Essays on dynamic macroeconomics

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Publication date:
2007

Chapter 4

Ageing, Pensions, and Retirement

4.1 Introduction

Population ageing is playing havoc with the public pension schemes of many western countries. In a celebrated sequence of international comparative studies, Gruber and Wise (1999, 2004, 2005) and their collaborators have established a number of stylized facts pertaining to a subset of OECD countries. These facts are:

(SF1) For most developed countries, the pay-as-you-go social security system includes promises that cannot be kept without significant system reforms. In the absence of reform, the current systems are *fiscally unsustainable*.

(SF2) Over the last four decades, the trend is for older people to leave the labour force at ever younger ages. For most countries there is a clear downward trend in the labour force participation of pension-age males. Retirement is a *normal good* in the sense that the demand for years of retirement rises as agents' income rises (Barr and Diamond, 2006, p. 27).

(SF3) Only a very small fraction of the labour force retires before the earliest age at which public retirement benefits are available, the so-called *early eligibility age* (EEA hereafter). For males, the EEA in 2003 typically falls in the range of

This chapter is based on joint work with Ben Heijdra, 'Retirement, Pensions, and Ageing', mimeo

60-62 years of age. Similarly, only very few people work until the *normal retirement age* (NRA hereafter), which is typically 65 for most countries (Duval, 2003, p. 35) Together this implies that most people retire either at the EEA or somewhere in between the EEA and the NRA.

(SF4) Most social security programs contain strong incentives for older workers to leave the labour force. In most countries it simply does not pay to work beyond the EEA because *actuarial adjustments* are less than fair. As Gruber and Wise put it, 'once benefits are available, a person who continues to work for an additional year will typically receive less in social security benefits over his lifetime than if he quit work and started to receive benefits at the first opportunity' (2005, p. 5). The present value of expected social security benefits declines with the retirement age, and there is a high *implicit tax* on working beyond the EEA.

(SF5) In many European countries disability programs and age-related unemployment provisions essentially provide early retirement benefits, even before the EEA.

A formal analysis of issues surrounding ageing, retirement, and pensions should accommodate at least some, but preferably all, of these stylized facts. In this chapter we study the consumption, saving, and retirement decisions of individual agents facing lifetime uncertainty, or longevity risk. In addition, we also determine the macroeconomic consequences of individual behaviour and policy changes. We use the extended-Blanchard-Yaari framework developed in Chapter 2. We maintain the assumption that the country is small in world capital markets and thus faces an exogenous world interest rate which we take to be constant. To analyse the effects of ageing societies, we use the generalised demographic framework of Chapter 3. By allowing the mortality rate to depend on age and time of birth, the model can be used to investigate the micro- and macroeconomic effects of a reduction in *adult mortality*. We still assume that finitely lived agents fully insure against the adverse affects of lifetime uncertainty by purchasing actuarially fair annuities.

The second building block of our analysis concerns the labour market participation decision of individual agents. Following the seminal contribution by Sheshinski (1978) and much of the subsequent literature, we assume that labour is indivisible (the agent either works full time or not at all), that the retirement decision is irreversible, and that the felicity function is additively separable in consumption and leisure. All agents are blessed with perfect foresight and maximize an intertemporal utility function subject to a lifetime budget constraint. Workers choose the optimal retirement age, taking as given the time- and age profiles of wages, the fiscal system, and the public pension system. Not surprisingly, like Mitchell and Fields and many others we find that 'the optimal retirement age ... equates the marginal utility of income from an additional year of work with the marginal utility of one more year of leisure' (1984, p. 87).

The two papers most closely related to the analysis in this chapter are Sheshinski (1978) and Boucekkine et al. (2002).[1] We extend the analysis of Sheshinski (1978) in two directions. First, as was already mentioned above, we incorporate a realistically modelled lifetime uncertainty process, rather than a fixed planning horizon. Second, we embed Sheshinski's *microeconomic* model in the context of a small open economy and are thus able to study the *macroeconomic* repercussions of ageing and pension reform. We generalize the analysis of Boucekkine et al. (2002) by including a concave, rather than linear, felicity function, and by modelling a public pension system with realistic features such as an EEA which differs from the NRA and non-zero implicit tax rates. Furthermore, we conduct our analysis with a general description of the demographic process, whereas they use a specific functional form for this process throughout their paper.

The remainder of this chapter is organized as follows. In Section 4.2 we present the model and demonstrate its main properties. Consumption is proportional to total wealth, consisting of financial and human wealth. With a realistic demography, the marginal propensity to consume out of wealth is increasing in the agent's age because the planning horizon shortens as one grows older and the agent does not wish to leave any bequests. We derive the first-order condition for the optimal retirement age and show that it depends not only on the mortality process but also on the features of the fiscal and pension systems. The mortality process, in combination with the birth rate, also determines a unique path for the population growth rate.

In Section 4.3 we abstract from the public pension system and study the comparative static effects on the optimal retirement age of various age related shocks. A reduction in the disutility of working leads to an increase in the optimal retirement age. In contrast, an upward shift in the age profile of wages causes a negative wealth effect but a positive substitution effect, rendering the total effect on the optimal retirement age ambiguous. A reduction in adult mortality increases the

[1] In the interest of brevity, we refer the interested reader to the literature surveys on retirement and ageing by Lazear (1999); Hurd (1990, 1997) and D. N. Weil (1997). For a recent literature survey on pension reform, see Lindbeck and Persson (2003).

expected remaining lifetime for everyone, though more so for older agents. The effect of increased longevity on the optimal retirement age is ambiguous in general because the lifetime-income effect cannot be signed a priori. For realistic scenarios, however, the increased longevity only starts to matter quantitatively at ages exceeding the NRA so that the lifetime-income effect works in the direction of increasing the optimal retirement age.

Section 4.3 also presents the graphical apparatus that we use throughout the chapter. We demonstrate that the optimal retirement decision is best studied in terms of its consequences for lifetime income and the *transformed retirement age*. The transformed retirement age is a monotonically increasing transformation of the calender age of retirement and captures the notion of an agent's economic (rather than biological) age. Our graphical apparatus has the attractive feature that indifference curves are convex and that, with an age invariant wage rate, actuarially fair adjustment leads to a linear budget constraint. We believe that our graphical representation is more intuitive than the conventional one based on biological years.

In Section 4.4 we re-introduce the public pension system (including disability programs and age-related unemployment provisions, see SF5) and determine its likely consequences for the retirement decision of individual agents. Using data from Gruber and Wise (1999) for nine OECD countries, we compute conservative estimates for standardized lifetime income profiles and find that these profiles are concave in the transformed age domain. For at least six of these countries, the lifetime income profile features a kink at the EEA as a result of non-trivial implicit tax rates. Combined with convex indifference curves, it is not surprising that many agents choose to retire at the EEA, conform stylized facts (SF3) and (SF4).

In Section 4.5 we take the concavity of lifetime income profiles for granted and discuss the comparative static effects on the optimal steady-state retirement age of various changes in the tax system or the public pension system. We restrict attention to interior solutions because an optimum occurring at the kink in the lifetime income profile is insensitive to infinitesimal changes. An increase in the lumpsum[2] tax leads to a reduction in lifetime income and an increase in the optimal retirement age. Retirement is thus a normal good in our model, conform stylized fact (SF2). Not surprisingly, an increase in the labour income tax has an ambiguous effect on the retirement age because the substitution effect is negative and the wealth effect is positive. Holding constant the slope of the pension benefit curve, an increase in its level unambiguously leads to a decrease in the retirement age—the wealth effect

[2] In the public economics literature, a lumpsum tax is usually named 'poll' tax. Since we used the term 'lumpsum' in the previous chapters, we will continue using it here.

and the substitution effect operate in the same direction. In contrast, an increase in the slope of the benefit curve, holding constant its level, leads to an increase in the optimal retirement age as a result of the positive substitution effect.

In Section 4.6 we compute and visualize the general equilibrium effects of various large demographic shocks and several assumed policy reform measures. Conform stylized fact (SF1), we postulate that in the initial steady state individuals are stuck at the early retirement kink. Because both the shocks and the policy reform measures are infra marginal, we simulate a plausibly calibrated version of our model to compute the impact-, transitional-, and long-run effects on the macroeconomy.

Finally, in Section 4.7 we present some concluding thoughts and give some suggestions for future research. The chapter also contains a brief Appendix containing some additional material on the retirement age transformation as well as data on replacement rates and implicit taxes for nine OECD countries.

4.2 The model

4.2.1 Households

An individual values both consumption and leisure. The (remaining) lifetime utility function at time t for an agent born at time $v(v \leq t)$ is written as:

$$\Lambda(v,t) \equiv e^{M(u)} \int_t^{\infty} \left[U(\bar{c}(v,\tau)) - \mathbf{I}(\tau - v, R(v))D(\tau - v)\right] e^{-\theta\cdot(\tau-t)-M(\tau-v)} d\tau, \quad (4.1)$$

where $u \equiv t - v$ is the agent's age in the planning period and $\mathbf{I}\left(\tau - v, R(v)\right)$ is an indicator function capturing the agent's labour market status:

$$\mathbf{I}(\tau - v, R(v)) = \begin{cases} 1 & \text{for } 0 < \tau - v < R(v) \\ 0 & \text{for } \tau - v \geq R(v) \end{cases} . \quad (4.2)$$

In Equation (4.1), $U(\cdot)$ is a concave consumption-felicity function (to be discussed below), $\bar{c}(v,\tau)$ is goods consumption, $D(\cdot)$ is the age-dependent disutility of working, $R(v)$ is the retirement age (see below), θ is the constant pure rate of time preference ($\theta > 0$), and $e^{-M(\tau-v)}$ is the probability that the agent is still alive at time τ. The cumulative mortality rate is defined as $M(\tau - v) \equiv \int_0^{\tau-v} m\,(s)\, ds$, where $m(s)$ is the instantaneous mortality rate of a household of age s (see the discussion in Section 2.3 for details). Two features of the lifetime utility function are worth noting.

First, following the standard convention in the literature, the instantaneous utility function is assumed to be additively separable in goods consumption and labour supply.[3] Previous to retirement the agent works full time, and inelastically supplies its unitary time endowment to the labour market. After retirement the agent does not work at all. Hence, we model the labour market participation decision (rather than an hours-of-work decision). Leaving the labour force is assumed to constitute an irreversible decision.[4] As a result, the age at which the agent chooses to withdraw from the labour market, which we denote by $R(v)$, can be interpreted as the *voluntary retirement age*. Second, the disutility of working is non-decreasing in age, i.e. $D'(\tau - v) > 0$. This captures the notion that working becomes more burdensome as one grows older (cf. Boucekkine et al. (2002, p. 346)).

The budget identity is given by:

$$\begin{aligned}\dot{\bar{a}}(v,\tau) = [r + m(\tau - v)]\bar{a}(v,\tau) + \mathbf{I}(\tau - v, R(v))\bar{w}(\tau - v)[1 - t_L(\tau)] \\ + [1 - \mathbf{I}(\tau - v, R(v))]\bar{p}(v,\tau,R(v)) - \bar{c}(v,\tau) - \bar{z}(\tau), \quad (4.3)\end{aligned}$$

where $\bar{a}(v,\tau)$ is real financial wealth, r is the exogenously given (constant) world rate of interest, $\bar{w}(\tau - v)$ is the age-dependent before-tax wage rate, t_L is the labour income tax, $\bar{p}(\cdot)$ is the public pension benefit, and $\bar{z}$ is the lumpsum tax (see below). As usual, a dot above a variable denotes that variable's time rate of change, e.g. $\dot{\bar{a}}(v,\tau) \equiv d\bar{a}(v,\tau)/d\tau$. Like in the previous chapter, we follow Yaari (1965) and Blanchard (1985), and postulate the existence of a perfectly competitive life insurance sector which offers actuarially fair annuity contracts. As a result, the annuity rate of interest facing an agent of age $\tau - v$ is given by $r + m(\tau - v)$.

The public pension system is modelled as follows. The government cannot force people to work, i.e. the voluntary retirement age, $R(v)$, is chosen freely by each individual agent. However, there exists an *early eligibility age* (EEA hereafter), which we denote by R_E. The EEA represents the earliest age at which social retirement benefits can be claimed. An agent who chooses to retire before reaching the EEA ($R(v) < R_E$) will only get a public pension benefit from age R_E onward, i.e. this agent will have no non-asset income during the age interval $[R(v), R_E]$. The pension

[3] See, for example, Sheshinski (1978); Burbidge and Robb (1980); Mitchell and Fields (1984); Kingston (2000); Boucekkine et al. (2002); Kalemni-Ozcan and Weil (2002), and d'Albis and Augeraud-Véron (2005)

[4] Apart from lifetime uncertainty there are no other stochastic shocks in our model and agents are blessed with perfect foresight. The empirical literature models retirement under uncertainty using the option-value approach. See, for example, Stock and Wise (1990b, 1990a); Lumsdaine et al. (1992) and the recent survey by Lumsdaine and Mitchell (1999).

benefits someone ultimately receives depends solely on that person's retirement age:[5]

$$\bar{p}(v,\tau,R(v)) = \begin{cases} 0 & \text{if } \tau - v < R_E \\ B(R(v)) & \text{if } \tau - v \geq R_E \end{cases} \tag{4.4}$$

where $B(R(v))$ is non-decreasing in the retirement age, i.e. $B'(R(v)) \geq 0$. Note that $B(R(v))$ might be discontinuous at some retirement ages, but if it exists such a jump is positive by assumption.

Lifetime income (or human wealth) is defined as the present value of after-tax non-asset income using the annuity rate of interest for discounting. For a working individual, whose age in the planning period falls short of the desired retirement age ($t - v < R(v)$), lifetime income is given by:

$$\bar{li}(v,t,R(v)) \equiv e^{ru+M(u)} \left[\int_u^{R(v)} \bar{w}(s) e^{-rs-M(s)} ds - \int_u^{\infty} \bar{z}(v+s) e^{-rs-M(s)} ds \right] + SSW(v,t,R(v)), \tag{4.5}$$

where $SSW(v,t,R(v))$ represents the value of *social security wealth*:

$$SSW(v,t,R(v)) = e^{ru+M(u)} \left[B(R(v)) \int_{\max\{R_E,R(v)\}}^{\infty} e^{-rs-M(s)} ds - \int_u^{R(v)} t_L(v+s) \bar{w}(s) e^{-rs-M(s)} ds \right]. \tag{4.6}$$

Intuitively, social security wealth represents the present value of retirement benefits minus contributions, again using the annuity rate of interest for discounting. Lifetime income is an important wealth component for each agent. Indeed, by integrating the budget identity (4.3) for $\tau \in [t,\infty)$ and imposing the No-Ponzi-Game (NPG) condition, we obtain the lifetime budget constraint:

$$e^{ru+M(u)} \int_t^{\infty} \bar{c}(v,t) e^{-r\cdot(\tau-v)-M(\tau-v)} d\tau = \bar{a}(v,t) + \bar{li}(v,t,R(v)). \tag{4.7}$$

The present value of current and future consumption is equated to total wealth,

[5] We thus assume a *pure defined benefit system*, i.e. previous payments into to the pension system do not influence the benefit. Sheshinski (1978, p. 353) assumes that pension benefits also depend on characteristics of the worker's wage profile before retirement, e.g. the arithmetic average wage, $\bar{w}_R \equiv (1/R) \int_0^R \bar{w}(s) ds$, or the maximum earned wage, $\bar{w}_R \equiv \max\{\bar{w}(s)\}$ for $0 \leq s \leq R$. We have abstracted from this dependency to keep the analysis a simple as possible.

which equals the sum of financial wealth and human wealth.

The agent of vintage v chooses a time path for consumption $\bar{c}(v,\tau)$ (for $\tau \in [t,\infty)$) and a retirement age $R(v)$ in order to maximize lifetime utility (4.1) subject to the lifetime budget constraint (4.7). Unfortunately, the optimisation procedure used in Chapter 3 does not work here. In the previous chapter, an individual had to choose an optimal schooling period and picked that schooling period that maximised lifetime income. Here the lifetime income maximising retirement age does not maximise lifetime utility, since leisure increases utility. However, due to the separability of preferences, the optimization problem can be solved in two steps. In the first step, we solve for optimal consumption conditional on total wealth. As before we postulate an iso-elastic consumption-felicity function (see Equation (2.6) on page 15):

$$U(\bar{c}(v,\tau)) \equiv \begin{cases} \dfrac{\bar{c}(v,\tau)^{1-1/\sigma}-1}{1-1/\sigma} & \text{for } \sigma \neq 1 \\ \ln \bar{c}(v,\tau) & \text{for } \sigma = 1 \end{cases} \tag{4.8}$$

where σ is the intertemporal substitution elasticity ($\sigma > 0$). The level and time profile for consumption are given by:

$$\bar{c}(v,t) = \frac{\bar{a}(v,t) + \bar{l}i(v,t,R(v))}{\Delta(u,r^*)}, \tag{4.9}$$

$$\bar{c}(v,\tau) = \bar{c}(v,t)e^{\sigma\cdot(r-\theta)(\tau-t)}, \quad \text{for } \tau \geq t, \tag{4.10}$$

where $r^* \equiv r - \sigma\cdot(r-\theta)$.[6] The Δ-function is defined in (2.12) and the important properties of this function are stated in Proposition 2.1.

Equation (4.9) shows that consumption in the planning period is proportional to total wealth, with $1/\Delta(u,r^*)$ representing the marginal propensity to consume. It follows from Proposition 2.1(v) that the consumption propensity is an increasing function of the individual's age in the planning period. Old agents face a relatively short expected remaining lifetime, due to increasing mortality rates, and thus consume a larger fraction of their wealth in each period. Equation (4.10) states the time path for consumption. As in the previous chapters, we assume throughout this chapter that $r > \theta$, i.e. we study a small nation populated by patient agents. It follows from (4.10) that the desired consumption profile is exponentially increasing over time.

[6] The derivation of Equations (4.9)–(4.10) is explained in detail in Chapter 2

In the second step of the maximization problem the optimal retirement age is chosen. This in turn determines optimal lifetime income. The retirement decision is only relevant for a working individual, because labour market exit is an absorbing state. By substituting (4.9)–4.10 into (4.1) we obtain the expression for lifetime utility of a working individual:

$$\bar{\Lambda}(v,t) \equiv e^{\theta u + M(u)} \int_u^{\infty} \left[U\left(\frac{\bar{a}(v,t) + \bar{l}i(v,t,R(v))}{\Delta(u,r^*)} e^{\sigma\cdot(r-\theta)(s-u)} \right) e^{-\theta s + M(s)} ds \right.$$
$$\left. - \int_u^{R(v)} D(s) e^{-\theta s - M(s)} ds \right], \qquad \text{for } u < R(v). \quad (4.11)$$

Borrowing terminology from econometrics, we refer to $\bar{\Lambda}(v,t)$ as the *concentrated* utility function, i.e. it is a transformation of the original lifetime utility function with the maximized solution for the consumption path incorporated in it. As a result, the concentrated utility function only depends on total wealth (including lifetime income) and on the retirement age. Every working individual maximizes (4.11) by choosing $\bar{l}i(v,t,R(v))$ and $R(v)$ subject to the definition of lifetime income (4.5), taking as given the stock of financial assets in the planning period.[7] This is a simple two-dimensional optimization problem with a single constraint. The optimal retirement age, $R^*(v)$, is the implicit solution to the following first-order condition:[8]

$$D(R(v)) e^{-\theta R(v) - M(R(v))} = \left[\frac{\bar{a}(v,t) + \bar{l}i(v,t,R(v))}{\Delta(u,r^*)} \right]^{-1/\sigma} \frac{d\bar{l}i(v,t,R(v))}{dR(v)}. \quad (4.12)$$

The comparative static effects of the optimal retirement age with respect to ageing and pension shocks are studied in detail in Sections 4.3 and 4.5 below. One important property of the solution is immediately apparent from (4.12): no rational agent will choose a retirement age at which lifetime income is downward sloping. Because the disutility of working is strictly positive, the optimal solution must be situated on the upward sloping part of the $\bar{l}i(v,t,R(v))$ function. A direct corollary to this argument is as follows. If there exists a lifetime-income maximizing retirement age, say R_I, then this age is an upper bound for the utility-maximizing retirement age, i.e. it is never optimal to retire after age R_I.[9]

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[7] After retirement, $R(v)$ is fixed and lifetime income is no longer a choice variable. Each individual simply chooses consumption such that the lifetime budget constraint is just satisfied.

[8] Similar expressions can be found in Sheshinski (1978, p. 354) and Burbidge and Robb (1980, p. 424)). Our expression differs from theirs because we allow for lifetime uncertainty, whereas they assume that agents have fixed lifetimes.

[9] See also Footnote 14 below. As is pointed out by Kingston (2000, p. 834f5), Lazear (1979) assumes that

4.2.2 Demography

We use the demographic framework presented in Section 3.2.1 on page 60. The instantaneous mortality rate is $m(\alpha, \psi_m(v))$, and the corresponding cumulative mortality by $M(u, \psi_m(v)) = \int_0^u m(\alpha, \psi_m(v))d\alpha$, where $\psi_m(v)$ is a parameter that only depends on the time of birth.

The birth rate varies over time, but is still exogenous by assumption. In Section 3.2.1 we showed that the size of a newborn generation at time v is proportional to the current population at that time, i.e. $L(v, v) = b(v)L(v)$, with $b(v)$ the birth rate and $L(v)$ is the population size at time v. The size of cohort v at some later time τ is given by:

$$L(v, \tau) = L(v, v)e^{-M(\tau - v, \psi_m(v))} = bL(v)e^{-M(\tau - v, \psi_m)}. \tag{4.13}$$

The population shares are given by

$$l(v, t) \equiv \frac{L(v, t)}{L(t)} = b(v)e^{-N(v,t) - M(t-v, \psi_m)}, \qquad t \geq v, \tag{4.14}$$

and the population growth rate is implicitly defined by

$$\frac{1}{b(v)} = \int_{-\infty}^{t} e^{-N(v,t) - M(t-v, \psi_m)} dv. \tag{4.15}$$

Box 3.1 shows how to calculate the population growth rate from Equation (4.15) by rewriting this equation as a Volterra equation of the second kind.

4.2.3 Firms

Perfectly competitive firms use physical capital and efficiency units of labour to produce a homogeneous commodity, $Y(t)$, that is traded internationally. The technology is represented by the following Cobb-Douglas production function:

$$Y(t) = K(t)^{\varepsilon} [A_Y H(t)]^{1-\varepsilon}, \qquad 0 < \varepsilon < 1, \tag{4.16}$$

where A_Y is a constant index of labour-augmenting technological change, $K(t)$ is the aggregate stock of physical capital, and $H(t)$ is employment in efficiency units. Following Blanchard (1985, p. 235) and Gomme et al. (2005, p. 431) we assume

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the disutility of labour is zero, so that retirement occurs at the point where lifetime income is maximized. Since this typically occurs late in life, Lazear uses this result to rationalize the existence of mandatory retirement.

that labour productivity is age dependent, i.e. a surviving worker of age $\tau - v$ is assumed to supply one unit of 'raw' labour and $E(\tau - v)$ efficiency units of labour. The efficiency profile is exogenous.[10] Aggregate employment in efficiency units is thus given by:

$$H(t) \equiv \int_{-\infty}^{t} L(v,t)E(t-v)\mathbf{I}(t-v,R(v))dv. \tag{4.17}$$

Following the same steps as in Section 3.2.2 we obtain the usual factor demand equations

$$r+\delta = \varepsilon\left(\frac{A_Y h(t)}{k(t)}\right)^{1-\varepsilon} = \frac{\partial Y(t)}{\partial K(t)}, \tag{4.18}$$

$$w(t) = (1-\varepsilon)A_Y\left(\frac{A_Y h(t)}{k(t)}\right)^{-\varepsilon} = \frac{\partial Y(t)}{\partial H(t)}, \tag{4.19}$$

where $h(t) \equiv H(t)/L(t)$ and $k(t) \equiv K(t)/L(t)$. For each factor of production, the marginal product is equated to the rental rate. Since the fixed world interest rate pins down the ratio between $h(t)$ and $k(t)$, it follows from (4.19) that the rental rate on efficiency units of labour is time-invariant, i.e. $w(\tau) = w$. Hence, both physical capital and output are proportional to employment at all time:

$$k(t) = A_Y\left(\frac{\varepsilon}{r+\delta}\right)^{1/(1-\varepsilon)} h(t), \tag{4.20}$$

$$y(t) = A_Y\left(\frac{\varepsilon}{r+\delta}\right)^{\varepsilon/(1-\varepsilon)} h(t), \tag{4.21}$$

where $y(t) \equiv Y(t)/L(t)$. Finally, since efficiency units of labour are perfectly substitutable in production, cost minimization of the firm implies that the wage rate for a worker of age u is equal to:

$$\bar{w}(u) = wE(u). \tag{4.22}$$

Despite the fact that w is constant, the wage facing individual workers is age-dependent because individual labour productivity is.

[10] The comparative static effects of changes in the $E(\tau - v)$ function on the retirement decision are studied in Section 4.3 below. Note that there exists a large literature on life-cycle labour supply and human capital accumulation. See, for example, Ben-Porath (1967), Razin (1972), Weiss (1972), Heckman (1976), Driffill (1980), Gustman and Steinmeier (1986) , Heckman et al. (1998), and Mulligan (1999).

4.3 Retirement and ageing in the absence of pensions

In this section we study the comparative static effect on the optimal retirement age of various ageing shocks. In order to build intuition, we abstract from a public pension system and restrict attention to a comparison of steady states. A supplementary aim of this section is to introduce the graphical apparatus with which the effects of pensions and ageing can be visualized in an intuitive manner.

4.3.1 The retirement decision

In the steady state, we have $t_L(s) = t_L$, $\bar{z}(s) = \bar{z}$, $\bar{a}(v,t) = \bar{a}(u)$, $R(v) = R$, $\bar{l}i(v,t,R(v)) = \bar{l}i(u,R)$, and the concentrated lifetime utility function and the expression for lifetime income can both be written in terms of the individual's actual age, u, and the planned retirement age, R:

$$\bar{\Lambda}(u) \equiv e^{\theta u + M(u)} \left[\int_u^\infty U\left(\frac{\bar{a}(u) + \bar{l}i(u,R)}{\Delta(u,r^*)} e^{\sigma\cdot(r-\theta)(s-u)} \right) e^{-\theta s - M(s)} ds - \int_u^R D(s) e^{-\theta s - M(s)} ds \right], \quad (4.23)$$

$$\bar{l}i(u,R) = e^{ru+M(u)} \int_u^R \bar{w}(s) e^{-rs-M(s)} ds - \bar{z}\Delta(u,r), \quad (4.24)$$

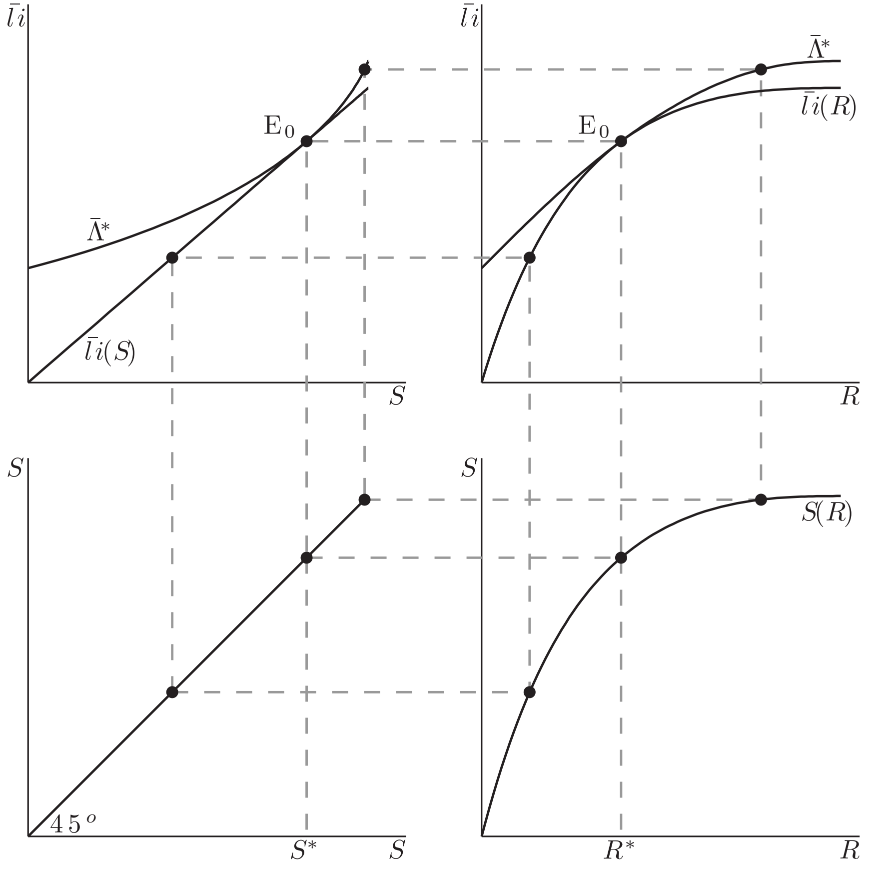
where $\bar{z}\Delta(u,r)$ represents the present value of lumpsum tax payments for an agent of age u.

In principle, it is possible to analyse the steady-state optimization problem directly in $(\bar{l}i, R)$-space, but the solution is not easy to visualize because both indifference curves and the budget constraint are not well behaving, e.g. indifference curves are S-shaped or concave—see Appendix 4.A. This is not a problem, in and of itself, because it can be shown that, under mild restrictions, the budget constraint is always more curved in an interior solution than the indifference curves are. However, for the sake of simplicity and to facilitate the graphical exposition, it is more convenient to use a monotonic transformation of the retirement age (rather than R itself) as the retirement choice variable. In particular, we define the auxiliary variable S, which we refer to as the *transformed retirement age*, as follows:

$$S(u,R) = e^{ru+M(u)} \int_0^R e^{-rs-M(s)} ds, \qquad \text{for } 0 \le u \le R. \quad (4.25)$$

Clearly, S is a continuous, monotonically increasing transformation of R for a given

Figure 4.1. Optimal retirement and the transformed retirement age



age u, which ensures that the inverse function, $R = R(u, S)$, also exists. In the lower right panel of Figure 4.1 the transformation from R to S for a newborn (i.e. $S(0, R)$) is illustrated, using the Gompertz-Makeham mortality process fitted to the cohort born in the Netherlands in 1920 as in Chapters 2 and 3. See table 2.1 for details and parameters values of the Gompertz-Makeham mortality function. The concave shape of the transformation stretches the S intervals for young ages and compacts these intervals for old ages.

For a general demography, such as the Gompertz-Makeham process, the inverse function, $R(u, S)$, is only defined implicitly by Equation (4.25).[11] The derivative of

[11] For the Blanchard (1985) case the instantaneous mortality rate is constant and equal to μ_0. Equation

this inverse function is given by

$$\frac{\partial R}{\partial S} = e^{-ru-M(u)} e^{rR(u,S)+M(R(u,S))} > 0. \tag{4.26}$$

Where no confusion arises we drop the dependency of R on S and u from here on. For future reference we note that the EEA, utility-maximizing, and lifetime-income maximizing values for S are given by, respectively, $S_E = S(u, R_E)$, $S^* = S(u, R^*)$, and $S_I = S(u, R_I)$.

The slope and curvature of the indifference curves in $(\bar{l}i, S)$-space are obtained by implicit differentiation of Equation (4.23):

$$\left.\frac{d\bar{l}i}{dS}\right|_{\bar{\Lambda}_0} \equiv -\frac{\partial\bar{\Lambda}/\partial R}{\partial\bar{\Lambda}/\partial\bar{l}i} \times \frac{\partial R}{\partial S} = e^{(r-\theta)(R-u)} D(R) \left[\frac{\bar{a}(u)+\bar{l}i}{\Delta(u,r^*)}\right]^{1/\sigma} > 0, \tag{4.27}$$

$$\left.\frac{d^2\bar{l}i}{dS^2}\right|_{\bar{\Lambda}_0} = \left[\frac{1}{\sigma\cdot(\bar{a}(u)+\bar{l}i)} \left.\frac{d\bar{l}i}{dS}\right|_{\bar{\Lambda}_0} + \left(\frac{D'(R)}{D(R)} + r - \theta\right)\frac{\partial R}{\partial S}\right] \left.\frac{d\bar{l}i}{dS}\right|_{\bar{\Lambda}_0} > 0. \tag{4.28}$$

The indifference curves are upward sloping, since postponing retirement causes additional disutility of labour which must be compensated with a higher lifetime income. By assumption $D'(R) \geq 0$ and $r > \theta$, so the indifference curves are convex. In the upper left panel of Figure 4.1 an indifference curve for a newborn is illustrated—see the curve labelled $\bar{\Lambda}^*$.

By differentiating (4.24), noting (4.22) and (4.26), we find that the slope and curvature of the $\bar{l}i(u, S)$ curve are given by:

$$\frac{d\bar{l}i}{dS} = \bar{w}(R) = wE(R) > 0, \tag{4.29}$$

$$\frac{d^2\bar{l}i}{dS^2} = \bar{w}'(R)\frac{\partial R}{\partial S} = wE'(R)\frac{\partial R}{\partial S} \lesseqgtr 0. \tag{4.30}$$

By increasing the (transformed) retirement age slightly, lifetime income is increased by an amount equal to the wage rate facing an agent of age R. Depending on the age profile of wages, the budget constraint may contain convex segments (for

(4.25) simplifies to:

$$S(u,R) \equiv \frac{e^{(r+\mu_0)u}}{r+\mu_0}\left[1 - e^{-(r+\mu_0)R}\right], \qquad \text{for } R \geq 0,$$

and the $R(u, S)$ function is given by:

$$R(u,S) \equiv -\frac{1}{r+\mu_0}\ln\left[1-(r+\mu_0)Se^{-(r+\mu_0)u}\right], \qquad \text{for } 0 \leq S < \frac{e^{(r+\mu_0)u}}{r+\mu_0}.$$

$\bar{w}'(R) > 0$), linear segments (for $\bar{w}'(R) = 0$), and concave segments (for $\bar{w}'(R) < 0$). The relevant case, however, appears to be that the wage is either constant or declining with age around the optimal age of retirement—see OECD (1998, p. 133) for empirical evidence on OECD countries. To streamline the discussion, we adopt the following assumption.

Assumption 4.1. *The wage schedule is non-increasing around the optimal retirement age and beyond, i.e.* $\bar{w}'(R) \leq 0$ *around and beyond* R^*.

In the upper left panel of Figure 4.1 we illustrate the linear budget constraint that results for the special case of an age-invariant wage rate ($\bar{w}'(R) = 0$ for all R). The optimum is located at point E_0, where there exists a tangency between the lifetime budget line and an indifference curve. The upper right panel shows the same equilibrium in $(\bar{l}i, R)$-space.

4.3.2 Ageing effects

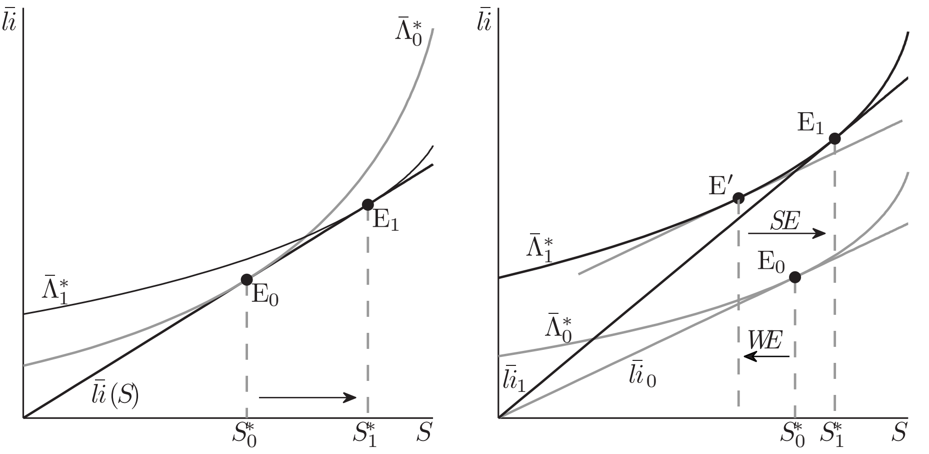
Our model distinguishes both *biological* and *economic* age dependencies. A biological ageing effect involves changes in the mortality structure, as captured by the mortality function $M(u, \psi_m)$, where ψ_m is the shift parameter introduced in Chapter 3. Economic ageing, on the other hand, refers to changes in the disutility of working or in the efficiency of labour over the life cycle, as captured by the functions $D(u, \psi_d)$ and $E(u, \psi_e)$, respectively, where ψ_d and ψ_e are the associated shift parameters. In the remainder of this section we focus on the retirement decision of a newborn, i.e. we set $u = \bar{a}(u) = 0$ in Equations (4.23)–(4.24). This entails no loss of generality because the agent's plans are dynamically consistent, i.e. the optimal retirement age is age-invariant.

Economic ageing

In Figure 4.2(a) we illustrate the effect on lifetime income and the optimal retirement age of a decrease in the disutility of labour, i.e. $\partial D(u, \psi_u)/\partial \psi_d \leq 0$ for all u, with strict inequality around $u = R^*$. Such a preference shock leaves the budget constraint unchanged, but changes the slope of the indifference curves. Indeed, it follows from (4.27) that:

$$\frac{\partial}{\partial \psi_d} \left[\frac{d\bar{l}i}{dS} \right]_{\bar{\Lambda}_0} = e^{r-\theta)(R-u)} \left[\frac{\bar{l}i}{\Delta(0, r^*)} \right]^{1/\sigma} \frac{D(R, \psi_d)}{\partial \psi_d} < 0. \tag{4.31}$$

Figure 4.2. Optimal retirement and economic ageing shocks



(a) Reduced disutility of working (b) Increase in labour productivity

The indifference curves become flatter and the agent chooses a higher retirement age as a result—see the move from E$_0$ to E$_1$ in Figure 4.2(a).

In Figure 4.2(b) we depict the comparative static effect of an increase in the age profile of labour efficiency, i.e. $\partial E(u, \psi_e)/\partial\psi_e \geq 0$ with strict inequality for $u = R^*$. Indifference curves are not affected by this shock but the budget constraint is. Indeed, the effects of such a shock are complicated because there are offsetting wealth- and substitution effects. It follows from (4.24) that the budget constraint shifts up:

$$\frac{\partial \bar{li}}{\partial \psi_s} = w \int_0^R \frac{\partial E(s, \psi_e)}{\partial \psi_e} e^{-rs-M(s)} ds > 0, \tag{4.32}$$

and from (4.29) that it becomes steeper:

$$\frac{\partial}{\partial \psi_s} \left[\frac{d\bar{li}}{dS} \right] = w \frac{\partial E(R, \psi_e)}{\partial \psi_e} > 0. \tag{4.33}$$

In Figure 4.2(b) we illustrate the case for which the optimal retirement increases. The total effect is the change from E$_0$ to E$_1$, consisting of a negative wealth effect (from E$_0$ to E$'$) and a positive substitution effect (from E$'$ to E$_1$).

Biological ageing

Two types of demographic shocks are considered in our analysis, namely a change in the birth rate and a change in the mortality process. Clearly, in view of (4.23)–(4.24), the birth rate does not directly affect the retirement choice of individual agents.[12] The mortality process, however, affects the $\Delta(u,\lambda)$ function and thus the optimal retirement choice. In this chapter we will make the same assumptions regarding the effect of a change of ψ_m on the mortality process as in Chapter 3 (see Assumption 3.1 on page 65). These assumptions assure that the expected remaining lifetime increases for all ages and that the mortality profile shifts more downward for old ages than for young ages, it is a so-called adult mortality shock.

In order to compute the effect of increased longevity on retirement, we write the first-order condition for the optimal transformed retirement age, $d\bar{\Lambda}/dS = 0$, as follows:[13]

$$\Gamma(R^*,\psi_m) \equiv \bar{w}(R^*) - D(R^*)e^{(r-\theta)R^*}\left[\frac{\bar{l}i(0,R^*,\psi_m)}{\Delta(0,r^*,\psi_m)}\right]^{1/\sigma} = 0, \tag{4.34}$$

where the second-order condition for utility maximization implies that $\partial\Gamma/\partial R^* < 0$, and where $\bar{l}i(0,R^*)$ is given by:

$$\bar{l}i(0,R^*,\psi_m) \equiv \int_0^{R^*} \bar{w}(s)e^{-rs-M(s,\psi_m)}ds - \bar{z}\Delta(0,r,\psi_m). \tag{4.35}$$

In Equation (4.35), lifetime income depends on the mortality parameter ψ_m because both wage income and lumpsum taxes are discounted using the actuarially fair annuity rate of interest, $r + m(s,\psi_m)$. In addition, in Equation (4.34) the mortality parameter affects the marginal propensity to consume out of total wealth.

It follows from (4.34) that:

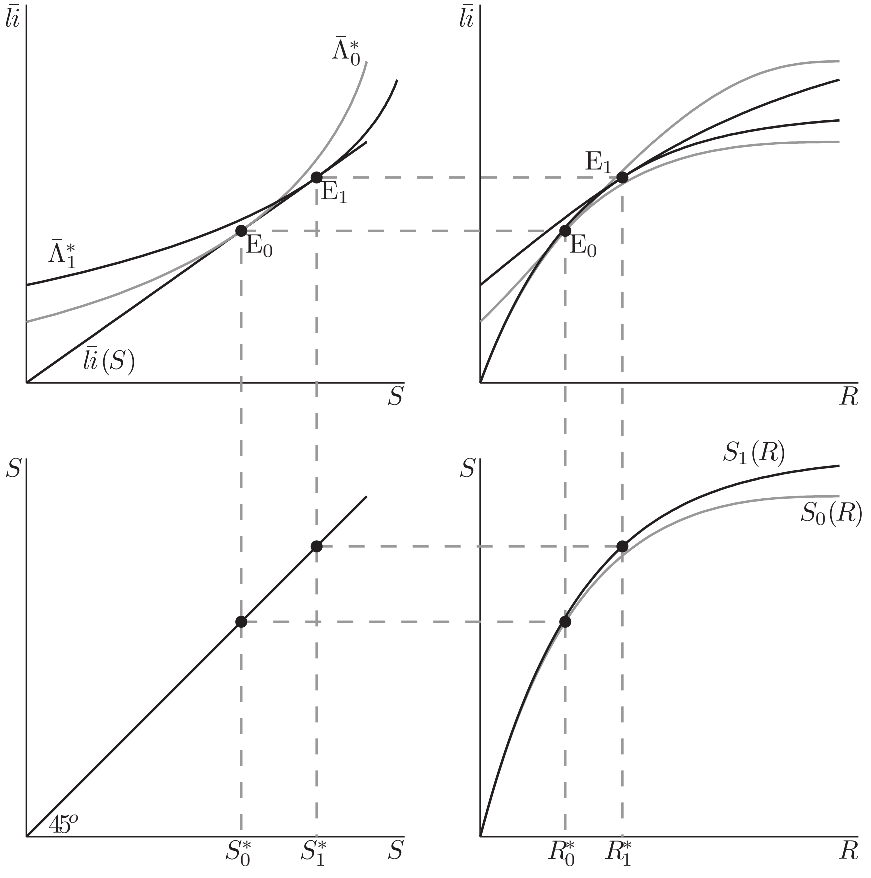
$$\frac{dR^*}{d\psi_m} = \frac{\partial\Gamma/\partial\psi_m}{|\partial\Gamma/\partial R|} = \frac{\bar{w}(R^*)}{\sigma|\partial\Gamma/\partial R^*|}\left[\frac{\partial\Delta(0,r^*,\psi_m)/\partial\psi_m}{\Delta(0,r^*,\psi_m)} - \frac{\partial\bar{l}i(0,R^*,\psi_m)/\partial\psi_m}{\bar{l}i(0,R^*,\psi_m)}\right] \gtreqless 0. \tag{4.36}$$

Clearly, the sign of the comparative static effect is determined by the term in square brackets on the right-hand side of (4.36). Using Proposition 3.1(iii) we find that $\partial\Delta(0,r^*,\psi_m)/\partial\psi_m > 0$ so the *propensity effect* operates in the direction of increasing

[12] Of course, in general equilibrium the birth rate may affect the retirement choice via the fiscal system. See Section 4.6 for a further analysis.

[13] This expression is obtained by combining Equations (4.27) and (4.29) and setting $u = \bar{a}(u) = 0$.

Figure 4.3. Optimal retirement and increased longevity



the retirement date. We find from (4.35) that the *lifetime-income effect* is ambiguous in general:

$$\frac{\partial \bar{li}(0, R^*, \psi_m)}{\partial \psi_m} = -\int_0^{R^*} \bar{w}(s) \frac{\partial M(s, \psi_m)}{\partial \psi_m} e^{-rs - M(s,\psi_m)} ds - \bar{z} \frac{\partial \Delta(0, r, \psi_m)}{\partial \psi_m} \gtreqless 0. \quad (4.37)$$

The first term on the right-hand side is positive (see Proposition 3.1(i)), i.e. as a result of reduced discounting of wage income, lifetime income increases. But lighter discounting also increases the lifetime burden of the lumpsum tax, i.e. the second term on the right-hand side is also positive. As a result, the wage effect moves in the opposite direction of the tax effect and the net effect of ageing on lifetime income cannot be signed a priori. Of course, in the absence of lumpsum taxes, the lifetime-income effect is positive and thus works in the direction of decreasing the retirement age. There is a strong presumption, however, that the first term on the right-hand side of (4.37) is rather small. Indeed, as can be gleaned from Figure 3.2(a) on page 67, an adult mortality shock starts to matter quantitatively for age levels at which most agents have already retired in advanced countries. Hence, the retirement age is likely to increase as longevity increases because the tax effect is dominant, i.e. $dR^*/d\psi_m > 0$ in realistic scenarios.

In Figure 4.3 we illustrate the comparative static effects of increased longevity. In panel (d), the mortality shock increases the transformed retirement age at all values of R, though more so for higher ages. Intuitively, by making the transformation curve steeper, a post-shock octogenarian is 'younger' than his/her pre-shock counterpart. As a result, the indifference curves in panel (a) flatten out so that, with a linear budget constraint (with $\bar{w}'(R) = 0$), the equilibrium shifts from E_0 to E_1. In panel (b) the same comparative static effect is shown in $(\bar{l}i, R)$-space.

4.4 Realistic pension system

In this section we re-introduce the public pension system and investigate its likely consequences for the trade-offs facing workers in advanced economies. As in the previous section, we continue to assume that the pension system is in a steady state. As a result, social security wealth (4.6) can be written as follows:

$$SSW(u,R) = e^{ru+M(u)}\left[B(R)\int_{\max\{R,R_E\}}^{\infty} e^{-rs-M(s)}ds - t_L\int_u^R \bar{w}(s)e^{-rs-M(s)}ds\right]. \tag{4.38}$$

By incorporating social security wealth into the steady-state budget constraint (4.24) and differentiating with respect to the transformed retirement age we obtain:

$$\frac{d\bar{l}i}{dS} = \begin{cases} (1-t_L)\bar{w}(R) + B'(R)\Pi(R,R_E,\infty,r) > 0 & \text{for } S < S_E \\ (1-t_L)\bar{w}(R) + B'(R)\Delta(R,r) - B(R) \geq 0 & \text{for } S_E \leq S \leq S_I \end{cases} \tag{4.39}$$

where R_E and R_I (S_E and S_I) are, respectively, the (transformed) EEA and lifetime-income maximizing retirement age.[14] The Π-function appearing in the upper branch

[14] In the presence of a public pension system, R_I is defined implicitly by $\Delta(R_I,r) = B(R_I) - (1 - t_L)w(R_I)$. Since $B'(R_I) \geq 0$, $\bar{w}'(R_I) \leq 0$ (Assumption 4.1) and $\partial\Delta(R_I,r)/\partial R_I < 0$ (Proposition 2.1(v)), it follows that there exists a unique value for R_I.

of (4.39) is defined in general terms as:

$$\Pi(u, \underline{u}, \bar{u}, \lambda) = e^{\lambda u + M(u)} \int_{\underline{u}}^{\bar{u}} e^{-\lambda s - M(s)} ds. \tag{4.40}$$

$\Pi(u, \underline{u}, \bar{u}, \lambda)$ is the present value of an annuity that one receives during the age interval $(\underline{u}, \bar{u})$, evaluated at age u, using the discount rate λ. The 'regular' Δ-function, is a special case of the Π-function, with $\underline{u} = u$ and $\bar{u} = \infty$.

As is evident from (4.39), the shape, slope, and curvature of the budget constraint is complicated by the existence of the EEA. If $B(R)$ and $B'(R)$ are both continuous at $R = R_E$, then the budget constraint is continuous but features a kink at that point equal to $-B(R_E)$. The kink represents the retirement benefit that is foregone by not retiring at R_E but at some later age.

The curvature of the lifetime income function is ambiguous in general, i.e. it cannot be inferred from theoretical first principles whether or not it is concave in the relevant region. Our reading of the empirical comparative-institutional literature for OECD countries, however, gives us enough confidence to formulate the following assumption.

Assumption 4.2. *In the relevant calender age domain of 55 to 70, the lifetime income function is concave in the transformed retirement age S. It may feature a single kink at the EEA.*

Our defence for this assumption takes up the remainder of this section and proceeds as follows. In the literature (e.g. Gruber and Wise (1999, 2004), and OECD (2005)) retirement incentives are typically summarized with the EEA, the NRA, the replacement rate, the pattern of benefit accrual, and the implicit tax rate. Using these incentive indicators, it is possible to derive the shape and slope of the lifetime income function.

The *replacement rate* is defined as the ratio of the retirement benefit to net wages. In terms of our theoretical framework, the replacement rate RR for someone retiring at or after the EEA is given by:

$$RR(R) \equiv \frac{B(R)}{(1 - t_L)\bar{w}(R)} \qquad \text{for } R \geq R_E. \tag{4.41}$$

This replacement rate differs greatly between countries, but also between ages. As can be seen from Table 4.B.1 in Appendix 4.B, the replacement rate for France starts at 92% at the EEA (59 years) and slowly increases to 96% thereafter. In contrast, in

Canada the replacement rate starts at 18% at age 59, after which it increases to 91% for a 69 year old.

The *benefit accrual* is the nominal change in social security wealth if one postpones retirement by one year (i.e., it is $\partial SSW/\partial R$ in terms of our model). The benefit accrual depends on the age of the individual. To compare accrual levels, we can either hold constant the age at which social security wealth is evaluated or evaluate social security wealth at the actual retirement age. Both methods have their advantages and drawbacks. The first makes it easier to track social security wealth over time, the second allows for easier comparison of retirement incentives at the retirement age. In this chapter we will use the second definition because it allows for easier mathematics in the transformed retirement age S-space. By differentiating Equation (4.38) with respect to R and evaluating at age $u = R$ we obtain:

$$ACC(R) \equiv \left.\frac{\partial SSW(u,R)}{\partial R}\right|_{u=R} = \begin{cases} B'(R)\Pi(R,R_E,\infty,r) - t_L\bar{w}(R) & \text{for } R < R_E \\ B'(R)\Delta(R,r) - t_L\bar{w}(R) - B(R) & \text{for } R > R_E \end{cases} \tag{4.42}$$

The level of benefit accrual is closely connected to the slope of the lifetime income function (as a function of S). Indeed, by combining (4.39) and (4.42) we obtain:

$$\frac{d\bar{li}}{dS} = ACC(R) + \bar{w}(R). \tag{4.43}$$

In this context, actuarial adjustment of the pension benefit is called *fair* if and only if $ACC(R) = 0$ for all R.

The benefit accrual depends on the monetary units in which social retirement benefits are measured and the age at which the social security wealth is evaluated. Most studies standardize the benefit accrual either with the level of social security wealth or with the present value of net wages. The first measure is the accrual rate, the second measure is the implicit subsidy.

The negative of the implicit subsidy is the *implicit tax rate* (IT), measuring the additional tax rate one 'implicitly' faces over and above the normal taxes. A negative accrual is an additional tax on labour and discourages work. Conversely, a positive accrual is an implicit subsidy on labour and encourages the individual to work an additional year. Since we evaluate the accrual level at the retirement age, we should not discount the net wage rate, so the implicit tax can be written in terms of our model as:[15]

$$IT(R) \equiv -\frac{ACC(R)}{(1-t_L)\bar{w}(R)}. \tag{4.44}$$

By substituting this expression for the implicit tax rate into Equation (4.43), we can write the slope of the lifetime income function as:

$$\frac{d\bar{li}}{dS} = (1-t_L)\bar{w}(R)\left[\frac{1}{1-t_L} - IT(R)\right]. \tag{4.45}$$

Under the maintained assumption that gross wages are constant for higher ages (typically in the range 55–70), Equation (4.45) can be used to compute the shape of the lifetime income function. Dividing lifetime income by net wages gives a 'standardized' measure of lifetime income which is more easily comparable between countries. The only caveat is that we do not have data on the relevant labour income tax, so we cannot estimate $1/(1-t_L)$. This is not a problem, however, because we are only interested in the curvature of the lifetime income function (its convexity or concavity). The term $1/(1-t_L)$ only influences the slope of the lifetime income profile, but it has no effect on its curvature. To get an idea of the shape of the lifetime income profile we proceed as if there are no labour income taxes. Since the fraction $1/(1-t_L)$ has a lower bound of 1 (because in reality taxes are positive), we thus obtain a conservative estimate for lifetime income.

Figure 4.4 shows the lifetime income profiles for nine OECD countries, as we computed them using the implicit tax rates published in Gruber and Wise (1999). For convenience these tax rates have also been reported in Table 4.B.2 in Appendix 4.B. The lifetime income profiles are normalized at age 54 to enable comparison between countries. The graphs contain two horizontal axes. The main (lower) horizontal axis measures the transformed retirement age, S, whilst the secondary (upper) axis shows the corresponding values for the untransformed retirement age, R. The effect of the non-monotonic scaling is clearly visible.

Figure 4.4(a) characterizes the retirement systems in continental Europe. Lifetime income profiles are increasing in the retirement age, more or less concave and usually have a clear kink at the EEA (which is 60 years in most countries, but only 55 in Italy) or at the NRA (65). A notable exception is formed by the Netherlands. Its profile has a sharp spike at age 59 and decreases until the NRA of 65. The pen-

[15] Some contributors to Gruber and Wise (1999) do not provide information concerning the age at which they evaluate the present value of social security wealth. This is not a problem, however, provided we do not use the accrual levels, but the accrual rates or implicit tax rates.

sion system in the Netherlands is such that there exists an implicit tax of more than 141% of net earnings. The pension benefits someone receives hardly increases if someone retires after age 59, but one still has to pay contributions to the pension system. Moreover, replacement rates are very high due to the usually generous, but mandatory, company pension systems. It is not surprising that most employees in the Netherlands retire at age 60.[16]

Figure 4.4(b) characterizes the retirement systems in Canada, Japan, the United Kingdom, and the United States. A feature of these systems is the rather low implicit tax rates. A low implicit tax is a symptom of either (i) a near-actuarially fair system, or (ii) a rather poorly developed pension system. The replacement rates in Table 4.B.1 indicate that the former is the case in Canada and the United States, whereas the latter is relevant for the United Kingdom, leaving Japan as a somewhat mixed case. As a result of the small implicit tax rates, the wage effect in the lifetime income function (4.45) is dominant and the standardized lifetime income profiles are roughly the same in these four countries.

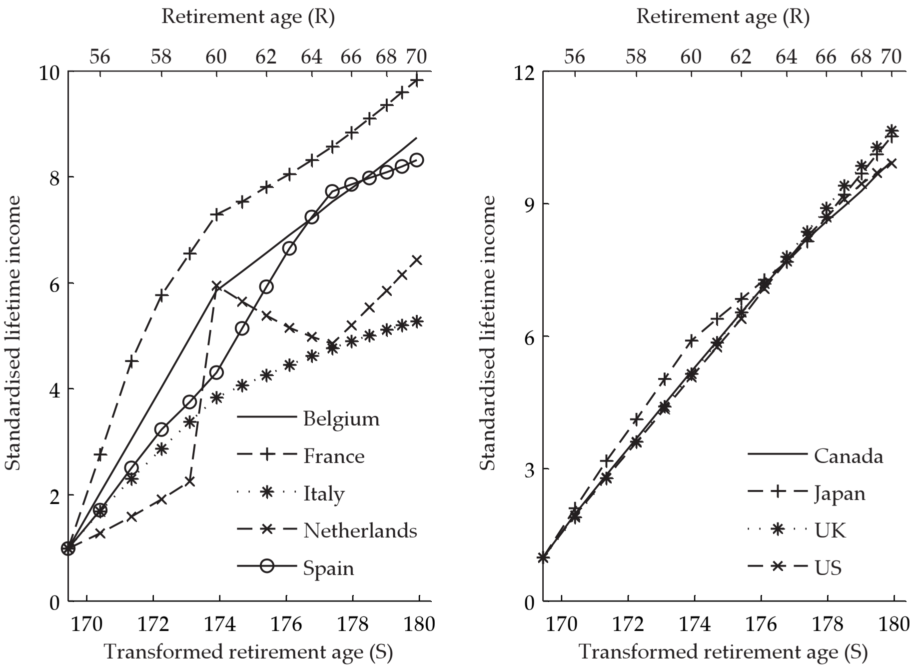
Although Figure 4.4 only shows conservative estimates for the lifetime income profiles, it does give an accurate picture concerning the shape of these profiles. Apart from Spain and the Netherlands, the income profiles are concave and may feature a kink at the EEA. Even though the profile for the Netherlands is not concave, there is a pronounced kink at age 60 which precludes individuals from working beyond that age.

4.5 Ageing and pension shocks

In this section we study the comparative static effects on the optimal steady-state retirement age of various marginal changes in the tax system or the public pension scheme. In view of Assumption 4.2 and because indifference curves are convex in $(\bar{l}i, S)$-space, the optimum retirement age is unique. If there is no kink in the lifetime income profile, then there will be an interior solution. In the presence of a single kink, however, there are three possible outcomes. First, if the agent's disutility of labour is high, and indifference curves are relatively steep, then the interior optimum occurs to the left of the kink, i.e. the agent chooses $R^* < R_E$, contra stylized fact (SF3). Second, if labour disutility is moderate, then indifference curves are relatively flat and there will be a corner solution at the kink, i.e. $R^* = R_E$. Third,

[16] The graph is based on retirement schemes as they existed in the late 1980s. More recent figures published in Gruber and Wise (2004) provide a qualitatively similar picture.

Figure 4.4. Lifetime income profiles for nine OECD countries (lower bound)



(a) Continental Europe (b) Other countries

Source: Gruber and Wise (1999) and own calculations.

if labour disutility is very low then there will be an interior solution to the right of the EEA, i.e. $R^* > R_E$. The second and third cases are not inconsistent with reality.

In this section we focus on the interior solutions and we assume that the retirement age is strictly larger than the EEA ($R^* > R_E$). By combining Equations (4.27) and (4.39) and setting $u = \bar{a}(u) = 0$ we obtain the first-order condition which implicitly defines a unique solution for R^*:

$$\Gamma(R^*) \equiv (1 - t_L)\bar{w}(R^*) + B'(R^*)\Delta(R^*, r) - B(R^*) - e^{(r-\theta)R^*} D(R^*) \left[\frac{\bar{li}(0, R^*)}{\Delta(0, r^*)}\right]^{1/\sigma} = 0, \quad (4.46)$$

where $\bar{li}(0, R^*)$ is obtained by adding $SSW(0, R^*)$ to the right-hand side of Equation (4.35) above. The second-order condition of utility maximization implies that

$\partial\Gamma/\partial R^* < 0$. For future reference we define the following partial derivative:

$$\left|\frac{\partial\Gamma(R^*)}{\partial\bar{l}i}\right| \equiv e^{(r-\theta)R^*}\frac{D(R^*)}{\sigma\Delta(0,r^*)}\left[\frac{\bar{l}i(0,R^*)}{\Delta(0,r^*)}\right]^{(1-\sigma)/\sigma} > 0. \tag{4.47}$$

Changes in the tax system affect the optimal retirement age in the following way. First, an increase in the lumpsum tax leads to a reduction in lifetime income and an increase in the retirement age:

$$\frac{dR^*}{d\bar{z}} = \frac{\partial\Gamma/\partial\bar{z}}{|\partial\Gamma/\partial R^*|} = \Delta(0,r)\left|\frac{\partial\Gamma(R^*)}{\partial\bar{l}i}\right| > 0. \tag{4.48}$$

Intuitively, the tax change induces a pure wealth effect. Because consumption and leisure are both normal goods, labour supply is increased, i.e. the agent retires later in life. Second, a change in the labour income tax rate has an ambiguous effect:

$$\frac{dR^*}{dt_L} = \frac{\partial\Gamma/\partial t_L}{|\partial\Gamma/\partial R^*|} \equiv -\frac{\bar{w}(R^*)}{|\partial\Gamma/\partial R^*|} + \left|\frac{\partial\Gamma(R^*)}{\partial\bar{l}i}\right|\frac{\int_0^{R^*}\bar{w}(s)e^{-rs-M(s)}ds}{|\partial\Gamma/\partial R^*|} \lesseqgtr 0. \tag{4.49}$$

The first term on the right-hand side of (4.49) represents the substitution effect, which is negative. A higher tax discourages working and thus encourages retiring earlier in life via that effect. The second term is the positive wealth effect. The tax increase makes the agent poorer and thus provides incentives to retire later in life. In summary, the labour income tax increase operates qualitatively like a decrease in labour efficiency (see Equations (4.32)–(4.33) and Figure 4.2(b)).

Changes in the pension system affect the retirement decision as follows. First, holding constant the slope of the retirement benefit curve, the effect of a change in its *level* is negative:

$$\frac{dR^*}{dB(R)} = \frac{\partial\Gamma/\partial B(R)}{|\partial\Gamma/\partial R^*|} = -\frac{1}{|\partial\Gamma/\partial R^*|} - \left|\frac{\partial\Gamma(R^*)}{\partial\bar{l}i}\right|\frac{\int_{R^*}^{\infty}e^{-rs-M(s)}ds}{|\partial\Gamma/\partial R^*|} < 0. \tag{4.50}$$

In this case the wealth- and substitution effects operate in the same direction. The first term on the right-hand side of (4.50) is the negative substitution effect: by increasing the public retirement benefit the rewards to working longer are reduced, i.e. the relevant branch of the budget constraint (4.39) is rotated in a clockwise fashion. The second term on the right-hand side of (4.50) is the negative wealth effect. The benefit increase boosts lifetime income and thus induces agents to work less and to retire earlier in life. In graphical terms, the wealth effect leads to an upward shift of the lifetime budget constraint.

Second, ceteris paribus the level of the benefit function, a change in its *slope* $B'(R)$ causes a positive substitution effect:

$$\frac{dR^*}{dB'(R)} = \frac{\partial\Gamma/\partial B'(R)}{|\partial\Gamma/\partial R^*|} = \frac{\Delta(R^*, r)}{|\partial\Gamma/\partial R^*|} > 0. \tag{4.51}$$

Intuitively, the steeper slope of the benefit function induces agents to postpone retirement somewhat. In graphical terms, the budget constraint rotates counter-clockwise and the optimal retirement age shifts to the right.[17]

4.6 Demographic change and policy reform

In this section we compute and visualize the general equilibrium computational results of various demographic shocks and their assumed fiscal reform measures. We restrict attention on measures characterizing the aggregate economy. The per capita aggregate variables are calculated as in the previous chapters. Per capita consumption, for example, is computed as $c(t) \equiv \int_{-\infty}^{t} l(v,t)\bar{c}(v,t)dv$, where the relative cohort weight, $l(v,t)$, is defined in Equation (4.14) above, and individual consumption, $\bar{c}(v,t)$, is given in (4.9).

In accordance with stylized fact (SF4), we calibrate the model in such a way that the initial optimum retirement age is at the EEA, i.e. the budget constraint features a kink at the EEA and individual agent are 'stuck' in this corner solution. The main demographic and economic features of the calibrated model are as follows. The mortality process is as in the previous chapters. It represents the fitted G-M process for the cohort born in 1920 in the Netherlands. Life expectancy at birth for this cohort is 65.5 years. The crude birth rate is set at $b = 0.0237$, a value that lies in between the observed birth rates of 1920 and 1940. In combination, the demographic parameters imply an initial steady-state population growth rate equal to $\hat{n}_0 = 1.34\%$. For households we assume in this chapter that the world interest rate facing them equals $r = 0.05$, the rate of time preference is $\theta = 0.03$, and the intertemporal substitution elasticity is $\sigma = 0.8$. In combination, these parameter values imply an annual consumption growth for individuals of $\dot{\bar{c}}(v,t)/\bar{c}(v,t) = \sigma\cdot(r-\theta) = 0.016$. Disutility of labour and labour efficiency are both age-invariant and set at, respectively, $D(u) = 0.15$ and $E(u) = 10$. On the production side, we set the share of capital in the production function at $\varepsilon = 0.4$,

[17] Following Sheshinski (1978, pp. 357-8), we write the pension benefit as $B(R, \psi_b)$, where ψ_b is a shift parameter. The first pension shock assumes $\partial B/\partial\psi_b > 0$ and $\partial^2 B/\partial\psi_b\partial R = 0$. The second shock sets $\partial B/\partial\psi_b = 0$ and $\partial^2 B/\partial\psi_b\partial R > 0$.

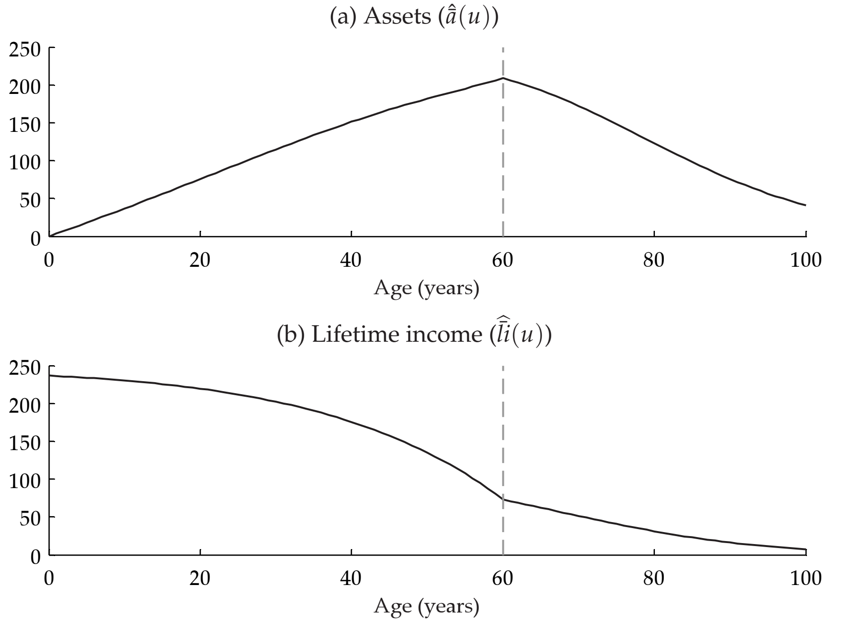
the technology index is $A_Y = 1$, and the depreciation rate of capital is $\delta = 0.06$. For the policy parameters we use the following values. The labour income tax is $t_L = 0.1$, the lumpsum tax is $\bar{z}_0 = -0.166$, the initial debt level is $\hat{d}_0 = 10$, and the EEA is set at $R_E = 60$. For somebody who retires before the EEA, pension benefits are zero until the EEA and equal to $\beta_0 = 7.094$ from age EEA onward (this value for β_0 amounts to 50% of an agent's gross wage). For somebody retiring at or after the EEA, pension benefits are zero until actual retirement, and equal to $\beta_0 + \beta_1(R - R_E)$ from age R onward, where $\beta_1 = 0.05$.

Figure 4.5 shows the steady-state age profiles of financial assets and lifetime income. Panel (a) shows that the profile for assets is inverse U-shaped and reaches a peak at $u = R_E$. After retirement, the agent slowly decumulates its assets. Panel (b) shows that there is a kink in the profile for lifetime income at $u = R_E$. The initial steady state has the following aggregate features: $\hat{w} = 14.2$, $\hat{a}_0 = 100.5$, $\hat{li}_0 = 184.4$, $\hat{h}_0 = 9.0$, $\hat{y}_0 = 21.2$, $\hat{c}_0 = 16.0$, $\hat{i}_0 = 5.7$, $\hat{k}_0 = 77.0$, and $\hat{f}_0 = 13.5$. The output shares of consumption, investment, net exports, and the government primary surplus are, respectively, 75.6%, 26.7%, −2.3% and 1.73%. We summarize the initial steady state in column (1) of Table 4.1. Our model economy is clearly not a banana republic. It is a wealthy country ruled by a fiscally responsible government (that is running a primary surplus), and populated by long-lived and patient citizens (who as a group hold a net claim on the rest of the world).

The comparative dynamic exercises performed throughout this section take the following form. Starting from the initial steady state, the economy is hit by one of two types of demographic change occurring at time $t = 0$, namely a baby bust or an increase in longevity (reduced adult mortality). The demographic shocks are the same as in the previous chapter, a 24% drop in the birth rate and a longevity shock that increases expected remaining lifetime from 65.5 years to 77.6 years. In both cases, the demographic shock renders the public pension system fiscally unsustainable in the long run, conform stylized fact (SF1). At time $t = 0$, however, the policy maker announces a policy reform—to be implemented at some later date, $T_R > 0$—which restores fiscal sustainability. The announcement is believed by individual agents as the policy maker has been credible in the past.

We study the effects of three types of policy reform. In Section 4.6.1 we assume that the policy maker engineers a once-off change in the lumpsum tax, $\bar{z}$, at time $t = T_R$ which restores government solvency. The policy response is the same for the two types of demographic change. In keeping the lumpsum tax time-invariant, both before and after the reform, the government engages in tax smoothing.

Figure 4.5. Individual steady-state wealth profiles with optimal retirement



In Section 4.6.2 we assume that the policy maker uses different instruments to address the two types of demographic change. For the baby bust, the policy response consists of a once-off increase in the labour income tax rate, t_L, occurring at time $t = T_R$. This is again a tax smoothing scenario as t_L is time-invariant both before and after the shock. For the longevity shock, the policy response consists of a permanent increase in the EEA, occurring at time $t = T_R$, which restores solvability without any further tax changes.

BOX 4.1

The government budget constraint

The key to solving the model is to determine the policy instrument that keeps the government finances sustainable. It is impossible to determine government debt in the infinite future, we therefore cut off the problem by postulating that the government debt must be stable at some finite future date, T. Government debt at date T is determined by the outstanding debt at time t and the budget deficits in between $G(\tau)$, which is determined by the policy instrument mix.

$$d(T) = d(t)e^{r\cdot(T-t)-N(T)+N(t)} + \int_t^T G(\tau)e^{r\cdot(T-\tau)-N(T)+N(\tau)}d\tau, \tag{4.52}$$

$$G(\tau) = \int_{-\infty}^{\tau} b(v)Q(v,\tau)e^{N(v)-N(\tau)-M(v,\tau)}dv \tag{4.53}$$

where $Q(v,\tau)$ are the net payments to the system at time τ of someone born at time v,

$$Q(v,\tau) = \begin{cases} t_L(\tau)\bar{w}(\tau - v) + \bar{z}(\tau) & \text{if working, } \tau - v \le R(v) \\ \bar{z}(\tau) - \bar{p}(v,\tau,R(v)) & \text{if retired, } \tau - v > R(v) \end{cases} \tag{4.54}$$

Equation (4.52) shows the first problem, any approximation errors in the initial debt level $d(t)$ and the government deficits $G(\cdot)$ explode because of the multiplication with exploding exponential terms. To solve this we solve the problem backwards in time. Multiply both sides with $e^{-r\cdot(T-t)+N(T)-N(t)}$ and rearrange

$$d(t) = d(T)e^{-r\cdot(T-t)+N(T)-N(t)} - \int_t^T G(\tau)e^{r\cdot(t-\tau)-N(t)+N(\tau)}d\tau. \tag{4.55}$$

If we pick T large enough that debt is in the new steady state, we determine the stable debt level implied by the government deficit, that is fully determined by the policy instruments. From this new steady state level and the deficits, we can calculate the implied government debt at time $t = 0$. The whole problem translates into a simple one dimensional root finding problem: find for level of the policy instrument for which $d(0)$ as defined in (4.55) equals the predetermined debt level. Any root finding algorithm works.

There is however one more caveat. We must numerically evaluate both integrals in Equations (4.52) and (4.53). The problem is that the net payments $Q(v,\tau)$ are not continuous in v, $Q(\cdot)$ jumps at the retirement age. Before retirement, the agent has to pay the lumpsum tax and labour tax, after retirement, he only has to pay the lumpsum tax and if the agent passed the EEA, he also receives a retirement benefit. The usual solution to this problem is to split the integral, such that the integrand is continuous on each part.

Although we faced the same problem in the previous chapter, the problem here is slightly more complicated. In the previous chapter, the policy instrument was a neutral lumpsum tax that did not affect the schooling decision. The discontinuity in net payments in Chapter 3 always occurred at $\tau = v + s(v)$ (s in the previous chapter is the schooling period) and we knew where to split the integral. Here we do not know this beforehand. In principle, we could first determine the retirement ages for the specific policy instrument combination, then split the integral in parts, and continue, but this is too time consuming.

Much faster and ultimately simpler to implement is to write the integral in (4.55) in terms of the generational accounts (see Auerbach et al., 1994)

$$\begin{aligned} GA(v,t) &= e^{r\cdot(t-v)+M(t-v)} \int_t^\infty Q(v,\tau) e^{-r\cdot(\tau-v)-M(\tau-v)} d\tau \\ &= SSW(v,t) + e^{r\cdot(t-v)+M(t-v)} \int_{t-v}^\infty z(v+u) e^{-ru-M(u)} du \end{aligned}$$

The generational accounts are the present value of net payments to the system and are easy to calculate for stepwise policy changes in the system. For sake of readability write the integral in (4.55) as

$$D_A(t) = \int_t^T \int_{-\infty}^\tau b(v) Q(v,\tau) e^{r\cdot(t-\tau)+N(v)-N(t)-M(v,\tau)} dv d\tau$$

Now split the inner integral at $v = t$ and change the order of integration

$$\begin{aligned} D_A(t) = &\int_{-\infty}^t b(v) e^{N(v)-N(t)} \int_t^T Q(v,\tau) e^{r\cdot(t-\tau)-M(v,\tau)} d\tau dv \\ &+ \int_t^T b(v) e^{N(v)-N(t)} \int_v^T Q(v,\tau) e^{r\cdot(t-\tau)-M(v,\tau)} d\tau dv \end{aligned}$$

Some tedious, but otherwise straightforward math shows that the inner integrals can be written in terms of the generational accounts

$$\begin{aligned} D_A(t) = &\int_{-\infty}^t b(v) e^{N(v)-N(t)} \left[e^{-M(v,t)} GA(v,t) - e^{-r\cdot(T-t)-M(v,T)} GA(v,T) \right] dv \\ &+ \int_t^T b(v) e^{N(v)-N(t)} \left[e^{r\cdot(t-v)} GA(v,v) - e^{-r\cdot(T-t)-M(v,T)} GA(v,T) \right] dv \end{aligned}$$

The integrand in the first term has a discontinuity at $v = 0$ caused by the a possible demographic shock, so we should split that one. This leaves us with three separate integrals, all with a continuous integrand, that we can evaluate with any numerical integration method. With an efficient method to calculate $D_A(0)$, we can solve the whole model within a number of seconds.

4.6.1 Tax reform

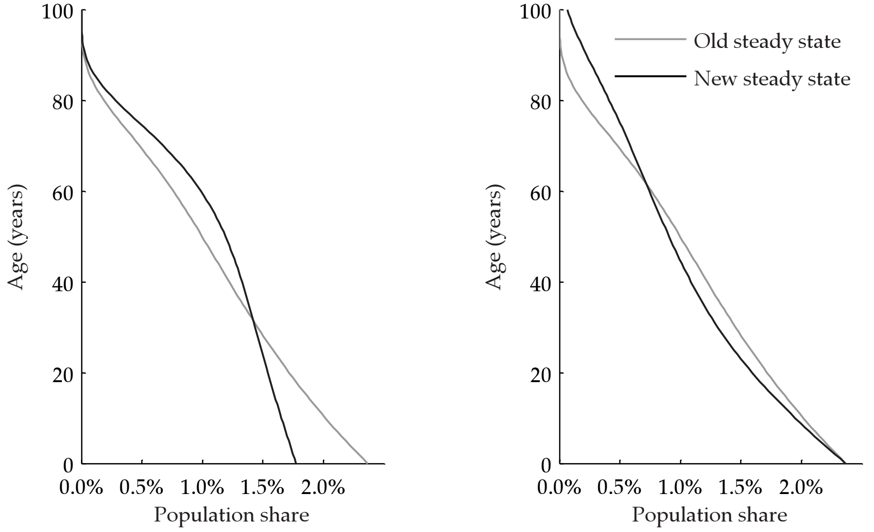
Throughout this subsection the announced policy reform consists of a once-off change in the lumpsum tax which makes government finances healthy again.

Baby bust The effects of a once-off decrease in the birth rate occurring at time $t = 0$ are visualized in Figures 4.6(a), 4.8 and 4.7(a). The baby bust causes a twenty-five percent decrease in the birth rate, from $b_0 = 0.0237$ to $b_1 = 0.0177$. It is assumed that policy reform is implemented twenty years after the baby bust, i.e. $T_R = 20$ in these figures. Since this reform has no effect on the kink in the lifetime income profile, individuals continue to retire at the EEA. Figure 4.6(a) depicts the change in the steady-state age composition of the population. The mass of the distribution is moved from younger to older ages. Figure 4.8(a) shows the demographic transition due the baby bust. There is an immediate drop in the population growth rate because the arrival rate of new agents has decreased permanently, i.e. $n(0) - \hat{n}_0 = b_1 - b_0$ (see the last paragraph in Box 3.1). Following the initial jump, $n(t)$ adjusts in a non-monotonic fashion to the new demographic equilibrium at $\hat{n}_1 = 0.43\%$.

Figure 4.8(b) illustrates the transition path for per capita employment in efficiency units (Recall that the paths for per capita output and physical capital are both proportional to $h(t)$—see Equations (4.20) and (4.21) above). Employment declines in a non-monotonic fashion, from $\hat{h}_0 = 9.0$ to $\hat{h}_1 = 8.5$. There is a gradual decline in $h(t)$ from $t = 0$ until $t = 60$ both because fewer workers enter the labour force than before the shock and because the larger pre-shock cohorts retire. At time $t = 60$, the path for $h(t)$ starts to rise again because the flow of retirees consists entirely of relatively small post-shock cohorts. Beyond $t = 60$, the path for employment converges in a cyclical fashion to the new steady state.

Figure 4.8(c) depicts the adjustment path for per capita consumption. At impact, consumption falls because all pre-shock generations anticipate the future lumpsum tax increase and cut their consumption level accordingly. During the first half cen-

Figure 4.6. Steady state population composition before and after a birth rate shock (a) and a longevity shock (b)



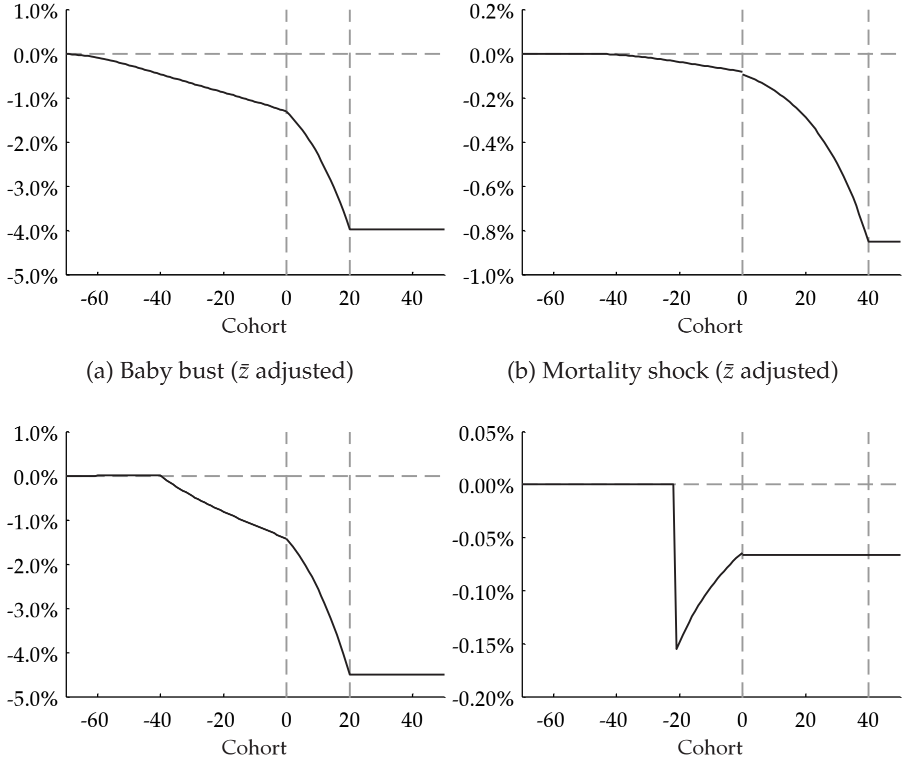
(a) Baby bust (b) Reduced adult mortality

Notes: Mortality process is a Gompertz-Makeham (see Table 2.1 for parameter values), birth rate is 2.36%. Baby bust is a 25% downward jump of the birth rate to 1.78%. Reduced adult mortality is a 50% decrease of μ_1 and 10% decrease of μ_2.

tury following the shock consumption rises due to a strong numerator effect caused by the reduction in the population growth rate. Consumption reaches a peak at the point where the weight of the relatively rich pre-shock cohorts starts to dwindle as a result of mortality. Consumption declines thereafter because post-shock generations have a lower consumption level due to the heavier lumpsum tax burden they are faced with during their lifetimes. The path of asset income, depicted in Figure 4.8(d) shows the strong savings response that occurs during the time period $0 < t < T_R$. Agents anticipate the higher taxes from T_R onward and save more than before the shock. At time T_R, the slope of the asset path is reduced because the tax increase is implemented. Eventually, the last of the relatively large pre-shock cohorts enter retirement and start to dissave so that aggregate assets fall somewhat. The long-run effect of the baby bust is an increase in per capita assets from $\hat{a}_0 = 100.5$ to $\hat{a}_1 = 107.9$.

Figure 4.8(e) illustrates the path of per capita government debt. The baby bust destabilizes the public pension system and leads to a gradual build up of gov-

Figure 4.7. Welfare effects of demographic shocks and the policy reforms



(a) Baby bust ($\bar{z}$ adjusted) (b) Mortality shock ($\bar{z}$ adjusted)

(c) Baby bust (t_L adjusted) (d) Mortality shock (EEA adjusted)

Notes: Mortality process is a Gompertz-Makeham (see Table 2.1 for parameter values), birth rate is 2.36%. Baby bust is a 25% downward jump of the birth rate to 1.78%. Mortality shock is a 50% decrease of μ_1 and 10% decrease of μ_2.

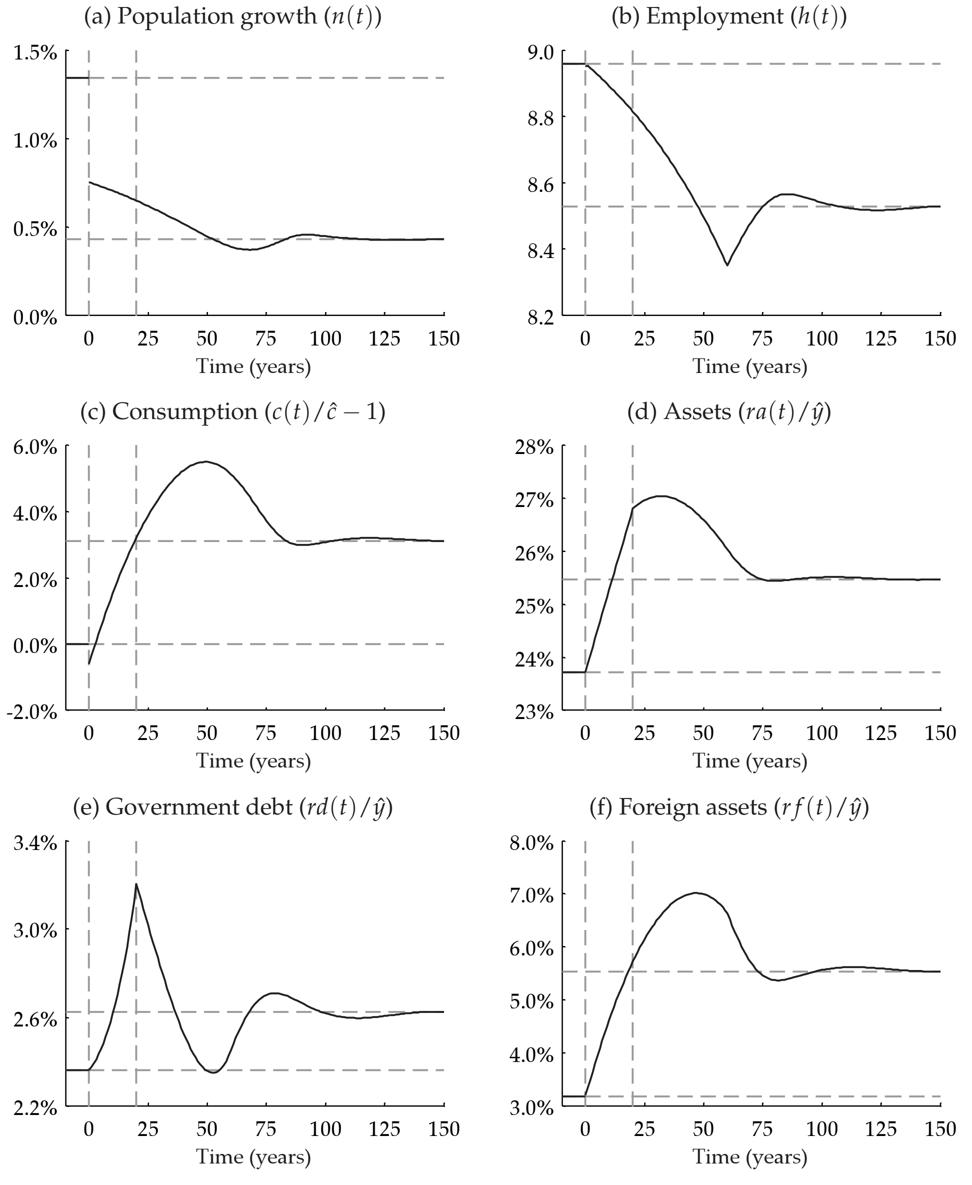
ernment debt in the pre-reform period, $0 < t < T_R$. At time T_R, the lumpsum tax is increased, solvency is restored, and the government can redeem some of its outstanding debt obligations. Interestingly, the post-reform transition path is non-monotonic because the relatively large pre-shock cohorts die and thus stop paying taxes. In the long run, the baby bust leads to an increase in per capita debt from $\hat{d}_0 = 10$ to $\hat{d}_1 = 11.1$.

Finally, in Figure 4.8(e) we plot the adjustment path for net foreign assets. Obviously, since $a(t) = k(t) + d(t) + f(t)$, the path for net foreign assets mirrors that of total assets, the capital stock, and government debt. During the first half century of adjustment, agent's strong savings response (panel (d)) coincides with the accumulation of net foreign assets. Note that at time T_R the government starts to redeem public debt, i.e. both $k(t)$ and $d(t)$ are falling immediately after T_R. Total assets are still rising, however, so it follows that foreign asset accumulation continues quite vigorously even after the tax reform has taken place. The long-run effect of the baby bust consists of an increase in net foreign assets from $\hat{f}_0 = 13.5$ to $\hat{f}_1 = 23.5$. For convenience we summarize the quantitative results of the baby bust in column (2) of Table 4.1.

In Figure 4.7(a) we illustrate the change in welfare experienced by the different generations. To facilitate the interpretation of the effects, we present equivalent-variation (EV) measures expressed in terms of initial wealth level. For pre-shock generations ($v \leq 0$) we compute the change in lifetime utility from the perspective of the shock period ($t = 0$), i.e. we plot the EV-value of $d\bar{\Lambda}(v,0)$ for $v \leq 0$. In contrast, for post-shock generations ($v > 0$), we compute the welfare change from the perspective of their birth date, i.e. we plot the EV-value of $d\bar{\Lambda}(v,v\$v > 0$ in Figure 4.7. The welfare effects of the baby bust are straightforward. All generations lose out as a result of the lumpsum tax increase. For old pre-shock generations the welfare effect is small. These generations have a very short time horizon and for them the tax increase that will occur only at time $T_R = 20$ hardly poses any burden at all. The younger the pre-shock generations are, the heavier the burden of the anticipated tax increase become. Similarly, for post-shock generations the welfare loss becomes larger the closer they are born to the time at which the tax increase takes place. Worst off are those generations born at or after T_R: the welfare loss is about 4 percent of initial wealth for them.

Increased longevity The effect of an embodied longevity shock occurring at time $t = 0$ are visualized in Figures 4.6(b), 4.9 and 4.7(b) (welfare effects). The effect on the mortality rate itself is illustrated Figure 3.2(a) on page 67. The μ_1-parameter of the G-M process is reduced by 50% and the μ_2 parameter by 10%, leading to an increase of the expected lifetime at birth from $R_0(0) = 65.45$ to $R_1(0) = 77.57$ years. Figure 4.6(b) depicts the long-run effect on the age composition of the population. The population pyramid is squeezed for ages up to about 62, but is thickened for higher ages. Figure 4.9(a) shows that the demographic transition, following an embodied longevity shock, is rather slow. Indeed, even 30 years after the shock the population growth rate is virtually at its initial steady-state level. For that reason we assume that the policy reform is implemented 40 years after the longevity shock, i.e. $T_R = 40$ in Figures 4.9 and 4.7(b). Just as for the baby bust, the tax reform has

Figure 4.8. Aggregate effect of a baby bust ($\bar{z}$ balances the budget)



Notes: Mortality process is a Gompertz-Makeham (see Table 2.1 for parameter values), birth rate is 2.36%. Baby bust is a 25% downward jump of the birth rate to 1.78%. Policy change is announced at $t = 0$, implementation is at $t = 20$.

Table 4.1. Initial steady state and long-run effects of demographic shocks

	(1) Initial st. st.	(2) Baby bust $\bar{z}$ adjusts	(3) Mortality $\bar{z}$ adjusts	(4) Baby bust t_L adjusts	(5) Mortality R_E adjusts
μ_1	5.61×10^{-5}	5.61×10^{-5}	2.80×10^{-5}	5.61×10^{-5}	2.80×10^{-5}
μ_2	0.09616	0.09616	**0.0867**	0.09616	**0.0867**
$\Delta(0,0)$	65.5	65.5	81.6	65.5	81.6
b	0.02365	**0.01774**	0.02365	**0.01774**	0.02365
R^*	60	60	60	60	62.13
$\hat{n}$	0.0134	0.0043	0.0163	0.0043	0.0163
T_R		20	40	20	40
t_L	0.100	0.100	0.100	*0.142*	0.100
R_E	60	60	60	60	*61.7*
$\bar{z}$	−0.166	*0.343*	*−0.0573*	−0.166	−0.166
$\hat{d}$	10.0	11.1	−1.4	10.9	−1.2
$\hat{a}$	100.5	107.9	134.9	105.8	134.6
$\widehat{li}$	184.4	165.4	188.1	166.0	191.6
$\hat{h}$	9.0	8.5	8.5	8.5	8.6
$\hat{y}$	21.2	20.2	20.0	20.2	20.3
$\hat{c}$	16.0	16.5	16.4	16.4	16.6
$\hat{i}$	5.7	4.7	5.5	4.7	5.6
$\hat{f}$	13.5	23.5	60.75	21.5	59.7
$\hat{k}$	77.0	73.3	72.7	73.3	73.7
$\hat{c}/\hat{y}$	0.756	0.820	0.820	0.815	0.812
$\hat{i}/\hat{y}$	0.267	0.278	0.280	0.234	0.278

Notes: Exogenous shocks are indicated by bold text, policy instruments by italic text

no effect on the retirement choice, i.e. pre-shock and post-shock agents all retire at the EEA. It follows that post-shock agents expect a much longer retirement period than pre-shock agents do.

The quantitative long-run effects of the longevity shock have been reported in column (3) of Table 4.1. The key features of the transition paths in Figure 4.9 are as follows. In Figure 4.9(b), employment is virtually constant until the tax reform takes place and rises slightly thereafter. People live longer so the inflow into the labour market exceeds the outflow. For $R_E < t < 90$ there is a sharp decrease in employment because the post-shock cohorts start to retire. Because their longevity is higher than for the pre-shock cohorts, the retiring cohorts are relatively large and the outflow from the labour market is huge. In the new steady state, employment is permanently lower because the weight of retirees is larger than before. People live longer but they do not work for a longer period of time. As a result, per capita employment falls.

Figure 4.9(c) depicts the adjustment path for consumption. For $t < R_E$, per capita consumption falls because post-shock newborns consume less than pre-shock newborns, i.e. the negative horizon effect dominates the positive lifetime-income effect. Consumption rises again for $R_E < t < 90$. Pre-shock generations have all passed away but post-shock generations—who live longer lives—have a relatively high consumption level later on in life. In the new steady state per capita consumption is higher as a result. Figure 4.9(d) shows that per capita assets rise during the transition. As is shown in Figure 4.5(a), the individual age profile for assets is increasing up to age $u = R_E$. The longevity shock implies that larger population fractions ultimately reach the EEA and beyond. As result, per capita assets increase.

Figure 4.9(e) shows that public debt is virtually constant for $0 < t < T_R$. This is because the longevity shock takes a long time before it starts to seriously affect the government finances. Were the government to do nothing, debt would ultimately explode, conform stylized fact (SF1). However, our fiscally responsible government slightly increases thelumpsuml tax from T_R onward, thus making room for higher future outlays on pension payments. Figure 4.9(f) shows that net foreign assets rise during the transition.

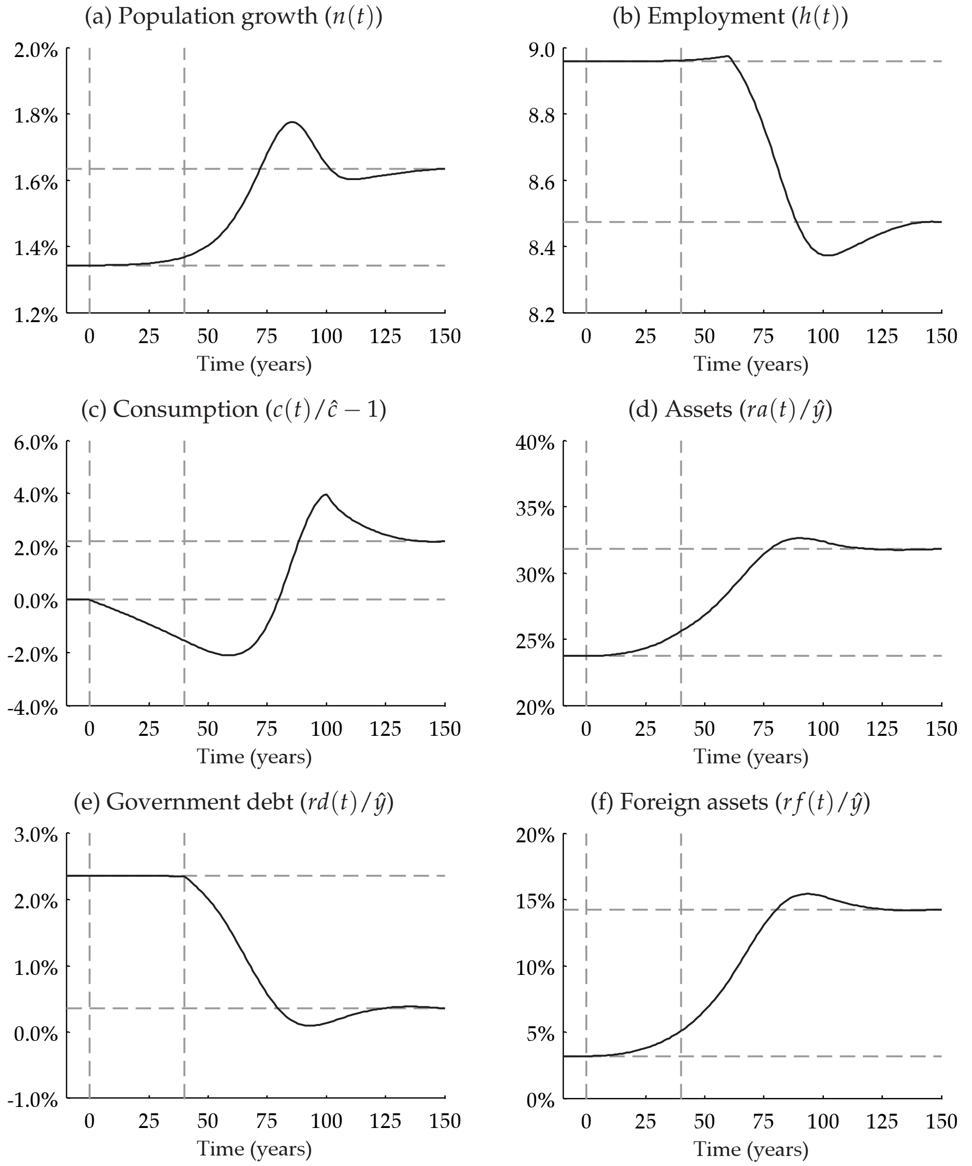
The welfare effects of the longevity shock are visualized in Figure 4.7(b). Just as for the baby bust, (a) all generations lose out as a result of the lumpsum tax increase and (b) welfare losses are increasing in the generations index, v. Because the tax increase is much smaller than for the baby bust scenario, the welfare losses are smaller for all generations.

4.6.2 Pension reform

In this subsection the announced pension reform is assumed to be specific to the type of demographic shock hitting the economy. Indeed, we assume that t_L is increased following a baby bust, whereas the EEA is increased in reaction to increased longevity.

Baby bust The quantitative long-run effects of the baby bust have been reported in column (4) of Table 4.1. A crucial feature of the solution is that the increase in the

Figure 4.9. Aggregate effect of reduced adult mortality ($\bar{z}$ balances the budget)



Notes: Mortality process is a Gompertz-Makeham (see Table 2.1 for parameter values), birth rate is 2.36%. Reduced adult mortality is a 50% decrease of μ_1 and 10% decrease of μ_2. Policy change is announced at $t = 0$, implementation is at $t = 40$.

labour income tax is not sufficiently large to induce individuals to retire at an age beyond the EEA. Indeed, both pre-shock and post-shock agents continue to retire at the EEA, and as a result the labour income tax operates just like a lumpsum tax. The only difference between the two scenarios is that retirees do not have to pay the labour income tax, whereas they do pay the lumpsum tax. For this reason, the welfare profiles are slightly different for the two scenarios. Comparing Figures 4.7(a) and (c) we find that the welfare loss is zero for all pre-shock cohorts older than $R_E - T_R$ in the labour tax scenario. These generations will be retired from the labour force by the time the tax reform is implemented.

Increased longevity In column (5) of Table 4.1, and Figures 4.10 and 4.7(d) we characterize the effects of the longevity shock. The EEA is increased at time T_R in such a way that the government maintains solvency. This implies that the EEA rises from $R_{E0} = 60$ to $R_{E1} = 61.7$. For $0 < t < T_R$ agents continue to retire at age R_{E0} but thereafter agents retire almost a year later in life, at R_{E1}. Comparing Figures 4.9 and 4.10 we find that the main difference between thelumpsuml and EEA scenarios is found in the adjustment path for employment (panel (b) in these figures). In Figure 4.10(b) there is a sharp increase in employment at time T_R because nobody retires at that time. Some pre-shock generations delay their retirement by 0.9 years. Since new cohorts continue to enter the labour market, employment rises sharply. The remainder of the adjustment path is similar as for the lumpsum tax case: there is a sharp decline at $t = R_{E1}$ as the first of the post-shock cohorts retire.

Comparing Figures 4.7(b) and (d) we find that the welfare effects are rather different for the two scenarios. Five groups of cohorts can be identified in Figure 4.7(d). Group 1 consists of cohorts whose generations index satisfies $v < T_R - R_{E1}$. These cohorts have either already retired at the time of the shock ($t = 0$) or will be just old enough at the time of the policy reform (T_R) to retire at that time and receive benefits immediately. This means that at time $t = T_R$ such agents must be at least R_{E1} years of age. For these generations there is no welfare loss as a result of the anticipated EEA perform. They continue to retire at age R_{E0}.

Groups 2 and 3 are cohorts for which $T_R - R_{E1} < v < T_R - R_{E0}$. Agents in this group face a choice. Option 1: they can either retire early at age R_{E0} (the old EEA) and be without income for a brief period of time because they retire too early under the new regime. Option 2: they can adjust their planned retirement age from R_{E0} to R_{E1}. It turns out that the oldest generations will choose option 1 whereas the youngest generations will choose option 2, with the pivotal generation index being at $v^* = -20.5$. Agents in both groups experience a welfare loss as a result of the reform. Interestingly, the welfare loss is increasing in v for $T_R - R_{E1} < v < v^*$ but decreasing in v for $v^* < v < T_R - R_{E0}$.

Group 4 consists of cohorts for which $T_R - R_{E0} < v < 0$. People in this group did not have any real choice. At time T_R they are too young to retire with benefits under the under the old regime and thus have to retire at age R_{E1}. Their delayed pension is compensated partially by higher a level of lifetime income because they have a longer working life. The welfare loss for agents in this group is decreasing in v.

Finally, group 5 consists of post-shock cohorts for which $v > 0$. Agents in this group are all affected equally. They all choose the retirement age R_{E1} and they all face the same initial conditions in life.

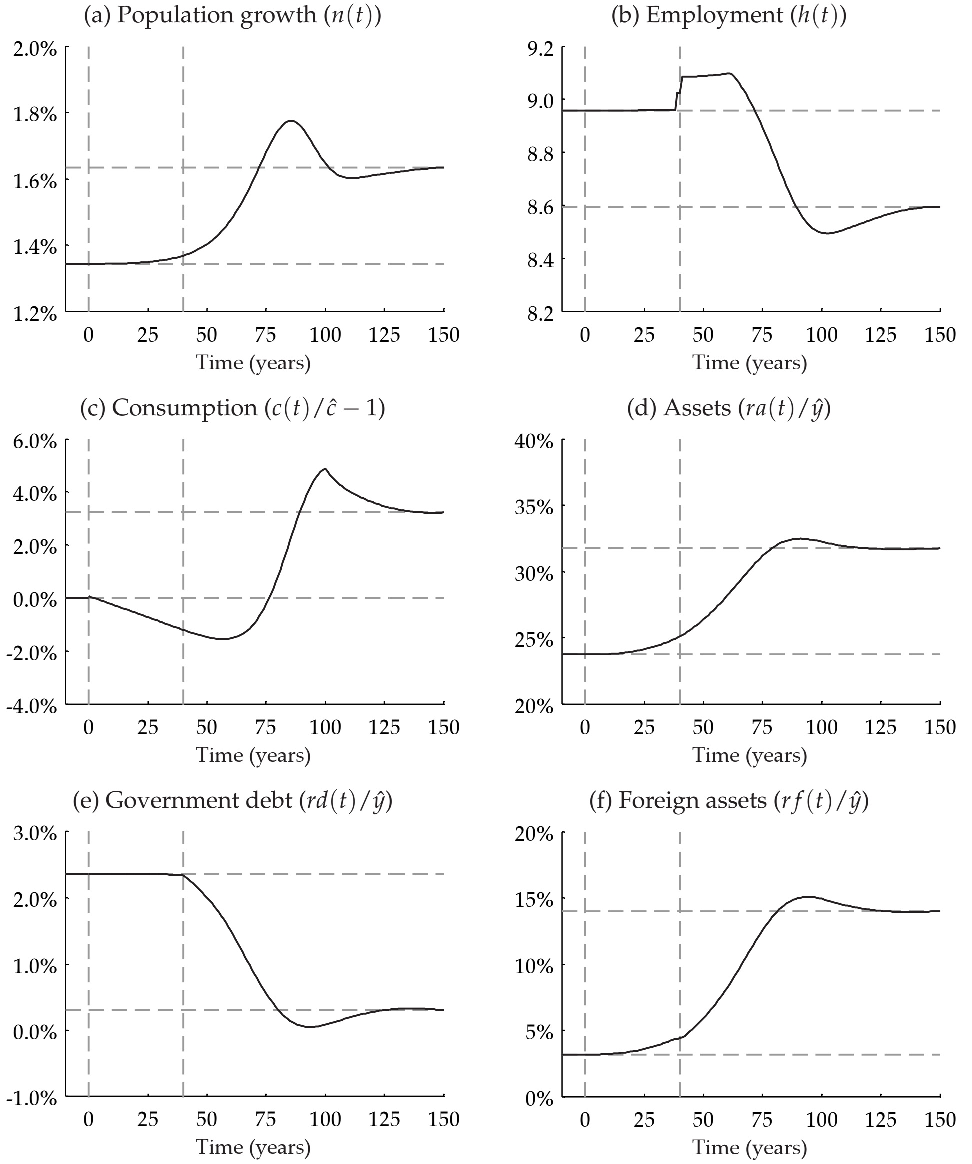
4.6.3 Discussion

The key findings of this section are as follows. First, although both a baby bust and a longevity boost have an adverse effect on the government's budget, there is a striking difference in the speed with which such effects become apparent. Indeed, for the baby bust the adverse effects show up immediately. Government debt starts to rise sharply immediately after the shock because the flow of tax payers dwindles. In contrast, for the longevity shock it takes a very long time before any effect on the government's balances can be observed.

Second, even though we simulated very large demographic changes, wealth effects are simply too weak to get agents to move out of the kink and to postpone retirement beyond the EEA. For a realistic calibration, the implicit tax rates are rather high, ranging from 11.1% until age 60, jumping to 62.8% at that age, and subsequently rising to 67% at age 70. The kink in the lifetime income profile acts as a kind of early retirement trap. Changes in the lumpsum tax or the labour income tax are insufficiently powerful instruments to get agents out of the trap. The welfare costs of the tax increase are non-trivial. Indeed, our simulations show that post-shock agents experience a welfare loss that is the equivalent of more than 4% of initial wealth!

Third, an increase in the EEA itself constitutes a rather good policy measure. By increasing the EEA, the kink in the lifetime income profile is shifted to right, and agents retire later on in life despite the existence of high implicit tax rates. We show that the welfare effects of the EEA increase are tiny: post-shock agents experience a

Figure 4.10. Aggregate effect of reduced adult mortality (EEA balances the budget)



Notes: Mortality process is a Gompertz-Makeham (see Table 2.1 for parameter values), birth rate is 2.36%. Reduced adult mortality is a 50% decrease of μ_1 and 10% decrease of μ_2. Policy change is announced at $t = 0$, implementation is at $t = 40$.

welfare loss that is the equivalent of less than 0.1% of initial wealth as a result of the EEA increase! Agents not only work longer but they also get a higher consumption level as a result.

4.7 Conclusions

In this chapter we have studied the microeconomic and macroeconomic effects of ageing in the context of a small open economy populated by disconnected generations of finitely-lived agents facing age-dependent mortality and constant factor prices. From a policy perspective, our main finding is as follows. Most actual pension systems induce a kink in the lifetime income function which acts as an early retirement trap. Fiscal changes are not potent enough to get individuals out of the trap. Increasing the early entitlement age appears to be a low cost policy measure to counteract the adverse effects of the various demographic shocks.

Our analysis is subject to a number of potentially important limitations, some of which we wish to address in the near future. First, in this chapter the age profile of labour efficiency is exogenous, i.e. there is no endogenous human capital accumulation decision. A possible solution is to combine the models of this chapter Chapter 3. This is quite feasible, but it will blur the two results and this makes the interpretation of effects of ageing on various macroeconomic variables harder.

Second, we only consider once-off changes in the demographic processes. In reality, demographic changes occur only gradually over time. The main complication lies in the calculation of the population dynamics, i.e. the population growth rate. The macroeconomic block of the model (individual optimisation, production, saving) remains the same since the interest rate is constant in the small open economy. The main difference with the current stepwise shock is that the transition periods are longer and the costs are spread out over more generations.

Third, we have focused attention of mortality and have ignored the equally important issue of morbidity. One of the main functions of a retirement system is to support people that are not capable of working due to old age related diseases. Identification problems arise if health is not perfectly observable.The risk exists that either the retirement system becomes too expensive because too many people use it, while they are perfectly capable of working, or that people that cannot work are kept out of the system.

4.A Years-of-retirement transformation

Burbidge and Robb (1980, p. 424) use a linear space transformation. Instead of using the retirement age directly, they reformulate their model in terms of years or retirement, $T - R$, where T is the fixed planning horizon. Two things are worth noting. First, their linear transformation does not solve the problem of non-convex indifference curves—see below. Second, in our model, T is a stochastic variable and the expected planning horizon at birth is given by $\Delta(0,0)$, where $\Delta(u,\lambda)$ is defined in (2.12) in Chapter 2. Transforming our model in terms of expected years of retirement (from the perspective of birth), $\Delta(0,0) - R$ suffers from the same defects.

The basic point is that a linear transformation does not guarantee well-behaved indifference curves. This can easily be demonstrated in the context of our model. The steady-state concentrated utility function is given by (4.23) in the text. We write it as $\bar{\Lambda}(u, \bar{l}i, R)$ but hold u constant. To determine the slope and curvature of the indifference curves, we need the following building blocks:

$$\begin{aligned}
\bar{\Lambda}_{\bar{l}i} &\equiv \frac{\partial \bar{\Lambda}}{\partial \bar{l}i} = \left[\frac{\bar{a}(u) + \bar{l}i}{\Delta(u, r^*)}\right]^{-1/\sigma} > 0, \\
\bar{\Lambda}_R &\equiv \frac{\partial \bar{\Lambda}}{\partial R} = -D(R)e^{\theta\cdot(u-R)+M(u)-M(R)} < 0, \\
\bar{\Lambda}_{\bar{l}i,\bar{l}i} &\equiv \frac{\partial^2 \bar{\Lambda}}{\partial \bar{l}i^2} = -\frac{1}{\sigma\cdot\left[\bar{a}(u) + \bar{l}i\right]}\bar{\Lambda}_{li} < 0, \\
\bar{\Lambda}_{\bar{l}i,R} &\equiv \frac{\partial^2 \bar{\Lambda}}{\partial \bar{l}i \partial R} = 0, \\
\bar{\Lambda}_{R,R} &\equiv \frac{\partial^2 \bar{\Lambda}}{\partial R^2} = \left[\frac{D'(R)}{D(R)} - \theta - m(R)\right]\bar{\Lambda}_R \gtrless 0.
\end{aligned}$$

The slope of the indifference curve in $(R, \bar{l}i)$-space is

$$\left.\frac{d\bar{l}i}{dR}\right|_{\bar{\Lambda}_0} \equiv -\frac{\bar{\Lambda}_R}{\bar{\Lambda}_{\bar{l}i}} = D(R)e^{\theta\cdot(u-R)+M(u)-M(R)}\left[\frac{\bar{a}(u) + \bar{l}i}{\Delta(u, r^*)}\right]^{1/\sigma} > 0.$$

Hence, the indifference curves are always upward sloping.

To compute the curvature of the indifference curve in $(R, \bar{l}i)$-space we must take into account the dependency of $\bar{l}i$ on R along a given indifference curve. After some manipulation, we find:

$$\left.\frac{d^2\bar{l}i}{dR^2}\right|_{\bar{\Lambda}_0} \equiv -\frac{d\left(\bar{\Lambda}_R/\bar{\Lambda}_{\bar{l}i}\right)}{dR} = -\frac{1}{\bar{\Lambda}_{\bar{l}i}^2}\left[\bar{\Lambda}_{\bar{l}i}\bar{\Lambda}_{R,R} - \bar{\Lambda}_R\bar{\Lambda}_{\bar{l}i,\bar{l}i}\left.\frac{d\bar{l}i}{dR}\right|_{\bar{\Lambda}_0}\right]$$

$$= \left.\frac{d\bar{l}i}{dR}\right|_{\bar{\Lambda}_0} \left[\frac{1}{\sigma \cdot \left[\bar{a}(u) + \bar{l}i\right]} \left.\frac{d\bar{l}i}{dR}\right|_{\bar{\Lambda}_0} + \frac{D'(R)}{D(R)} - \theta - m(R) \right] \gtrless 0.$$

Equation (4.7) is a rather intractable expression, and the sign is ambiguous in general. However, numerical simulations reveal that for realistic parameter values the indifference curves are either concave in R or S-shaped (i.e., convex for small R and concave for large R). Similar results can be derived for the specification used by Burbidge and Robb (1980, p. 425), so their assumption that the indifference curves are convex in the relevant region is problematic.

The key point to note is that a linear transformation of the retirement age is unhelpful. Hence, transforming our model in terms of expected years of retirement, $\Delta(0,0) - R$, is not useful either.

4.B Data

In Section 4.4 of we use data on replacement rates and implicit tax rates that were gathered from the various chapters in Gruber and Wise (1999). For convenience we present an overview of these data here. The figures refer to data taken from Tables 1.4, 2.2, 3.5, 5.4, 6.1, 7.1, 8.7, 9.2, 10.4, and 11.1. Note that we report the retirement age in the first column of Tables 4.B.1 and 4.B.2. In contrast, Gruber and Wise (1999) report the last age of active employment. Our entries for age 60 are thus equivalent to their entries for age 59.

Table 4.B.1. Replacement rates in nine OECD countries

Age	Belgium	Canada	France	Italy	Japan	Neth's	Spain	UK	US
55				0.726					
56				0.744					
57				0.761					
58				0.780					
59	0.749	0.182	0.920	0.798	0.552	0.910	0.590		
60	0.771	0.202	0.910	0.799	0.800	0.906	0.661		
61	0.794	0.217	0.920	0.804	0.799	0.900	0.730		0.403
62	0.817	0.245	0.910	0.805	0.802	0.902	0.816		0.440
63	0.839	0.270	0.920	0.805	0.801	0.892	0.895		0.476
64	0.863	0.508	0.920	0.809	0.438	0.909	0.996		0.703
65	0.874	0.518	0.930	0.809	0.549	0.909	0.998	0.464	0.749
66	0.882	0.527	0.940	0.809	0.547	0.909	0.996	0.491	0.798
67	0.890	0.850	0.950	0.809	0.716	0.909	0.988	0.519	0.845
68	0.898	0.881	0.960	0.809	0.608	0.909	0.981	0.549	0.872
69	0.905	0.914	0.960	0.809	0.607	0.909	0.973	0.581	0.898

Source: Gruber and Wise (1999)

Table 4.B.2. Implicit tax rates in nine OECD countries

Age	Belgium	Canada	France	Italy	Japan	Neth's	Spain	UK	US
55	-0.129	-0.049	-0.910	0.245	-0.195	0.687	0.216	0.020	-0.022
56	-0.134	0.003	-0.970	0.308	-0.202	0.650	0.108	0.010	0.046
57	-0.145	0.037	-0.460	0.338	-0.106	0.612	0.153	0.030	0.060
58	-0.148	0.038	0.040	0.372	-0.112	0.578	0.362	0.030	0.069
59	-0.157	0.040	0.050	0.401	-0.138	-3.777	0.286	0.030	0.072
60	0.496	0.063	0.670	0.697	0.338	1.410	-0.149	0.030	0.071
61	0.497	0.066	0.600	0.711	0.340	1.384	-0.120	0.020	0.064
62	0.491	0.064	0.630	0.718	0.342	1.339	-0.112	0.020	-0.028
63	0.489	0.071	0.560	0.729	0.340	1.280	0.046	0.020	-0.005
64	0.473	0.169	0.560	0.746	0.204	1.222	0.160	0.020	0.031
65	0.529	0.285	0.520	0.756	0.000	0.357	0.757	0.010	0.188
66	0.519	0.323	0.480	0.772	0.000	0.347	0.767	0.020	0.225
67	0.476	0.259	0.460	0.787	0.000	0.337	0.777	0.030	0.269
68	0.463	0.203	0.450	0.803	0.000	0.327	0.741	0.050	0.439
69	0.440	0.229	0.430	0.818	0.000	0.315	0.705	0.070	0.455

Source: Gruber and Wise (1999)