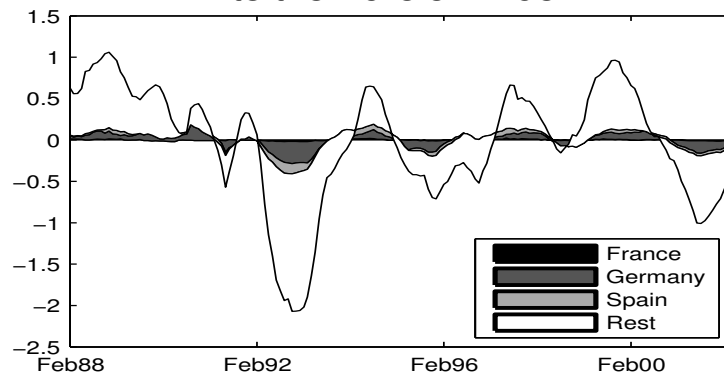
**B, Variable contributions to the EuroIJR index**



**C, Contribution of most important country variables to EuroIJR**



**D, Contribution of industrial production to the EuroIJR index**



Panel B shows the contributions from the group of coincident variables and from the set of leading variables. This figure reveals that especially in the recession of 1992-1994 coincident variables played a dominant role. Visual inspection does not reveal important signals from the leading variables, but as noted before, a formal evaluation of the forecasting performance of the index would proceed along different lines. Finally, Panels C and D show the contribution from individual variables to the index. Panel C shows the contribution from the most important variable from each country in terms of its contribution to the total index. These three variables are certainly not driving all movement in the index, as especially the recession in 2001 reveals: after the index had turned negative, it took a number of months before these variables also made a negative contribution. In the recession of the early-1990s, the three variables are more synchronized with the rest of the variables.

It is striking to look at the relative importance of similar variables for different countries. Panel D shows the contribution of industrial production of each country to the EuroIJR index. German industrial production is clearly the most important, even though German manufacturing is only about 30 percent of euro area manufacturing (French and Spanish manufacturing are about 20 and 10 percent of euro area manufacturing, respectively). This difference is confirmed when looking at the weight of each series in Table 2.4. While German industrial production receives a weight of more than 10 percent, French and Spanish industrial production get a weight of 1.8 and 4.1 percent, respectively. This seems to indicate that German manufacturing plays a much more important cyclical role than is indicated by its size. This may be partly due to the greater importance of more cyclical capital equipment manufacturing in Germany, but international linkages may also be important. Once again, however, firm conclusions are hard to come by due to the limited set of countries that is studied.

## **2.6 Concluding remarks**

A timely and up-to-date picture of economic circumstances is invaluable for decision makers in both government and business. Since GDP is only released once a quarter and with a considerable lag, earlier and more frequent indexes of the state of economic activity are useful, especially in turbulent economic times. Such indexes, constructed by



statistical analysis, can be a useful complement to models that have to make assumptions about structure of the economy or behaviour by consumers and firms.

This chapter compared the performance of two different methods for constructing business cycle indexes, namely the NBER method and the generalized dynamic factor model. In the NBER method, variables are selected for inclusion in the index based on a researcher's judgment of how closely the cyclical behaviour of a variable matches that of an index of economic activity such as GDP. The generalized dynamic factor model of Forni *et al.* (2000) uses statistical criteria to give a variable a larger or smaller weight.

An advantage of the generalized dynamic factor model compared to the NBER method is that business cycle indexes can be constructed with a smaller number of (judgmental) choices about the components and their weight in the index, since many of those choices are determined by the statistical model. One advantage of the NBER-type indexes is that the business cycle index is constructed from only a limited number of variables. As a result, changes in the index can be more easily traced back to the component or components that drive this change. This allows analysts and users to evaluate which variables have contributed to a recession, say a slowdown in the industrial production or a drop in employment. If the number of components of the index grows too large, this insight is much harder to get.

Using both the NBER method and the generalized dynamic factor model, three business cycle indexes were compared. The first index is based on euro area GDP and the components of the coincident indexes for France, Germany and Spain from The Conference Board, selected according to the NBER method. The second is the euro area index of Altissimo *et al.* (2001), constructed by applying the generalized dynamic factor model to a dataset with nearly 1000 economic variables. The third index is a hybrid index that uses the 37 components of the Conference Board's coincident and leading indexes for France, Germany and Spain as well as euro area GDP and applies the generalized dynamic factor model to weigh and combine these variables into a business cycle index.

One of the most important uses of a business cycle index is to signal peaks and troughs of business cycles. Therefore, the three indexes are compared based on that criterion. Although none of the three indexes perfectly matches the turning points of euro area GDP, all three are reasonably close. The indexes identify the two classical turning

points of GDP in the 1990s, but have more difficulties with growth rate cycle turning points. This is at least partly because the Bry-Boschan algorithm that is used for dating cyclical turning points was designed for dating classical cycles, not growth rate cycles. Differences between the monthly and quarterly chronology of growth rate cycle turning points suggests more research into this issue would be useful.

Overall only a limited number of variables is necessary to capture the salient features of the euro area business cycle. Including more variables adds little extra information, but the added noise may make the index harder to identify (Boivin and Ng, 2003). A business cycle index based on a limited number of variables also makes it easier to identify some of the driving forces of the euro area cycle. The analysis of the hybrid EuroIJR index reveals some important differences between the contributions of France, Germany and Spain to the euro area growth rate cycle. Most notably, the same variable from different countries seems to play very different roles. For example, while German industrial production is one of the variables with the largest impact on the euro area cycle, industrial production in France and Spain make only small contributions. This suggests that German industrial production moves more 'in tune' with the overall euro area economy. This brings up the issue to what extent a common monetary policy for the euro area is suitable for each of the individual countries. As this question has important implications for the (political) sustainability of the common currency, it is the topic of the next chapter.

### **Appendix 2.A The Generalized Dynamic Factor Model**

In recent years increasingly large datasets on economic time series have become available. However, many of the existing statistical tools were not well suited to analyze such datasets. One of the tools that is frequently used to extract relevant information from large datasets is factor analysis. The basic idea behind factor analysis is that movements in a large number of series are driven by only a limited number of (latent) common shocks.

In factor models, the  $N$  series in a dataset with  $T$  observations each are decomposed into a common component ( $\chi$ ) that is driven by only  $Q < N$  common shocks ( $u$ ) and an idiosyncratic component ( $\xi$ ):  $x_{it} = \chi_{it} + \xi_{it}$ . In the standard static factor model the implicit assumption is made that all series are only contemporaneously affected by the common shocks. The Generalized Dynamic Factor Model (GDFM) of Forni *et al.* (2000) is both dynamic and it allows for limited cross-correlation between the idiosyncratic components. For most time series analysis the dynamic character of the model is especially important as common shocks may not have an impact on a series contemporaneously but with a lead or lag. The GDFM allows for a decomposition of the common component in a cyclical  $\chi^C$  and non-cyclical  $\chi^{NC}$  component so the complete decomposition becomes:

$$(A2.1) \quad x_{it} = \chi_{it} + \xi_{it} = \chi_{it}^C + \chi_{it}^{NC} + \xi_{it}.$$

The generalized dynamic factor model proposed by Forni *et al.* (2000) can be written as:

$$(A2.2) \quad X = UB^C(L) + UB^{NC}(L) + \Xi,$$

which is the matrix notation of Equation (2.1). Uppercase characters denote the matrix of corresponding lowercase variables. The series  $x_{it}$  are normalized to have a mean of zero and a variance of one.

The factor loadings  $b$  and common shocks  $u$  are not uniquely identifiable, but Forni *et al.* (2000) prove that under four assumptions the common component of each series can be uniquely identified and consistently estimated as both  $N$  and  $T$  go to infinity. First of all, the common shocks are white noise and the idiosyncratic components are

stationary processes, uncorrelated with past, present and future values of the common shocks. Second, the spectral density matrix of the observation matrix  $X$  exists. Third, the first  $Q$  eigenvalues go to infinity as the number of series  $N$  goes to infinity and finally, all remaining eigenvalues remain bounded.

The proposed estimation scheme consists of four steps. In the first step, the information in the time domain is transformed to the frequency domain, to easily incorporate information of leading and lagging relationships. Second, a filter is constructed that maximizes the variance explained by the common component using principal component analysis. Third, the filter is transformed back to the time domain and fourth, applied to the time series to obtain the common component of each series.

More precisely, the estimation scheme is as follows. The first step is to calculate a series of auto-covariance matrices of the data matrix  $\Gamma_m$ ,  $m = -M, \dots, M$ . The integer  $M$  represents the number of leading and lagging observations that contain information on the current common component. To obtain a consistent estimator,  $M$  must go to infinity and the quotient  $M/T$  must go to zero as  $T$  tends to infinity. Forni *et al.* (2000) propose to use  $M = \text{round}(2/3)T^{1/3}$ . The second step is to use a Fourier transformation on the auto-covariance matrices to estimate the spectral density using the Bartlett kernel estimator:

$$(A2.3) \quad \Sigma(\theta_t) = \sum_{m=-M}^M \Gamma_m \omega_m e^{-im\theta_t}, \quad t = 0, 1, \dots, F,$$

with  $\omega_m = 1 - [|m|/(2M + 1)]$  the Bartlett kernel. This means the spectrum,  $\Sigma(\theta_t)$ , is evaluated at some predetermined number of frequencies,  $F$ , given by  $\theta_t = 2\pi t/(F + 1)$ .

In the third step the  $Q$  largest (real) eigenvalues are calculated as well as the corresponding (complex) eigenvectors  $p_q(\theta_t)$  of the spectral density matrix at frequency  $\theta_t$ . If we stack the eigenvectors in a matrix  $V(\theta) = [p_1(\theta), \dots, p_Q(\theta)]$ , the weights of the filter in the frequency domain are given by:

$$(A2.4) \quad \mathbf{K}(\theta_m) = V(\theta_m)V(\theta_m)', \quad m = -M, \dots, M,$$

with  $V'$  the transposed complex conjugate of  $V$ .

To select only the cyclical part of the common component, the inverse Fourier transform is applied in the third step using only the frequencies associated with the cyclical frequencies and obtain the two-sided filter:

$$(A2.5) \quad \mathbf{K}_k^C = \sum_m \mathbf{K}(\theta_m) e^{ik\theta_m}, \quad k = -M, \dots, M,$$

with  $m$  in the summation such that  $|\theta_m| < (2/14)\pi$ , so  $\theta_m$  is part of the cyclical interval.

The final step involves applying the filter to the dataset  $\mathbf{X} = (x_{it}, \dots, x_{Nt})$  for all  $t = 1, \dots, T$ , to get an estimator of the cyclical common component:

$$(A2.6) \quad \hat{\boldsymbol{\phi}}^C = \frac{1}{2M+1} \sum_{k=-M}^M L^k(\mathbf{X}) \mathbf{K}_k^C.$$

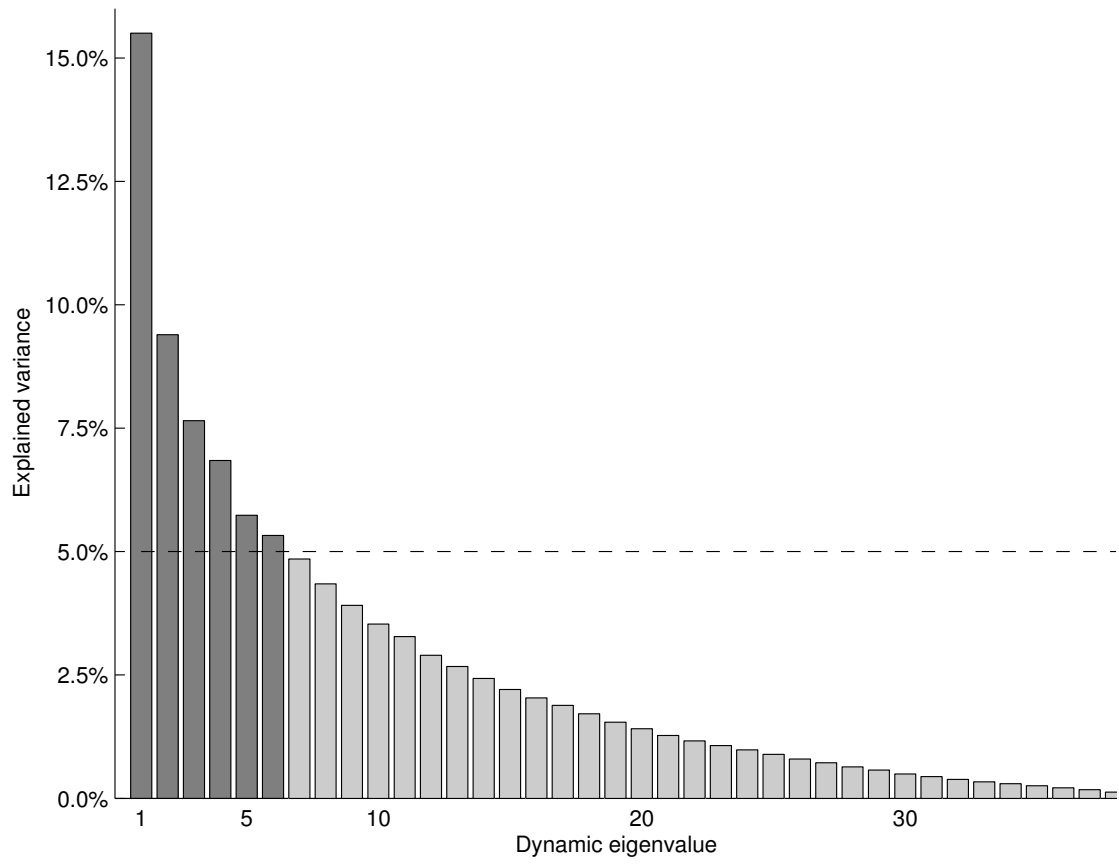
That is, each common component is a moving average ( $M$  lags and leads) of the series in question. The weights of this moving average are determined by the eigenvectors of the spectral densities. For  $M=0$  this model reduces to the standard static principal component model.

### Choice of $Q$

Until now the number of common shocks  $Q$  was assumed to be known. In practice of course,  $Q$  must be chosen or ideally be estimated based on the dataset at hand. As Forni *et al.* (2000) point out, there is no formal statistical test to help identify the number of factors in their GDFM. However, Forni *et al.* (2000) propose to relate the choice of  $Q$  to the variance explained by the  $i^{\text{th}}$  eigenvalue (averaged over all frequencies). If the model assumptions are fulfilled, there is a substantial gap between the variance explained by the  $Q^{\text{th}}$  and the  $(Q+1)^{\text{th}}$  eigenvalue. Forni *et al.* (2000) propose to include factors as long as they explain 5 percent of total variance and this rule of thumb is followed here.

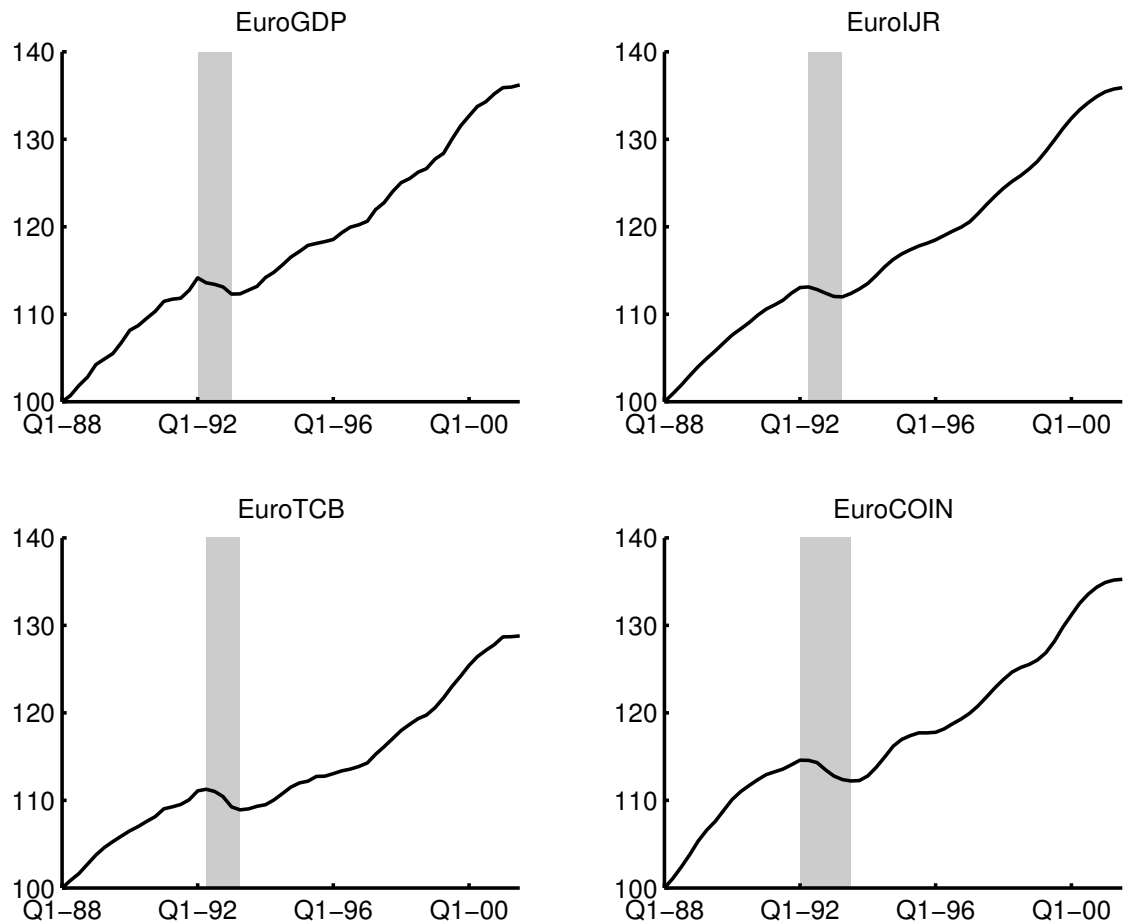
Appendix Figure 2.1 shows the percentage of total variance explained by the different eigenvalues. As the figure shows, only the first 6 eigenvalues explain more than 5 percent, so  $Q=6$  is chosen. The figure also shows that the choice of  $Q=6$  is somewhat arbitrary, as the difference in explained variance with eigenvalues 7 and 8 is quite small. Some robustness analysis, however, suggests that the resulting business cycle index is not very sensitive to the exact choice of  $Q$ .

**Appendix Figure 2.1 Percentage of variance explained by each eigenvalue (38 series, 189 observations)**



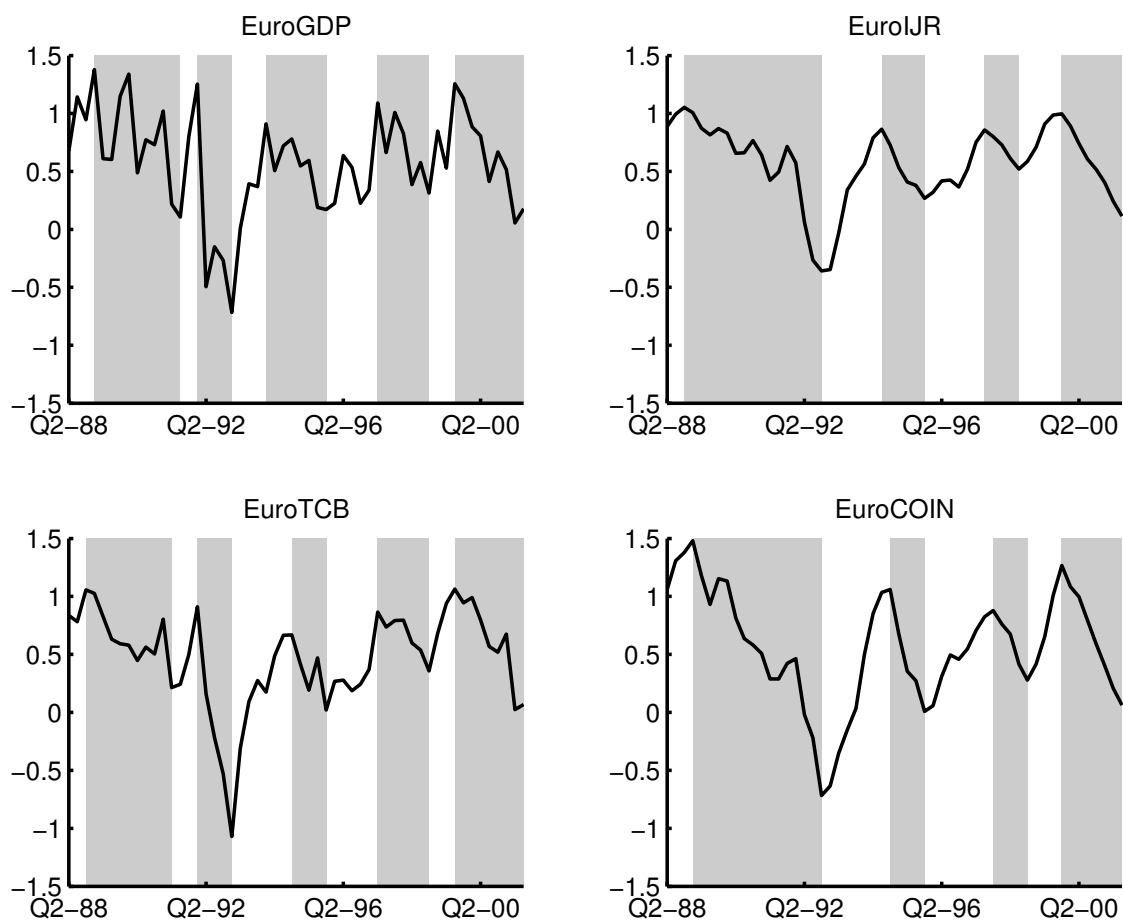
*Appendix 2.B Plots of Quarterly GDP and Business Cycle Indexes*

**Appendix Figure 2.2 Euro GDP and quarterly business cycle index levels, 1988-2002, January 1988=100**



Note: shaded areas mark business cycle recessions with absolute declines in economic activity.

**Appendix Figure 2.3 Euro GDP and quarterly business cycle index growth rates, 1988-2002**



Note: shaded areas mark growth rate cycle recessions with decreasing growth rates of economic activity.



