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Network public goods with asymmetric information about cooperation preferences and network degree

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ABSTRACT

We propose a game theoretical model of one-shot network public goods formalizing the 'closure argument' that cooperation is more frequent in denser groups or networks. Equilibrium analyses show that (i) an 'inefficiency problem' exists: players all preferring mutual cooperation need not all cooperate; (ii) in dyads, groups and networks with degree independence, first order stochastic dominance shifts of the distribution of cooperation preferences or the degree distribution (weakly) increases cooperation, and (iii) the latter result does not hold for networks with degree dependence. Hence the closure argument always holds in networks satisfying degree independence but not in other networks.

1. Introduction

Social dilemmas are situations in which individually rational behavior leads to collectively undesirable outcomes, posing the problem of cooperation among selfish individuals. Explaining cooperation between genetically unrelated individuals in social dilemmas is a central issue in the behavioral sciences (e.g., Buchan et al., 2002; Dawes, 1980; Fehr and Gächter, 2002; Fehr and Gintis, 2007; Kollock, 1998; Willer, 2009). Arguably, the 2-person prisoner's dilemma (PD) game is the social dilemma that is most used to examine factors affecting cooperation in social dilemmas (e.g., Axelrod, 1984). The PD is actually a binary form of the more general public goods (PG) game (e.g., Ledyard, 1995); we will use the term 'binary PG' to refer to the PD and to bring out the link between the PG and the PD.

Famous examples of social dilemmas that have acquired literary renown are the *tragedy of the commons* (Hardin, 1968; Ostrom, 1990) and trench warfare in WWI as described and analyzed by Axelrod (1984). As social scientists have long recognized, everyday life is rife with social dilemmas, from efforts reducing environmental pollution, maintaining a valuable community resource (Bouma et al., 2008), or overthrowing an oppressive regime (Opp et al., 1995), to attempts at establishing a lobbying association (see the examples in Marwell and Oliver, 1993). In all these cases, all individuals would prosper if the collective goals were reached, but no individual by himself has a sufficiently strong incentive to contribute to their achievement.

Groups differ in their ability to produce and sustain cooperation, and hence in the welfare they produce. In real-life groups and social networks we often see a certain 'spatial' structure of cooperation: Some groups or parts of the network succeed in establishing high rates of cooperation in PDs and investment in PGs, whereas other groups or parts of the network fail to do so, leading to large differences in welfare between groups and different parts of the network.

In the sociological literature, a group's ability to overcome social dilemmas and organize sufficient cooperation has been defined as an element of that group's *social capital* (e.g., Coleman, 1990; Dijkstra, 2013; Putnam, 1995). Large differences in this form of social capital are observed across different communities (see Halpern, 2005 for empirical cases). An explanation of these between-community differences is contained in the 'closure argument' (e.g., Burt, 2000; Coleman, 1990). The principal consequence of the closure argument is that cooperation rates are higher in densely knit communities than in more loosely connected ones. This is the case because of two distinct mechanisms. First, members of dense communities quickly obtain information about each other's behavior and can thus more easily monitor and sanction each other. Moreover, since potential sanctioners have social relations with each other, they can support each other and coordinate their sanctioning behavior (Coleman, 1990: 270–278). Second, members of dense communities have many social relations with others (i.e., fellow community members) who also have many social relations (with others who also have many social relations,

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etcetera). Thus, each community member has a high stake in cooperative social relations and has connections with others who do, too.

The first mechanism underlying the closure argument involves monitoring, learning, and sanctioning, and can only operate in repeated settings: Community members have to meet again, and identify others' past behaviors. Monitoring, learning and sanctioning opportunities in repeated interactions are at the heart of the distinction between 'trust' and 'assurance' proposed by Yamagishi and Yamagishi (1994; see also Yamagishi, 2003). In particular, 'assurance' is defined as the expectation of cooperation based on incentives. For instance, the possibility of being ostracized or punished in future interactions can induce cooperative behavior in the present. Relatedly, Buskens and Raub (2002) show how actors can come to trust others to behave cooperatively in repeated interactions through direct (i.e., dyadic) and indirect (i.e., network-mediated) learning, and direct (i.e., dyadic) and indirect (i.e., third-party) sanctioning (see also Buskens et al., 2010). The second mechanism underlying the closure argument is operative in both repeated and one-shot settings. This mechanism entails that more is at stake in dense networks than in sparser ones because members of dense networks have more social relations with others who also have more social relations, etc. Any single decision to cooperate or defect thus has larger payoff consequences for individual players in denser networks.

The main goal of the current paper is to examine analytically the hypothesis that cooperation rates are higher in more dense networks or in more dense parts of networks. Our analysis will focus on one-shot interactions and will thus only address the second mechanism underlying the closure argument outlined above. The closure hypothesis we examine is that dense networks represent a large source of potentially valuable social relationships for all members, with cooperation increasing in the density of the network.

We focus on one-shot interactions for four reasons. The first concerns mathematical tractability of the model. The one-shot case is analytically tractable and less complex than the repeated case. Second, explaining observed cooperation in one-shot social dilemmas is theoretically distinct from explaining cooperation in repeated ones. More specifically, some explanations of cooperation in repeated social dilemmas (e.g., Folk Theorems (e.g., Fudenberg and Tirole, 1991) and sequential equilibrium (Kreps et al., 1982) cannot work in the one-shot case. In the one-shot case we are forced to look more closely at the 'micro model of the actor'. Our third reason to focus on one-shot interactions is that the proposed mechanism for the one-shot case will also operate in repeated settings. Thus, going from one-shot to repeated interactions, the 'monitoring and sanctioning mechanism' is *added to* the 'valuable social relationships mechanism', but the latter remains operative. Therefore it is important to independently model and understand the latter mechanism. Our final reason to focus on one-shot interactions is that the closure argument is understudied in this context, being commonly associated with repeated interactions.

The closure hypothesis is examined using both a micro model of the actor and a game theoretic model of the one-shot binary PG game. Our micro model based on the *social exchange heuristic* (Kiyonari et al., 2000) assumes that actors over-evaluate mutual cooperation. We will argue below that there is considerable empirical evidence for the plausibility of the social exchange heuristic. Our micro model extends the social exchange heuristic in three important ways. First of all, we parameterize the heuristic in a way that allows modeling heterogeneity between players. Thus, our model allows different players to apply the heuristic to different degrees. There is empirical evidence that individuals indeed differ in this respect (Simpson, 2004), with important consequences for the likelihood of cooperation. Second, we relax the standard assumption of complete information about the preferences of other players. Concretely, this means that in our model players are uncertain about the extent to which other players apply the social exchange heuristic. Third, we are the first to apply the social exchange heuristic to a network game.

In our game theoretical analysis we apply Bayesian Nash equilibrium (BNE) (e.g., Fudenberg and Tirole, 1991) to three social environments. These are (i) the dyad, or 2-person game, (ii) the completely connected social group, or multi-person game, and (iii) the social network. While the dyad and multi-person game represent the two ends of the continuum, the main focus will be on the network context. We will first consider networks satisfying *degree independence*, i.e., networks in which an actor's own degree carries no information about the degrees of his neighbors. Then we'll consider the more general case of *degree dependence*.

There are four reasons why we focus on the distinction between networks with and without degree independence. First, since we assume players know only their own degree and the overall degree distribution of the network, players cannot condition their strategic choices on any network properties other than degree (in)dependence in the current model. This is not to say that these other network properties such as clustering, transitivity, and reciprocity are irrelevant to the issue of the production of public goods on networks. However, *under the informational assumptions of our model*, these network properties are not relevant. Second, degree (in)dependence may be an important determinant for an actor's cooperation decision. For example, if degree dependence is such that actors with a high degree are especially connected to others with a high degree, a focal actor with many neighbors knows that his neighbors likely have many neighbors, too. Therefore, in deciding whether to cooperate the focal actor will above all things consider the expected behavior of highly connected others, as their behavior especially impacts him. The expected behavior of more sparsely connected actors is much less relevant to his decision. Under degree independence, however, the focal actor must consider the expected behavior of any type of other actor, regardless of degree.

Third, we demonstrate that the distinction between degree independence and degree dependence is crucial for understanding the spatial structure of cooperation in networks, in our model. Specifically, we show that in the case of degree independence denser networks always exhibit (weakly) higher rates of cooperation than less dense ones, conditional on N (the number of players, or alternatively the maximum degree in the network) and the distribution of preferences for mutual cooperation. Since the model concerns one-shot decisions, this result is solely driven by the fact that individuals value good social relations, and that those who have many of them in the same community or network thus have a lot at stake. For social networks with degree dependence we show that in general there is no positive association between network density and cooperation rate. Hence the closure hypothesis is guaranteed to hold for networks satisfying degree independence but not for other networks.

Fourth, many real-life social networks exhibit considerable degree dependence. Hence our paper would have limited practical value if it remained silent on cooperation in networks with degree dependence.

2. Relation to existing literature

The social exchange heuristic is consistent with the distinction made in the literature between two types of social dilemma players. When facing a binary PG a proportion of players is consistently found to display so-called assurance game preferences, and preferring mutual cooperation over defecting on a cooperating partner. The remainder of the players display standard 'prisoner's dilemma game preferences', having the reverse preference (e.g., Ahn et al., 2003; Hayashi et al., 1999; Dijkstra, 2012).

Our current study is related to three strands of network literature, namely, (i) experimental and observational studies of cooperation on static networks, (ii) experimental studies of cooperation on dynamic networks, and (iii) formal models of cooperation on static networks. We start with briefly describing three empirical studies of cooperation on static networks (first strand), followed by a discussion of a few studies of the other two strands.

Reanalyzing the experimental data of Fehr and Gächter (2002), Fowler and Christakis (2010) show how the random matching procedure employed in these experiments implies a static social network between the participants, in which a social tie means shared group membership. They then show how cooperative behavior (i.e., public good contributions) is 'contagious' up to three degrees of separation in this network. Thus, a focal participant's contribution affects the contributions of her neighbor, her neighbor's neighbor, and her neighbor's neighbor's neighbor. Note that regardless of the implied network structure, the random matching procedure ensures that strategically each decision is effectively one-shot. In a web-based, finitely repeated networked public goods experiment Suri and Watts (2011) find no effects of static network topology (for instance small-world, isolated cliques, and random regular networks) on average contributions. These authors do find that contributions are contagious up to one degree (i.e., only direct neighbors). In our one-shot model an actor's (equilibrium) behavior likewise depends on the behavior of her neighbors, leading to patterns of 'contagion'. The extent of contagion in our model depends on both the network structure and the actual equilibrium played.

In a cross-sectional observational network study combined with a field experiment, Apicella et al. (2012) find assortativity on cooperation in a public goods game in social networks of the Hadza hunter-gatherers in Tanzania. Two types of social networks were measured and cooperative behavior was measured in a separate one-shot public goods experiment. Ties in both networks were more likely between people who contributed the same amount in the public goods game and public good contributions were contagious up to two degrees of separation in one of the networks. Since the study is cross-sectional the observed assortativity can be due both to the network being dynamic (e.g., cooperators seeking each other out) or to behavioral influence.

Wang et al. (2012) is an example of an experimental study of cooperation on dynamic networks. It shows that preferential attachment in dynamic networks may have a large positive impact on cooperation. The possibility to propose and delete links in a repeated game boosted cooperation and the average payoffs players earned, and significantly increased assortativity between cooperators. Since our current model is one-shot we do not model such network dynamics.

We mention three studies of formal modeling of cooperation on static networks which are closely related to our paper. Raub and Weesie (1990) study cooperation in the infinitely repeated PD game played on a social network. Their model demonstrates how the extent to which information can flow freely through the network impacts cooperation, the condition most conducive to cooperation being the one with unimpeded information flow. Bramoullé and Kranton (2007) study the one-shot provision of locally nonexcludable goods on networks. They show how specialization (some players contributing fully, others completely free-riding) and holes in the network (i.e., absent ties) can increase overall welfare. Finally Galeotti et al. (2010) analyze a broad class of one-shot network games under the assumption of asymmetric information about degree. In this respect their paper provides the analytical framework for our current model.

3. Organization of the paper

In the next section we will explain the basic binary PG game and our micro model of the individual agent. In the subsequent section we introduce the game theoretic equilibrium concept we employ. We then apply the concept to the 2-person game in the ensuing section and show that cooperation in the dyad can occur but is not guaranteed. In the following section we apply the micro model to a multi-person game, which we interpret as an $(N+1)$-person completely connected network. In the subsequent section we apply the micro model to social networks. In the social network version of the model, not only preferences for mutual cooperation but also *network degree* is private information. We then investigate how network structure, modeled by means of the (conditional) degree distributions, affects the equilibrium patterns of investment in the binary network-PG. We first study networks characterized by degree independence, and then analyze networks with degree dependence. In the main body of the text we analyze the case where players' payoffs from mutual cooperation depend linearly on the number of neighbors with whom they cooperate. In Appendix B we show that our analytical results also hold for the more general case of marginal payoffs decreasing in the number of neighbors with whom they cooperate. The model in Appendix B encompasses the special case where players over-evaluate mutual cooperation with a single (i.e., at least one) neighbor. The paper closes with a discussion of the results and the scope conditions of the model.

4. One-shot two-person PG and micro-model

In our one-shot 2-person PG each of the two players simultaneously decides whether or not to invest in the public good. Investment costs 1, and yields a return of a for both players, with $a<1$. Given these incentives, not investing in the one-shot PG is the only rational thing to do; if the other player invests, investing yields $2a$ which is less than $1+a$, and if the other player does not invest an own investment yields a, which is less than 1. Hence not investing is the dominant strategy. However, in experimental research utilizing this one-shot game, unexpectedly high rates of investment are found in a wide range of situations (e.g., Sally, 1995).

Many authors (e.g., Bolton and Ockenfels, 2000; Fehr and Schmidt, 1999; Levine, 1998) explain investment in the one-shot PG by assuming that players have some kind of 'social preferences'. An individual having social preferences not only cares about the payoff consequences of the outcome for herself, but also for (some of) the other player(s). The extent to which such social preferences can explain contributions to public goods is debated, however. In many instances in which individual contributions have a perceptible impact on neither the public good produced nor the welfare of any subset of other individuals, considerable contributions are nonetheless made (e.g., voting in general elections). Additionally, Burton-Chellew and West (2013) offer experimental evidence contradicting the social preference hypothesis for public goods.

A different approach, the *social exchange heuristic*, according to which players cognitively transform the binary PG (or PD) into an Assurance Game, is proposed by Kiyonari et al. (2000), Simpson (2004) and Yamagishi et al. (2007). In an Assurance Game, as opposed to a binary PG, a player prefers to invest when the other player invests, and prefers not to invest when the other does not invest. The social exchange heuristic determines individuals' preference orders over outcomes, and is supposed to be part of the human cognitive make-up as shaped through evolution (Kiyonari et al., 2000). It is important to realize that in the social exchange heuristic the preference order is defined over the *outcomes* themselves, and not over either just the payoff consequences, or

Table 1
2-Person binary PG with premiums on mutual investment.

		Player 2	
		Invest (C)	Not invest (D)
Player 1	Invest (C)	$2a+\theta_1$, $2a+\theta_2$	a, $(a+1)$
	Not invest (D)	$(a+1)$, a	1, 1

the individual's behavior (i.e., her choice of investment or non-investment). The social exchange heuristic thus makes a distinction between the material outcomes associated with the binary PG game and the effective preferences over these outcomes. This is in line with the distinction between the 'given matrix' and the 'effective matrix' introduced by Kelley and Thibaut (1978). After the cognitive transformation of the ('given') binary PG into the ('effective') Assurance Game, players are assumed to play the game rationally.

Kiyonari et al. (2000) cite evidence showing that many human participants in experiments indeed rate the outcome of mutual investment as more desirable than the outcome in which the focal individual does not invest, while her partner does. Similarly, Rilling et al. (2002) offer evidence from questionnaire data and additional neurological evidence from a PD experiment showing that mutual cooperation/investment is evaluated as the most desirable outcome by many participants, and is associated with consistently more activation in brain areas related to reward processing. Thus, there appears to be ample scientific evidence underpinning the social exchange heuristic.

We assume that whether a transformation of the binary PG into an Assurance Game occurs depends on both the attractiveness of the payoffs associated with the outcomes and an *individually specific premium* for mutual investment. This premium θ_i represents the individual psychological reward a player experiences when mutual investment is the outcome, in addition to the material payoffs already specified. Table 1 presents the PG game after including θ_i, with 'C' (from 'cooperation') denoting investment and 'D' (from 'defection') denoting not investing. Note that the PG is transformed into an Assurance Game if $\theta_i > 1 - a$ for both players, in which not investing is no longer the dominant strategy.

5. Game theoretic model: Bayesian Nash equilibrium

We assume that information about θ_i is private and that the θ_i are drawn independently from cumulative distribution function $P(\cdot)$ on the set $\Theta_i = (-\infty, \infty)$. $P(\cdot)$ is common knowledge, and is also referred to as the players' *beliefs*. A player's pure strategy in this game is a map from the set of her premiums to her actions, $s_i : \Theta_i \to \{C, D\}$[2]. We focus on symmetric pure strategy equilibria, meaning that the equilibrium strategy map is the same for all players. Considering that (i) the game involves players with private information about their payoff functions, and (ii) the uncertainty about the payoffs of other players is formalized in a common knowledge cumulative distribution function, the relevant equilibrium concept is Bayesian Nash equilibrium (BNE) (e.g., Fudenberg and Tirole, 1991). A BNE is a Nash equilibrium of the 'extended game' in which nature first determines the players' premiums according to $P(\cdot)$, after which the players observe their own premium only, and then simultaneously decide on their actions. The BNE is applied to the 2-person game, the multi-person game, and the social network in the subsequent sections.

[2] One could also argue that a player's pure strategy is a map from her premiums *and her beliefs* to the set of her actions. However, since we assume $P(\cdot)$ is common knowledge, and a player's posterior beliefs (i.e., beliefs after she learns her premium) are entirely determined by her premium, we decide to put the beliefs in the description of the game, and not of the strategies.

6. The 2-person PG

Both players never investing (i.e., $s_i(\theta_{\theta_i}) = \{D\}$, for any θ_i, $i = 1, 2$) is a BNE, since $\{D, D\}$ is a combination of best replies, no matter the values of the premiums. In addition, there could be BNE with a positive probability of investment in the PG. Suppose that the equilibrium probability of cooperation by player j is y. Then player i should play C whenever,

$$(2a+\theta_i)y + a(1-y) \geq (a+1)y + (1-y). \tag{1}$$

Eq. (1) makes clear that BNE with positive probabilities of investment have a 'cutoff value' character: player i plays D whenever $\theta_i < \theta^*$, and C otherwise, with $(2a+\theta^*)y + a(1-y) = (a+1)y + (1-y)$. This monotonicity property enables us to characterize the BNE as

$$\theta^* = \frac{1-a}{y}, \tag{2}$$

where

$$y = 1 - P(\theta^*). \tag{3}$$

Thus, Eqs. (2) and (3) together define symmetric, pure-strategy BNE that have positive investment probabilities. Whether such equilibria exist depends on $P(\cdot)$, as will be shown below.

Although investment is possible in equilibrium, this does not imply that the 'cooperation problem' typically modeled with PGs is gone. Not only is mutual non-investment $\{D,D\}$ always an equilibrium, there is also an 'inefficiency problem' associated with the BNE having positive investment probabilities.

Proposition 1 ((inefficiency problem in the 2-person PG)). *Whenever* $P(1-a) > 0$, $\theta^* > 1 - a$.

Proof. All proofs of propositions are in Appendix A.

The inefficiency problem reflects that two players may exist who strictly prefer mutual investment over unilateral defection on their part but do not invest in equilibrium (i.e., players i with premiums $\theta^* > \theta_i > 1 - a$). Since for both these players i, $2a + \theta_i > 1$, there is a welfare loss and the outcome is Pareto inefficient. Inefficiency occurs because of the lack of certainty the players experience about each other's premiums (i.e., if $P(1-a) > 0$).

We will illustrate the BNE and the inefficiency problem with Examples 1–3. Working out these examples in the 2-person PG provides the intuition for our analyses in the more intricate multi-person and network cases. All examples have $a = 3/4$, but have different $P(\cdot)$. The $P(\cdot)$ of the three examples can be ordered with respect to first order stochastic dominance (FOSD), i.e., $P_3(x) \leq P_2(x) \leq P_1(x)$, for all $x \geq 1 - a$. Substantively, the FOSD ordering reflects that players' beliefs concerning the premium of the other player under $P_3(\cdot)$ are more positive than they are under $P_2(\cdot)$. That is, for any premium x above the defection threshold $1 - a$, the probability that the other player's premium is at least x is higher under $P_3(\cdot)$ than under $P_2(\cdot)$. $P(\cdot)$ in all three examples is beta distributed, with (α,β) equal to (2,5), (1,1), (2,1), for Examples 1, 2, 3, respectively. Characteristic of the beta distribution is that $\theta_i = [0, 1]$ with mean equal to $\alpha/(\alpha+\beta)$. All three distributions are shown in Fig. 1; FOSD is evident from the fact that the distribution functions do not cross for $x \geq 1 - a = 1/4$.

In example 1 no BNE with positive investment probability exists. In example 2 one BNE exists that allows cooperation at $\theta^* = 1/2$, with 50% of the population cooperating. The inefficiency problem of example 2 is reflected by the fact that 25% of the players, those with $1 - a = 1/4 \leq \theta_i \leq 1/2 = \theta^*$, (weakly) prefer $\{C,C\}$ to $\{D,C\}$ but play D. In Example 3 two BNE with positive investment probability exist. The first equilibrium has $\theta^* \cong 0.27$, $y \cong 0.93$, the second has $\theta^* \cong 0.84$, $y \cong 0.29$. Again, there is an inefficiency problem since both $\theta^* > \frac{1}{4}$. The solutions of the examples are represented in Fig. 1 by the

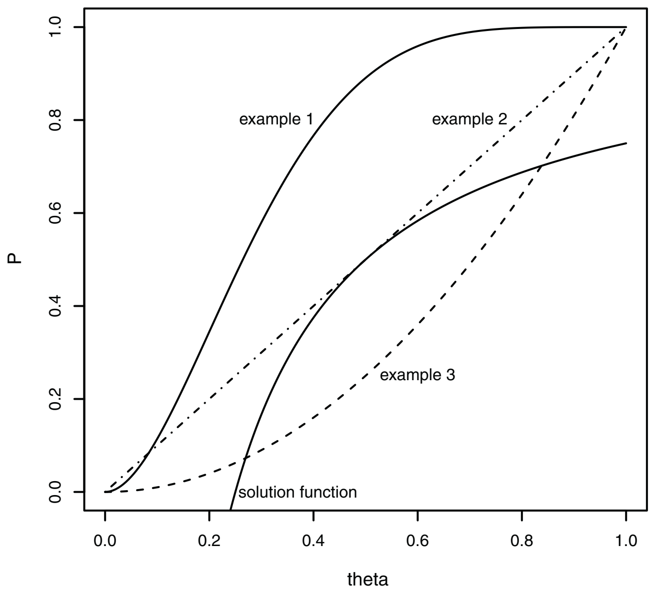

Fig. 1. Three beta cumulative distribution functions of θ and the solution function. Upper solid line—example 1 [$(\alpha,\beta)=(2,5)$]; lower solid line—solution function; upper dotted line—example 2 [$(\alpha,\beta)=(1,1)$]; lower dotted line—example 3 [$(\alpha,\beta)=(2,1)$].

intersections of $P(\cdot)$ and the solution function $P = 1 - \frac{1-a}{\theta} = 1 - \frac{1}{4\theta}$ derived from substitution of (3) in (2).[3]

The three examples suggest that the conditions for investment, represented by the value of θ^*, improve when the distribution of θ_i shifts to the right (i.e., if the $P(\cdot)$ can be ordered with respect to FOSD). This relationship is formalized in the next proposition.

Proposition 2 ((relation between $P(\cdot)$ and θ^*)). *If for two continuous distributions of premiums A and B, A FOSD B for $\theta \geq 1-a$, and at least one BNE exists for B, then $\theta^*_A \leq \theta^*_B$ and $y_B \leq y_A$.*

Proposition 2 reflects that if beliefs become more positive (in the FOSD sense) the equilibrium premium threshold decreases and the equilibrium investment probability increases. A consequence of Proposition 2 is that if a BNE exists for P_B, then a BNE also exists for P_A if P_A FOSD P_B for $\theta \geq 1-a$. The intuition of Proposition 2 can be grasped by looking at Fig. 1. The rightward shift of cumulative distribution $P_3(x)$ compared to cumulative distribution $P_2(x)$ leads to two intersections between the solution function $P = 1-(1-a)/\theta$ and distribution $P_3(x)$, instead of the single tangent point of this curve with $P_2(x)$. One of the intersections of the solution function with $P_3(x)$ is associated with a smaller θ than at the tangent point with $P_2(x)$.

6.1. Multi-person PG

In our multi-player extension of the PG we assume that there are $N+1$ players in the game. Thus, from the perspective of any player i there are N other players. In this and the next section we develop the linear case, assuming that a cooperating player i experiences the same marginal premium of mutual investment θ_i for each additional partner with whom she cooperates. The assumption of a constant marginal premium is arguably not very realistic, as it implies that players are 'infinitely gregarious', equally valuing the first and the 100th cooperating partner. Therefore, in Appendix B we show that the results of our model (Propositions 3–5) also follow under the more realistic assumption of premiums decreasing in the number of cooperating partners. The model in Appendix B encompasses both the linear case and the (extreme) situation in which players receive the premium for only 1 cooperating neighbor. We focus on the linear case in the main text for expositional clarity.

In the linear case the utility of investing when k others invest is $u_i(C)=(a+\theta_i)k+a$, and the utility of not investing is $u_i(D)=ak+1$. Then player i's expected utilities of investing and not investing are,

$$E\{u_i(C)\} = (a+\theta_i)Ny + a, \tag{4}$$

and

$$E\{u_i(D)\} = aNy + 1, \tag{5}$$

respectively.

Eqs. (4) and (5) show that symmetric pure-strategy BNE with positive investment probabilities in the multi-person game also have the monotonicity property: given any probability of investment y, players play D when $\theta_i < \theta^*$, and C otherwise. By equating (4) and (5) it can be shown that these BNE are characterized by

$$\theta^* = \frac{1-a}{Ny}, \tag{6}$$

and

$$y = 1 - P(\theta^*)$$

((7), (3) repeated).

The inefficiency problem still exists, as Proposition 3 shows.

Proposition 3 ((inefficiency Problem in the multi-person PG)). *Suppose $P\left(1-(a/N)\right) > 0$. Then $\theta^* > (1-a)/N$.*

The inefficiency problem again reflects that $N+1$ players i may exist in equilibrium with premiums satisfying $\theta^* > \theta_i > (1-a)/N$, and who thus strictly prefer all $N+1$ players (including themselves) to invest, but who still choose not to invest. Similarly, a proposition equivalent to Proposition 2 can be proved, i.e., it also holds in the N-person game that $\theta^*_A \leq \theta^*_B$ and $y_B \leq y_A$ whenever A FOSD B for $\theta \geq (1-a)/N$. In other words, making beliefs more positive in the multi-person PG also decreases the equilibrium premium threshold and increases the equilibrium investment probability.

Depending on $P(\cdot)$, the multi-person game allows for the dissipation of the inefficiency problem. That is, θ^* decreases if N increases, meaning that all players who strictly prefer mutual investment over unilateral defection do in fact invest in equilibrium for infinite N. As an example of the positive effect of N on cooperation, reconsider the 2-person PG of the third example from the last section with $P(\cdot)$ beta distributed with $(\alpha,\beta)=(2,1)$. The lower BNE for $N+1=2$ was $\theta^* \cong 0.27$ with $y \cong 0.93$. If $N+1=3$, the lower BNE is shifted downward to $\theta^* \cong 0.13 < 1-a$, with $y \cong 0.98$. The higher BNE is shifted upward to $\theta^* \cong 0.93$, with $y \cong 0.14$.[4] The new solutions for $N=3$ are the intersections of $P(\cdot)$ of Example 3 with the function $P = 1 - \frac{1}{(N/(1-a))\theta} = 1 - \frac{1}{8\theta}$ derived from (6).

6.2. Network PG

The network PG assumes that players have different 'neighborhoods' of others with whom they play the PG. Thus, there exists a

[3] Assuming that $P(\cdot)$ is differentiable everywhere, with density $p(\cdot)$, and letting θ^* be a BNE according to Eqs. (2) and (3), it can be shown that this BNE is asymptotically stable if and only if $(1-a/y^2)\,p(\theta^*) < 1$, using the stability notion of Nash tâtonnement (e.g., Bramoullé and Kranton 2007: 484). In Example 2 the equilibrium is neutrally stable. In Example 3 the equilibrium with the lowest threshold is stable, and the other is unstable.

[4] Just as in the 2-person case, it can be shown that BNE with positive investment probabilities in the multi-person game are asymptotically stable if and only if $(1-a)/Ny^2\,p(\theta^*) < 1$. The equilibrium with the lowest premium is then stable, whereas the highest is unstable.

social network of relations that is composed of a set of 'overlapping local' PGs. Each player i has a set of undirected social relations to others, the number of which is called player i's *degree*, d_i. We let the number of players in the network game again be $N+1$ so that the maximum degree is N. Instead of interpreting the model as an $(N+1)$-person network game, we can also interpret the games as being played on a network of arbitrary size, but with a maximum degree of N. Finally, in all models in this section we assume that d_i and θ_i are uncorrelated, i.e., a player's number of neighbors is not associated with his premium for mutual cooperation.

Let n_i denote the set of players that are neighbors of i, excluding i herself. Let $G(\cdot)$ denote the degree density, and $G^1(d_j) = G(d_j|d_j \geq 1) = G(d_j)/\sum_{k=1}^{n_i} G(k)$ the conditional degree density for neighbor j of player i, i.e., the degree density for j given that i and j are connected. Following Galeotti et al. (2010), we assume that players' degrees are private information. Thus, a player will know her own degree but is uncertain about the degree and premium of each of her neighbors with whom she plays the game. It is assumed that the discrete degree density $G^1(\cdot)$ is common knowledge, just like $P(\cdot)$. Each player i now has a 'type' consisting of the pair (θ_i, d_i). A pure strategy for player i in this network game is a map from her type to the set $\{C,D\}$, $s_i : (\theta_i, d_i) \rightarrow \{C, D\}$.

6.3. Degree independence

Let the vector of degrees of i's neighbors be $\vec{n}_i$, and denote the discrete probability density over these vectors by $G^1_{\vec{n}_i}(\cdot)$. Let the joint discrete probability density of d_i and $\vec{n}_i$ be denoted by $G^1_{\vec{n}_i,d_i}(\cdot,\cdot)$. Degree independence is then defined as

$$G^1_{\vec{n}_i,d_i}(\vec{n}_i, d_i) = G^1_{\vec{n}_i}(\vec{n}_i)G(d_i) = G(d_i)\prod_{j \in n_i} G^1(d_j)$$

for all i, all d_i and all $\vec{n}_i$. Note how degree independence implies two things: (i) the conditional degree density over the degrees of player i's neighbors is independent of the degree of player i for any set of neighbors of i (first equality), and (ii) the degrees of any set of distinct neighbors of player i are mutually independent random variables (second equality). That is, player i's degree informs i of how many neighbors she has but does not allow her to adjust her assessment of the degrees of those neighbors beyond what she already learned from $G^1(\cdot)$. Thus, under degree independence $G^1(\cdot)$ represents all the information (or alternatively, the beliefs) players have about the degrees of their neighbors. A network formation mechanism that is guaranteed to give rise to degree independence is the Poisson random graph (e.g., Newman, 2003), often referred to as the Erdös–Rényi random graph model (Erdös and Rényi, 1959).

Degree independence entails that knowing d_i and θ_i does not inform player i about the degrees and premiums of her neighbors. Hence, from the perspective of each player it is as if all his neighbors independently played a mixed strategy with the same probability of investment, y. Thus, for all players i, regardless of their degree, we can write the expected utility of playing C and D as

$$E\{u_i(C)\} = (a + \theta_i)d_i y + a, \tag{8}$$

and

$$E\{u_i(D)\} = a d_i y + 1, \tag{9}$$

respectively.

The BNE with strictly positive investment probabilities again satisfy the monotonicity property, but now both in premium and degree. Considering degree, if both $\theta_i > 0$ and $y > 0$ then $E\{u_i(C)\}$ increases faster in d_i than does $E\{u_i(D)\}$. Thus, given $y > 0$ there is a cutoff d^* such that player i with fixed $\theta_i > 0$ plays D when $d_i < d^*$ and C otherwise, with

$$d^* = \frac{1-a}{\theta_i y}. \tag{10}$$

Similarly, given player i's degree d_i and a BNE with $y > 0$, there is a threshold premium $\theta^* > 0$, such that player i plays D when $\theta_i < \theta^*$, and C otherwise, with

$$\theta^* = \frac{1-a}{d_i y}. \tag{11}$$

To conclude, under degree independence both higher density and higher premiums favor investing. The BNE is then characterized by

$$\theta^*_k = \frac{1}{k}\frac{(1-a)}{y}, \tag{12}$$

and

$$y = \sum_{k=1}^{N} G^1(k)(1 - P(\theta^*_k)), \tag{13}$$

with equilibrium premiums decreasing hyperbolically in k:

$$\theta^*_1 > \cdots > \theta^*_k = \tfrac{1}{k}\theta^*_1 > \cdots > \theta^*_N = \tfrac{1}{N}\theta^*_1.$$

For illustration consider the following example of network game NG_1 with maximum degree $N = 5$, degree distribution $G^1(1) = 0.1613 = G^1(4)$, $G^1(2) = 0.3226 = G^1(3)$, $G^1(5) = 0.0323$, uniform $P(\cdot)$ on the unit interval, and again $a = 3/4$. Applying Newton's method with starting values equal to threshold premiums of zero to find the equilibrium with the highest y yields $(\theta^*_1, \theta^*_2, \theta^*_3, \theta^*_4, \theta^*_5) = (3/10,\ 3/20,\ 3/30,\ 3/40,\ 3/50)$, with $y \cong 0.83$. More specifically, 70%, 85%, 90%, 92.5%, 94% of the actors invest with degrees 1 to 5, respectively.[5]

Proposition 4 relates the proportion of cooperation y_A obtained with premium distribution P_A and degree distribution G^1_A to (i) y_B obtained with P_B FOSB P_A and G^1_A, and (ii) y_C obtained with P_A and G^1_C FOSD G^1_A.

Proposition 4 ((relation between $P(\cdot)$ and θ^* and d^* under degree independence)). (4i) *If for $\theta \geq (1-a)/N$ both P_A and P_B are continuous and P_B FOSB P_A, $G^1_A = G^1_B = G^1$, and at least one BNE exists for A, then $\theta^*_{Bk} \leq \theta^*_{Ak}$ and $y_A \leq y_B$.*

(4ii) *If G^1_C FOSD G^1_A, $P_A = P_C = P$, and at least one BNE exists for A, then $\theta^*_C \leq \theta^*_A$ and $y_A \leq y_C$.*

Proposition 4i corresponds to the results we derived for the 2-person (Proposition 2) and multi-person games, that making the beliefs about the premiums of other players more positive (in the FOSD sense) decreases the equilibrium premium thresholds (for all degree players) and increases the equilibrium investment probability. Proposition 4ii reflects that if the beliefs about the neighbors are that their degrees increase (in the FOSD sense), then the equilibrium premium thresholds decreases (for all degree players) and the equilibrium investment probability increases.

As an illustration of Proposition (4ii) consider network game NG_2 with degree distribution $G^1(1) = 0.0813$, $G^1(2) = 0.3426 = G^1(3)$, $G^1(4) = 0.1813$, and $G^1(5) = 0.0523$, and the same distribution of premiums as in NG_1. The degree distribution of NG_2 FOSD the degree distribution of NG_1.We then find the equilibrium with the highest investment probability $(\theta^*_1, \theta^*_2, \theta^*_3, \theta^*_4, \theta^*_5) \cong$

[5] A BNE in the network game is asymptotically stable if and only if $\sum_k G^1(k)p(\theta^*_k|k)(1-a)/ky^2 < 1$. The equilibrium in this example is then stable with $\sum_k G^1(k)p(\theta^*_k|k)(1-a)/ky^2 = 0.14 < 1$.

$(7/25, 7/50, 7/75, 7/100, 7/125)$, with $y \cong 0.88$, compared to $(\theta_1^*, \theta_2^*, \theta_3^*, \theta_4^*, \theta_5^*) = (3/10, 3/20, 3/30, 3/40, 3/50)$, with $y \cong 0.83$ for NG_1.

Two implications of Proposition 4 are that (i) the proportion of players investing in the multi-person game is higher than in the network game, and (ii) if a BNE with positive investment probability exists in a network game, then a BNE also exists for another network game in which P or G FOSD that of the original network game. As a specific case, if a BNE exists for the network game then BNE also exist for the corresponding multi-person game. Consider again the network game with six players. The solution in the 6-person game is $\theta^* \cong 0.053$, with $y \cong 0.947$. Not only is the probability of investment y lower in the network game, but even the highest degree players have a lower conditional probability to invest (.94) than those in the completely connected 6-person game (.947).

The analysis to this point has shown that in dyads, groups and networks with degree independence, FOSD shifts of the distribution of cooperation preferences or the degree distribution improves the conditions for cooperation. Thus, the one-shot version of the closure argument we are investigating here is valid for networks satisfying degree independence.

6.4. Degree dependence

Degree dependence means that the neighbors' degree distribution of a player is dependent on his own degree. Contrary to the situation of degree *in*dependence, this implies that a player learns something about the degrees of his neighbors through knowing his own degree, beyond the information contained in the marginal degree density. In a situation of degree dependence we can thus no longer work with the marginal discrete degree density, but have to work directly with joint density $G^1_{\vec{n}_i, d_i}(\cdot, \cdot)$. We will assume this density is 'anonymous': for given d_i and $\vec{n}_i$, any permutation of the elements of $\vec{n}_i$ yields the same joint probability. Thus, the neighbors' 'identities' consist only of their degrees. A player i's pure strategy is still a map from her 'type' (degree and premium) to the actions, C and D. Since the joint density is anonymous, from the perspective of player i all her neighbors have the same investment probability. However, since degrees are now dependent the investment probability depends on the degree of player i herself. Therefore, we let y_k denote the equilibrium probability of investment by an arbitrary neighbor of player i, when $d_i = k$.

Proposition 5 ((relation between θ^* and d^* under degree dependence)). *In networks with degree dependence there is no necessary relationship between the equilibrium premium thresholds and degrees.*

We prove Proposition 5 by way of one example with positive degree dependence (NG_3) and one example with negative degree dependence (NG_4), and thereafter interpret the consequences of Proposition 5.

The marginal degree density of NG_3 with $N = 5$ is uniform (i.e., $G^1(k) = 0.2$ for $k = 1,2,\ldots,5$), with conditional degree densities shown in Table 2. Note that degree dependence is positive in NG_3. More particularly, it holds that a conditional degree density for a certain degree FOSD the conditional density for any lower degree, which results in a positive relation between degree (Table 2, first column) and conditionally expected degree (last column). The network is characterized by players with degree 1 almost exclusively linked to each other, and similarly for players with degree 5, whereas players with degrees 2, 3, and 4 are more evenly linked to each other. Hence NG_3 mainly consists of three types of substructures: (i) 'dyads' of pairs of players linked to each other with very few other links, (ii) clusters of 5 players that are basically fully connected and have very few outside links, and (iii) a more regular substructure with internal degree independence linking players with degrees 2, 3, and 4.

Table 2
Conditional degree probability densities of network game NG_3 with positive degree dependence.

$G^1_{\vec{n}_i \mid d_i}(\cdot \mid d_i)$						
Degree	1	2	3	4	5	Conditionally expected degree
1	0.96	0.01	0.01	0.01	0.01	1.1
2	0.01	0.33	0.34	0.31	0.01	2.93
3	0.01	0.33	0.32	0.33	0.01	3
4	0.01	0.32	0.32	0.34	0.01	3.02
5	0.01	0.01	0.01	0.01	0.96	4.9

Assuming that $P(\cdot)$ is uniform on the unit interval, and as before that degrees and premiums are stochastically independent and $a = 3/4$, it can be shown that a BNE for NG_3 is the vector of thresholds $(\theta_1^*, \theta_2^*, \theta_3^*, \theta_4^*, \theta_5^*) \cong (0.45, 0.14, 0.09, 0.07, 0.98)$ (see Appendix A). Note that the thresholds in this BNE are not monotonous in degree. Investment probabilities rise from .55 for players with degree 1 to .93 for players with degree 4. Players with a degree of 5, however, have an equilibrium investment probability of only about 0.02. By the way, NG_3 also has at least one BNE satisfying the monotonicity property, e.g., the BNE with thresholds $(\theta_1^*, \theta_2^*, \theta_3^*, \theta_4^*, \theta_5^*) \cong (0.61, 0.14, 0.09, 0.07, 0.05)$.

Network game NG_4 also has $N = 5$ and a uniform marginal degree density with conditional degree distribution shown in Table 3. Degree dependence is negative in NG_4. More particularly, it holds that a conditional degree density for a certain degree FOSD the conditional density for any higher degree, which results in a negative relation between degree and conditionally expected degree (Table 3, last column). The network structure of NG_4 is characterized by players with degree 1 who are almost always linked to players with degree 5, and vice versa. Players with degrees 2, 3, and 4 are more evenly linked to each other. Thus, NG_4 mainly consists of two types of substructures: (i) 'stars' in which one player is connected to mostly five others who seldom have others links, and (ii) substructures with internal degree independence, linking players with degrees 2, 3, and 4.

Under the same assumptions as in NG_3, a non-monotonous BNE exists in NG_4 with thresholds $(\theta_1^*, \theta_2^*, \theta_3^*, \theta_4^*, \theta_5^*) \cong (0.96, 0.14, 0.09, 0.07, 0.76)$, with relatively low investment probabilities for players with degree 1 or 5. That this is indeed a BNE can be verified in the same way as is done for NG_3 in Appendix A. In addition to this non-monotonous BNE, there is also at least one monotonous BNE with thresholds $(\theta_1^*, \theta_2^*, \theta_3^*, \theta_4^*, \theta_5^*) \cong (0.27, 0.14, 0.09, 0.07, 0.068)$.

Proposition 5 and the examples prove that the one-shot version of the closure argument that is guaranteed to be valid under degree independence, is not necessarily operative under degree

Table 3
Conditional degree probability densities of network game NG_4 with positive degree dependence.

$G^1_{\vec{n}_i \mid d_i}(\cdot \mid d_i)$						
Degree	1	2	3	4	5	Conditionally expected degree
1	0.01	0.01	0.01	0.01	0.96	4.9
2	0.01	0.32	0.33	0.33	0.01	3.01
3	0.01	0.32	0.33	0.33	0.01	3.01
4	0.01	0.34	0.32	0.32	0.01	2.98
5	0.96	0.01	0.01	0.01	0.01	1.1

dependence. That is, increasing the degree distribution (in the FOSD sense) does not necessarily improve the conditions for cooperation.

7. Discussion

Cooperation in social dilemmas such as the binary PG or PD depends to an important extent on the social structure in which such dilemmas are embedded. The well-known closure argument explicitly links the structure of social relationships to cooperation, contending that in tightly knit communities cooperation rates are higher than in more loosely knit ones. One of the mechanisms producing this purported effect is that in dense networks players simply have many positively valued social relations. That humans indeed value cooperative social relations over and above the associated material payoff consequences is the central claim of the social exchange heuristic. In the current paper we therefore proposed a micro model of the individual player based on the social exchange heuristic. Elaborating on previous formulations, we assume that players put an individually specific premium on mutual cooperation, which is private information. We then employ this micro model in the analyses of three common social structures that impact upon cooperation rates: (i) the dyad, (ii) the fully connected multi-person group, and (iii) the social network. With respect to social networks we distinguish between those with and without degree independence.

Symmetric Bayes–Nash equilibrium analyses of the one-shot game in the three social structures found that: (i) although cooperation is possible in equilibrium there remains an 'inefficiency problem' in the sense that players who all prefer mutual cooperation over defecting on a cooperating partner need not all to choose to cooperate in equilibrium (Propositions 1 and 3); (ii) increasing the number of group members (in the fully connected group), and applying a FOSD shift to the distribution of cooperation premiums or the degree distribution (in the dyad and in the network with degree independence, respectively), (weakly) decreases the equilibrium premium threshold (Propositions 2 and 4) and (weakly) increases rates of cooperation, and (iii) the latter result does not hold for networks with degree dependence (Proposition 5).

Our theoretical results imply that, under the assumptions of our model, the particular version of the closure mechanism we scrutinized is guaranteed to work under degree *in*dependence. One could increase conditions for cooperation in the binary PG by increasing the density of the network or by applying a FOSD shift to the distribution of premiums. Under degree dependence, however, dense cliques can spin off into inefficient equilibria with hardly any investment in the PG, whereas more loosely and evenly connected parts of the network sustain higher levels of investment. In general there need not be the kind of monotonicity proposed by the closure argument under degree dependence: denser communities do not necessarily reach higher rates of cooperation.

Our model has scope limits following from its formal description. These scope limits indicate directions in which the model could be expanded. First of all, the basic PG investment decisions in the current model are one-shot. Extending this to (finitely or infinitely) repeated play implies that individuals learn about the structure of the network and the premiums of their neighbors in the course of the game. This allows investigation of issues such as how players' beliefs are updated, how and whether reputations are formed, etc. Second, in a repeated setting we could also make the network dynamic, letting individuals choose their network connections based on what they have learned from previous play. Natural questions are then which network structures are stable, and to what extent these networks are efficient and satisfy equity (e.g., Jackson and Wolinsky, 1996; Doğan et al., 2009). A third main direction in which the current model could be extended concerns the information individuals have about the network. In the current model individuals knew only their own degree and the neighbors' degree distribution. In future research one could investigate different information structures in which individuals for instance have more certainty about the degrees of their own neighbors, than about the degrees of individuals further removed in the network.

The immediate result of any one of the previously discussed extensions is that the model becomes more complex than the one-shot model with local information analyzed in this paper. Repeating the game, making the network dynamic and endowing players with finer-grained degree information all have the effect of greatly enlarging the game's strategy space. Characterizing the equilibria of such a model is by no means straightforward, and would likely require computational as opposed to analytic methods.

A different extension of the current analysis would be the application of the social exchange heuristic model to games other than the PD/PG. Many real-world situations have a 'coordination game' structure in which a player's best response in terms of material payoffs depends on the strategies of others, even in a one-shot game. Our analysis could be extended to coordination games such as the assurance game or the step-level public good game (Van de Kragt et al., 1983). A model similar to the social exchange heuristic employed in the current paper would give each player a premium for 'successful coordination' and make that premium private information. The relation between beliefs about this (private information) premium and players' investment behavior seems intuitively not straightforward, as very positive beliefs about the size of the premiums of other players could easily induce a given player to defect. Investigating such a model in a network context appears meaningful.

Finally, we could interpret the model results as hypotheses and design experiments to test them. Gächter and Thöni (2011) argue forcefully for the use of laboratory experiments to test questions of the kind we have addressed theoretically in the current paper. Most relevant to the one-shot version of the closure argument would be an investigation of the effects of increasing network density (through a sequence of degree distributions ordered by FOSD) on investment behavior under both degree dependence and degree independence. Under degree independence the model predicts a positive relation between density and investment rates, whereas under degree dependence no such positive relation is explicitly predicted. Empirically we may thus expect more deviations from a positive relation between density and investment rates under degree dependence than degree independence. The web-based methodology employed by for instance Suri and Watts (2011) seems very promising for conducting experiments to test such hypotheses.

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Appendix A. Proofs of propositions

Proposition 1 ((inefficiency problem in the 2-person PG)). *Whenever $P(1-a)>0$, $\theta^*>1-a$.*

Proof. Consider a player j with $\theta_j=1-a$. For player j the expected utility of playing C is smaller than or equal to the expected utility of playing D, with equality only when $y=1$. But $y=1$ implies that player i with $\theta_i<1-a$ should play C, which contradicts $P(1-a)>0$. Q.E.D.

Proposition 2 ((relation between $P(\cdot)$ and θ^*)). *If for two continuous distributions of premiums A and B, A FOSD B for $\theta\geq 1-a$, and at least one BNE exists for B, then $\theta^*_A\leq\theta^*_B$ and $y_B\leq y_A$.*

Proof. $y_A=1-P_A(\theta^*_A)\geq 1-P_B(\theta^*_A)$ since $P_A\leq P_B$ for $1-a\leq\theta^*_A$. Eqs. (2) and (3) show that θ^* is decreasing in y, and because y is again increasing when θ^* is decreasing, we obtain: $\theta^*_B = (1-a)/(1-P_B(\theta^*_B)) \geq (1-a)/(1-P_A(\theta^*_B))$, and there must exist a θ^*_A such that $(1-a)/(1-P_A(\theta^*_B)) \geq (1-a)/(1-P_A(\theta^*_A)) = \theta^*_A$. Hence $\theta^*_A \leq \theta^*_B$ and $y_B = 1-P_B(\theta^*_B) \leq 1-P_A(\theta^*_A) = y_A$. Q.E.D.

Proposition 3 ((inefficiency problem in the multi-person PG)). Suppose $P\left((1-a)/N\right) > 0$. Then $\theta^* > (1-a)/N$.

Proof. Consider player j with $\theta_j = (1-a)/N$. Then, $E\{u_j(C)|\theta_j = (1-a)/N\} = aNy + (1-a)y + a \leq aNy + 1 = E\{u_j(D)|\theta_j = (1-a)/N\}$, with equality only if $y=1$. But $y=1$ implies that player i with $\theta_i < (1-a)/N$ should play C, which is a contradiction. Q.E.D.

Proposition 4 ((relation between $P(\cdot)$ and θ^* and d^* under degree independence)). (4i) If for $\theta \geq (1-a)/N$ both P_A and P_B are continuous and P_B FOSB P_A, $G^1_A = G^1_B = G^1$, and at least one BNE exists for A, then $\theta^*_{Bk} \leq \theta^*_{Ak}$ and $y_A \leq y_B$.

(4ii) If G^1_C FOSD G^1_A, $P_A = P_C = P$, and at least one BNE exists for A, then $\theta^*_C \leq \theta^*_A$ and $y_A \leq y_C$.

Proof. (4i) The proof is similar to the proof of Proposition 2.

$$y_B = \sum_{k=1}^{N} G^1(k)(1-P_B(\theta^*_{Bk})) \geq \sum_{k=1}^{N} G^1(k)(1-P_A(\theta^*_{Bk})) \text{ since } P_A \leq P_B$$

for $\theta \geq \frac{1-a}{N}$. Eq. (12) shows that θ^*_k is decreasing in y, and because y is again increasing when θ^*_k is decreasing, we obtain from (12): $\theta^*_{Ak} = \frac{1-a}{k\sum_{k=1}^{N} G^1(k)(1-P_A(\theta^*_{Ak}))} \geq \frac{1-a}{k\sum_{k=1}^{N} G^1(k)(1-P_B(\theta^*_{Ak}))}$, and there must exist θ^*_{Bk} such that $\frac{1-a}{k\sum_{k=1}^{N} G^1(k)(1-P_B(\theta^*_{Ak}))} \geq \frac{1-a}{k\sum_{k=1}^{N} G^1(k)(1-P_B(\theta^*_{Bk}))} = \theta^*_{Bk}$. Hence $\theta^*_{Bk} \leq \theta^*_{Ak}$ for all k and $y_A \leq y_B$. Q.E.D.

Proof. (4ii) Since θ^*_k decreases in degree k, $1-P(\theta^*_k)$ increases in k. Hence G^1_C FOSD G^1_A implies $y_C = \sum_{k=1}^{N} G^1_C(1-P(\theta^*_{Ak})) \geq \sum_{k=1}^{N} G^1_A(1-P(\theta^*_{Ck}))$. From here, the proof follows the proof of (4i), resulting in $\theta^*_{Ck} \leq \theta^*_{Ak}$ for all k and $y_C \leq y_B$. Q.E.D.

Proposition 5 ((relation between θ^* and d^* under degree dependence)). *In networks with degree dependence there is no necessary relationship between the equilibrium premium thresholds and degrees.*

Proof. That the vector of thresholds $(\theta^*_1, \theta^*_2, \theta^*_3, \theta^*_4, \theta^*_5) \cong (0.45, 0.14, 0.09, 0.07, 0.98)$ is indeed a BNE in NG_3 can be verified by computing the expected payoffs of playing C and playing D for each possible degree, using the conditional degree distributions in the rows of Table 2 in the main text. For example, player i with $d_i=3$ faces $y_3 = 0.01 \times 0.55 + 0.33 \times 0.86 + 0.32 \times 0.91 + 0.33 \times 0.93 + 0.01 \times 0.02 = 0.8876$. This yields expected payoffs of cooperation and defection of $E\{u_i(C)\} = \left(3/4 + \theta_i\right) \times 3 \times 0.8876 + (3/4) = 2.7471 + 2.6628\theta_i$, and $E\{u_i(D)\} = (3/4) \times 0.8876 \times 3 + 1 = 2.9971$, respectively. Solving for θ_i yields $\theta^*_3 = 0.09$ (rounded to two digits).

Appendix B. Marginal premium of mutual cooperation decreasing in the number of cooperating neighbors.

Suppose all players have the same parameter $\delta \in [0, 1)$ discounting each additional instance of mutual cooperation. In the main text we presented the results for the special (linear) case $\delta = 1$.

B.1. Multi-person game

First consider the multi-person game. Suppose again that the equilibrium investment probability is y, and suppose k other players invest. Then the payoff of cooperation for player i is $u_i(C, k) = a(k+1) + \theta_i \sum_{j=1}^{k} \delta^{j-1}$, and the payoff of defection is $u_i(D, k) = ak + 1$. Taking the expectation of both payoffs yields

$$E\{u_i(C)\} = \sum_{k=0}^{N} \binom{N}{k} y^k (1-y)^{N-k} u_i(C, k) = a(Ny+1) + \frac{\theta_i}{1-\delta}(1-[1-y(1-\delta)]^N) \tag{B1}$$

and

$$E\{u_i(D)\} = aNy + 1 \tag{B2}$$

respectively.

(B1) can be proved as follows:

$$\begin{aligned}
E\{u_i(C)\} &= \sum_{k=0}^{N} \binom{N}{k} y^k (1-y)^{N-k} u_i(C, k) \\
&= \sum_{k=0}^{N} \binom{N}{k} y^k (1-y)^{N-k} [a(k+1) + \theta_i \sum_{j=1}^{k} \delta^{j-1}] \\
&= \sum_{k=0}^{N} \binom{N}{k} y^k (1-y)^{N-k} [a(k+1) + \theta_i \frac{1-\delta^k}{1-\delta}] \\
&= aNy + a + \frac{\theta_i}{1-\delta}[1 - \sum_{k=0}^{N} \binom{N}{k} (\delta y)^k (1-y)^{N-k}] \\
&= a(Ny+1) + \frac{\theta_i}{1-\delta}[1 - (1-y)^N \sum_{k=0}^{N} \binom{N}{k} (\frac{\delta y}{1-y})^k] \\
&= a(Ny+1) + \frac{\theta_i}{1-\delta}[1 - (1-y)^N (1 + \frac{\delta y}{1-y})^N] \\
&= a(Ny+1) + \frac{\theta_i}{1-\delta}(1 - [1-y(1-\delta)]^N)
\end{aligned}$$

Q.E.D.

Eqs. (B1) and (B2) show that BNE again have the monotonicity property. Since $E\{u_i(C)\}$ strictly increases in θ_i, it holds that for given y players play D when $\theta_i < \theta^*$, and C otherwise. We can characterize the BNE as,

$$\theta^* = \frac{1-a}{1-[1-y(1-\delta)]^N}(1-\delta), \tag{B3}$$

and

$$y = 1 - P(\theta^*). \tag{B4}$$

If $\delta = 0$, we have the extreme special case in which players receive the premium for only 1 cooperating neighbor. The inefficiency problem (cf. Proposition 3) also exists in the general case with discounting, stated in Proposition 1.

Proposition 1 ((inefficiency problem in the multi-person PG with discount parameter)). Suppose $P\left((1-a)\frac{1-\delta}{1-\delta^N}\right) > 0$. Then $\theta^* > (1-a)(1-\delta)/(1-\delta^N)$.

Proof. Consider player j with $\theta_j = (1-a)(1-\delta)/(1-\delta^N)$. Substituting this value in (B1) and comparing to (A1) yields $E\{u_j(C)\} \leq E\{u_j(D)\}$, with equality only if $y=1$. But $y=1$ implies that player i with $\theta_i < (1-a)(1-\delta)/(1-\delta^N)$ should play C, which is a contradiction. Q.E.D.

B.2. Network PG

In networks with degree independence the expected payoffs of cooperation and defection are,

$$E\{u_i(C)\} = a(d_i y + 1) + \frac{\theta_i}{1-\delta}(1-[1-y(1-\delta)]^{d_i}), \tag{B5}$$

and

$$E\{u_i(D)\} = ad_i y + 1, \quad \text{(B6)}$$

respectively.

The BNE with strictly positive investment probabilities again satisfy monotonicity in premium and degree. Since $1 - [1 - y(1-\delta)]^{d_i}/(1-\delta) > 0$, monotonicity in θ_i is straightforward. Fixing both θ_i and $y > 0$, we can rewrite $E\{u_i(C)\} \geq E\{u_i(D)\}$ as $[1 - y(1-\delta)]^{d_i} \leq \theta_i - (1-a)(1-\delta)/\theta_i$. Thus, if $\theta_i > (1-a)(1-\delta)$ there is a cutoff d^* such that player i with fixed $\theta_i > (1-a)(1-\delta)$ plays D when $d_i < d^*$ and C otherwise, with d^* defined as

$$[1 - y(1-\delta)]^{d^*} = \frac{\theta_i - (1-a)(1-\delta)}{\theta_i}. \quad \text{(B7)}$$

To conclude, under degree independence both higher density and higher premiums favor investing. The BNE is then characterized by

$$\theta_k^* = \frac{1-a}{1 - [1 - y(1-\delta)]^k}(1-\delta), \quad \text{(B8)}$$

and

$$y = \sum_{k=1}^{N} G^1(k)(1 - P(\theta_k^*)), \quad \text{(B9)}$$

for any degree k.

It follows from (B8) that $\theta_k^* > (1-a)(1-\delta)$, reflecting monotonicity in degree. If $\delta = 0$, we again have the case in which players receive the premium for only 1 cooperating neighbor. By (B8) θ_k^* is decreasing in y, and by (B9) y is again increasing when θ_k^* is decreasing. Thus, we can prove the equivalent of Proposition (4i). Finally note that by (B8) θ_k^* decreases in degree k, and thus $1 - P(\theta_k^*)$ increases in k. Thus, we can also prove the equivalent of Proposition (4ii). That is, cooperation rates (weakly) increase whenever either the distribution of premiums or of degree shifts to the right.

To conclude we have shown that for a decreasing marginal premium of cooperation Propositions 3 and 4 from the paper also hold. Under degree dependence this model has the same characteristic as the linear model, namely that the equilibrium investment probability of the neighbors of player i generally depend on player i's degree.

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