Asset liability management for pension funds using multistage mixed-integer stochastic programming

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To Barbara

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Preface

Towards the end of my study econometrics I had the feeling that I stood at the crossroads: how would I use the acquired knowledge? The wish to do scientific research became stronger and stronger. Therefore, I discussed the idea to become a Ph.D. student with prof. dr. W.K. Klein Haneveld. During that talk, it turned out that he was the right person at the right time at the right place, because he had already the desire to supervise a Ph.D. student for research on Stochastic Linear Programming for Asset Liability Management.

As I understood from many people, a Ph.D. path is characterized by ups and downs. By now, I also belong to the group who subscribes to that viewpoint.

This Ph.D. thesis is accomplished under the supervision of prof. dr. W.K. Klein Haneveld and dr. M.H. van der Vlerk. They have learned me a lot during the four years I worked with them. Especially the structured way of thinking of prof. dr. W.K. Klein Haneveld made a profound impression on me. Furthermore, I would like to thank prof. dr. R.A.H. van der Meer, dr. H.A. Klein Haneveld, and ir. H. Stam for discussions regarding some modeling aspects.

I would also thank the people of the University of Groningen who were directly or indirectly involved with the realization of this Ph.D. thesis. Especially, I would thank dr. D.P. van Donk.

I will also thank my family and friends. They were interested in my research and supported me where possible. I am especially indebted to my wife Barbara. She gave me room to finish this dissertation, even directly after the birth of our daughter Esther. Therefore, I dedicate this thesis to Barbara.

Zuidhorn, July 2005.

Contents

List of notations

General

Time is denoted by an index t. All periods considered are periods of one year. Year t is the time span from time $t-1$ to time t. The initial decision moment is denoted by $t=0$, and the horizon is denoted by time T. The set of all decision moments is denoted by $\mathcal{T} := \{0,\ldots,T\}$. In addition we have the subsets $\mathcal{T}_0 := \{0,\ldots,T-1\}$, and $\mathcal{T}_1 := \{1,\ldots,T\}$.

Scenarios are denoted by a superscript s. The total number of scenarios is given by S. The set of all scenarios is denoted by $\mathcal{S} := \{1,\ldots,S\}$. Moreover, i_t denotes the index of the branch at a node at time t. At time t, the state (of the world) is indicated as (t,s).

The total number of asset classes is N, and index j refers to asset class j. Its values 1,. . . ,4 refer to stocks, bonds, real estate, and cash, respectively.
Moreover, all financial quantities are denominated in million euros.

ω_t is the vector of random parameters whose values are revealed in year t, ω_t^s is its value in scenario s. In addition, x_t^s denotes the decision vector at time t in scenario s.

In addition, we have the following:

$\mathbb{R}$	Set of real numbers.
e	$e = 2.71828\ldots$
$\log$	Natural logarithm.
$\mathbb{E}$	Expectation operator.
$P(.)$	Probability operator.
$\min$	Minimum operator.
$\sum$	Summation operator.
$\prod$	Multiplication operator.
Δ	Symbol which denotes a change.
$\lvert x \rvert$	Absolute value of x.
$(x)^+$	$\max\{0,x\}$.
$(x)^-$	$\max\{0,-x\}$.
p_t^s	Probability of scenario s at time t.
p^s	Probability of scenario s.
M	Sufficiently large number ('big M').
$\mathcal{N}(\mu,\sigma^2)$	Normal distribution with mean μ and variance σ^2.

$branch_t$	Number of branches from each state at time t.
Ξ_t	Cardinality of the bundle of scenarios through any node at time t.
$\mathcal{K}_t^s$	Set which contains those $s' \in \mathcal{S}$, such that the pension fund may end up in state (T, s'), given state (t, s).
$\mathcal{K}_t^s(q)$	Set which contains all $s' \in \mathcal{S}$, such that state (q, s') can be reached with strict positive probability, and no $s'' < s'$ exists with the same history up to and including time t, given time t.
$\mathcal{S}_t$	Set which contains those $s' \in \mathcal{S}$, such that no $s'' < s'$ exists with the same history up to and including time t, given time t.
$I(1)$	Integrated process of the first order.

Decision variables

Continuous decision variables

Assets

A_t^s	Value of the assets at time t in scenario s.
X_{jt}^s	Value of investments in asset class j, at the beginning of year t in scenario s.
XI_{jt}^s	Value of assets in class j bought at time t in scenario s.
XD_{jt}^s	Value of assets in class j sold at time t in scenario s.

Liabilities

B_t^s	Benefit payments in year t in scenario s.
L_t^s	Value of the liabilities at time t in scenario s.

Underfunding and overfunding

Z_t^s	Remedial contribution by the sponsor at time t in scenario s, used to restore the level of the funding ratio α.
ZI_t^s	Remedial contribution by the sponsor at time t in scenario s, as far as it surpasses the lower bound τW_t^s.
DZ_t^s	Direct cash flow by the sponsor, because of a funding ratio below the level θ.
V_t^s	Restitution to the sponsor at time t in scenario s.
$Sur\alpha_t^s$	Surplus with respect to the level α at time t in scenario s.
$Sho\alpha_t^s$	Shortage with respect to the level α at time t in scenario s.
$Sur\Lambda_T^s$	Surplus with respect to the level Λ at time T in scenario s.
$Sho\Lambda_T^s$	Shortage with respect to the level Λ at time T in scenario s.

Contribution rate

c_t^s	Contribution rate for year $t+1$, determined in state (t, s).
ci_t^s	Increase in the contribution rate (with respect to c_{t-1}^s) greater than ρ at time t in scenario s.
cd_t^s	Decrease in the contribution rate (with respect to c_{t-1}^s) greater than η at time t in scenario s.
cdu_t^s	Deviation of the contribution rate from its upper bound.

Binary decision variables

u_t^s	Binary variable which indicates whether the funding ratio is less than α ($u_t^s = 1$) or not ($u_t^s = 0$) at time t in scenario s.
z_t^s	Binary variable which indicates whether a remedial contribution is made by the sponsor ($z_t^s = 1$) or not ($z_t^s = 0$) at time t in scenario s.
o_t^s	Binary variable which indicates whether the funding ratio is higher than β ($o_t^s = 1$) or not ($o_t^s = 0$) at time t in scenario s.
v_t^s	Binary variable which indicates whether a restitution is made to the sponsor ($v_t^s = 1$) or not ($v_t^s = 0$) at time t in scenario s.
m_t^s	Binary variable which indicates whether or not the participants of the fund receive full compensation for the increase in the general wage level in year t in scenario s.
l_t^s	Binary variable which indicates whether the participants of the fund receive full compensation for the increase in the general wage level up to and including year t in scenario s.

Parameters

Bounds with respect to funding ratios

α	Minimum required level of the funding ratio considered in mid-term risk constraints.
θ	Level of the funding ratio which is used to judge whether the sponsor has to make an immediate payment to the fund.
β	Level of the funding ratio considered for restitutions.
Λ	Minimum desired level of the funding ratio at the horizon.

Counting years

a	Number of consecutive years after which the sponsor has to make a remedial contribution if in these years the funding ratio is less than α.
b	Number of consecutive years after which the fund has to make a restitution to the sponsor if in these years the funding ratio is higher than β.

Asset allocation

A_0	Value of the assets at time 0.
X_{j0}	Initial investment in asset class j.
$\underline{f}_j$	Lower bound on the fraction of asset class j in the asset portfolio.
$\overline{f}_j$	Upper bound on the fraction of asset class j in the asset portfolio.
k_j	Proportional transaction cost for asset class j.
u_i^s	Indicator whether in year i the funding ratio was less than α ($u_i^s = 1$) or not ($u_i^s = 0$) in scenario s, $i = 1-a, 2-a, \ldots, 0$.
o_i^s	Indicator whether in year i the funding ratio was higher than β ($o_i^s = 1$) or not ($o_i^s = 0$) in scenario s, $i = 1-b, \ldots, 0$.

Contribution rate

c_{-1}	Contribution rate in year 0.
$\underline{c}$	Lower bound on the contribution rate.
$\overline{c}$	Upper bound on the contribution rate.
c^*	Minimum required contribution rate in case of a remedial contribution.
ρ	Maximum increase in the contribution rate between two consecutive years such that no penalties are incurred.
η	Maximum decrease in the contribution rate between two consecutive years such that no penalties are incurred.

Large remedial contributions

τ	Bound on a remedial contribution as a fraction of the liabilities such that no additional penalties ζ_{ZI} are incurred.

Risk

ψ	Fraction of the liabilities, such that ψL_t^s gives an upper bound on the maximum allowed expected next year's shortage.
ϕ	Prescribed probability in long-term chance constraints.
ϕ_t	Minimum required reliability corresponding to decisions at time t, used in one-year chance constraints.

Fixed costs

λ_u	Fixed costs associated with underfunding with respect to the level α.
λ_z	Fixed costs associated with a remedial contribution from the sponsor to the fund.
λ_o	Fixed benefits associated with overfunding with respect to the level β ($\lambda_0 \leq 0$).
λ_v	Fixed benefits associated with a restitution ($\lambda_v \leq 0$).
λ_m	Fixed costs associated with not giving full compensation for the increase in the general wage level in a year.

Unit costs

ζ_{ci}	Unit cost associated with an increase in the contribution rate in two consecutive years greater than ρ.
ζ_{cd}	Unit cost associated with a decrease in the contribution rate in two consecutive years greater than η.
ζ_Z	Unit cost associated with a remedial contribution Z_t^s.
ζ_{ZI}	Additional unit cost associated with a remedial contribution above the threshold value τW_t.
ζ_{DZ}	Unit cost associated with a direct remedial contribution DZ_t^s.
ζ_V	Unit benefit associated with a restitution ($\zeta_V \leq 0$).
ζ_L	Unit cost associated with a value of the liabilities below its upper bound.
$\zeta_{\Lambda d}$	Unit cost associated with a shortage with respect to the level Λ at the horizon.
$\zeta_{\Lambda i}$	Unit benefit associated with a surplus with respect to the level Λ at the horizon ($\zeta_{\Lambda i} \leq 0$).

Scenario tree

r_{jt}^s	Return (expressed as a fraction) on asset class j in year t in scenario s.
w_t^s	Change (expressed as a fraction) in the general wage level in year t in scenario s.
$\underline{L}_t^s$	Lower bound on the value of the liabilities at time t in scenario s.
$\overline{L}_t^s$	Upper bound on the value of the liabilities at time t in scenario s.
$\underline{B}_t^s$	Lower bound on the value of the benefit payments in year t in scenario s.
$\overline{B}_t^s$	Upper bound on the value of the benefit payments in year t in scenario s.

φ_t^s	Change in the liabilities from time $t-1$ to time t, not due to changes in the general wage level.
$PSC_t^s(q)$	Pension spot curve at time t in scenario s for discounting expected benefit payments which are due q years from year t.
φ_t^s	Percentage change in the liabilities in year $t+1$ in scenario s.
W_t^s	Total level of the pensionable wages of the active participants in year t in scenario s.
γ_t^s	Discount factor associated with cash flows at time t in scenario s.

Heuristic

$NCP\alpha_t^s$	Net capital position with respect to the level α at time t in scenario s.
$\Delta A_t^s(subtree)$	Level of change in payment in state (t,s), which affects the asset values in the subtree of (t,s).

Scenario generation

$\tilde{r}$	Continuously compounded return or log return.
ν_j	Autocorrelation coefficient for the returns in asset class j, $j = 1, 3$.
χ	Parameter in the error-correction model, which describes the long-run relationship between r_4 and w.
ϵ_{4t}	Disturbance term in the error-correction model, associated with the returns on the bank account.
ϵ_{wt}	Disturbance term in the error-correction model, associated with the change in the general wage level.
ϑ_1	Parameter in the error-correction model which serves as a measure for the speed of adjustments.
ϑ_2	Parameter in the error-correction model which serves as a measure for the speed of adjustments.
$\sigma_{\epsilon 4}^2$	Variance of the disturbance terms of the bank account in the error-correction model.
$\sigma_{\epsilon w}^2$	Variance of the disturbance terms of the changes in the general wage level in the error-correction model.
$y_t^s(q)$	Yield corresponding to a risk-free zero-coupon bond maturing q years from time t, given the current state (t,s).
a_1	Difference between the yield on bonds with the longest and shortest term to maturity.
a_2	Parameter which controls the shape of the yield curve.
a_3	Parameter which controls the shape of the yield curve.
a_{4t}^s	Yield on bonds with the longest terms to maturity in state (t,s).
$C_t^s(q)$	Coupon payments of the bond portfolio q years from time t, given state (t,s).

$PrB_t^s(q)$	Principal payments of the bond portfolio q years from time t, given state (t, s).
PB_t^s	Value of the bond portfolio in state (t, s).
PS_t^s	Value of the stock portfolio in state (t, s).
μ_1	Mean of simple net stock return.
σ_1	Standard deviation of simple net stock return.
$\tilde{\mu}_1$	Mean of continuously compounded stock return.
$\tilde{\sigma}_1$	Standard deviation of compounded stock return.
$\varrho_{j,t+1}$	Innovation in a GARCH model for asset class j, $j = 1, 3$.
d_{j1}	Parameter in a GARCH model, which denotes a constant term in next year's volatility for asset class j, $j = 1, 3$.
h_{j1}	Measure of the extent to which a volatility shock in one year feeds through into next year's volatility in a GARCH model for asset class j, $j = 1, 3$.
h_{j2}	Parameter which serves as a measure of the rate at which previous year's volatility shocks feed through into next year's volatility in a GARCH model for asset class j, $j = 1, 3$.
D_{t+q}	Dividend payment q years ahead, given time t.
R_j	Internal rate of return on asset class j, $j = 1, 3$.
g_j	Growth rate of dividend payments for asset class j, $j = 1, 3$.
$earp_t^s$	Ex-ante risk premium at time t in scenario s.
$\underline{earp}$	Lower bound on the ex-ante risk premium.
$\overline{earp}$	Upper bound on the ex-ante risk premium.
δ_{jt}^s	Indicator which denotes whether r_j outperforms r_2 from time 0 to time t in scenario s or not for asset class j, $j = 1, 3$.
$P_t^*(r_j \geq r_2)$	Historical probability of outperformance of returns of asset class j over bond returns over a period of t years.
π_t^s	Risk neutral probability in state (t, s).
$B_t^*(q)$	Expected benefit payments q years ahead, given time t.

Output variables

F_t^s	Funding ratio at time t in scenario s.
f_{jt}^s	Fraction of asset class j in the portfolio at time t in scenario s.
I_t^s	Degree of change of indexation at time t in scenario s.
r_{pt}^s	Return on the portfolio in year t in scenario s.

Definitions of the output variables

$$F_t^s := \frac{A_t^s}{L_t^s}$$

$$f_{jt}^s := \frac{X_{jt}^s}{\sum_{i=1}^{N} X_{it}^s}$$

$$r^s_{pt} := \sum_{j=1}^{N} f^s_{jt} r^s_{jt}$$

$$I^s_t := \frac{L^s_t}{(1+\varphi^s_t)(1+w^s_t)L^s_{t-1}}.$$

Chapter 1

Introduction and Summary

Recently, my parents received a letter from the pension fund from which they get their pension payments every month. Good news. The benefit rights of this fund are fully indexed with respect to increases in the general wage level. My parents belong to the lucky ones that received a letter with good news: not every pension fund compensates the rights of retired people for increases in prices or wages over the previous year. Not only retired people may suffer from weak financial positions of pension funds, also some active participants are far from happy. They have to pay a larger fraction of their pensionable salary to the fund. This means less purchasing power for them. Moreover, also the supervisor shows his teeth: pension funds with very low funding ratios (ratios of the values of the assets and the liabilities) have to take corrective actions, such as an increase of the contribution rate, to strengthen the financial position of the fund. Moreover, funds that invest a lot in stocks, need additional buffers in order to make the pension fund less vulnerable to unfavorable financial developments.

What is the reason of the recent low funding ratios of many funds? Bad management? Too high restitutions a few years ago? Too low contribution rates? Too optimistic future expectations?

At least one thing is certainly true: the financial positions of almost all funds weakened, because of decreasing stock prices in the last years. From 1995 to September 2000, stock returns were exceptionally high, see for example Figure 1.1, where the development of the broadly diversified MSCI World-index is presented. These data are derived from Datastream [20]. Because daily data were only available from July 1998, the first part of the figure looks less smooth than the latter part. Before July 1998, monthly data were used.

The value of this index increased from 458 (on January 1, 1995) to 1160 (in March 2000). This means a return (even without dividends) of more than 150 percent in 5 years and 3 months. Encouraged by such very high returns, many pension funds invested an increasing fraction of their assets in stocks, see for example the website of the Dutch central bureau of statistics, CBS [16].

It is not surprising that funding ratios of pension funds increased in those years. Some funds had generated such high reserves, that participants and sponsors had premium holidays. This means that active participants did not pay regular contri-

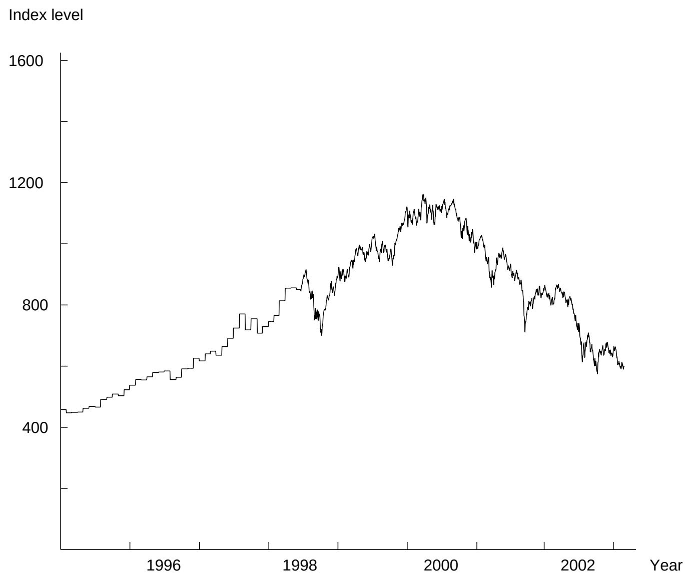

Figure 1.1: MSCI World-index from January 1, 1995 to March 1, 2003.

butions to the funds, whereas pension rights were still built-up. Moreover, some funds even made restitutions to their sponsor.

As we can also see in Figure 1.1, the MSCI World-index decreased gradually from March 2000 on. On March 1, 2003, its value was 598. This means that the index lost approximately 50 percent of its value in 3 years. Other stock indices showed similar performances, see Table 1.1. These data are also derived from Datastream [20]. From this table, it is clear that in all parts of the world stock prices declined.

Country	Index	Value on Jan. 1, 1995	Value on Sept. 1, 2000	Value on March 1, 2003	% change from 1995 to 2000	% change from 2000 to 2003
Great Britain	FTSE 100	3,062	6,672	3,553	+118	-47
The Netherlands	AEX	187	692	257	+270	-63
Switzerland	SMI	2,628	8,234	4,148	+213	-50
United States	Dow Jones	3,834	11,215	7,837	+193	-30
United States	Nasdaq	751	4,234	1,320	+464	-69
Japan	Nikkei	19,723	16,861	8,363	-15	-50
Hong Kong	Hang Seng	8,188	17,210	9,111	+110	-47

Table 1.1: Developments in stock indices in different parts of the world.

The combination of high fractions of assets invested in stocks and very low returns on them, eroded the financial position of many funds. The pension funds in The Netherlands lost approximately €20 billion in 2002, see the website of CBS [16]. The financial position of Dutch pension funds deteriorated so fast, that it attracted a lot of attention in the press.

To get an indication of recent development of the financial position of pension funds in The Netherlands, Figure 1.2 is added. This figure is based on a similar figure, which appeared in NRC Handelsblad [95]. The numbers in this figure are based on those of the supervisor of Dutch pension funds, PVK.

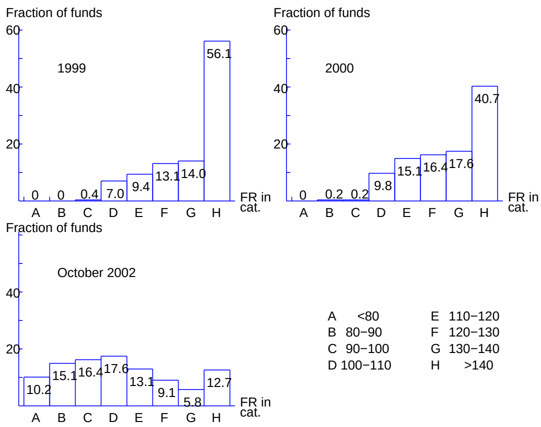

Figure 1.2: The development of the funding ratio (FR) of the Dutch pension funds in recent years.

Also according to NRC Handelsblad [96], in October 2002, 300 pension funds were underfunded. Together, these 300 funds had a shortage of €23 billion. In 2003 these problems were not solved. De Volkskrant [98] wrote that the funding ratio of a quarter of the 'bedrijfstakpensioenfondsen' (pension funds related to companies in the same branch of industry) was too low.

From this brief financial history it is clear that pension funds face one major source of risk: uncertainties with respect to future developments of financial markets. One can imagine that there are many more uncertainties the board of pension funds have to deal with. Some of them are described in Section 1.2.4.

To manage risks (and to better understand them), pension funds and their advisors have developed financial models with which they compute the impact of future capital market developments on their financial position. These so-called ALM models focus on the decision making problem of pension funds. In this thesis, we present the ALM model we have developed. In this model detailed risk measures are incorporated. Many of such ingredients were not considered before in ALM models described in the literature. We consider not only short-run risks (by which we mean unfavorable developments which will be revealed within one year), also risks associated with longer time periods are taken into account. Moreover, flexibility plays a key role: the board of the fund can periodically change its decisions with respect to investments, contributions, and indexation. In this way, they can react on observed developments of financial markets and on developments associated with the participants of the fund. In order to describe the uncertainties in the model (like future returns on assets), we have developed a *scenario generator* to find future developments of all uncertain parameters.

Our ALM model is an optimization model, so its aim is to specify the conditions that the decision variables have to satisfy, together with the consequences by which their numerical values are judged. That is, from a mathematical point of view, such a model specifies precisely the set of feasible solutions by means of constraints, and the subset of optimal solutions by means of the objective function. Of course, for practical applications, model building should be followed by numerical calculation of solutions. Indeed, there exist algorithms to find an optimal

solution of our ALM model. However, for realistic sized instances they need astronomically large solution times. One of the reasons is the complication due to the flexibilities. Therefore, in order to find good (but not necessarily optimal) solutions in reasonable time, we developed a heuristic approach.

Before we describe our ALM model, we first consider the Asset Liability Management (ALM) problem for pension funds. In Section 1.1 we describe what a pension is, which types of pensions exist, and the various ways in which pension rights are accumulated. Also historical developments are presented, not only in The Netherlands, but also in some large countries. At the end of the section, also expected future developments will be considered. In the second section the various aspects are discussed, which are directly related to ALM problems: interested parties, instruments which are at the disposal of the boards of pension funds, and the supervisor. Also several types of risk are discussed. In the last section of this chapter, we describe ALM models and solution techniques. Finally, we describe the key characteristics of our ALM model.

1.1 Pensions and pension funds

In this section, some fundamental concepts with respect to pensions are described. We also present figures to show how pensions are actually arranged, especially in The Netherlands, but also in some major countries. Moreover, we present some general information on pension funds and the relevant legislation in The Netherlands. The legislation used in this thesis was found on the website of the PVK [75] in April 2003. At the end of this section, we pay attention to future challenges.

1.1.1 Pensions as second pillar facility

Pension is a generic term for periodic payments which replace the former salary in case of reaching a certain age, disability or death of the employee. Many types of pension exist. Moreover, several ways to build up pension rights exist. There are also many types of pension funds. We will describe these types of pension funds in the next subsection.

The basis of the existence of pension funds is solidarity between generations and between participants of a pension fund. After all, some participants will never profit from the contributions they made, because they die early. On the other hand, other participants live longer than average. As a result, they will receive more money from the fund than they have actually saved by themselves. Because many funds have a large number of participants, risks can be reduced.

One can distinguish three pillars concerning old age, disability, and surviving relatives provisions. The first pillar involves the provisions by the government. In The Netherlands, these are the Algemene Ouderdomswet (AOW, an old age provision), the Algemene Nabestaandenwet (ANW, a surviving relatives provision), both social insurances, and the Wet op de Arbeidsongeschiktheidsverzekering (WAO, a disability provision), an insurance by employees.

The second pillar covers the pension scheme in the relationship between the employer and the employee. The third pillar consists of individual life insurances, which each individual can take out by a life insurance company. They are independent from labor relations.

1.1.2 Types of pension funds

On January 1, 2002, there were 889 pension funds in The Netherlands. These funds can be categorized in funds related to a single company, funds related to companies in the same branch of industry, and funds for individuals who have the same occupation. We describe these three types of funds briefly below.

- **Pension funds related to a single company**
  In this type of fund, participating employees are all employed in the same company. Participation for all employees is mandatory. Examples of pension funds in The Netherlands that belong to this category are the funds of Akzo Nobel, Philips, Shell, and Unilever.

- **Pension funds related to companies in the same branch of industry**
  Participating employees are all employed in companies in the same branch of industry. Also in this type of fund, participation is mandatory. Examples in The Netherlands are Algemeen Burgerlijk Pensioenfonds (ABP), and Pensioenfonds voor de Gezondheidszorg, Geestelijke en Maatschappelijke Belangen (PGGM).

- **Pension funds for individuals who have the same occupation**
  Participants in these funds are all professionals who have their own practice, and all work in the same discipline. In this case, no relationship employee-employer exists. Participation can be mandatory. Examples of professions which fall into this category are medical specialists, dentists, and physiotherapists.

- **Other types of funds**
  Most pension funds fall in one of the first three classes mentioned above. In addition, there are some saving funds for companies and one pension fund that is provided by law (the notarial pension fund).

Type of fund	2002		1998	
	Number	Percentage	Number	Percentage
Related to a single company	779	87.6	870	90.0
The same branch of industry	92	10.3	79	8.2
Individuals	11	1.2	11	1.1
Other	7	0.8	7	0.7
Total	889	100	967	100

Table 1.2: Numbers of pension funds in The Netherlands in 2002 and 1998, split-up according to the type of fund.

In Table 1.2 an overview is given of the numbers of pension funds in 2002 and 1998, for every type of pension fund described above. We see that most funds are related to a single company. Because of mergers, their total number decreased the last four years. At the same time, we see an increasing number of pension funds related to companies in the same branch of industry.

1.1.3 Types of pensions

Every type of pension provides the participant with an income after some event has happened. In this section, we discuss the most important types of pensions.

- **Retirement pension**
  This is a pension for the financial care of a person, after the in the pension rules described pensionable age is reached. Generally, this payment is made lifelong.

- **Widow's pension**
  This is a form of surviving relatives pension, that is paid to the widow(er) of a participant of the pension regulation. Generally, this payment is also made lifelong.

- **Partner pension**
  This is the equivalent for the above described widow(er) pension. This pension applies for people who live together without being married, and satisfy a number of conditions.

- **Orphan pension**
  This is a form of surviving relatives pension, that is paid to the child(ren) of a participant of the pension regulation. This type of payment is made, till the child(ren) has (have) reached a prespecified age.

- **Pension in case of disability**
  This type of pension is made after the participant of the fund has become incapacitated for work.

Not all pension funds have all types of pension payments. In Table 1.3 we give an overview of both the absolute numbers and the percentages of the pension funds, which offered in 2002 and 1998 the above discussed types of pensions to their participants.

	2002		1998	
Type of pension	Number	Percentage	Number	Percentage
Retirement pension	835	93.9	932	96.4
Widow's pension	846	95.2	940	97.2
Partner pension	651	73.2	660	68.3
Orphan pension	832	93.6	919	95.0
Pension in case of disability	426	47.9	407	42.1
Number of funds	889		967	

Table 1.3: Numbers of the types of pensions offered by pension funds in The Netherlands in 2002 and 1998.

We conclude that almost all pension funds offer retirement, widow's, and orphan pensions. Roughly three quarters also have a partner pension and approximately halve of the funds give a pension in case of disability.

Moreover, we see that the different types of pensions which are recorded in the pension regulation, are not much changed in the last five years.

1.1.4 Pension systems

In the Dutch law, three pension systems (i.e. systems to build up pension rights) are distinguished: a system based on the final salary, a system based on the average salary, and the so-called *defined contribution system*. The first two systems are also called *defined benefit systems*. In principle, the employer decides which of the systems is used. All these systems assume that pension rights will be built-up in 40 years. Now, we describe these three systems briefly, and also some variants of them.

Final pay systems

We distinguish two variants of the system based on final salaries.

- **Actual final pay system**
  In this system, every wage increase not only affects the rights which will be built-up in the remaining years of service, but also in the previous built-up rights.

- **Moderate final pay system**
  This system only differs from the system described above, in the sense that wage increases in the last years of service do not result in a higher pension.

This prevents that (extreme) wage increases in the last years of service result in a very high pension.

Systems based on the average earned salaries

Also for systems based on the average earned salaries, two variants are distinguished.

- **A system based on the actual average earned wage**
  In this system, every wage increase influences the pension that will be built-up in the remaining years of service. The pension over previous years of service remains unaltered.

- **An indexed system based on the earned salaries**
  This system is characterized by the fact that the pension based on past years of service are corrected for increases in prices or wages. Indexing is discussed in more detail in Section 1.2.3.

Defined contribution system

In a defined contribution system, the employer yearly transfers money (usually a percentage of the pensionable salary) to purchase a part of the employees' pension. The level of the pension depends on the number of years the pension contributions have been paid, the realized return in the years the pension has been built up, and the interest rate at the moment of retirement. This pension system generally also has fiscal consequences for the employee.

Systems to accumulate pension rights used in practice

In Table 1.4 we give an overview of the absolute and relative number of pension funds which use the different pension systems, both for 2002 and 1998.

	2002		1998	
System	Number	Percentage	Number	Percentage
Actual final pay	218	24.5	317	32.8
Moderate final pay	252	28.3	282	29.2
Average earned salaries	29	3.3	36	3.7
Indexed based salaries	126	14.2	90	9.3
Defined contribution	76	8.5	84	8.7
Other	188	21.1	158	16.3
Number of funds	889	100	967	100

Table 1.4: Numbers and percentages of the pension systems used by pension funds in The Netherlands in 2002 and 1998.

We see that the percentage of funds that uses the system based on the actual final salaries has decreased the last five years. Especially a shift towards indexed systems based on earned wages can be seen.

1.1.5 Indexation

When benefit payments are only expressed in nominal payments, and are not corrected for increases in prices or wages, the purchasing power of retired people is harmed considerably. To prevent this, nominal benefit payments are often increased in line with inflation. This is called *indexing benefit payments*.

In Table 1.5 we have presented, for a number of possible ways to index pension rights, the absolute and relative number of pension funds that made use of each of these ways in 2002 and 1998. In this table, only the funds are stated which had an old age pension. The category 'Other' contains for example the minimum, maximum, and average of increases in prices and wages.

	2002		1998	
Index	Number	Percentage	Number	Percentage
General price level	319	38.2	305	32.7
General wage level	38	4.6	42	4.5
Development wages employer	33	4.0	34	3.6
Development wages branch of industry	37	4.4	44	4.7
Periodic decision by management	116	13.9	129	13.8
No compensation	114	13.7	158	17.0
Other	178	21.3	220	23.6
Number of funds	835	100	932	100

Table 1.5: Numbers and percentages of bases used to index pension rights by pension funds in The Netherlands in 2002 and 1998.

Most funds provide indexation in line with the general price level. The percentage of funds that uses this base increased slightly the last five years. At the same time, the percentage of funds that do not index pension rights at all, decreased in those years.

1.1.6 Developments

In this section, we briefly describe the historical development of the size of the total asset value and the number of participants related to pension funds in The Netherlands. Then, developments up to 2002 are discussed in more detail.

Total asset value

In the last decades, the total asset value of all pension funds together has increased enormously. In Table 1.6 we present figures of the total asset values, split-up in type of pension fund for 2002 and 1998. These figures are all in billion euros. Note that the percentages of each type of fund in the total asset value remained constant in those years.

	2002		1998	
Type of fund	Amount	Percentage	Amount	Percentage
Related to a single company	143.2	29.8	88.9	29.9
The same branch of industry	320.4	66.8	198.9	66.8
Individuals	15.5	3.2	9.1	3.1
Other	0.8	0.2	0.7	0.2
Total asset value	479.9	100	297.6	100

Table 1.6: Total asset value in billion euros for every type of pension funds in The Netherlands in 2002 and 1998.

To get an even better understanding of the increase in asset values over time: the total asset value of all funds together in 1950 was approximately €1.4 billion. This number is derived from H.A. Klein Haneveld [51].

Number of participants

In Table 1.7, the total number of participants of Dutch pension funds is presented. These participants are also split-up in active members, deferred members, and retired persons.

	2002		1998	
Group	Number	Percentage	Number	Percentage
Active members	5,413,217	39.1	4,693,249	38.5
Deferred members	6,438,196	46.5	5,662,113	46.5
Retired persons	2,005,217	14.5	1,819,371	14.9
Total	13,856,630	100	12,174,733	100

Table 1.7: Total number of participants in pension funds in The Netherlands in 2002 and 1998, split-up in different groups.

We see that the total number of participants increased with more than 1.5 million people from 1998 to 2002. However, it is possible that individuals have built-up pension rights in more than one pension fund.

Recent developments (up to 2002)

At the beginning of this chapter, we have already described the recent problems many pension funds in The Netherlands have to deal with. The current situation of pension funds in The Netherlands can be summarized as follows. First of all, the interest rates are very low, see for example the website of De Nederlandsche Bank [26]. These low rates lead to a high value of the liabilities, since these interest rates are used to discount expected future cash flows.

Also expectations with respect to asset returns decreased. Moreover, Dutch pension funds have to conform to new standards with respect to recoveries in case of underfunding and to create buffers to avoid unfavorable future circumstances. The supervisor also sets bounds on parameter settings which are used in ALM studies. These new requirements by the Dutch supervisor of pension funds can be found in the circular of September 30, 2002 [74]. Finally, new international accounting standards result in more pressure on the company related to the pension fund.

Even though these circumstances look far from ideal, the financial position of pension funds can be improved in various ways:

- **Increase contributions**
  An increase of contributions by active participants means that cash inflows are higher for funds. This results in a strengthened financial position. Many funds have increased the contribution rate in 2002 and 2003, for example the two largest funds in The Netherlands, ABP and PGGM, see their websites [1] and [76].

- **Remedial contribution**
  The sponsor of the funds can also pay a lump-sum to the fund. A number of companies in The Netherlands have used (or consider to use) this instrument to support their pension fund. Examples of companies that (consider to) use this instrument are ABN Amro, Ahold, Akzo Nobel, Heineken, KPN, and TPG, see the website of Vereniging van Gepensioneerden Elsevier-Ondernemingen [93].

- **Incomplete indexing**
  Instead of higher cash-inflows, one can also choose to give incomplete compensation (or no compensation at all) for increases in prices or wages to retired people. Of course, these retired people oppose such proposals vehemently. For that reason, retired people claim more influence in the decision making process within pension funds, see for example NRC Handelsblad [97]. According to Trouw [99], the pension fund related to the metallurgical industry breaks in 2003 with the habit of fully indexing pension rights.

- **Economize on the pension regulation**
  This approach results also in a lower value of the liabilities. A possibility is to switch from a system based on final wages to a system based on average wages. In Table 1.5 we have seen such a shift. According to Trouw [100], even the regulator of Dutch pension funds considers this to be a serious option to improve the financial position of the funds.

As we have seen, the boards of pension funds have various instruments at their disposal to bring the funding ratio up to the required standard. Without doubt, a number of these instruments will meet resistance of some interested parties. This will be explained in more detail in Section 1.2.

1.1.7 International perspective

Large discrepancies exist in the field of pensions between different countries. We will discuss a few aspects for some large countries. Successively, we describe the amount of capital of pension funds, how the second pillar is financed, and the fraction of working population covered by the second pillar. Aspects with respect to supervision and regulation will be discussed in Section 1.2.3.

Capital of pension funds

In Section 1.1.6 we have seen that in 1998, pension funds in The Netherlands managed approximately €300 billion. This is more than 113 percent of the Dutch gross domestic product (GDP) in that year. The ratio of assets managed by pension funds over GDP is a measure of how much is saved for old age provisions.

In some other countries, this fraction is much lower. In Table 1.8 these ratios are presented for some large countries. These figures are from 1997 and are derived from Laboul [58]. The main reason why these numbers differ so much from country to country is the way pensions are regulated. In some countries one saves in order to build up rights. In other countries the current working population has to finance the pension payments of the old aged.

Country	Fraction assets/GDP
France	0.07
Germany	0.15
Italy	0.02
The Netherlands	1.13
Spain	0.04
United Kingdom	0.79

Table 1.8: Fraction of assets in pension funds over GDP in some European countries in 1997.

We conclude from Table 1.8 that in many countries one has hardly saved for old age provisions. This may have serious consequences in the (near) future, not only with respect to the payment of benefit payments, but also for the interest rate in the capital market. This rate may increase when countries have to borrow money in order to be able to make benefit payments. This also has macroeconomic consequences.

Pension systems

First of all, not all countries use the same way to finance the second pillar. For some major countries, the way of financing this pillar is presented in Table 1.9. These figures are based on Laboul [58].

Second-pillar schemes are usually funded, and thus generate own resources. These are based on the principle of accumulated reserves. In a pay-as-you-go system, no reserves are accumulated over time. This type of funding is more exposed to demographic risks than funded systems, see Blommestein [6]. In addition, pay-as-you-go schemes are more exposed to political risks. Most pension funds in the United Kingdom and the United States use a defined contribution system. This implies that the participant is exposed to all types of risk. Moreover, there is only limited solidarity in this system.

Country	System used to finance the second pillar
France	Pay-as-you-go for the compulsory part. Funded or pay-as-you-go for occupational pensions. Funded for the part of pensions above mandatory minimum.
Germany	Funded. Pay-as-you-go for public servants.
Japan	Funded.
United Kingdom	Funded.
United States	Funded.

Table 1.9: The way the second pillar is financed in some major countries.

Percentage of working population covered by second pillar

The percentage of working population covered by the second pillar differs from country to country. To get an idea of how much they differ, we have presented these numbers for some countries in Table 1.10. These numbers are derived from Davis [22].

Country	Percentage	Country	Percentage
Denmark	80	Portugal	15
Germany	42	Spain	15
Greece	5	Sweden	90
Italy	5	United Kingdom	50
Japan	37	United States	46
The Netherlands	90		

Table 1.10: Percentage of working population covered by second pillar schemes.

The numbers presented in Table 1.10 can mainly be explained by the fact for which percentage of the working population it is mandatory to be affiliated to a pension system. For many people in Denmark, Sweden, and The Netherlands, participation is mandatory. On the other hand, especially in southern Europe, the percentage of working population covered by the second pillar is very low.

Even in the presence of pension schemes, individual entitlement may be subject to numerous conditions, and some categories of employees may be excluded. Forms of discrimination include age restrictions, salary restrictions, and restrictions based on sex. Not only restrictions on who can join the scheme exist in many countries, also discrimination between the sexes regarding retirement age, benefits, and mortality tables are often made.

1.1.8 Challenges

In the next few years, new challenges arise due to the aging populations in almost all major countries in the world. This is shown in Table 1.11, where OECD projections of the percentage of people of 65 and older to the population aged 15 to 64 are presented for 2010 and 2030 for some European countries. To get a feeling for these numbers, also the percentages in 1990 are given. The percentages presented in Table 1.11 are obtained from Laboul [58].

Country	1990	2010	2030
France	20.8	24.6	39.1
Germany	21.7	30.3	49.2
Italy	21.6	31.2	48.3
The Netherlands	19.1	24.2	45.1
Spain	19.8	25.9	41.0
United Kingdom	24.0	25.8	38.7

Table 1.11: Percentage of elderly over working population: estimates for 2010 and 2030 and actual data for 1990.

Of course, these figures do not imply that pension funds will be faced with problems. If everyone saves for his or her own old age provision, and assets are managed appropriately, pension funds may be able to fulfill all their liabilities, even if many people retire at the same time. However, if current active participants have to finance the pensions of the old aged, as is the case in some countries, serious problems may arise in the (near) future.

An important issue is whether also in the future the solidarity between generations and participants is guaranteed. Moreover, the question whether pensions remain affordable payable in the future will attract much attention.

1.2 ALM for pension funds

Asset Liability Management for pension funds is a risk management approach, which takes into account the assets, the liabilities, and also the interactions between the different policies which the board of a pension fund can apply. The board of a pension fund should find acceptable policies that guarantee with large probability that the *solvency* of the fund is sufficient during the planning horizon and, at the same time, all promised benefit payments will be made. The solvency is the ability of the pension fund to fulfill all promised payments in the long-run. Usually, the solvency at a certain time moment is measured as the *funding ratio*. Recall that this is the ratio of assets and liabilities.

Underfunding occurs when the funding ratio is less than one. Another way of characterizing underfunding is by saying that the *surplus* is negative, where the surplus is the difference between the value of the assets and the value of the liabilities. The surplus is the part of the reserves of the pension fund that is not needed for paying benefit payments. The funding ratio changes over time, mainly because of fluctuations in the liabilities and in the assets. Therefore, a pension fund rebalances its asset portfolio and adjusts for example its contribution rate regularly, in order to control changes of the funding ratio over time. In case of distress, the sponsor of the fund may have to help out with a remedial contribution.

In the ALM decision process, conflicting interests of different parties exist. In the next section, we will look in more detail at the interests of different parties. In Section 1.2.2 we discuss the policies and instruments which are at the disposal of the board of pension funds. In Section 1.2.3, the way supervision is organized is described, in particular the situation in The Netherlands, but also in some other countries. The last two sections are devoted to risks and (recent) developments.

1.2.1 Interested parties in the policy of pension funds

At least five parties are involved in the decision making process by the board of a pension fund, or are interested in its results. First of all, the *active participants* are (or should be) interested. They are especially concerned about the level of the contribution rate. In particular older active participants are also interested in the degree of indexation: they would like to be compensated for inflation in all years. Active participants make contributions on a regular basis to the fund to build up rights concerning (some of) the different types of pensions described in Section 1.1.3. If the contribution rate increases for example, the active participants have to make a larger contribution to the pension fund, which results in a lower disposable income.

A second interested group consists of *retired persons* and surviving relatives of them. For this group, especially the indexing policy is important. Of course, they would like to receive full compensation for increases in prices or wages.

Also *deferred members* have interests, since they have vested rights. Therefore, they are for example concerned about the indexing policy of the fund. This interest will be explained in Section 1.2.3.

The *sponsor* of the fund is also involved. Not only does the sponsor pay a part of the regular contributions, also in case of financial distress the sponsor plays an important role. If the funding ratio drops below a certain threshold, the sponsor of the fund may contractually be forced to restore the funding ratio. On the other hand, in case of financial prosperity, the sponsor may also benefit. Note, however, that not all pension funds have a sponsor. Every pension fund related to a single company has a sponsor. Moreover, also the government may act as a sponsor of the fund of civil servants. Other funds related to companies in the same branch of industry, or funds for individuals with the same occupation, may not have a sponsor. Next to concerns about the level of the contribution rate and the level of remedial contributions and restitutions, the sponsor is also interested in the costs associated with carrying out the pension administration.

The last party discussed here is the *supervisor* of the fund. Pension funds have to justify and report their activities to the supervisor. The role of the supervisor differs from country to country. The Dutch situation will be discussed further in Section 1.2.3.

Although all parties discussed here will be satisfied in case of financial prosperity, tensions between (some of) these groups are to be expected if the financial position of the fund is weak. Pensioners would like to receive an index-linked pension. However, this may result in even more pressure on the funding ratio, and in addition, on higher contributions by active participants or even a remedial contribution by the sponsor of the fund. On the other hand, this field of tension makes ALM problems challenging.

1.2.2 Policies and instruments

The board of a pension fund has many instruments to its disposal to control the funding ratio. These are discussed in this section. The board should take into account the interests of all parties involved in the decision making process, to find the 'best' policy mix. We stress here that the ALM process is considered from the perspective of the pension fund. Figure 1.3 shows the major policies and rules by which the fund can control the funding ratio.

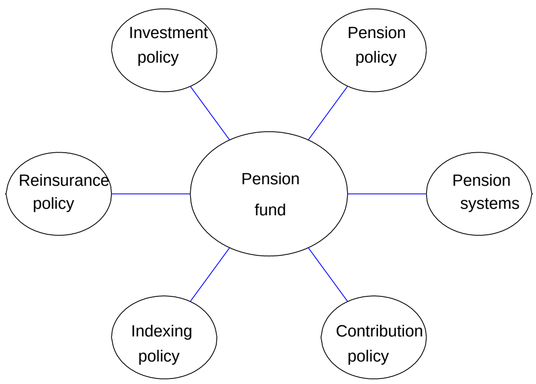

Figure 1.3: Policies and rules of a pension fund.

- **Pension policy**
  The pension policy deals with decisions with respect to the different types

of pensions that the fund includes in the pension regulation. These were described in Section 1.1.3. Active participants, deferred members and retired people are interested in the pension policy, because they are the ones who will receive money from the pension fund.

- **Pension system**
  The rules with respect to the benefit payments are registered in the pension rules. In these rules, the pension system is described. These were described in Section 1.1.4. Especially the sponsor and the active participants are interested in the pension rules, because they have to finance the system.

- **Indexing policy**
  The indexing policy is important in valuing the liabilities and (future) benefit payments. In Section 1.1.5 we have explained what indexing is. The board of a fund has to decide which base to use, for example a consumer price index, or a wage index. Moreover, generally every year again it has to be decided whether the financial position of the fund suffices to give (full) compensation. An actuary plays a key role in this decision. Retired people, deferred members and active participants all would like to be compensated for increases in prices or wages. These are the parties who benefit from indexing pension rights.

- **Reinsurance policy**
  Pension funds can sublet certain risks, like the risk of decease or disability, partially or entirely to an insurance company. This is called reinsurance and is part of the reinsurance policy of the pension fund. Reinsurance is compulsory for small pension funds in The Netherlands. The supervisor judges the reinsurance policy of pension funds. The supervisor tries to avoid that pension funds are exposed to much risk.

- **Contribution policy**
  The board of a pension fund can not only manage its liabilities, also the assets can be managed. One of the instruments to manage the assets is by means of the contribution policy. In the contribution policy, the system is chosen on which the level of the contribution rate is determined. Most pension funds use a dynamic contribution rate. In this system, the level of the contribution rate can be modified in the course of time. However, it is also possible that the different interested parties involved in the decision process agree about a fixed contribution rate. The active participants and the sponsor are the parties who are mainly interested in the level of the contribution rate, because they have to finance the system. Details about the different contribution systems that exist are beyond the scope of this thesis.

- **Investment policy**
  The value of the assets is also influenced by the *investment policy*. In this policy, the board of the pension fund decides in which asset classes the fund invests its assets. Also the levels of the lower and upper bounds on the fraction of the total assets invested in each asset class, and rules concerning rebalancing are part of the investment policy. For example, it is possible that

investments are made in indices, or that assets are actively managed. Also investments to reduce risks, like currency hedging, are considered. The supervisor is concerned about the investment policy, because investments directly influence the risk of underfunding. Pension funds should invest their assets such that this risk is small. To do so, rules exist with respect to levels of buffers which pension funds need if they invest in certain asset classes, see for example the circular [74].

1.2.3 Supervision

In this section, we discuss the (developments of the) tasks of the Dutch supervisor of pension funds. Moreover, the regulations for pension funds is considered. Finally, we consider some regulation issues from an international perspective.

Situation in The Netherlands (2002)

The supervisor of the Dutch pension funds is the Pensioen- & Verzekeringskamer (PVK). This authority was set up in 1923, initially to supervise the policies of insurance companies. Until 2001, this authority was named Verzekeringskamer. Only since 1952, the tasks of supervision expanded to pension funds. At the same time, the Pensioen- en Spaarfondsenwet (PSW) came into effect. The PSW is a collection of laws and a number of instructions based thereupon. In the early years, the PSW contained only a few regulations. Since the mid 1980s, the legislator interfered more and more in the contents of the pension schemes. They have done this to protect the interests of employees.

Every year, pension funds have to submit an annual report that has to be written in a prespecified way. Besides, the PVK inspects the daily affairs at the office of the funds regularly.

Because the supervision of pension funds was considered to be inadequate, the PVK got more rights on January 1, 2000. Since then, the PVK can give pension funds an instruction to bring their policies or the execution of them into conformity with legislative provisions. The PVK also has the right to impose penalties and to report to the Counsel for the Prosecution if a fund is in breach of the law. Moreover, the PVK is allowed to force pension funds to reinsure their liabilities when she considers this to be necessary for the sake of the participants.

The PSW contains rules on different fields. These are now described briefly.

- **Investment policy**
  Assets should be invested in a solid way. In addition, for pension funds related to a single company, rules exist with respect to the fraction of assets which may be invested in the own company. However, no further restrictions on the composition of asset portfolios exist for Dutch pension funds. In addition, no rules concerning currency matching exist.

- **Valuation**
  Liabilities are valued using a fixed discount rate. The level of this rate may not exceed 4 percent. Below we discuss recent developments in valuing liabilities.

- **Indexing**
  Although the purchasing power of retired people will decrease enormously if nominal pension rights are not corrected for increases in prices or wages, the PSW does not contain any obligation to index pension rights. Therefore, indexing is based on voluntariness. However, it is possible that a fund is compelled to index benefit payments, because such a provision is part of the statutes of the fund. Generally, in the statutes is written that the board of directors of the fund every year again decides whether or not the financial position of the fund suffices to index the rights.

  The only prescription with respect to the indexing of pension rights that is contained in the PSW, is a commitment of equal treatment: if retired people get a compensation, a corresponding compensation has to be given to deferred members.

Recent developments (end nineties to 2002)

To be able to judge the financial position of pension funds well, not only the assets should be valued using observed market prices, but also a market value of the liabilities should be found. This is the result of discussions between pension funds, the supervisor and consultants. However, it is far from trivial how to find a good market value for the liabilities. For a discussion of this issue, we refer to H.A. Klein Haneveld [51]. We come back to this issue in Section 5.4.

The PVK also introduces new rules pension funds have to satisfy. These rules, which concentrate on the risks with short-term and mid-term duration, are called *Financieel Toetsingskader (FTK)* and are described in [73]. The supervisor expects that these rules will come into effect from January 1, 2006. In Section 3.1 we describe these solvency tests of the supervisor in more detail.

Financial distress

In case of underfunding, the board of a pension fund immediately has to inform the PVK about this situation, see the circular [74]. Moreover, within three months the PVK should receive a recovery plan from the fund. The funding ratio should be sufficiently high again within one year.

If the funding ratio is greater than 1, but the buffers (needed for investments in risky assets) are not sufficiently large, the board also has to inform the PVK. Also in this case, the board should formulate a plan to obtain sufficiently large buffers. In this case, recovery may last two to eight years.

International perspective

In the last years, more and more attention is paid to pensions. This follows for example from the fact that two committees are established with an international character. The first is the OECD Working Party on Private Pensions, which is founded in 1999, and discusses themes like solvability, pension fund governance, and investment issues. In 2000, the International Network of Pension Regulations and

Supervisors is funded. The most important theme this committee discusses is the development of principles and best practices on the field of pensions.

In spite of these collaborations, large discrepancies exist between regulations in different countries. This will be discussed now. We concentrate again on regulations with respect to investments, valuation, and indexing.

- **Regulations concerning investments**
  Requirements with respect to investments of pension funds differ much from country to country. In some countries, only guidelines are present. In other countries, rather stringent demands have to be satisfied. Although in many countries investments have to be made in compliance with rules regarding diversification and liquidity, no additional rules exist with respect to investments within one asset class.

  In most countries, no restrictions are imposed with respect to currency matching. Only in very few countries, like Germany, guidelines are present.

- **Valuation methods**
  The valuation of assets and liabilities generally also differs from country to country. In Table 1.12 different valuation bases are presented for different asset classes in some countries. These results are from Laboul [58].

Country	Shares (quoted)	Shares (un-quoted)	Bonds	Real estate
France	1	2	4	4
Germany	3	3	3	1
Japan	1	2	1	1
United Kingdom	5	6	5	5

Table 1.12: Valuation bases for different asset classes. The numbers have the following meaning: 1 = lower of purchase price and market value, 2 = purchase price, 3 = lowest value ever, 4 = amortized value, 5 = market value, 6 = adjusted market value.

  In an international context, people try to find a consensus with respect to valuation and accounting. The International Accounting Standards Board (IABS) considers the concept *fair value* in more detail. This aim of valuing both assets and liabilities follows the International Accounting Standards no. 19 (IAS19). In IAS19, market values are used to value assets, and AAA-rated bonds are used to value liabilities.

- **Indexing**
  In some countries, indexing is mandatory by law, for example in Australia and Germany. In some other countries, like Canada and Mexico, indexing is rare. In Table 1.13 the legal status in some major countries are presented. These figures are derived from Davis [22].

Country	Existence/legal status
Germany	Mandatory indexation.
Japan	Rare, except for pensions substituting to public schemes.
United Kingdom	Benefits indexation.
United States	Discretionary indexation.

Table 1.13: The existence and legal status of indexing in some major countries.

1.2.4 Risks

Pension funds are exposed to many sources of risks. As explained above, the funding ratio is very important in determining the financial soundness of a fund. As a result, one of the greatest concerns of the board of a pension fund (and also of the sponsor) is the *risk of underfunding*: the risk that the value of the liabilities is higher than the value of the assets. How the supervisor of pension funds deals with the risk of underfunding is explained in more detail in Chapter 3.

Risks are involved with the specification of every policy that is discussed in Section 1.2.2. In this section, some of these risks are mentioned. However, this list is not intended to be exhaustive. We will only stress that many sources of risk exist.

- **Risks regarding the asset portfolio**
  One type of risk corresponding to making investment decisions is *currency risk*. Large pension funds usually invest their assets in internationally diversified portfolios. Currency risk is created by investments which are made in other currencies than the one in which the liabilities of a pension fund are expressed. A second source of risk with respect to the asset portfolio is the *risk of default*. Pension funds usually invest a fraction of their assets in bonds. There is always the risk that the issuer of the bond is not able to make the promised payments, which is called the risk of default. The last type of risk with respect to the asset portfolio discussed here, is the so-called *volatility risk*. This type of risk is present if the returns on the asset classes fluctuate more than expected. Risks regarding the asset portfolio are related to the investment policy of pension funds. The supervisor is concerned about this type of risk.

- **Actuarial risks**
  One type of actuarial risk is the *longevity risk*. This is the risk that a participant of the fund lives longer than may be expected on the basis of mortality rates. This type of risk is concerned with old age pensions. On the other hand, also *risk of short life* is an actuarial risk. This is the risk that a participant lives shorter than expected. In this case, more benefit payments may have to be made to surviving relatives. Another type of actuarial risk arises if the liabilities are valued using a fixed discount rate, and the current interest rate in the financial markets is lower than this fixed rate. This is a risk, because the market value of the liabilities is higher in this case. The supervisor is interested in the actuarial risks pension funds are exposed to.

- **Risks with respect to contributions**
  The sponsor of the fund may not be able to make its part of the contributions, or to make a remedial contribution if the financial position of the sponsor is bad. This may result in a situation in which the sponsor is contractually forced to pay, but is not able to do so. Therefore, *risk of default of the sponsor* is a source of risk from the perspective of the pension fund. Active participants may have to contribute more to the fund in this case. As a result, they are also concerned about this type of risk.

- **Risks regarding reinsurance**
  Risk of default is also present if reinsurance is considered. This is the case if the insurance company is not able to make its promised payments.

- **Risks with respect to indexing**
  The fund may also face risks with respect to the indexing of pension rights. For example, if the benefit payments have to be corrected for inflation by contract, high inflation rates may lead to higher than expected benefit payments, and therefore also to a higher value of the liabilities. Active participants, deferred members and retired people are concerned about the risks with respect to indexing, because they benefit from indexing pension rights.

Reducing risks

We have seen different types of risk pension funds are concerned with. However, it is also possible to reduce risks. Broadly diversified asset portfolios may reduce volatility risk for example. In addition, wise investments in derivatives reduce risks. Although pension funds are exposed to many sources of risk, they cannot go bankrupt.

1.2.5 Developments

In this section we consider some developments over the last decades in the policies, both in The Netherlands and in some other countries. First, we describe changing compositions of asset portfolios. Then, we discuss developments in valuation fundamentals.

Composition of asset portfolios

After the second world war, the composition of the asset portfolios changed dramatically. In the early years after this war, almost all assets were invested in bonds, which were not actively managed.

In the seventies, more active strategies were used to get some additional return. Also in this period, most of the assets were invested in bonds. Probably because of low returns on bonds and high inflation rates in the seventies, pension funds searched for alternative investment opportunities. Since 1983, larger fractions of the assets are invested in stocks. The changing composition of the asset portfolios are presented in Table 1.14. In this table, the category 'Other' consists of commodities,

Country	Year	Stocks	Bonds	Real estate	Cash	Other	Total
The Netherlands	1983	16	71	13	0	0	100
The Netherlands	1993	29	58	13	0	0	100
The Netherlands	2000	41	34	11	1	13	100
France	1997	12.6	43.1	7.9	6.5	29.9	100
Germany	1997	9.0	75.0	13.0	3.0	0.0	100
Italy	1997	4.8	76.4	16.7	2.0	0.0	100
Spain	1997	11.3	60.0	3.7	11.5	13.5	100
United Kingdom	1997	72.9	15.1	5.0	7.0	0.0	100

Table 1.14: Portfolio compositions of pension funds.

for instance. The data for 1983 and 1993 are derived from H.A. Klein Haneveld [52]. The data for 2000 are derived from the PVK [75].

Nowadays, most assets are actively managed. Also the geographical diversification improved: a shift has taken place in The Netherlands from mainly investing in Dutch stocks and bonds to internationally diversified portfolios. In addition, investments in derivatives, like options and futures increased. They are used to manage risks.

In many countries, the fraction of assets invested in bonds dominates the fractions invested in other asset classes. The United Kingdom is, together with The Netherlands as we have seen, an exception. Investing a large fraction of assets in a risk-free way generally leads to unnecessarily high funding costs. In Table 1.14 the composition of asset portfolios in some European countries are presented. These figures, which are derived from Laboul [58], are from 1997.

Valuation

As we have argued above, the valuation of both the assets and the liabilities is important in order to be able to give relevant information regarding the financial position of a pension fund. In addition, the valuation is important to compare the performance of one fund through time, and also to compare the position of different funds at the same time.

Nowadays, almost all assets are valued according to observed market prices. To be able to judge the financial position of pension funds appropriately, also the value of the liabilities should be based on observed market prices. However, to find a market value for the liabilities is far from trivial. For details about discussions with respect to this theme we refer to H.A. Klein Haneveld [51].

1.3 ALM models for pension funds

Asset Liability Management problems are nowadays tackled in a very different way than some decades ago. In this section we will discuss developments with respect to tackling ALM problems. First of all, we consider the earliest ALM models. Then, we examine two techniques in more detail: simulation and stochastic programming. At the end of this section, we list the main characteristics of the ALM model presented in this thesis.

1.3.1 Earliest ALM models

The earliest Asset Liability Management models in the literature were deterministic models and duration matching techniques were applied to find the best portfolio. The stream of future benefit payments was assumed to be known in advance with certainty. Examples of these models are those of Macaulay [62], Redington [81] and Bierwag et al. [4]. These models, in which only bonds were considered as possible investments, were used until the mid 1980s. By then, bond models were used in which the future stream of benefit payments were stochastic. Examples of these models are those by Fabozzi and Fabozzi [32], Cox et al. [18], Jacob et al. [44] and Norris and Epstein [69]. Alternative portfolios were again found by duration matching techniques.

However, duration matching techniques have some major drawbacks, as is discussed by Hiller and Schaack [40]. Problems are to be expected if interest rates change unexpectedly, reinvestment risk has to be considered, and these type of models are extremely sensitive to the specific term structure model used.

1.3.2 Simulation

Only in the late 1980s, some large pension funds used the first integrated analyses for ALM problems, see Frijns and Goslings [35] and Van der Meer [64]. The first integrated analyses were made by using simulation models. H.A. Klein Haneveld and Boender were the first ones who made simulation models for ALM problems for pension funds in The Netherlands. In the literature, such a model is described by Boender et al. [7]. Because of the ability to use a lot of scenarios, simulation models for ALM problems are popular.

With simulation, the financial position of a pension fund can be calculated in many possible future circumstances. This is the big advantage of simulation techniques: a relatively large number of scenarios can be used. However, simulation techniques also have a major drawback: many choices with respect to policies have to be kept fixed. For ALM problems, this means that one has to formulate explicitly decision rules with respect to a fund's contribution policy, investment policy and indexing policy. It is very well possible that other policies than the one which is chosen, lead to better solutions, for example to lower funding costs.

1.3.3 Stochastic Linear Programming

To overcome the drawbacks of simulation, one can formulate stochastic linear programming models (SLP) to tackle ALM problems. Instead of exogenous variables (as in simulation), decisions now become endogenous. This also implies that stochastic programming is more difficult than simulation. Simulation is based on evaluation, while stochastic programming is based on optimization: SLP searches for the *best* solution, given bounds on the variables, the constraints of the problem, and the objective function. SLP for ALM problems takes into account the following characteristics:

- **Uncertainty**
  In ALM problems, many sources of uncertainty appear. For example, future developments of financial markets and inflation levels are all unknown yet. SLP takes these uncertainties explicitly into account (although the specification has to be given by the user of course).

- **Dynamics**
  For ALM problems, dynamics (more time periods) is essential. At each specified decision moment in time, SLP takes into account both previous decisions (like the composition of the asset portfolio and the level of the contribution rate) and the possibility to adjust these decisions at a later decision moment, based on revealed values of uncertain parameters.

- **Linear constraints**
  As we will see in the next two chapters, constraints for ALM problems can be written as linear constraints. Moreover, many details can be described in this way, as we will also see in Chapter 2.

These characteristics make SLP very attractive to use in solving ALM problems. For a general survey of stochastic programming we refer to Prékopa [80], Birge and Louveaux [3] and Kall and Wallace [47]

The major constraint of this solution technique is its relatively long solution time. This is the reason why in practice ALM problems are 'solved' by simulation. However, due to algorithmic progress and technological developments, nowadays relatively large models can be solved by SLP in reasonable time.

In the academic world, stochastic linear programming for finance problems were developed by Kallberg et al. [48], and Kusy and Ziemba [57]. SLP attracted a lot of attention by the paper of Cariño et al. [14]. They used this solution technique for an ALM problem for a large Japanese insurance company. Another ALM model for insurance companies is the one by Mulvey et. al. [67]. Also in the banking industry stochastic programming is used. Examples of ALM models for banks are those by Klaassen [49], Bradley and Crane [12], Lane and Hutchinson [59], Dempster and Ireland [23], and Mulvey and Vladimirou [68].

ALM models for pension funds appeared in Consigli and Dempster [17], Dert [24], Kouwenberg [55], and Hilli et al. [41]. We can solve ALM models with more scenarios and states than is done in Consigli and Dempster [17]. It is hard to compare the sizes of the problems Dert considered, with the ones presented in this thesis. This follows from the fact that Dert, who is the only one who uses binary decision variables in his ALM model, uses additional states (which do not have successors). This will be explained in more detail in Chapter 3. Kouwenberg solved models with more scenarios and states than we can in reasonable time. He used many processors at the same time to solve problems, while we solved the problems on a single machine. Other applications of stochastic programming in ALM for pension funds are for example those by Dupačová and Polívka [29] and Bogentoft et al. [8].

1.3.4 Main characteristics of our ALM model

The ALM model presented in this thesis is much more detailed than the ones presented in the literature. The key ingredients of our ALM model are described briefly now.

- **Contribution rate**
  The contribution rate has to satisfy lower and upper bounds. Moreover, large deviations in two consecutive years are penalized.

- **Risk constraints**
  The expected next year's shortage with respect to a certain level of the funding ratio may not exceed a prespecified value, which depends on the value of the liabilities.

- **Indexing**
  Indexing is considered to be an instrument of the board of pension funds. Therefore, if the financial position of the fund is weak, the board may decide not to compensate (or to compensate only partially) for increases in prices or wages.

- **Underfunding and remedial contributions**
  If in a prespecified number of consecutive years the fund faces underfunding, the sponsor of the fund has to restore the funding ratio to a prespecified level by means of a remedial contribution.

- **Horizon effects**
  At the horizon of the model, which is the last moment at which decisions are modelled, surplusses and shortages with respect to certain levels of the funding ratio are rewarded and penalized, respectively.

This list does not contain all characteristics of the ALM model; it gives an idea of some important aspects. For a detailed overview of the ingredients of our model, we refer to Chapter 2.

One of the properties of our model is its *flexibility*, in two ways. First of all, there is a flexible way of modeling *solvency risk*:

- The level of the funding ratio is compared with different standards. Moreover, we compare these levels at different times.

- Short term risks are considered with a fixed upper bound (which is to be set by the user of the model).

- Mid-term risks are taken into account by means of a remedial action of the sponsor after a number of consecutive years (to be specified by the user) in which underfunding is registered.

- Long term risks are considered by means of penalizing underfunding with respect to a prespecified level of the funding ratio at the horizon.

Second, there is a flexible way of modeling interactions between parties involved in the decision making process. This is done by introducing soft constraints and penalties. The interaction between the parties can be represented by choosing appropriate parameter settings with respect to:

- Contribution rates (the levels of the lower and upper bounds, and penalty parameters associated with changes in the levels of the contribution rates).

- Penalties associated with underfunding with respect to a prescribed level of the funding ratio at the horizon.

- Fixed penalty costs. These are important to penalize the unfavorable events. In our model, these fixed penalty costs are used next to proportional penalty costs (to penalize the level of unfavorable events like the amount of underfunding).

In the above listing, we mentioned that some values of the parameters are to be specified by the user of this model. Of course, some of these levels may for example be prescribed by the supervisor.

1.4 Summary

In the next two chapters, we focus on the formulation of our ALM model. To be able to present this model in the context of SLP, we first introduce scenarios and the decision structure in Chapter 2. Moreover, in that chapter the largest part of our ALM model will be built. Special attention is paid to (model) indexations and flexible risk measures. These risk measures require that if the funding ratio is too low in a number of consecutive years, the sponsor is forced to make up the deficit. In Chapter 3 we describe newly proposed risk criteria introduced by the Dutch supervisor of pension funds and how these criteria are linked to the risk constraints we incorporate in our ALM model. Especially the risk of underfunding in one year will be considered.

As a result of the introduced flexible risk constraints, and introduced fixed penalty costs for unfavorable events, binary decision variables (i.e. variables which have either the value 0 or 1) are unavoidable. Therefore, we obtain a multistage mixed-integer stochastic program, which is a very difficult optimization problem in general (see e.g. Römisch and Schultz [82]). It is therefore not to be expected that optimal solutions can be found in reasonable time for realistically sized instances. This is the reason why we consider a heuristic approach in finding solutions. This heuristic, which is described in Chapter 4, iteratively searches for improvements such that all constraints are satisfied. In this heuristic, insights into the problem are used. The numerical results show that heuristic solutions can be found for these large-scale mixed-integer stochastic programs.

In the formulation of the multistage SLP, scenario trees are used to model the uncertain future. In Chapter 5 we describe how numerical values for the returns on the asset classes and the changes in the general wage level are found. Also future changes in the (market value of the) liabilities and discount rates are considered.

In Chapter 6 the results of some numerical experiments are described. In that chapter, we consider an illustrative case in detail. We also describe some impressions obtained by considering some alternative modeling choices, and other scenario trees.

As we will see, the ALM model described in this thesis closely fits the developments and interests in society. Indeed, we incorporate the laws as prescribed by the Dutch supervisor of pension funds in our model. Moreover, relative positions of the interested parties can be represented by choosing appropriate parameter values. However, it is not easy to find a suitable setting for the parameter values. Moreover, more research is needed to analyze the source of the (extreme) sensitivity with respect to the set of scenarios used. These (and other) conclusions will be described in Chapter 7.

Chapter 2

ALM model

In the previous chapter, we have described what a pension fund is, and what ALM for pension funds is. In this chapter (the main part of) an ALM model is described. After describing the decision process and the characteristics of our model, scenarios and decisions are introduced. Then, the mathematical modeling is described in detail. This chapter focuses on the objective function and the constraints. Additional constraints will be discussed in detail in Chapter 3; these so-called risk constraints deal with the shortage after one year. The complete mathematical model is contained in Appendix A.

2.1 The ALM decision process

The goal of pension funds is to fulfill all obligations towards the participants. In this section we describe the decision process in the way it will be incorporated in the ALM model to be discussed.

In such models, the user has to fix a *planning horizon* which specifies the total number of years which are considered in the decision making process. The planning horizon is split into subperiods of one year. In every year, benefit payments are made, premiums are received, and changes in the status of the participants are recorded appropriately. At the end of the year, the board also knows the return of the asset portfolio. The value of the assets is determined using market prices at that moment. Moreover, at that time the fund makes an actual estimate of its liabilities. Once these two numbers are known, the level of the *funding ratio* (the ratio between the values of the assets and the liabilities) is determined. This funding ratio is an important performance measure: it indicates the actual financial position of the fund. It is compared to the development in the previous years to judge the effects of the actual strategy.

When all last year's information is revealed, the board looks forward: what are the expectations with respect to the future? For example, expectations with respect to future returns or developments of the inflation level may be adjusted.

Given the financial position of the fund at the end of a year, and possibly adjusted expectations, the board should make certain decisions. These (adjustments of) decisions aim at a sufficiently high future funding ratio, given the situation at the decision moment. One possible adjustment is to change the composition of the asset portfolio. For example, when the funding ratio increased last year to a relatively high level, the fund may consider to invest a larger fraction of its assets in asset classes with a high expected return (even though the associated risk may also be higher). In this case, the restrictions with respect to the composition of the asset portfolio should be kept in mind. Such restrictions may be imposed by the regulator, or may be established statutorily.

Another possible adjustment is to change the contribution rate. If the funding ratio decreased last year, the (board of the) pension fund may consider to increase the contribution rate. On the other hand, if the solvency of the fund increased last year, a decrease in the contribution rate may be considered. In the decision to change the level of the contribution rate, lower and upper bounds on this rate should be taken into account. Moreover, in determining the level of next year's contribution rate, it should be kept in mind that a rapidly changing contribution rate may be undesirable.

If at a decision moment the funding ratio is below a minimum required level, it is assumed that the sponsor has to pay a remedial contribution. This may for example be the case if the supervisor orders the board to undertake action on behalf of the participants of the fund. Such a remedial contribution in case of underfunding may also contractually be determined between the fund and the sponsor.

At the end of a year, also the level of last year's inflation is known. Given this level, and the solvency position of the fund at that moment, the board may decide to adjust future benefit payments entirely (full indexation), only partially, or not at all. This decision immediately influences the level of the benefit payments of the current old aged.

The decisions the board of a fund has to make are influenced by the interests of different parties involved in the decision making process. Moreover, decisions should be made such that unfavorable circumstances will be avoided as much as possible in the future. Decisions have to satisfy constraints on the level of the contribution rate, on the composition of the asset portfolio, on the values of the liabilities and benefit payments, and on an upper bound on the expected next year's shortage.

In determining which decisions to make, most recent information with respect to uncertain future circumstances will be taken into account. It is also kept in mind that in future years adjustments of the decisions can be made, when new information is revealed.

Optimization with respect to the decision variables in models which take into account multiple decision moments and uncertainty is called *multistage stochastic programming*. To formulate such models, *scenario trees* have to be used. A tree gives a collection of possible future developments of uncertain elements, like returns on assets and inflation.

Unfavorable circumstances, which the board would like to avoid, are for example large changes in the contribution rate for active participants, remedial contributions and not giving full indexation. These decisions are undesirable, but they are not ruled out: they may be necessary to avoid an even worse event: *underfunding*. Constraints which allow unfavorable events are called *soft constraints*, as opposed to *hard constraints*, which always have to be satisfied. To make these soft constraints meaningful, so-called *penalty parameters* are introduced. These parameters serve to penalize undesirable events. The penalty costs incurred by such undesirable events are balanced in the objective function with 'real' funding costs, as will become clear in the description of our ALM model.

As will also become clear later in this chapter, the ALM model described in this thesis is a *multistage mixed-integer stochastic program*. The integer variables appear into the model as indicators of the unfavorable events mentioned above, needed for a correct introduction of the penalty parameters in the model. Moreover, the integer variables are used to model flexible risk measures: the sponsor is obliged to make a remedial contribution if the funding ratio is below a minimum required level in a number of consecutive years.

Before we describe the ALM model mathematically, we will describe the characteristics of the model in more detail in the next section.

2.2 Characteristics of the ALM model

The ALM model described in this thesis is an optimization model, and therefore it is formulated as a set of constraints and an objective function. It describes the decision process the pension fund has to deal with. The user, which is assumed to be the board of a pension fund can (and should) specify certain preferences.

In our ALM model, some constraints serve for a correct bookkeeping. In these constraints, cash inflows and outflows are registered. Besides, constraints on the portfolio mix are present in the ALM model. These constraints deal with the composition of the asset portfolio: the fraction of the assets invested in each asset class has to satisfy lower and upper bounds specified by the board or regulator.

The model also contains constraints which deal with underfunding and a possible remedial contribution by the sponsor of the fund. Both underfunding and remedial contributions are penalized by means of fixed penalty costs. These (fictitious) fixed penalty costs are incorporated into the model to express the undesirability of certain events.

In our ALM model, we use the following policy rules. If the funding ratio is below a prespecified level, fixed penalty costs are incurred. If this observed underfunding means that this ratio is below the minimum required level in a (prespecified) number of consecutive years, we assume (as one of the decision rules) that the sponsor is forced to restore the funding ratio. If the funding ratio falls even below a predefined level (which is lower than the threshold value considered in defining underfunding), an immediate contribution from the sponsor is required. Next to these rules, we also introduce the policy rule that the marginal costs associated with large remedial contributions (which are payments above a certain fraction of the total pensionable salaries) are higher than the corresponding costs associated with low ones. Moreover, in our ALM model the sponsor is only willing to make a remedial contribution if the funding ratio is below a minimum required level.

The model described in this thesis also uses policy rules with respect to the contribution rate. It is assumed that the level of the contribution rate, which is expressed as a fraction of the total pensionable salaries, has to satisfy lower and upper bounds. Because a highly volatile contribution rate is not appreciated (at least by participants in case of large increases and by the supervisor in case of large decreases), large increases and decreases are penalized (although they are allowed if the lower and upper bounds are satisfied). In case of a remedial contribution, a minimum level of the contribution rate is required.

Our ALM model also takes into account the indexation of pension rights. The decision whether or not to increase the pension rights of participants of the fund for increases in wages or prices will be an outcome of the model. Since not giving full indexation is undesirable, (fixed) penalty costs will be imposed in this case. However, all contractually determined minimum benefit payments have to be made in time.

Because the ability to fulfill obligations is a central issue in asset liability management, additional constraints which deal with the *risk of underfunding* are considered. These constraints impose restrictions on the contribution rates and compositions of asset portfolios, such that the expected next year's shortage is sufficiently small. These constraints will be discussed in Chapter 3.

In our model also *overfunding* is considered. This is a situation in which the pension fund has a large surplus. In case of overfunding, the board of the fund may consider to transfer money back to the sponsor. Such a *restitution* is forced if overfunding is present in a prespecified number of consecutive years.

We assume that the board of the pension fund under consideration makes decisions, while keeping in mind the long-run desire to stay (or become) solvable. To do so, a target level of the funding ratio at the horizon of our decision model is introduced. Surplusses and shortages with respect to this level at the horizon are rewarded and penalized respectively.

Before we describe these characteristics of our ALM model in more detail, we first introduce scenarios. In addition, we explain what the decision variables in our ALM model are.

2.3 Scenarios and decisions

We assume that the ALM model has a horizon T years from now. The resulting years are denoted by an index t, where time 0 is the current time. By year t ($t = 1, \ldots, T$), we mean the span of time $[t-1, t)$.

At each time $t \in \mathcal{T}_0 := \{0, 1, \ldots, T-1\}$, the pension fund is allowed to make decisions, based on the actual knowledge of parameters. For example, given that last year's returns on the different asset classes are known, the fund may change its asset mix. Time t is assumed to be the end of the financial year t. We assume that a financial year coincides with a calendar year.

One way of modeling uncertainty of parameters in an optimization model is through a large but finite number S of scenarios. Each scenario represents a possible realization of all random parameters in the model. To be specific, let ω_t represent the vector of random parameters whose values are revealed in year t. Then, the set of all scenarios is the set of all realizations $(\omega_1^s, \ldots, \omega_T^s)$, $s \in \mathcal{S} := \{1, 2, \ldots, S\}$, of $(\omega_1, \ldots, \omega_T)$. Scenario s has probability p^s, where $p^s > 0$ and $\sum_{s=1}^{S} p^s = 1$. Since in a dynamic model information on the actual value of the random parameters is revealed in stages, a suitable representation of the set of scenarios is given by a

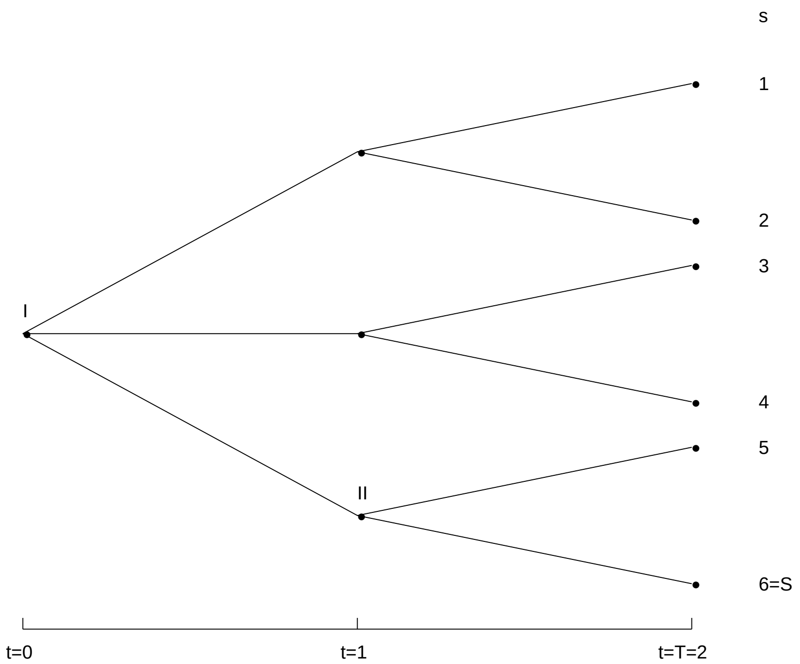

Figure 2.1: Example of a scenario tree.

scenario tree. In Figure 2.1 an example of a scenario tree is presented; in this tree, $T = 2$ and $S = 6$. Each path from $t = 0$ to $t = T$ represents one scenario. Any node of the tree, corresponding to time t, symbolizes a possible *state* at time t, represented by the observed values of $\omega_1, \ldots, \omega_t$. The *branches* directly to the right of it symbolize the various values of ω_{t+1}, given the realization of $\omega_1, \ldots, \omega_t$. Obviously, all scenarios passing this node have the same history in the years $1, \ldots, t$.

Now, we will explain the representation of states in the scenario tree. We denote the number of branches from each state at time t to states at time $t + 1$ by $branch_t$. The structure of the scenario tree is completely determined by the values of $branch_t$, $t = 0, \ldots, T - 1$. Given these values, the total number of scenarios, S, is known. This number is given by

$$S := \Pi_{t=0}^{T-1} branch_t,$$

and gives the total number of end-nodes at time T. Every end-node corresponds to one path starting at time 0, hence, with one scenario.

We now describe how the S scenarios in the tree are numbered, given the structure of the tree. Every branching from a parent node (also called predecessor) to the $branch_t$ successors gets a certain order. This order is determined arbitrary. As a result, every end-node (and therefore also every scenario) is uniquely indicated by the series of branching indices $(i_0, i_1, \ldots, i_{T-1})$, with $i_t \in \{1, \ldots, branch_t\}$, $t = 0, \ldots, T - 1$, and where i_t is the index of the branch at the node at time t. Now, the scenarios are numbered $1, \ldots, S$, corresponding to the lexicographical ordering of the successive branchings. In Table 2.1 the relation between the scenario-index and the branching structure is given.

Scenario index s	branching indices $(i_0, \ldots, i_{T-1})$
1	$(1, 1, \ldots, 1)$
2	$(1, 1, \ldots, 2)$
$\vdots$	$\vdots$
$branch_{T-1}$	$(1, 1, \ldots, 1, branch_{T-1})$
$branch_{T-1} + 1$	$(1, 1, \ldots, 2, 1)$
$\vdots$	$\vdots$
$S - 1$	$(branch_0, branch_1, \ldots, branch_{T-2}, branch_{T-1} - 1)$
S	$(branch_0, branch_1, \ldots, branch_{T-2}, branch_{T-1})$

Table 2.1: Relation between the scenario-index and the branching structure.

Example 2.1

We will explain the above introduced notation by means of the scenario tree which is depicted in Figure 2.1. In this tree, we have $branch_0 = 3$, $branch_1 = 2$, $T = 2$, and $S = 6$. In Table 2.2, for every scenario $s = 1, \ldots, S$ its series of indices (i_0, i_1) are given. $\square$

Scenario index s	branching (i_0, i_1)
1	$(1, 1)$
2	$(1, 2)$
3	$(2, 1)$
4	$(2, 2)$
5	$(3, 1)$
6	$(3, 2)$

Table 2.2: Relation between the scenario-index and the branching structure in Example 2.1.

At this point, we will describe how the states at time t are indicated. The node of scenario s at time t is uniquely determined, and can be denoted by (t, s), $t \in \mathcal{T}$, $s \in \mathcal{S} = \{1, \ldots, S\}$. Often, this notation is sufficient, but sometimes it is not. Especially when for all nodes at time t something has to be calculated, duplicate work can be avoided if one only considers different nodes. Indeed, two scenarios have the same node at time t, exactly when they share the same path from time 0 to t. Therefore, at time t ($t \in \mathcal{T}$), there are

$$\Pi_{q=0}^{t-1} branch_q$$

different nodes, each with a different history before time t. When we choose for every node the scenario with the lowest scenario index, we obtain the set $\mathcal{S}_t$ for $t = 0, \ldots, T$:

$$\mathcal{S}_t = \{s \in \mathcal{S} : s' \in \mathcal{S}', s' < s \Rightarrow (t, s') \neq (t, s)\}.$$

For example, $\mathcal{S}_0 = \{1\}$, $\mathcal{S}_T = \mathcal{S}$, and $|\mathcal{S}_t| = \Pi_{q=0}^{t-1} branch_q$.

From each of the nodes (t, s), $s \in \mathcal{S}_t$, where t is fixed, Ξ_t different scenarios develop, where

$$\Xi_t = \frac{S}{\Pi_{q=0}^{t-1} branch_q} = \Pi_{q=t}^{T-1} branch_q.$$

For example, $\Xi_0 = S$, and $\Xi_T = 1$ (empty products are by definition equal to 1). The parameter Ξ_t has a clear interpretation: it defines the cardinality of the bundle of scenarios through any node at time t. The set of scenarios which develop via (t, s) are denoted by $\mathcal{K}_t^s$. It holds that $|\mathcal{K}_t^s| = \Xi_t$.

More generally, a representation of all nodes at time $q \in \{t+1, \ldots, T\}$ of scenarios with the same history up to and including time t, is given by

$$\{(q, s') : s' \in \mathcal{K}_t^s(q)\},$$

with $\mathcal{K}_t^s(q) = \mathcal{K}_t^s \cap \mathcal{S}_q$. Indeed, for $q = T$ it holds that $\mathcal{K}_t^s(T) = \mathcal{K}_t^s$.

Example 2.2

This example makes use of the scenario tree depicted in Figure 2.1, and is intended to clarify the notation introduced above. For the tree under consideration, we obtain $\Xi_0 = 6$, $\Xi_1 = 2$, and $\Xi_2 = 1$. Moreover, we have the following sets: $\mathcal{S}_0 = \{1\}$, $\mathcal{S}_1 = \{1, 3, 5\}$, $\mathcal{S}_2 = \{1, 2, 3, 4, 5, 6\}$.

In the node $(t, s) := (0, 1)$, represented by I in Figure 2.1, the set $\mathcal{K}_0^1$ is given by $\{1, 2, 3, 4, 5, 6\}$, since all nodes $(2, s')$, $s' \in \mathcal{K}_0^1$ can be reached from state I. Moreover, $\mathcal{K}_0^1(1) := \{1, 2, 3, 4, 5, 6\} \cap \{1, 3, 5\} = \{1, 3, 5\}$. We see that this gives the unique set of successors of node I, with the lowest scenario indices.

In the node $(t, s) = (1, 5)$, represented by II, we obtain $\mathcal{K}_1^5 = \{5, 6\}$, since only nodes $(2, 5)$ and $(2, 6)$ are accessible from this state. From node $(1, 5)$ only the nodes $\mathcal{K}_1^5(2) = \{1, 2, 3, 4, 5, 6\} \cap \{5, 6\} = \{5, 6\}$ are accessible. □

We will now introduce the random parameters. For $t \in \mathcal{T}_1 := \{1, 2, \ldots, T\}$, we define the realizations in scenario $s \in \mathcal{S}$ by

$$\omega_t^s = (r_{1t}^s, r_{2t}^s, \ldots, r_{Nt}^s, w_t^s, \underline{L}_t^s, \overline{L}_t^s, \underline{B}_t^s, \overline{B}_t^s, \gamma_t^s, W_t^s),$$

where

r^s_{jt}	=	return (expressed as a fraction) on asset class j in year t in scenario s, $\quad j = 1, \ldots, N$,
w^s_t	=	change (expressed as a fraction) in the general wage level in year t in scenario s,
$\underline{L}^s_t$	=	lower bound on the value of the liabilities at time t in scenario s,
$\overline{L}^s_t$	=	upper bound on the value of the liabilities at time t in scenario s,
$\underline{B}^s_t$	=	lower bound on the value of the benefit payments at time t in scenario s,
$\overline{B}^s_t$	=	upper bound on the value of the benefit payments at time t in scenario s,
γ^s_t	=	discount factor associated with cash flows at time t in scenario s,
W^s_t	=	total level of the pensionable wages of the active participants in year t in scenario s.

All financial quantities, except r^s_{jt} and w^s_t, are denoted in million euros, and N denotes the total number of asset classes.

In the description of the ALM decision process in Section 2.1, we have seen that the board of the pension fund has to make decisions at time 0 ('now'), based on the actual knowledge of the fund, and on given expectations with respect to uncertain future developments, like returns on assets and inflation. Once new information is revealed (i.e., realizations of the uncertain parameters become available), the fund will make new decisions, based on this information, and possible adjusted expectations.

Because decisions are made in every node of the scenario tree, *decision variables* are related to this tree, too. Basically, a decision at time t may depend on the observed part of the scenario at that time, but not on unknown values of parameters of future years. That is, for each possible history (i.e. for each node at time t in the scenario tree) there is precisely one vector of decision variables representing the decisions at hand.

However, in the model formulation it is convenient to introduce a complete set of decision variables for each scenario separately. Therefore, so-called *nonanticipativity* or *information constraints* have to be added, in order to guarantee that decisions do not depend on values of random parameters that will be revealed in later years. Denoting the vector of decision variables at time t in scenario s by x^s_t, the nonanticipativity constraints imply $x^s_t = x^q_t$ if scenarios s and q coincide up to and including year t. The decision vector x^s_t is defined as follows:

$$x^s_t = (XI^s_{1t}, \ldots, XI^s_{Nt}, XD^s_{1t}, \ldots, XD^s_{Nt}, c^s_t, L^s_t, B^s_t, Z^s_t, DZ^s_t, V^s_t),$$

where

XI_{jt}^s = value of assets in class j bought at time t in scenario s, $j = 1, \ldots, N$,
XD_{jt}^s = value of assets in class j sold at time t in scenario s, $j = 1, \ldots, N$,
c_t^s = contribution rate for year $t + 1$ in scenario s,
L_t^s = value of the liabilities at time t in scenario s,
B_t^s = value of the benefit payments at time t in scenario s,
Z_t^s = remedial contribution by the sponsor at time t in scenario s,
DZ_t^s = direct cash flow by the sponsor, because of a funding ratio which is (far) too low at time t in scenario s,
V_t^s = restitution to the sponsor at time t in scenario s.

At the time horizon $t = T$, only the decisions L_T^s, B_T^s, Z_T^s, DZ_T^s, and V_T occur. The precise meaning of the decision variables will become clear in the next sections. We stress here that some other decisions introduced in Section 2.2 follow from the values of the decision variables presented above. For example, the degree of indexation in a state is a result of the value of the liabilities in that state.

The following additional variables are important too. For each $t \in \mathcal{T}_1$ and each scenario $s \in \mathcal{S}$ we have:

A_t^s = total asset value at time t in scenario s,
X_{jt}^s = value of investments in asset class j, at the beginning of year t in scenario s.

These are state variables. They are determined by the parameters and the decision variables, but from an optimization point of view they are decision variables too, if one includes their definitions as constraints in the model, as we shall do. Next, we have to explain in more detail what we mean by 'time t' in the definition of A_t^s. We assume that at the end of year t, i.e., just before time t, the contribution of year t comes in (although it is common that contributions are paid monthly to the fund) and the benefit obligations of year t are paid. At the same time, the revenues of the assets of year t are revealed. At that time, the board of the fund also has to make a decision with respect to the level of the indexation. After this decision is made, the value of the liabilities is determined, and one knows whether underfunding is present or not. In case of underfunding, possibly a remedial contribution from the sponsor Z_t^s or DZ_t^s is made. In case of overfunding, a restitution is considered.

In Figure 2.2 we have depicted the decisions at time t graphically. Given the decisions at the previous decision moment (at time $t-1$), and the observed realization of the stochastic parameters, A_t^s is known after the benefit payments of year t are made. Given this asset value, decisions are made with respect to the values of the liabilities. A_t^s and L_t^s together determine also the level of the funding ratio of the fund. As a result, decisions with respect to remedial contributions and restitutions have to be made. Finally, the asset portfolio is rebalanced for the next year, and also the level of next year's contribution rate is determined.

In the next subsections, accounting and policy constraints and the objective function of the ALM model are discussed. Constraints which impose a restriction on next year's expected shortage are considered in the next chapter. In appendix A, the mathematical formulation of all constraints and the objective function of our

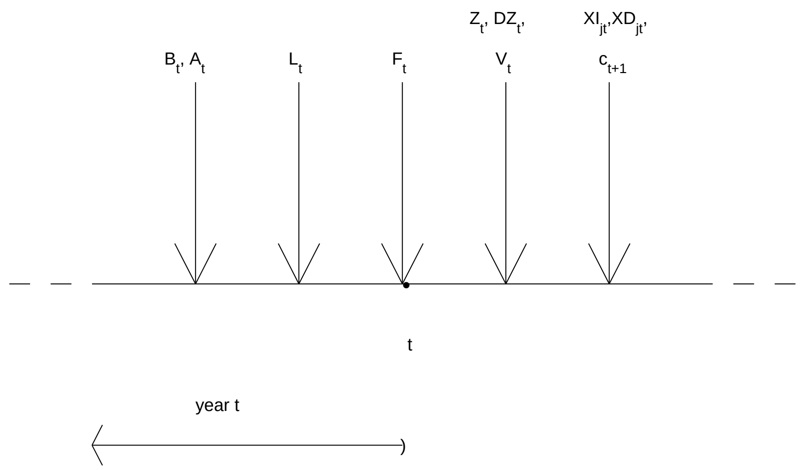

Figure 2.2: The order in which decisions and calculations are made at time t. Note that, since L_t is a decision variable in our model, the board of the fund can decide if rights are indexed or not (or only partially) at time t.

ALM model is given, too.

2.4 Accounting and policy constraints

In the previous section we defined the scenarios and the decision variables of our ALM model. Now we introduce constraints which deal with bookkeeping. Also, some policy constraints will be introduced in this section. Some other constraints were already mentioned before: the nonanticipativity constraints and the definitions of the state variables. In addition, nonnegativity is required for buying and selling assets, for cash flows from the sponsor, and restitutions:

$$XI^s_{jt} \geq 0, \quad XD^s_{jt} \geq 0, \quad j = 1, \ldots, N, t \in \mathcal{T}_0, s \in \mathcal{S},$$

$$Z^s_t \geq 0, \quad DZ^s_t \geq 0, \quad V^s_t \geq 0, \quad t \in \mathcal{T}, s \in \mathcal{S}.$$

The total value of the assets at time t in scenario s is given by the value of the asset portfolio, increased with contributions by active participants, and corrected for benefit payments which were paid in year t:

$$A^s_t = \sum_{j=1}^{N}(1 + r^s_{jt})X^s_{jt} + c^s_t W^s_t - B^s_t, \quad t \in \mathcal{T}_1, s \in \mathcal{S}, \tag{2.1}$$

where W_t^s denotes the total level of the pensionable salaries of the active participants in year t in scenario s. The value of the investments in asset class j, at the beginning of year $t+1$ in scenario s, is recursively defined by

$$X_{j,t+1}^s = (1 + r_{jt}^s)X_{jt}^s + XI_{jt}^s - XD_{jt}^s, \quad t \in \mathcal{T}_0. \tag{2.2}$$

After a possible remedial contribution by the sponsor of the fund at time t, Z_t, a direct cash flow from the sponsor because of a funding ratio which is far too low, DZ_t, or a restitution to the sponsor, V_t, the asset allocation has to be made, such that all assets are allocated, and transaction costs are taken into account appropriately:

$$\sum_{j=1}^{N} X_{j,t+1}^s = A_t^s + Z_t^s + DZ_t^s - V_t^s - \sum_{j=1}^{N} k_j (XI_{jt}^s + XD_{jt}^s), \qquad s \in \mathcal{S}, t \in \mathcal{T}_0, \tag{2.3}$$

where k_j denotes the proportional transaction costs for asset class j. These transaction costs, arising from the adjustment of the asset portfolio at time t, do not affect A_t^s, but they do influence the new asset portfolio. Equation (2.3) states that all assets have to be invested, and that transaction costs are considered, and can be interpreted as a *cash balance equation* for cash flows.

Constraint (2.3) is an accounting constraint, since incoming and outgoing cash flows are recorded appropriately. Similar constraints appear in all known ALM models (although Dert [24] does not take into account transactions costs), see for example the ALM models presented in Consigli and Dempster [17], Dert [24], and Kouwenberg [55].

In Table 2.3 an overview of incoming and outgoing cash flows at time t in scenario s is given. Recall that we assume that all cash inflows and outflows in year t are recorded at time t.

Incoming cash flows	Outgoing cash flows
$c_t^s W_t^s$	B_t^s
Z_t^s	V_t^s
DZ_t^s	$\sum_{j=1}^N (1 + k_j) XI_{jt}^s$
$\sum_{j=1}^N (1 - k_j) XD_{jt}^s$	

Table 2.3: Incoming and outgoing cash flows at time t in scenario s.

In Lemma 2.1 it is shown that definitions (2.1), (2.2), and accounting constraints (2.3) together imply that the cash inflow equals the cash outflow in state (t, s).

Lemma 2.1 *Constraints (2.1), (2.2), and (2.3) imply that for each state* (t, s)*, the cash inflow equals the cash outflow.*

Proof

From equality (2.2), we have:

$$\sum_{j=1}^{N} X_{j,t+1}^s = \sum_{j=1}^{N} (1 + r_{jt}^s) X_{jt}^s + \sum_{j=1}^{N} XI_{jt}^s - \sum_{j=1}^{N} XD_{jt}^s. \tag{2.4}$$

Substituting definition (2.1) in (2.3) gives

$$\sum_{j=1}^{N} X^s_{j,t+1} = \sum_{j=1}^{N}(1+r^s_{jt})X^s_{jt} + c^s_t W^s_t - B^s_t + Z^s_t + DZ^s_t - V^s_t - \sum_{j=1}^{N} k_j(XI^s_{jt} + XD^s_{jt}). \quad (2.5)$$

Because the right-hand sides of (2.4) and (2.5) must be equal, we obtain after rearranging terms

$$c^s_t W^s_t + Z^s_t + DZ^s_t + \sum_{j=1}^{N}(1-k_j)XD^s_{jt} = B^s_t + V^s_t + \sum_{j=1}^{N}(1+k_j)XI^s_{jt}. \quad (2.6)$$

On the left-hand side of (2.6), we have the cash inflows at time t, whereas the right-hand side of 2.6 represents the cash outflows at that time. These coincide with those presented in Table 2.3. □

Next to the equalities and inequalities presented above, there are also fund-dependent lower and upper bounds on the asset mix:

$$\underline{f}_j \sum_{i=1}^{N} X^s_{it} \leq X^s_{jt} \leq \overline{f}_j \sum_{i=1}^{N} X^s_{it} \quad j = 1,\ldots,N, t \in \mathcal{T}_1, s \in \mathcal{S},$$

where $\underline{f}_j$ and $\overline{f}_j$ are parameters that specify lower and upper bounds on the value of asset class j, as a fraction of the total assets.

Instead of fixed lower and upper bounds, these bounds may be time dependent. Given the current portfolio (just before time 0), the bounds may be functions of the current fractions and time. However, we use fixed values for $\underline{f}_j$ and $\overline{f}_j$ for every time $t \in \mathcal{T}_0$ and scenario $s \in \mathcal{S}$ in our ALM model.

For the initial asset portfolio, the following constraints are added:

$$X^s_{j0} = X_{j0} + XI^s_{j0} - XD^s_{j0} - k_j(XI^s_{j0} + XD^s_{j0}), \quad j = 1,\ldots,N, \quad s \in \mathcal{S},$$

where X_{j0} is the initial investment in asset class j, just before possible changes at time 0 can be made.

In the next sections, we describe some important extensions to the constraints mentioned above. These extensions are made to make the model flexible so that it can accommodate the policies of the pension fund.

2.5 Cash flows from the sponsor in case of financial distress

Pension funds want to avoid underfunding, because this implies that it cannot be guaranteed that all future benefit payments can be done. Formally, underfunding means that the funding ratio is less than 1. We will use this concept in a more general way, by saying that at time t a fund faces *underfunding with respect to the level* v (for some positive number v) if the funding ratio is less than v at that time ($A_t < vL_t$). In our ALM model, we distinguish various levels for underfunding, each with its own purpose. In this section we introduce two levels, θ and α ($\theta < \alpha$). They play a role in the policy rules for remedial payments of the sponsor to the fund.

In the circular [74], the PVK requires a minimum level of the funding ratio of 1.05. Therefore, we set $\alpha = 1.05$ in the numerical experiments presented in Chapter 6. As soon as the funding ratio is less then 1.05, the PVK requires a scheme how the board will tackle the problem to restore the funding ratio. In this study, we do not require such an immediate intervention. Only if the funding ratio falls even below the level θ, the sponsor should make a remedial payment immediately. The numerical value of θ may for example be 1 or 0.95. This value may also be the result of negotiations between the sponsor and the fund, or it may be prescribed by the supervisor.

If the sponsor has to make an immediate payment to the fund because the funding ratio is less than θ, this payment should at least be equal to the amount of the shortage with respect to the level θ. This immediate payment, which is denoted by DZ_t^s, should prevent that the financial position of the fund will erode completely.

If at any time the level of the funding ratio is at least α, then there is no direct financial distress, and a remedial contribution of the sponsor is not needed. In our model, it is forbidden in these circumstances.

If at any time the funding ratio is at least θ, but less than α, in our model a remedial payment of the sponsor is allowed, but not obliged. But the sponsor is obliged to restore the funding ratio to the level α if this ratio is below the minimum required level α in a consecutive years, where a is a parameter. For $a > 1$ and $\theta < \alpha$, we see that we introduced flexibility into our model. Requiring a remedial contribution as soon as the funding ratio is below α (i.e. $a = 1$) may lead to solutions which are very expensive. It is quite possible, that such a radical interference is not really necessary. For instance, if there is a quick recovery of the financial markets after a correction, it may not be necessary to have a remedial contribution from the sponsor to the fund. In this case, the total cost of funding is reduced.

Dert [24] formulated an ALM model in which a remedial contribution has to be made as soon as the funding ratio drops below a certain threshold value. In other ALM models, *shortages* are not even modeled (as in Consigli and Dempster [17]), or they are only recorded (as is done in Kouwenberg [55]).

On the basis of four possible future developments of the funding ratio, which are presented in Figure 2.3, we clarify these policy rules. In these examples, we make the assumption that the sponsor of the fund has to make a remedial contribution if the funding ratio is below α in two consecutive years ($a = 2$).

In case I, a remedial contribution has to be made at time 3. Moreover, we also see an advantage of the flexible modeling. If the sponsor was obliged to make a remedial contribution as soon as the funding ratio drops below the level α, the sponsor should have paid more at time 2.

In case II, the sponsor does not need not to restore the funding ratio, because underfunding is recorded only once. Here, the sponsor is allowed to make a remedial contribution (just as in case I on the second decision moment). Whether or not

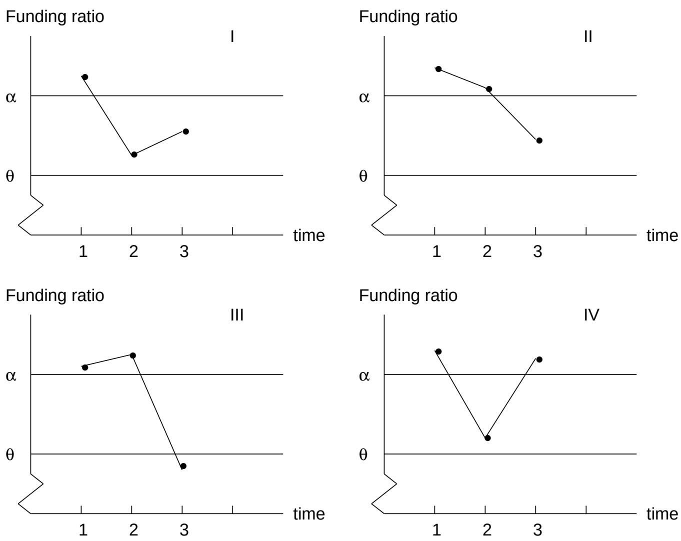

Figure 2.3: Four possible developments of the funding ratio, presented to clarify the decision rules with respect to underfunding in our ALM model.

a remedial contribution will be made at time 3, depends on future developments in the scenarios. If for example with very large probability a remedial contribution has to be made at time 4, the sponsor may prefer to pay at time 3. The reason of this will become clear in the next few pages.

Case III may be the result of a stock market crash. For the pension fund under consideration, the funding ratio drops below the level θ at time 3. Consequently, the sponsor has to interfere immediately. The funding ratio should at least be restored to the level θ.

Case IV emphasizes the advantage of the flexible modeling. After a decrease of the funding ratio under the critical level α, a recovery of the financial position of the fund occurs. Because the sponsor is not obliged to react at time 2, a feasible outcome is that no remedial contribution is made at all. The cause of the increase of the funding ratio may be favorable developments of financial markets, but also interventions by the board of the fund: at time 2, the asset portfolio may be changed, and/or the contribution rate may be increased.

This new modeling is important, since requiring a sufficiently high funding ratio at each balance date may be a too stringent perception of risk. Moreover, as a result of discussions in the beginning of the twenty-first century between the PVK and pension funds, resulted in the fact that the supervisor judges the solvency position of a fund partly on the basis of the funding ratio in successive years, see [74]. Another advantage of this modeling is already mentioned above: it may lead to a lower total cost of funding.

We model the payment of a remedial contribution after a consecutive years of underfunding as mixed-integer restrictions. We introduce binary variables u_t^s and

z_t^s to indicate underfunding and a remedial contribution respectively:

$$u_t^s = \begin{cases} 1 & \text{if } A_t^s < \alpha L_t^s \\ 0 & \text{otherwise,} \end{cases}$$

$$z_t^s = \begin{cases} 1 & \text{if } Z_t^s > 0 \\ 0 & \text{otherwise.} \end{cases}$$

From now on, we have $s \in \mathcal{S}$ and $t \in \mathcal{T}$, unless otherwise mentioned. The binary decision variables u_t^s and z_t^s get the correct values, because of the following 'definition inequalities':

$$A_t^s - \alpha L_t^s \geq -M u_t^s \tag{2.7}$$

$$A_t^s - \alpha L_t^s \leq M(1 - u_t^s) - \frac{1}{M} \tag{2.8}$$

$$Z_t^s \geq M(z_t^s - 1) - (A_t^s - \alpha L_t^s) \tag{2.9}$$

$$Z_t^s \leq M z_t^s \tag{2.10}$$

$$Z_t^s \geq 0$$

$$u_t^s \in \{0, 1\}, \quad z_t^s \in \{0, 1\}.$$

In these inequalities, M is a sufficiently large number ('big M'). Inequalities (2.7) and (2.8), together with the requirement $u_t^s \in \{0, 1\}$ provide the correct value for the binary decision variable u_t^s. If the funding ratio is below α, u_t^s is forced to become 1. Otherwise, this binary variable gets the value 0.

Inequalities (2.9) and (2.10), and the requirement $z_t^s \in \{0, 1\}$ force that in case of a remedial contribution $Z_t^s > 0$, z_t^s becomes 1, and otherwise it becomes 0. That is, although these two inequalities do not rule out that z_t^s becomes 1 if $Z_t^s = 0$, $z_t^s = 0$ is preferred if one considers the objective function (see Section 2.9). Moreover, if z_t^s is 1, the level of the remedial contribution is at least equal to the amount of underfunding. This forces the sponsor to restore the funding ratio at least to its minimum required level α.

The rules that a remedial contribution is only allowed in case of underfunding, and if at the last a decision moments the funding ratio is below α, a remedial contribution has to be made, also force to hold the following conditions:

$$z_t^s \leq u_t^s$$

$$z_t^s \geq \sum_{i=t-a+1}^{t} u_i^s - a + 1.$$

Here, for any $i \leq 0$, u_i^s is a given parameter, not depending on s, that indicates whether the funding ratio in year i was sufficiently high ($u_i^s = 0$) or not ($u_i^s = 1$).

If the sponsor of the fund has to pay a remedial contribution at time t, we still count year t as a year of underfunding. As a result, it is possible that the sponsor has to pay remedial contributions in successive years. This modeling makes sense, because of the weak financial position of the fund: even after a remedial contribution, the financial position is on the border of acceptability. Moreover, this way of modeling is also convenient from a mathematical point of view.

Drijver et al. [27] describe a more general way to model that the sponsor has to restore the funding ratio after some periods in which the funding ratio is too low. They model that the sponsor has to restore the funding ratio if in a of the last b periods ($b \geq a$) underfunding is present:

$$bz_t^s \geq \sum_{i=(t-b)^+ +1}^{t} u_i^s - a + 1.$$

For more details about this modeling, we refer to Drijver et al. [27].

Since the sponsor is generally not willing (and possibly not even able) to pay extremely large remedial contributions, a soft upper bound is given on this amount. This upper bound is defined as a fraction τ of the total level of the pensionable salaries W_t^s. Remedial contributions above τW_t^s are allowed, although the amount above this threshold, denoted by ZI_t^s, is penalized harder in the objective function. The decision variable $ZI_t^s = (Z_t^s - \tau W_t^s)^+$, is defined as follows in the ALM model:

$$ZI_t^s \geq Z_t^s - \tau W_t^s$$

$$ZI_t^s \geq 0.$$

Moreover, fixed penalty costs are incurred if the fund has to deal with underfunding and/or a remedial contribution is made, since these events are highly undesirable. As we have seen in the years 2002 and 2003, this makes sense: as soon as a fund announces that its financial position is weak, and possibly the sponsor of the fund has to make a remedial contribution, this is a hot issue in newspapers and magazines. All interested parties are far from happy: the supervisor because the fund is insolvent, the old aged because their benefit payments may not be indexed, the active participants because of a possible increase in the contribution rate, and the sponsor because it may have to pay a relatively large amount to the fund.

Figure 2.4 shows the relationship between the level of a remedial contribution to be paid by the sponsor of the fund, and the corresponding penalty. The fixed penalty costs due to this payment, is denoted by λ_z. The level of the remedial contribution is penalized by a factor ζ_Z. In addition, the level of the remedial contribution above τW_t, is penalized further by ζ_{ZI}.

The immediate cash flows from the sponsor to the fund in case of a shortage with respect to the level θ are modeled as follows:

$$DZ_t^s \geq \theta L_t^s - A_t^s - Mz_t^s \tag{2.11}$$

$$DZ_t^s \geq 0.$$

If $Z_t^s > 0$, and as a result, $z_t^s = 1$, the funding ratio is already restored to the level α. In this case, no additional immediate payment is required. The term $-Mz_t^s$ in (2.11) prevents a positive cash flow DZ_t^s in case of $Z_t^s > 0$. However, if $Z_t^s = z_t^s = 0$, and if $A_t^s < \theta L_t^s$, the sponsor is forced to pay at least $\theta L_t^s - A_t^s$.

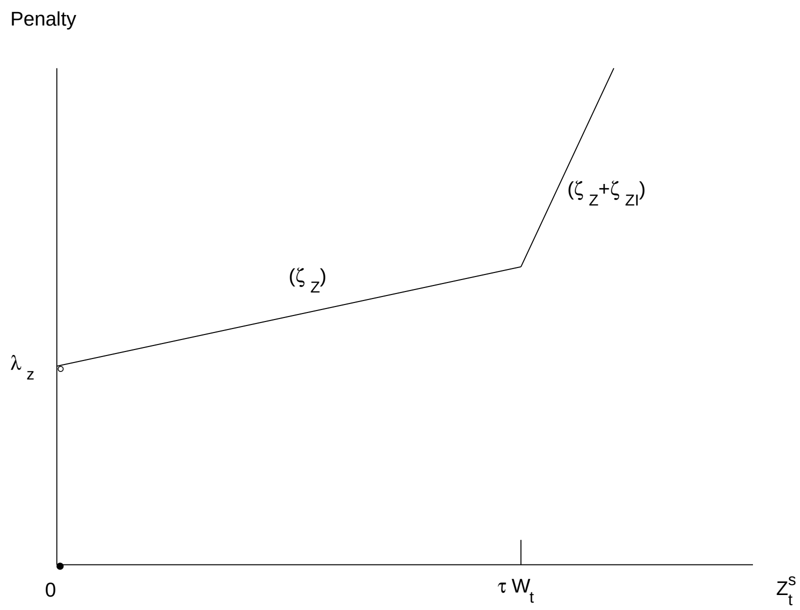

Figure 2.4: Relationship between the level of a remedial contribution by the sponsor, Z, and the corresponding penalty.

The user of the model has to specify the numerical value of ζ_{DZ} in relation with the numerical values of λ_z, ζ_Z, and ζ_{ZI} carefully. Indeed, if ζ_{DZ} would be chosen too low (relative to λ_z, ζ_Z, and ζ_{ZI}), optimization may result in states (t, s) with $DZ_t^s > 0$, and $Z_t^s = 0$, because then fixed penalty costs λ_z will be avoided. The value of ζ_{DZ} should be set sufficiently high to represent that such an immediate payment is extremely undesirable. Moreover, the numerical specification of the penalty parameters should be made, such that a correct representation is found of the wishes of the board of the fund under consideration (although this may not be easy), and also what is for example contractually determined between the sponsor and the pension fund.

2.6 Contribution rate

To finance the pension fund, employers and employees make on a regular basis payments to the fund. These payments are a percentage of a part of the total wages of the participants. This part is called the pensionable salary, and is denoted by W_t^s for year t. The board of the pension fund has to determine the level of the percentage of these pensionable salaries. For time t and scenario s, the level of this percentage is denoted by c_t^s. This level is determined at the beginning of every year. As a result, for year $t+1$, this level is specified at time t.

In determining next year's level of the contribution rate, the board of a fund not only has to take into account bounds on this level, it also considers the wish of a relatively stable contribution rate through time. Moreover, from the perspective of the sponsor it is reasonable to assume that the level of the contribution rate should be sufficiently high if the sponsor of the fund makes a remedial contribution, so that also active participants contribute to a better financial position of the fund. Details with respect to these policy rules and the mathematical formulations are discussed in this section.

Although the board can determine the level of the contribution rate, this level is bounded. Even if the funding ratio is so low, that not all future benefit payments can be guaranteed, c_t^s cannot increase indefinitely. This upper bound on c_t^s is denoted by $\overline{c}$, and is assumed to be time independent. Also a time independent lower bound on c_t^s, which is denoted by $\underline{c}$, exists. The numerical specification of $\underline{c}$ and $\overline{c}$ may be fund dependent, and may be stated in the fund's pension regulation. We assume that the following conditions hold: $0 \leq \underline{c} < \overline{c} < 1$.

The lower and upper bounds on the contribution rate c_t^s give rise to the following constraint in our ALM model:

$$\underline{c} \leq c_t^s \leq \overline{c}. \tag{2.12}$$

Not only the level of the contribution rate is important, but also its stability, since too much variability is undesirable.

We can model this refinement as

$$-\eta \leq c_t^s - c_{t-1}^s \leq \rho, \tag{2.13}$$

where $c_t^s - c_{t-1}^s$ represents the change in the contribution rate in two consecutive years (t and $t-1$) and η and ρ are fixed bounds for decreases and increases of the contribution rates. In the ALM model presented in Kouwenberg [55], these 'hard' constraints are used.

The numerical values for η and ρ also fund dependent. In addition, these values are expected to be not too low. On the other hand, if the values of η and ρ are large, (2.13) loses its meaning in modeling the undesirability of a contribution rate which changes rapidly.

However, if the funding ratio is relatively low for a number of years, it may be preferable to increase the contribution rate rather than to ask for a remedial contribution. This may lead to an increase in the contribution rate which exceeds ρ. Hence, it is better to specify (2.13) as a goal constraint: increases (decreases) greater than ρ (η) are allowed (although constraint (2.12) still has to be satisfied), but they are penalized in the objective function. Now, η and ρ denote the maximum decrease and increase in the contribution rate in two consecutive years, such that no penalties are incurred due to a rapidly changing contribution rate.

We model this in a linear programming formulation by the introduction of additional decision variables ci_t^s, representing the amount by which the increase in the contribution rate exceeds ρ at time t compared with the contribution rate at time $t-1$. The second inequality of (2.13) is replaced by

$$ci_t^s \geq c_t^s - c_{t-1}^s - \rho$$

$$ci_t^s \geq 0.$$

In the objective function, we penalize $ci_t^s = (c_t^s - c_{t-1}^s - \rho)^+$, which is positive if $c_t^s > c_{t-1}^s + \rho$.

We can use an analogous reasoning when the funding ratio is relatively high for a number of successive years. In this situation it may be preferable to lower the contribution rate in two successive years by more than η. This may be more desirable than, for example, making a restitution to the sponsor, because the board of the fund would like that the active participants profit from the financial prosperity of the fund.

We can model this by the introduction of additional decision variables cd_t^s, representing the amount by which the decrease of the contribution rate in year t exceeds η, as compared with the contribution rate at time $t-1$:

$$cd_t^s \geq c_{t-1}^s - c_t^s - \eta$$

$$cd_t^s \geq 0.$$

In the objective function, we penalize $cd_t^s = (c_{t-1}^s - c_t^s - \eta)^+$, which is positive if $c_t^s < c_{t-1}^s - \eta$.

We penalize ci_t^s and cd_t^s by positive parameters ζ_{ci} and ζ_{cd} (usually $\zeta_{cd} \leq \zeta_{ci}$), whereas no penalty is imposed if (2.13) is satisfied. Figure 2.5 shows an example of such a penalty function. In stochastic programming, this structure, that models

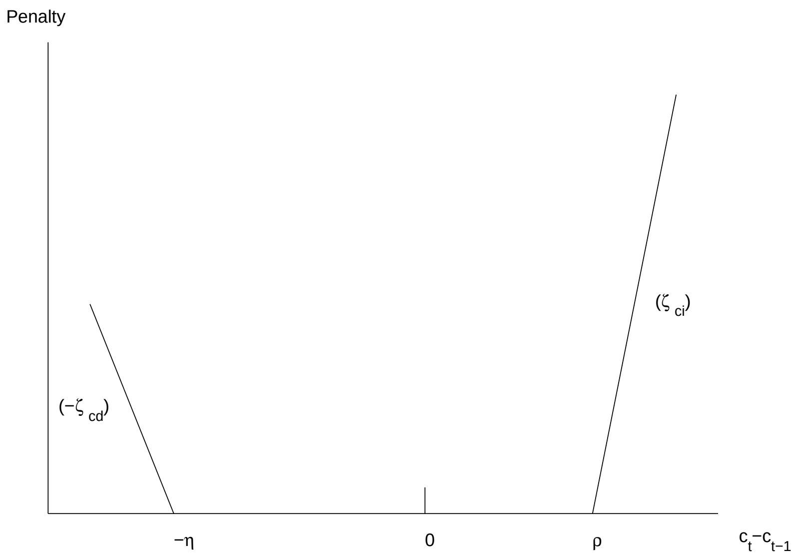

Figure 2.5: Penalty costs associated with a change in the contribution rate at time t, as compared with the contribution rate at time $t-1$.

piecewise linear increasing costs for shortages and surpluses, is known as multiple simple recourse. For details on multiple simple recourse, see Klein Haneveld [53] and Van der Vlerk [91]. We have never seen this structure in ALM models in the literature before.

The formulation described above makes sense in practise: relatively large increases in contribution rates are faced with a lot of resistance by active participants. As a result, if large deviations of the current level of the contribution rate could be avoided, this would be the best alternative for the parties involved in the decision making process. On the other hand, if large increases are needed, because otherwise the financial position would become too weak, such an increase can be considered.

Note that the above introduced formulation is also very flexible: if managers do not want to penalize large deviations in the contribution rate, they can choose to either specify $\zeta_{cd} = \zeta_{ci} = 0$, or to set $\eta = \rho \geq \overline{c} - \underline{c}$.

Contribution rate in case of a remedial contribution

As we have seen before, we included in our model the policy rule, that the sponsor of the fund has to restore the funding ratio if there is a shortage with respect to the level α in a number of consecutive years. It seems reasonable that if such a remedial contribution is made, the sponsor requires that the active participants also provide a sufficient contribution to the financing of the fund in the form of a sufficiently high contribution rate. Therefore, we will include this condition in a policy rule of our model, as follows. We denote the minimum required contribution rate in case of a remedial contribution by c^*. The numerical value of c^* may for example be the fund's actuarial premium. The constraint, which serves the rule mentioned above, is given by:

$$c_t^s - c^* \geq M(z_t^s - 1). \tag{2.14}$$

In this case, big M may be taken equal to $\overline{c} - \underline{c}$. If the sponsor of the fund has to make a remedial contribution, and, as a result, $z_t^s = 1$, (2.14) leads to the requirement $c_t^s \geq c^*$. On the other hand, if $z_t^s = 0$, no additional requirement with respect to the level of the contribution rate is made. Note that if one does not want to model this relationship between c_t^s and Z_t^s, one can simply eliminate this rule by choosing $c^* = \underline{c}$, since $c_t^s \geq \underline{c}$ always has to be satisfied.

Of course, there are a variety of alternative formulations for requirements with respect to the level of the contribution rate if the sponsor of the fund makes a remedial contribution. A few alternatives are presented below.

- Require that the contribution rate does not decrease if $z_t^s = 1$. This can be accomplished by the constraint

  $$c_t^s - c_{t-1}^s \geq M(z_t^s - 1).$$

  The disadvantage of this modeling is, however, that even if the sponsor of the fund makes a remedial contribution, it is allowed that the contribution rate may still be very low.

- Penalize the deviation of the contribution rate from its upper bound $\overline{c}$. This can be accomplished by the constraint

$$\overline{c} - c_t^s - cdu_t^s \leq M(1 - z_t^s), \tag{2.15}$$

and penalize the nonnegative decision variable cdu_t^s in the objective function. If the sponsor has to make a remedial contribution, (2.15) results in

$$cdu_t^s \geq \overline{c} - c_t^s,$$

and, as a result, the deviation of the contribution rate from its upper bound can be penalized. On the other hand, if $z_t^s = 0$, constraint (2.15) is non-binding.

A disadvantage of this modeling is that it may be difficult to find an appropriate value for the penalty parameter associated with cdu_t^s.

Which constraint(s) are added to the ALM model may depend on the contribution policy of the fund. Because we think formulation (2.14) is the most important for real world practice, we have chosen to include this constraint in our ALM model.

2.7 Indexation

Indexation of liabilities is the adjustment of the built-up rights to increases in prices or wages in a certain year. As we have seen in Chapter 1, pension funds are not obliged to adjust benefit payments. Generally, every year again, the board decides whether or not to increase pension rights. This decision may of course depend on the financial position of the fund, and also on the level of the increase in prices or wages. Therefore, this decision is made after the realization of the development in prices or wages is known. Typically, pension funds adjust the benefit payments for increases in the price or wage level fully if the solvency of the fund is sufficient. Moreover, if pension rights will be indexed only partially, or not at all, this leads to great dissatisfaction, especially of retired people. In this section, we describe how we have modeled indexation. Considering indexation as a decision instrument in an optimization model is new in the financial literature. In the remainder of this thesis, we use increases in the general wage level as the base to index rights.

Mathematical formulation

The basic idea of incorporating indexation as decisions in a linear programming structure is relatively easy. Consider the liabilities in state (t, s), L_t^s, not as a parameter, but as a decision variable, that may vary within a certain range. The bounds on this range are denoted by $\underline{L}_t^s$ and $\overline{L}_t^s$, and they are parameters in the scenario tree. The upper bound $\overline{L}_t^s$ represents the value of the liabilities if in all years $0, \dots, t$ full indexation is given to the participants of the fund. The lower bound $\underline{L}_t^s$ represents the nominal value of the liabilities in state (t, s). This means that from year

0 to year t the benefit payments are not adjusted at all for increases in the general wage level in those years. So we add the following constraint to our ALM model:

$$\underline{L}_t^s \leq L_t^s \leq \overline{L}_t^s. \tag{2.16}$$

Of course, more constraints are needed to model the indexation policy of a fund in a proper way. For instance, constraint (2.16) does not prevent $L_t^s = \underline{L}_t^s$ in all states (t, s), even if $L_t^s = \overline{L}_t^s$ would result in a sufficiently high funding ratio. In our model, we assume that the board of a pension fund has the following indexation goals:

- It strives to index liabilities with respect to last year's increase in the general wage level.
- If in a certain year the pension rights are not fully compensated for increases in the general wage level, it strives to give this compensation in a later year.

These goals are incorporated in our model by introducing penalties if they are not reached.

Incorporating indexation into the model, implies that the board of a pension fund gets an additional instrument to indicate what to do in case of financial distress. After all, if the funding ratio is sufficiently high, the board generally gives full compensation for increases in the wage level. Optimization of the model will also lead to this solution, because penalty costs are avoided in this case, However, the question remains what to do in case of less desirable financial circumstances. In that case, underfunding may only be avoided by giving up full indexation. This decision may also be influenced by the power of retired people, or by the financial soundness of the sponsor. In our ALM model, this balance of decisions will be the result of the numerical specifications of the penalty parameters.

Now, we will describe the constraints which are added to our model, which serves for a penalty in case the interests described above are not satisfied. We first describe the penalty associated with not giving full indexation in all years up to the current year. Adding the term

$$\sum_{s=1}^{S} \sum_{t=0}^{T} p_t^s \gamma_t^s (\zeta_L \left(L_t^s - \overline{L}_t^s \right)^-) \tag{2.17}$$

to the objective function, the total deviation is penalized. Here, the parameter ζ_L is a penalty parameter, and its numerical value should be specified by the user of the model.

We also want to include a (fixed) penalty if liabilities are not fully indexed. We have chosen this formulation, because the decision not to give this compensation is very undesirable and causes much commotion among interested parties.

To include such a penalty into a linear programming framework, we not only need to know the numerical value of w_t^s, but also what the change in the liabilities from time $t-1$ to time t is. In state (t, s) liabilities are fully indexed with respect to last years' increase in the general wage level w_t^s, if the following condition holds:

$$L_t^s \geq (1 + \varphi_t^s)(1 + w_t^s) L_{t-1}^s,$$

where

$$\varphi_t^s := \frac{\underline{L}_t^s - \underline{L}_{t-1}^s}{\underline{L}_{t-1}^s}$$

denotes the relative change in the nominal liabilities. Indeed, $\underline{L}_t^s$ generally differs from $\underline{L}_{t-1}^s$, for example because of changes in the age distribution of the participants of a specific pension fund. Note that φ_t^s is data in the scenario tree, since the lower bounds $\underline{L}_t^s$ and $\underline{L}_{t-1}^s$ are data, too. Because of the definition of φ_t^s, $L_t^s = (1+\varphi_t^s)L_{t-1}^s$ gives the value of the liabilities if the benefit rights are not indexed (but also not deteriorates). This number is multiplied by $(1+w_t^s)$ to get the value of the liabilities, such that future rights are indexed with respect to last years' increase in the general wage level.

Now, we introduce the '*degree of change of indexing*' in year t in scenario s. We denote it by I_t^s, and it is given by

$$I_t^s := \frac{L_t^s}{(1+\varphi_t^s)(1+w_t^s)L_{t-1}^s}.$$

The values of I_t^s can be found after the ALM model is solved, since then the values of L_t^s are known in all states (t,s). If I_t^s gets the value 1, liabilities are fully indexed, whereas values less than 1 indicate that benefit payments are only partially adjusted, or not adjusted at all. Note that a value of I_t^s greater than 1 is also possible. This may happen if in at least one year $q < t$ future benefit rights are not fully indexed. If at time t the benefit payments are also adjusted for increases in w_q^s, the numerical value of I_t^s exceeds 1. Note that I_t^s is not introduced as a decision variable, since then nonlinearities would have been introduced.

To include fixed penalty costs if the fund does not correct future benefit payments for w_t^s, we need binary decision variables. They indicate whether pension rights are fully indexed or not. These binary decision variables are denoted by m_t^s, and are defined by

$$m_t^s = \begin{cases} 1 & \text{if } L_t^s < (1+\varphi_t^s)(1+w_t^s)L_{t-1}^s \quad \text{i.e. } (I_t^s < 1) \\ 0 & \text{otherwise.} \end{cases}$$

This means that m_t^s gets the value 1 if the participants of the fund do not receive full compensation for last year's increase in the general wage level, and 0 if this compensation is given.

To find the correct values for the decision variable $m_t^s \in \{0,1\}$, the following constraints are added to the set of restrictions of our ALM model:

$$L_t^s - (1+\varphi_t^s)(1+w_t^s)L_{t-1}^s \geq -Mm_t^s,$$

$$L_t^s - (1+\varphi_t^s)(1+w_t^s)L_{t-1}^s \leq M(1-m_t^s) - \frac{1}{M}.$$

To penalize m_t^s, the following term is added to the objective function:

$$\sum_{s=1}^{S}\sum_{t=1}^{T} p_t^s \gamma_t^s (\lambda_m m_t^s),$$

where λ_m denotes the fixed costs associated with not giving full compensation. The numerical value of this penalty parameter has to be specified by the user of the model.

Benefit payments

The degree of indexing not only influences the value of the liabilities, but also the level of next year's benefit payments. If pension rights are never indexed, only nominal levels of these payments are made. On the other hand, if always full compensation is given, the nominal benefit payments are adjusted appropriately to reflect this policy. In this case, the benefit payments in state (t,s) are given by

$$\overline{B}_t^s = \underline{B}_t^s \Pi_{q=1}^t (1 + w_q^s),$$

where $\underline{B}_t^s$ denotes the nominal expected benefit payment in state (t,s). In our model, both $\underline{B}_t^s$ and $\overline{B}_t^s$ appear as parameters. They are the lower and upper bound, respectively, of the actual benefit payment B_t^s in year t and scenario s. The model adopts the following rule to find the value of B_t^s. The nominal expected benefit payment is increased (in the same proportion as) the fraction of the wage inflation participants of the fund received last year. These fractions are found by the values of the liabilities. Formally, this relationship between the benefit payments and the value of the liabilities is state (t,s) is given by

$$B_t^s = \underline{B}_t^s + \frac{L_{t-1}^s - \underline{L}_{t-1}^s}{\overline{L}_{t-1}^s - \underline{L}_{t-1}^s} (\overline{B}_t^s - \underline{B}_t^s). \tag{2.18}$$

Equality (2.18) is added to the set of constraints of our ALM model.

Implications of the modeling

Modeling indexing as described above has the following implications. First of all, if in a certain year t only partial compensation (or no compensation at all) is given, it is still possible that in the future compensation with respect to wage inflation of year t is given. However, the value of the binary decision variable m_t^s in state (t,s) remains 1, since at that moment, full compensation was not given.

On the other hand, it is also possible that once indexing is given, this decision is turned back at a later decision moment if the financial position of the fund is weakened. However, this does not change the values of the binary decision variable m in previous years.

2.8 Restitutions

In Chapter 1 we have seen that because of favorable developments of financial markets, the funding ratio may increase rapidly. In such a case, the question arises for the board of a fund, what to do with large surplusses. One possibility is that money is transferred back to the sponsor of the fund. These *restitutions* may also be established contractually: the sponsor makes a remedial contribution in case of underfunding, and benefits from large surplusses of the fund too. Recall that in our ALM model a restitution at time t in scenario s is denoted by V_t^s.

Our ALM model adopts the following rules for restitutions to the sponsor of the fund, in terms of two policy parameters β and b:

- A restitution to the sponsor can only occur if the funding ratio is greater than β, where β is a fixed level ($\beta \gg \alpha$).
- If a restitution is made, it should at least be equal to the amount of the surplus with respect to the level β.
- If in b consecutive years a surplus with respect to the level β is recorded, a restitution to the sponsor has to be made, where b is a fixed number.
- If a restitution has to be made, this payment is made as a lump-sum.
- A restitution in state (t, s) is only allowed if full indexing is given for increases in the general wage level in all previous years, i.e. if $L_t^s = \overline{L}_t^s$.

These rules are formulated as linear constraints in the decision variables, after adding the following binary decision variables:

$$o_t^s = \begin{cases} 1 & \text{if } A_t^s > \beta L_t^s \\ 0 & \text{otherwise,} \end{cases}$$

$$v_t^s = \begin{cases} 1 & \text{if } V_t^s > 0 \\ 0 & \text{otherwise.} \end{cases}$$

These binary variables get the correct values, because of the following 'definition inequalities':

$$\beta L_t^s - A_t^s \geq -M o_t^s$$

$$\beta L_t^s - A_t^s \leq M(1 - o_t^s) - \frac{1}{M}$$

$$V_t^s \geq M(v_t^s - 1) - (\beta L_t^s - A_t^s) \tag{2.19}$$

$$V_t^s \leq M v_t^s,$$

where $o_t^s \in \{0, 1\}$, $v_t^s \in \{0, 1\}$ and $V_t^s \geq 0$ for all $s \in \mathcal{S}$, and $t \in \mathcal{T}$.

Inequality (2.19) also forces the restitution to satisfy the second rule. The first and third rule are forced to hold by the conditions

$$v_t^s \leq o_t^s$$

$$v_t^s \geq \sum_{i=t-b+1}^{t} o_i^s - b + 1.$$

Here o_i^s ($i = 1 - b, \ldots, 0$) is a given parameter, not depending on s.

We still have to model the last rule, that a restitution is only allowed if full compensation is given for the increase in the general wage level in all previous years. In other words, a restitution in state (t, s) is only allowed if, next to conditions regarding the level of the funding ratio, $L_t^s = \overline{L}_t^s$ is satisfied.

To model this rule, an additional binary variable is needed to indicate whether $L_t^s = \overline{L}_t^s$ or not. This binary variable, denoted by l_t^s, is defined as follows:

$$l_t^s = \begin{cases} 1 & \text{if } L_t^s < \overline{L}_t^s \\ 0 & \text{otherwise.} \end{cases}$$

This binary variable gets the correct value by means of the following inequalities, together with the requirement $l_t^s \in \{0, 1\}$:

$$\overline{L}_t^s - L_t^s \leq M l_t^s \tag{2.20}$$

$$\overline{L}_t^s - L_t^s \geq M l_t^s - \frac{1}{M}. \tag{2.21}$$

The rule under consideration is modeled by adding the following inequality to the set of constraints:

$$V_t^s \leq M(1 - l_t^s).$$

If the binary decision variable l_t^s gets the value 0, i.e. if full compensation for increases in the general wage level is given up to and including year t, a restitution to the sponsor is allowed (or even forced because of other constraints). On the other hand, if l_t^s gets the value 1, no restitution will be made. Note that we could also have introduced fixed penalty costs associated with $l_t^s = 1$. However, from discussions with advisors of pension funds we concluded that not indexing liabilities is considered as the most important indicator.

2.9 Objective function

A pension fund wants to minimize the total cost of funding, i.e., the contributions made by the active participants of the fund and the remedial contributions. It also wants to avoid 'undesirable events'. Therefore, in our ALM model, we do not only include the funding costs, but also penalty costs and rewards in the objective function.

Fixed penalty costs due to underfunding, a remedial contribution, and a deterioration of indexation were introduced, because these events are highly undesirable. Moreover, large increases and decreases in the contribution rate in two consecutive years, the level of a remedial contribution, and deviations of the value of the liabilities from its upper bound are also penalized. On the other hand, a fixed reward is given for overfunding with respect to the level β and also for restitutions to the sponsor of the fund. In addition, the level of a restitution is rewarded, too.

At the horizon, surplusses and shortages with respect to the level Λ are rewarded and penalized, respectively. The parameter Λ serves as a minimum desired level of the funding ratio after T years.

The objective of the board of the fund is to minimize the total expected discounted costs (including penalty costs). As a result, the probability and the discount factor in each state (t, s), denoted by p_t^s and γ_t^s respectively, appear in the objective function.

All these components together constitute the objective function

$$
\begin{aligned}
&\sum_{s=1}^{S}\Big[\sum_{t=0}^{T} p_t^s \gamma_t (c_t^s W_t + Z_t^s) && \text{funding costs}\\
&\text{penalties:}\\
&+\sum_{t=1}^{T} p_t^s \gamma_t^s (\zeta_{ci} ci_t^s + \zeta_{cd} cd_t^s) W_t && \text{change in contribution rate}\\
&+\sum_{t=0}^{T} p_t^s \gamma_t^s (\lambda_u u_t^s) && \text{underfunding}\\
&+\sum_{t=0}^{T} p_t^s \gamma_t^s (\lambda_z z_t^s + (\zeta_Z - 1) Z_t^s + \zeta_{ZI} ZI_t^s) && \text{remedial contribution}\\
&+\sum_{t=0}^{T} p_t^s \gamma_t^s (\zeta_{DZ} DZ_t^s) && \text{cash payment}\\
&+\sum_{t=0}^{T} p_t^s \gamma_t^s (\lambda_m m_t^s) && \text{deterioration of indexation}\\
&+\sum_{t=0}^{T} p_t^s \gamma_t^s (\zeta_L (L_t^s - \overline{L}_t^s)^-) && \text{no full indexation}\\
&+\sum_{t=0}^{T} p_t^s \gamma_t^s (\lambda_o o_t^s) && \text{overfunding}\\
&+\sum_{t=0}^{T} p_t^s \gamma_t^s (\lambda_v v_t^s + \zeta_V V_t^s) && \text{restitution}\\
&+p_T^s \gamma_T^s (\zeta_{\Lambda d} (A_T^s - \Lambda L_T^s)^- + \zeta_{\Lambda i} (A_T^s - \Lambda L_T^s)^+)\Big]. && \text{horizon}
\end{aligned}
$$

At first sight, the above presented objective function may not seem appropriate to be used in a linear program, due to the presence of $(A_T^s - \Lambda L_T^s)^-$, $(A_T^s - \Lambda L_T^s)^+$ and $(L_t^s - \overline{L}_t^s)^-$. However, these terms can be taken into account in a linear programming framework. To do so, we replace $(A_T^s - \Lambda L_T^s)^-$ and $(A_T^s - \Lambda L_T^s)^+$ by nonnegative decision variables $Sho\Lambda_T^s$ and $Sur\Lambda_T^s$. Moreover, we add the following constraint to the set of restrictions:

$$A_T^s - \Lambda L_T^s = Sur\Lambda_T^s - Sho\Lambda_T^s \qquad s \in \mathcal{S}. \tag{2.22}$$

The requirement $\zeta_{\Lambda d} > -\zeta_{\Lambda i}$ has to be made, in order to obtain a bounded solution. In a similar way, the term $(L_t^s - \overline{L}_t^s)^-$ can be incorporated in a linear program.

In this chapter we have presented a large part of our ALM model. We have argued that indicators are useful. They are for example needed in appropriately modeling mid-term risks of pension funds. The introduction of indicators has consequences in an optimization model. From a computational point of view, such models become extremely difficult for reasonably sized problems. However, if one uses insights into the problem, one may obtain heuristic solutions to such ALM problems. This will be the subject of Chapter 4. Although considering mid-term risk is important in ALM problems, restrictions may also be imposed on next year's solvability of the fund. This type of constraint will be discussed in the next chapter.

Chapter 3

One-year risk constraints

In the model presented in Chapter 2, several flexible aspects are presented to maintain a sufficiently high level of the funding ratio: underfunding is penalized, and the sponsor has to restore the funding ratio if in too many consecutive years underfunding is recorded. In addition, also the level of such a payment is penalized. Finally, we have incorporated a target level of the funding ratio at the horizon.

The flexible aspects described above are all soft constraints in our model. However, as we will see in the next section, the supervisor of Dutch pension funds also imposes hard constraints with respect to the short-term solvency position. This is the reason why we also consider such constraints in our ALM model.

The question remains how to incorporate such short-term risk constraints. In the ALM model of Dert [24] decisions have to be made, such that the probability of underfunding in the next year is sufficiently small. However, we think that not only the probability of underfunding is important, but also the amount of a shortage.

In this chapter, these two possible ways to incorporate risk constraints which deal with underfunding in the ALM process are discussed. They are called chance constraints and integrated chance constraints respectively. They will not only be compared from an algorithmic point of view, but also their interpretations are discussed. As we will see, we prefer integrated chance constraints in our ALM model. Before we discuss these two types of risk constraints, we first describe (the developments of) the requirements pension funds have to comply with.

3.1 Solvency tests of supervisor (2002)

In Section 1.2.3 we have described how supervision is organized. We have also described which actions the board of a pension fund has to take in case of financial distress.

Currently, the financial position of Dutch pension funds are judged on the rules written in the *Actuariële Principes voor Pensioenfondsen (APP)*, which dates from 1997. According to the supervisor PVK, these principles have too many shortcomings to judge the solvency position of pension funds sufficiently. They are not dynamic enough and stress the current situation too much.

To gain more insight in a fund's financial position, the PVK has developed new rules, called *Financieel Toetsingskader (FTK)* in 2002. The central themes in the FTK are transparency, risks and results based on market values (not only for assets, but also for liabilities), and making methods explicit. In the FTK (such as formulated in 2002), three tests are described to judge the solvency position of a fund:

- A test which considers the solvency position of a pension fund in the long run, the so-called *continuïteitstoets*.

- A test of the financial position based on both the assets and the liabilities, corresponding to risks associated with the financial position in one year, called the *solvabiliteitstoets*. In this test, underfunding may occur with a prespecified acceptable, but small, probability. Both the assets and liabilities are valued using observed market prices.

- At the next balance date, the market value of the assets should at least be equal to the market value of the liabilities. This test is called the *minimumtoets*.

In formulating these three tests for pension funds, the PVK has considered developments in other sectors. Especially the regulation in the banking industry was an important reference point. The regulatory requirements for banks were introduced by the Bank of International Settlements (BIS), and started its work in 1988 (Basel Accord). Since then, it frequently updated these requirements (2000, new accord 2002). These regulations are followed by financial institutions all over the world. In the last accord, more emphasis is placed on the bank's own internal methodologies, supervisory review, and market discipline.

As we have seen in Chapter 2, funding ratios which are too low are penalized in our multiperiod model. Moreover, a remedial contribution is required if underfunding is recorded in too many consecutive years. In addition, we have incorporated a target level of the funding ratio at the planning horizon of our model. As a result, the 'continuïteitstoets' is taken into account in our model.

As will become clear in this chapter, we also incorporate one-year risk constraints in our model. However, we do not only consider the probability of underfunding in one year, but also the associated amounts. Therefore, we consider the 'solvabiliteitstoets' in an adjusted form.

In our model, the sponsor has to restore the funding ratio as soon as this ratio is less than θ. As a result, for $\theta = 1$ the requirement presented in the 'minimumtoets' would be satisfied. However, in the numerical experiments presented in Chapter 6, we have chosen to set $\theta = 0.90$, since we think that always requiring a funding ratio of at least 1 leads to solutions which are too expensive.

3.2 Chance constraints

In this section, we describe a first idea for representing risk constraints in ALM models, *chance constraints*. Incorporating chance constraints in ALM models was introduced by Dert [24].

Chance constraints serve as tools for modeling risk and risk aversion in stochastic programs. The board of a pension fund strives to satisfy the goal constraints

$$A_t^s \geq \alpha L_t^s \qquad \forall t \in \mathcal{T}_1, s \in \mathcal{S} \tag{3.1}$$

for some $\alpha \geq 1$. Incorporating constraint (3.1) in our model, might lead to excessively high funding costs or to infeasibilities. Instead, the board of a pension fund may formulate the condition that the probability of a sufficiently high funding ratio in the next T years is sufficiently large. This requirement can be modeled as

$$P(A_t^s \geq \alpha L_t^s, t \in \mathcal{T}_1) \geq \phi,$$

where ϕ denotes the prescribed probability. Although such a *long-term chance constraint* makes sense, it cannot be incorporated into a linear program. In the previous section we have seen that the supervisor in The Netherlands also considers the short-term financial position of pension funds. Therefore, we restrict ourselves here to one-year chance constraints. In these chance constraints, next year's level of the funding ratio should be sufficiently large with a prescribed probability ϕ_t:

$$P(A_t^s \geq \alpha L_t^s) \geq \phi_t, \quad t \in \mathcal{T}_1. \tag{3.2}$$

Condition (3.2) acts as a constraint *on the decisions at time t*, in terms of consequences at time $t+1$. As a matter of fact, although the representation in (3.2) does not show this explicitly, there are many chance constraints of this type at time t. In fact, there is a chance constraint for every node in the scenario tree corresponding to time $t \in \mathcal{T}_0$.

In the chance constraints (3.2), the probability distribution used is the conditional distribution of next year's random vector ω_t given the observed values of the past $\omega_1, \ldots, \omega_{t-1}$. The value of the parameter ϕ_t, the *minimum required reliability* at time t, is set by the decision makers. It should not be set too low, because then it will lose its meaning of modeling a goal. On the other hand, solving models with ϕ_t too large (e.g. approximately equal to one) may lead to expensive solutions or to infeasibilities as was the case with goal constraint (3.1). Also note that ϕ_t may be time dependent: in earlier years, it may be even more undesirable to have a low funding ratio.

In formulation (3.2), $P(A_t^s \geq \alpha L_t^s)$ is called the *reliability* and $1 - P(A_t^s \geq \alpha L_t^s)$ is called the *risk of next year's insolvency* and is closely related to Surplus-at-Risk as described by H.A. Klein Haneveld [51]. Decisions that are insufficiently reliable (with respect to the next decision moment) are not accepted. This restricts the feasible region.

It is well-known that chance constraints can be represented in a linear programming framework by introducing indicator variables. We will provide the details of this representation for (3.2). For reasons to be explained afterwards, we first replace the variable L_t^s in (3.2) by the upper bound parameter $\overline{L}_t^s$, so that the condition becomes stronger, potentially. Using the notation introduced in Section 2.3, we now explain how inequalities (3.2) can be written in a mixed-integer programming framework. At time t, we observe the realization of ω_t, and therefore know the actual state (t, s). Given this state, the conditional probability of each child node is given by $(branch_t)^{-1}$, since we assume that all child nodes are equally probable. The chance constraints can now be written as:

$$\frac{1}{branch_t} \sum_{s' \in \mathcal{K}_t^s(t+1)} I_{\{A_{t+1}^{s'} < \alpha \overline{L}_{t+1}^{s'}\}}(s') \leq 1 - \phi_t, \quad t \in \mathcal{T}_0, s \in \mathcal{S}_t,$$

where $I_{\{A_{t+1}^{s'} < \alpha \overline{L}_{t+1}^{s'}\}}(s') = 1$ if $A_{t+1}^{s'} < \alpha \overline{L}_{t+1}^{s'}$ and 0 otherwise. Given the definition of the binary variable u_t^s, which was introduced in the previous chapter, the chance constraints can be written as linear inequalities for each state (t, s), $t \in \mathcal{T}_0$, $s \in \mathcal{S}$:

$$Mu_{t+1}^{s'} \geq \alpha \overline{L}_{t+1}^{s'} - A_{t+1}^{s'}, \quad t \in \mathcal{T}_0, s \in \mathcal{S}_t, s' \in \mathcal{K}_t^s(t+1) \tag{3.3}$$

$$\frac{1}{branch_t} \sum_{s' \in \mathcal{K}_t^s(t+1)} u_{t+1}^{s'} \leq 1 - \phi_t, \quad t \in \mathcal{T}_0, s \in \mathcal{S}_t, \tag{3.4}$$

where, as before, M is a sufficiently large number.

Note that we have used the upper bound on the value of the liabilities in the chance constraints. Why not using their actual value $L_{t+1}^{s'}$? The reason is that, unlike $A_{t+1}^{s'}$, the level of these liabilities depend on decisions to be made at time $t+1$ rather than at time t for which (3.3) is formulated. At time t, the upper bound $\overline{L}_{t+1}^{s'}$ is a parameter and therefore its value is known.

If the number of child nodes is too small, the chance constraints coincide with the goal constraint (3.1). Assume for example that $\phi_t = 0.8$ and we have only two child nodes from a certain state, and the conditional probabilities associated with them are both $\frac{1}{2}$. In this case, the chance constraints lose their meaning of modeling risk, since the funding ratio is required to be greater than or equal to α in all states.

To obtain sufficiently detailed information about the probability distribution of the level of the funding ratio, one may introduce additional states, which do not have successors. As a result, a part of the scenario tree may look like the tree in Figure 3.1. In this figure, the additional states are described by the dots at the end of the dashed lines. They do not have successors. Given all the child nodes (both the ones which were already present and the new ones), sufficiently detailed information about the probability distribution of the funding ratio is present, so that the chance constraints become meaningful now. Although more subtlety is introduced, we did not succeed in working with these additional states.

We have seen that the chance constraints require that we should make decisions, such that only in a limited number of future states the funding ratio is less than α. This seems to be a nice way to model risk and it has a clear interpretation, too. And, since we already introduced the binary variables into the model to indicate whether the funding ratio is sufficiently high or not, these can be used for the chance constraints too.

Although this seems nice at first sight, we also have to deal with two less desirable properties in defining risk in this way. Chance constraints require that only in a limited number of future states the funding ratio may be less than its minimum required level α. But there are no direct restrictions on the amount of underfunding. Of course, if in a consecutive years a shortage with respect to the level α exists,

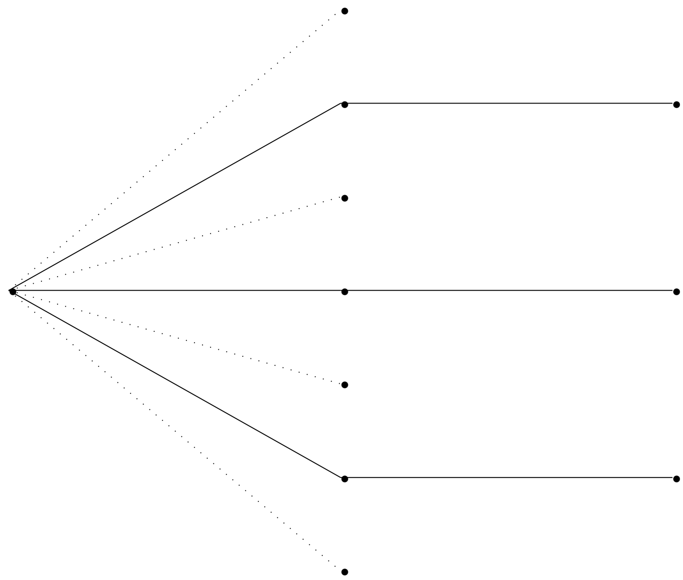

Figure 3.1: A small part of scenario tree in a model with chance constraints.

the sponsor of the fund has to make a remedial contribution, which is penalized in the objective function. But the chance constraints themselves do not impose limits on the amount of a possible shortage.

A second disadvantage of chance constraints is that for low values of $branch_t$ it is a rough way to model risk.

By means of the following example we will show that the induced feasible region may also be nonconvex in the continuous decision variables.

Example 3.1

Assume that the total asset value of a pension fund at time 0 is equal to 100, and the value of the liabilities is 90. The board of the fund requires a minimum funding ratio of 1.1. Suppose in addition that there are three states at time 1, and all conditional probabilities are $\frac{1}{3}$. In all these three states, we assume that the upper bound on the value of the liabilities equals 100.

If the minimum required reliability is set to $\frac{2}{3}$, we see that the chance constraints can be written as

$$\frac{1}{3}\sum_{s=1}^{3} u_1^s \le \frac{1}{3},$$

or,

$$\sum_{s=1}^{s} u_1^s \le 1,$$

that is, only in one of the successors underfunding is allowed.

Assume in addition that there are only two asset classes, stocks and bonds. The returns on these asset classes, which are denoted by r_1^s and r_2^s for stocks and bond respectively, are presented in Table 3.1. The investments in stocks and bonds at time 0 are denoted by X_1 and X_2 respectively. We assume that short selling is not allowed.

scenario	r_1^s	r_2^s
1	0.30	0.05
2	0.07	0.13
3	0.11	0.06

Table 3.1: Returns on stocks and bonds in the 3 scenarios of Example 3.1.

Given the description above, the pension fund has to make decisions, such that the following constraints are satisfied:

$$\begin{array}{lll} X_1 + X_2 & = 100 & \\ M u_1^s & \ge 110 - (1 + r_1^s) X_1 - (1 + r_2^s) X_2 & s = 1,2,3 \\ \sum_{s=1}^{3} u_1^s & \le 1 & \\ X_1 & \ge 0 & \\ X_2 & \ge 0 & \\ u_1^s & \in \{0,1\} & s = 1,2,3 \end{array}$$

The feasible portfolios, i.e. which satisfy all constraints, are depicted in Figure 3.2 by the solid line. These feasible portfolios are specified by $X_1 \in [20,50] \cup [80,100]$ and $X_2 = 100 - X_1$.

Note that if the minimum required reliability is set higher than $\frac{2}{3}$ in this example, the problem is infeasible. □

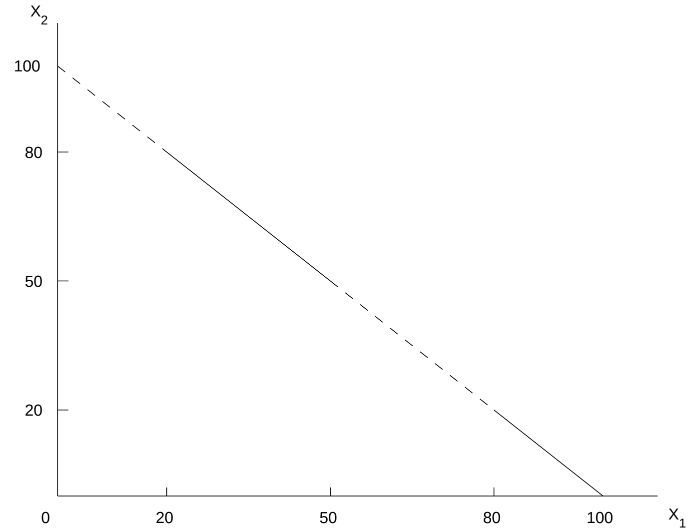

Figure 3.2: Feasible portfolios in the model with chance constraints using discrete distributions of Example 3.1.

From this example it is clear that the feasible regions of chance constraints are not convex. Of course, since we also have to deal with binary variables, we already had a nonconvex mathematical program. But also in the (X_1, X_2)-plane, we cannot expect to obtain convex feasible areas. Even here we might end-up in disjoint parts of the feasible region. This makes it very difficult to construct a feasible solution and to improve solutions. As we will see later, there is another way to model one-year risk, in which we end up with convex sets in the (X_1, X_2)-plane.

3.3 Integrated Chance Constraints

In this section, we describe a second way to incorporate one-year risk constraints into our ALM model: *integrated chance constraints* (ICCs). We formulate ICCs, give an interpretation, and describe their mathematical properties.

Integrated chance constraints are, just like the chance constraints, defined for every $t \in \mathcal{T}_0$, and $s \in \mathcal{S}_t$:

$$\mathbb{E}_{(t,s)}\left[\left(A_{t+1}^{s'} - \alpha \overline{L}_{t+1}^{s'}\right)^{-}\right] \leq q, \quad s' \in \mathcal{K}_t^s(t+1). \tag{3.5}$$

The ICCs state that the expected next year's shortage with respect to the level α and the upper bound $\overline{L}_{t+1}^{s'}$ may not exceed q. In this formulation we have chosen to use $\overline{L}_{t+1}^{s'}$ instead of $L_{t+1}^{s'}$ to emphasize the goal of the board of the fund to strive to give full indexation in every year (so that this upper bound is the desired level), although the board may deviate from this level due to unfavorable circumstances. In a linear programming framework, these constraints can be incorporated as follows:

$$\frac{1}{branch_t} \sum_{s' \in \mathcal{K}_t^s(t+1)} \left(A_{t+1}^{s'} - \alpha \overline{L}_{t+1}^{s'} \right)^- \leq \psi L_t^s, \tag{3.6}$$

where we have replaced the right-hand side q by ψL_t^s. We refer to Klein Haneveld [53] and Klein Haneveld and Van der Vlerk [54] for mathematical details on ICCs.

Integrated chance constraints have a property which is in accordance with what financial decision makers mean by avoiding risk: not only the probability of underfunding is important, but also the *amount* of the shortage. Therefore, ICCs more closely resemble the objectives of financial risk management than chance constraints do.

The right-hand side of the ICCs of (3.6) is the maximum accepted expected shortage with respect to the funding ratio α, and is specified as a fraction ψ of the actual value of the liabilities. This is reasonable, since in this way a relative measure is found which is related to the position of the pension fund under consideration. With respect to the numerical value of ψ, we propose to relate it to the duration of the liabilities. What we mean by this, and why we propose this, will be explained now. The *duration of the liabilities* is the weighted average maturity of the stream of benefit payments. The maturity of each benefit payment (i.e in how many years such a payment has to be made) is weighted by the fraction of L_t^s accounted for by the payment. Now, we will explain what this implies for pension funds. If the pension fund under consideration has relatively many young active participants and relatively few retired members, the duration of the liabilities is rather high. On the other hand, in case of funds with relatively many retired members, more weight is assigned to the benefit payments in the near future, and as a result, the duration is lower. For the first type of pension fund, a larger expected shortage is allowed. This makes sense, because this fund has more time to recover from a period of financial distress than the latter fund.

A nice mathematical property is that constraint (3.6) can be used in a linear programming framework without the need to introduce additional binary decision variables. This can be done by introducing additional nonnegative, continuous decision variables $Sho\alpha_t^s$. They measure the amount of shortage with respect to the level α in state (t, s). Adding the constraints

$$A_t^s + Sho\alpha_t^s \geq \alpha \overline{L}_t^s, \qquad t \in \mathcal{T}_1, s \in \mathcal{S}_t,$$

the integrated chance constraints (3.6) can be written as

$$\frac{1}{branch_t} \sum_{s' \in \mathcal{K}_t^s(t+1)} Sho\alpha_t^{s'} \leq \psi L_t^s \qquad t \in \mathcal{T}_1, s \in \mathcal{S}_t.$$

The inequalities above define convex, polyhedral feasibility sets. They are very attractive from an algorithmic point of view. Since the constraints defining the integrated chance constraints are all linear, they can be used in a linear programming framework, see also Klein Haneveld and Van der Vlerk [54]. We will illustrate this by means of the following example.

Example 3.2

In this example, we will use the same data as in Example 3.1. Assume in addition that the board of the pension fund has decided that the expected next year's shortage may not exceed 1.

The feasible region is now defined by the following set of linear (in)equalities:

$$\begin{array}{lll} X_1 + X_2 & = 100 & \\ Sho\alpha_1^s & \geq 110 - (1 + r_1^s)X_1 - (1 + r_2^s)X_2 & s = 1, 2, 3 \\ \frac{1}{3}\sum_{s=1}^{3} Sho\alpha_1^s & \leq 1 & \\ Sho\alpha_1^s & \geq 0 & s = 1, 2, 3 \\ X_1 & \geq 0 & \\ X_2 & \geq 0 & \end{array}$$

The resulting feasible portfolios are depicted in Figure 3.3. They are defined by $X_1 \in [20, 100]$ and $X_2 = 100 - X_1$. We see that the feasible set is convex in this case.

Note that if the maximum expected next year's shortage is less than 0.5, no feasible solution exists. □

We have seen that chance constraints only consider probabilities of underfunding, while integrated chance constraints take into account both probabilities and amounts of underfunding. In addition, from an algorithmic point of view, the ICCs have more attractive properties than chance constraints: ICCs can be incorporated in a linear program without additional binary variables. Moreover, if the risk aversion parameter is changed, the feasible region in case of ICCs changes smoothly, while this region changes in a rough way in case of chance constraints if the number of branches is low. Because integrated chance constraints have nicer properties than chance constraints, we use ICCs as one-year risk constraints in our ALM model.

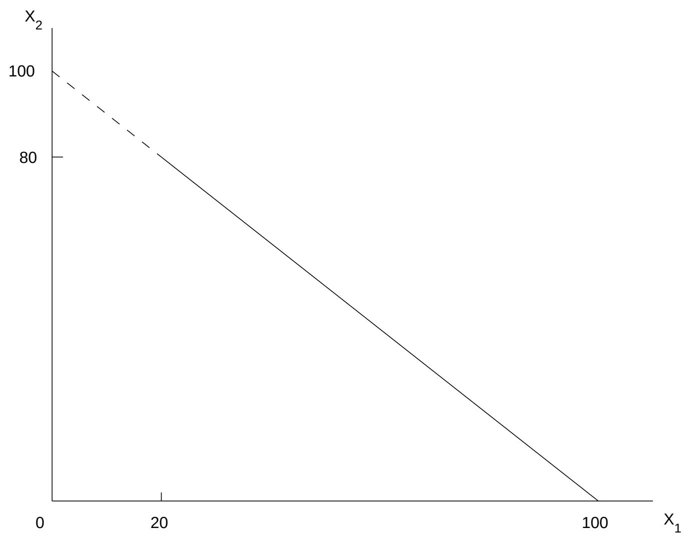

Figure 3.3: Feasible portfolios in the model with integrated chance constraints, presented in Example 3.2.

Chapter 4

Heuristic

In the previous two chapters, we have described our ALM model for pension funds in detail. We have seen that binary decision variables play an important role in this model. They are needed to model flexible risk measures and to penalize unfavorable events. As a result, our ALM model is a multistage stochastic program (MSLP) with both continuous and binary decision variables. It is well-known that in general mixed-integer problems are extremely difficult to solve, and that for very large problems (like realistically sized ALM problems), we may not expect to find an optimal solution in reasonable time, see for example Schrijver [86]. Because we still want to find good feasible solutions, we construct a heuristic.

This chapter is organized as follows. In Section 4.1 the background of the heuristic is presented: the conceptual ideas are discussed, the terminology is introduced, the order in which states are visited is clarified, and flowcharts of the heuristic are presented. In Section 4.2, the central section of this chapter, the heuristic is described. Finally, the details of some parts of the heuristic are given in Section 4.3. The reader who is only interested in the main ideas of the heuristic may skip this last section.

4.1 Background

Although the multistage mixed-integer stochastic program is extremely difficult to solve, from computational experiences with advanced SLP software OSL [71] we found that the MSLP (at least of the size we will calculate with, see Section 6.1) can be solved over the continuous decision variables. Thus, given a feasible solution, we can re-solve the problem relatively fast for changed values of a few binary decision variables. This is the result of so-called *hot starts*: the previous solution is used as a starting point to solve the problem again.

In the heuristic presented in this chapter it is tried to avoid (large) fixed penalty costs in case of unfavorable events: underfunding, a remedial contribution, or not indexing built-up rights with respect to increases in last year's general wage level. As a result, we do not consider the binary decision variables l_t^s, o_t^s, and v_t^s directly, although their definitions are taken into account appropriately in the heuristic.

In Section 4.1.1 we discuss the conceptual ideas of the heuristic. Section 4.1.2 focuses on the order of visiting nodes. In Section 4.1.3 flowcharts of the heuristic are presented. Finally, in Section 4.1.4 we discuss a more refined heuristic.

4.1.1 Conceptual ideas

In Chapter 2 we have seen that in three unfavorable events (large) fixed penalty costs are incurred: in case of underfunding, in case of a remedial contribution, and in case of a deterioration of indexation. The heuristic aims at avoiding these fixed penalty costs. Given a feasible solution, we try to improve this solution by considering changes of the value of some binary decision variables. Such potential improvements are inspired by insight in the model. To be specific, the following two steps are considered to improve a feasible solution.

1. Change the values of the binary decision variables, guided by *local targets*, using available *instruments* (based on insight of the problem under consideration). If a target is reachable, the corresponding node is called a *candidate* node (for improvement).

2. Given a candidate, update the binary variables according to the instruments used. With these updated fixed values of the binaries, resolve the MSLP. This gives us *optimal values for the continuous decision variables, given the values of the binary variables*. If this results in a *global improvement*, i.e. a lower value of the objective function, the candidate node is called a *suitable* one, and we keep the new values of the binaries. Otherwise the candidate is rejected and the values of the binaries remain the same.

These two steps are repeated until no suitable candidate is found anymore. How to find a suitable candidate is explained in detail in Section 4.2.

The *local targets*, mentioned in the first step, are to avoid one (or more) unfavorable event mentioned above: to avoid underfunding, to avoid a remedial contribution or to restore full indexation. The possible *instruments* to reach these targets are an increase of a contribution rate and/or a remedial contribution, and changed compositions of the asset portfolios in predecessor nodes of a candidate. Moreover, the value of the liabilities may be decreased in the state under consideration to avoid underfunding in that state.

Which instrument(s) are used to reach a target in a candidate node will be discussed in the next section. Moreover, in Section 4.3 the corresponding details are discussed.

4.1.2 Order of visiting nodes

In the search for candidates, we consider the nodes of the scenario tree in an increasing order of time. At each time, the nodes are considered according to the lexicographical ordering of the scenarios, as described in Section 2.3. This particular order of visiting nodes to search for candidates is chosen because of the following reasons.

First, as soon as a suitable candidate is selected, the corresponding effects in other states in the scenario tree have to be calculated in order to update the values of the binary decision variables according to their definitions. In this way, unfavorable events further down the tree may already be eliminated by considering the scenario tree in an increasing order of time. For example, if the contribution rate at time 0 is increased to avoid underfunding at time 1, usually all asset values in the scenario tree will increase, too (but not necessarily as we will see in Section 4.3.3).

Another reason why we consider the scenario tree in the order described above, is that the probabilities associated with states at early decision moments are larger than those corresponding to later decision moments, and cash flows are discounted in our ALM model. As a result, payments at early decision moments have a larger impact on the objective function value. Thus, one may expect that larger decreases in the value of the objective function can be found if decisions are adjusted early in the scenario tree.

4.1.3 Flowcharts

In this section, we present two flowcharts. The first one, which is shown in Figure 4.1, gives the main steps of the heuristic. The second one shows how the search for suitable candidates is organized. This second flowchart is presented in Figure 4.2. The steps presented in these two flowcharts are described in detail in Section 4.2.

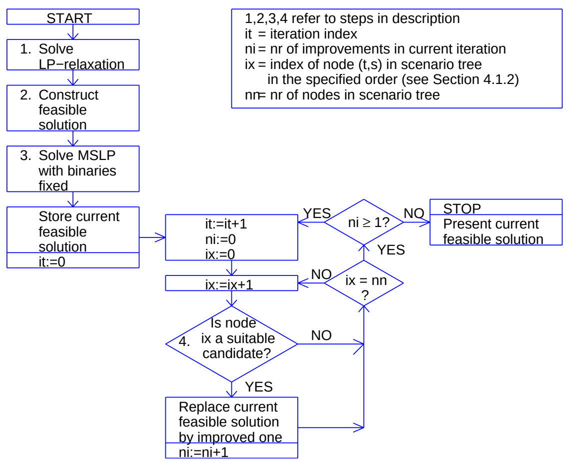

Figure 4.1: Flowchart of the main steps of the heuristic.

4.1.4 Refined heuristic

The heuristic presented in this chapter can be seen as a *greedy heuristic*: as soon as an improvement is found, it is implemented. The main advantage of such a greedy

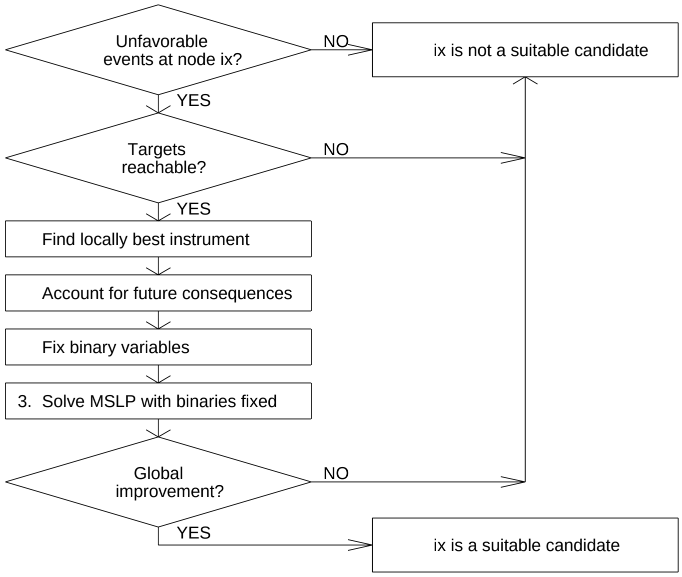

Figure 4.2: Flowchart of Step 4: identification of suitable candidates.

heuristic is that it is relatively fast.

Of course, it is possible to study the performance of a more refined heuristic. Such a refined heuristic could for example consider all binary variables with value 1 in the current feasible solution simultaneously. The changes associated with the one which leads to the largest reduction in the value of the objective function (if a reduction is found at all) may be implemented. Then, such a search is performed again, until no improvements can be found anymore in this way.

Although such a refined heuristic possibly will need more time, computational results might lead to the conclusion that this additional time is worthwhile. Recall that the ALM model is a model to support strategic decisions. From that point of view, a small increase in CPU time may not be disastrous. Unfortunately, the implementation of the refined heuristic described above cannot be presented in this thesis because of a time constraint.

4.2 Steps 1, 2, 3, and 4

In this section we will discuss the main ideas which lie behind the heuristic, and which are presented in the flowcharts in Figure 4.1 and Figure 4.2, in more detail.

Before we describe how the heuristic tries to avoid fixed penalty costs if a suitable candidate is found, an overview of the decisions which are kept fixed in the main steps of the heuristic, as presented in the flowcharts in Figure 4.1 and Figure 4.2, is given in Table 4.1. The fixed decisions are marked with a $\bullet$. Note that if the fractions of the investment in asset class j in the asset portfolio in state (t, s), denoted by f^s_{jt}, are fixed, the returns on the asset portfolios are also fixed. The assumption of fixed portfolio returns in Steps 2 and 4 is a simplification, which allows to carry out the necessary calculations described in this chapter.

Step	c_t^s	f_{jt}^s	L_t^s	u_t^s	z_t^s	m_t^s	l_t^s	o_t^s	v_t^s
1									
2	•	•	•						
3				•	•	•	•	•	•
4		•							

Table 4.1: Fixed components in each state (t, s) in the main steps of the heuristic.

4.2.1 Step 1: Initialization

In the first step, the linear programming (LP) relaxation is solved. This is done to find a good starting point to construct a feasible solution in Step 2.

The LP-relaxation is defined as the LP model that arises from the mixed integer model by replacing the binary variables u_t^s, z_t^s, m_t^s, l_t^s, o_t^s, and v_t^s by corresponding continuous decision variables with lower bound 0 and upper bound 1. The multistage stochastic linear program is now solved with only continuous decision variables, including the relaxed indicators.

4.2.2 Step 2: Construct a feasible solution

In the second step a feasible solution is constructed. We need a feasible solution as a starting point for improvements. However, the result of Step 1 is not feasible in general: its binary indicators may have fractional values.

In this step we fix the contribution rates c_t^s, the fraction of the amounts invested in the N asset classes in the portfolio ($f_{jt}^s, j = 1, \ldots, N$), and the value of the liabilities (L_t^s) in all states (t, s) in the scenario tree.

Given the values of the liabilities in each state of the scenario tree, the numerical values for m_t^s and l_t^s follow immediately from their definitions. Given the fixed levels of the contribution rates, the composition of the asset portfolios and the values of the liabilities, we apply the decision rules and the definition of the binary decision variables to find appropriate values for u_t^s, z_t^s, o_t^s and v_t^s. In the construction of a feasible solution, the states in the tree are considered in the order described in Section 4.1.2. The central issue in constructing a feasible solution is therefore to find appropriate values for DZ, Z and V in each state and adjust the asset values in the subtree correctly.

In Section 4.3.1 the details are described which are taken into account in the construction of a feasible solution.

4.2.3 Step 3: Continuous improvement

We assume that after the execution of Step 2 a feasible solution has been constructed. In Step 3, the multistage stochastic program is solved with the values of all binary decision variables fixed, as found in the previous step. Therefore, the multistage stochastic program is solved with respect to the continuous decision variables.

The result of Step 3, a feasible solution that is optimal for fixed values of the binary variables, is used as a starting point to search for improvements, in Step 4. The same Step 3 is to be executed in Step 4, each time a suitable candidate is found.

4.2.4 Step 4: Search for suitable candidates

When this step is executed, there is a current feasible solution, and a specific node (t, s) of the scenario tree has been selected. If the current feasible solution in this node does not describe unfavorable events, we are done. Otherwise, we check whether the state in which the unfavorable event (i.e. underfunding and/or no full indexation is given) is observed has the following properties.

- It is possible to avoid underfunding and/or it is possible to increase the value of the liabilities sufficiently, so that no fixed penalty costs are present any more.
- The resulting new feasible solution has a lower value of the objective function.

Given that we focus on u_t^s, m_t^s, and z_t^s in this heuristic (although the values of l_t^s, o_t^s, and v_t^s are taken into account appropriately), there are four possible combinations we have to consider to check if a different combination of the values of the binary variables can be found, such that the value of the objective function is decreased. These four cases are:

1. $u_t^s = 1, m_t^s = 0$ (and $z_t^s = 0$),
2. $u_t^s = 0, m_t^s = 1$ (and $z_t^s = 0$),
3. $u_t^s = 1, m_t^s = 1$ (and $z_t^s = 0$),
4. $z_t^s = 1$

Note that the possible combination, $u_t^s = 0$, $m_t^s = 0$ (and $z_t^s = 0$) does not need to be considered, since no fixed penalty costs can be removed in this case. Note also that states with $z_t^s = 1$ are considered separately, because we need to check whether a remedial payment is forced by the decision rules or not. We come back to this issue below.

All four possible combinations are considered now in detail.

Case 1. $u_t^s = 1$, $m_t^s = 0$ (and $z_t^s = 0$)

In scenario s at time t the funding ratio is below the minimum required level α. However, the participants of the fund do receive full compensation for last year's increases in the general wage level. In this case, the 'target' of Step 4 is to realize $u_t^s = 0$ by adjusting previous decisions.

To do so, we consider increases in the levels of the contribution rates, and/or increases in a remedial contribution if such a payment was already made before time

t in scenario s. If underfunding can not be avoided by means of these instruments, a decrease in L_t^s is considered.

If $u_t^s = 0$ is still not possible, the target $u_t^s = 0$ is not reachable. However, if the target is reachable by using one or more instruments, the best instruments to do so will be found. This is done by fixing the values of the binary decision variables and executing Step 3, i.e. solving the MSLP. If a global improvement is possible, state (t, s) is indeed a suitable candidate.

In order to determine if u_t^s is reachable by considering only increases in the levels of the contribution rate and an increase in a remedial contribution, we consider scenario s *backwards*, starting at time $t-1$. If the contribution rate in state $t-1$ is strictly below $\overline{c}$, this level is increased to

$$\min\{\frac{\alpha L_t^s - A_t^s}{W_t^s}, \overline{c}\}. \tag{4.1}$$

This implies that c_{t-1}^s is increased until either the shortage with respect to the level α disappears, or till the level of the contribution rate is set equal to its upper bound. If c_{t-1}^s cannot be increased sufficiently to avoid $u_t^s = 1$, we go to time $t-2$ (if $t-2 \geq 0$). This procedure is continued, where the shortage (the numerator in (4.1)) is adjusted appropriately each time. Moreover, the denominator is replaced by $W_q^s \Pi_{q'=q+2}^t (1 + rp_{q'}^s)$ if state (q, s), $0 \leq q \leq t-2$, is considered. This is necessary to find the appropriate increase in A_t^s. As soon as in the backward procedure we observe $z_q^s = 1$, the level of the corresponding remedial contribution is increased such that $u_t^s = 0$ is obtained:

$$\Delta Z_q^s = \frac{\alpha L_t^s - A_t^s}{\Pi_{q'=q+1}^t (1 + rp_{q'}^s)}.$$

This increase is always possible, since no (hard) upper bound on such a remedial payment exists.

As already mentioned above, if the target $u_t^s = 0$ is not reachable by considering increases in (remedial) contributions, a decrease in L_t^s is considered. First, L_t^s will be decreased such that its value equals $(1 + \varphi_t^s)(1 + w_t^s)L_{t-1}^s$ (if the current value is higher than this threshold value). This value is chosen, so that $m_t^s = 0$ still holds. If this decrease is insufficient to reach the target, L_t^s is set equal to its lower bound, $\underline{L}_t^s$. Indeed, by a deterioration of indexation, it may be possible to satisfy the minimum requirements with respect to the level of the funding ratio. Also in this case, the previous feasible solution is replaced by the new one if after optimization with respect to the continuous decision variables, the value of the objective function is decreased.

Remark

A backward search is performed because of the following reasons. First of all, the factor time is considered in our ALM model. This is done by discounting cash flows. As a result, the later payments have to be made, the cheaper it is. Moreover, in the stochastic program probabilities are taken into account. This implies also that the later certain payments can be made in scenario s to prevent underfunding in state (t, s), the smaller the corresponding probability. A third argument why we consider scenario s backwards, is that in this way increases in a regular contribution cannot lead to vanished opportunities to increase a remedial contribution. This would be the case if an increase in c^s_{q1} leads to $u^s_{q2} = z^s_{q2} = 0$, $q1 < q2 < t$, while before this increase a remedial contribution was made in state $(q2, s)$. Due to the higher regular contribution in state $(q1, s)$, $u^s_t = 0$ is not possible anymore if $Z^s_{q2} > 0$ is necessary to reach the local target.

Case 2. $u^s_t = 0, m^s_t = 1$ (and $z^s_t = 0$)

In the current feasible solution, the funding ratio is at least equal to α in state (t, s). However, in this state, the participants of the fund are not fully compensated for increases in the general wage level of the last year. The only instrument we have to consider in this case is an increase in L^s_t, such that $L^s_t = (1 + \varphi^s_t)(1 + w^s_t)L^s_{t-1}$. This increase is the minimum level of L^s_t such that $m^s_t = 0$. Then, all the consequences in the subtree are considered appropriately.

If u^s_t remains 0, we solve the MSLP with the values of the binary decision variables fixed and continue with the heuristic. If u^s_t becomes 1 due to the increase in L^s_t, we check whether $u^s_t = 0$ is possible. If this is possible, regular and remedial contributions are adjusted in scenario s before time t to prevent underfunding in state (t, s). This is done in the same way as described in the previous case. Again, we solve the multistage stochastic program.

If $u^s_t = 1$ cannot be avoided due to the fact that benefit rights are indexed, or the case $u^s_t = m^s_t = 0$ mentioned before did not lead to a better feasible solution, we check whether the combination $m^s_t = 0$, $u^s_t = 1$ leads to a lower value of the objective function after the MSLP is solved.

Case 3. $u^s_t = 1, m^s_t = 1$ (and $z^s_t = 0$)

If in the current feasible solution $u^s_t = 1$ and $m^s_t = 1$ in state (t, s), we first set $L^s_t = (1 + \varphi^s_t)(1 + w^s_t)L^s_{t-1}$, to obtain the target $m^s_t = 0$. Given this adjustment, the remainder of this case consists of the following two procedures:

(**A**) If the target $u^s_t = 0$ is reached by appropriate adjustments in regular and remedial payments, we check whether $u^s_t = m^s_t = 0$ leads to an improved solution. If this is indeed the case, B is not considered anymore.

If $u^s_t = 0$ is not possible due to the increase in the value of the liabilities, or if $u^s_t = 0$ is possible, but the value of the objective function is not decreased, go to B.

(**B**) We consider two possible targets: $u^s_t = 1$, $m^s_t = 0$ and $u^s_t = 0$, $m^s_t = 1$. The adjustments associated with both possible targets are

made as described under Case 1 and Case 2. Both combinations are considered, since we do not know which of these two cases leads to the best solution.

Case 4. $z^s_t = 1$ (hence $u^s_t = 1, m^s_t = 0$ or 1)

If in the current feasible solution a remedial contribution is made in state (t, s), it is checked whether this payment is forced by the modelling assumptions. This is the case if at the last a decision moments underfunding is recorded. Since such a payment is forced by the constraints of our ALM model in this case, we do not consider to remove such a payment.

However, if $Z_t^s > 0$ is not forced, it is considered whether it is attractive to remove such a payment. This may lead to an improved solution, since the fixed penalty costs λ_z are avoided now.

4.3 Details

In this section we describe some details of how we constructed a feasible solution. Moreover, we also consider the possible instruments to reach a target in more detail. Finally, Section 4.3.3 is devoted to the consequences of a change in an asset value for the states in its subtree.

4.3.1 Step 2: construction of a feasible solution

As already mentioned above, before we construct a feasible solution, the LP-relaxation is solved. This gives us compositions of the asset portfolios in each state before the horizon. As a result, we can also find the portfolio returns in each state (t, s), denoted by rp_t^s:

$$rp_t^s = \sum_{j=1}^{N} f_{jt}^s r_{jp}^s,$$

where

$$f_{jt}^s := \frac{X_{jt}^s}{\sum_{j=1}^{N} X_{jt}^s}$$

denotes the fraction of assets invested in asset class j at time t in scenario s, $j = 1, \ldots, N$.

In the construction of a feasible solution we use the portfolio returns found in the LP-relaxation. We use these returns, because the LP-relaxation gives us a good starting point and we don't know how to find better ones.

The construction of a feasible solution consists of the following three steps.

Step 2.1: Find numerical values for m_t^s and l_t^s

The values of the binary decision variables m_t^s and l_t^s only depend on the values of the liabilities in state (t, s). Therefore, given the values of the liabilities in each state in the scenario tree (as found in the LP-relaxation), we apply their definitions:

$$m_t^s = \begin{cases} 1 & \text{if } L_t^s < (1 + \varphi_t^s)(1 + w_t^s)L_{t-1}^s \\ 0 & \text{otherwise,} \end{cases}$$

$$l_t^s = \begin{cases} 1 & \text{if } L_t^s < \overline{L}_t^s \\ 0 & \text{otherwise.} \end{cases}$$

Step 2.2: Apply policy rules to find numerical values for u_t^s and z_t^s

In this second step, the nodes in the scenario tree are considered in the order described in Section 4.1.2. The reason of this order is that a change in the asset value in one state may have effects in all other states of the subtree of that state.

Recall from Section 2.3 that if a remedial payment is made, it is only made after the evaluation of the financial position of a fund. Therefore, if a remedial payment is observed in state (t, s), this payment does not affect A_t^s. However, such a payment does affect the asset value in all states in the subtree of (t, s). To indicate that asset values in the subtree of (t, s) are adjusted, because of a change in a payment in state (t, s) by an amount $x \in \mathbb{R}$, we use the notation $\Delta A_t^s(subtree) = x$. To be specific, $\Delta A_t^s(subtree) \neq 0$ means that A_t^s is not changed, but in all states in the subtree of (t, s), the asset values may be changed.

In the description of the actions to undertake, we assume that we are in state (t, s) of the scenario tree. We distinguish the following situations:

- **$A_t^s < \theta L_t^s$ and $0 < DZ_t^s < \theta L_t^s - A_t^s$.**
  Set $u_t^s = 1$, $z_t^s = 0$, $\Delta A_t^s(subtree) = \theta L_t^s - A_t^s - DZ_t^s$, and $DZ_t^s = \theta L_t^s - A_t^s$, and consider the effects in the subtree of (t, s).

- **$A_t^s < \theta L_t^s$, $DZ_t^s = 0$, and $0 \leq Z_t^s < \alpha L_t^s - A_t^s$.**
  Set $u_t^s = z_t^s = 1$, $\Delta A_t^s(subtree) = \alpha L_t^s - A_t^s - Z_t^s$, and $Z_t^s = \alpha L_t^s - A_t^s$ and consider the effects in the subtree of (t, s).

- **$\theta L_t^s \leq A_t^s < \alpha L_t^s$ and $DZ_t^s > 0$.**
  Set $u_t^s = 1$, $z_t^s = 0$, $\Delta A_t^s(subtree) = -DZ_t^s$, and $DZ_t^s = 0$ and and consider the effects in the subtree of (t, s). If in the last a years underfunding is recorded, set $z_t^s = 1$ $\Delta A_t^s(subtree) = \alpha L_t^s - A_t^s$, and $Z_t^s = \alpha L_t^s - A_t^s$ and consider the effects in the subtree of (t, s).

- **$\theta L_t^s \leq A_t^s < \alpha L_t^s$, $DZ_t^s = 0$ and $0 \leq Z_t^s < \alpha L_t^s - A_t^s$.**
  Set $u_t^s = z_t^s = 1$ $\Delta A_t^s(subtree) = \alpha L_t^s - A_t^s - Z_t^s$, and $Z_t^s = \alpha L_t^s - A_t^s$ and consider the effects in the subtree of (t, s).

- **$A_t^s \geq \alpha L_t^s$ and $DZ_t^s > 0$ and/or $Z_t^s > 0$.**
  Set $u_t^s = z_t^s = 0$, $\Delta A_t^s(subtree) = -Z_t^s - DZ_t^s$, and $Z_t^s = DZ_t^s = 0$ and consider the effects in the subtree of (t, s).

Step 2.3: Apply policy rules to find numerical values for o_t^s and v_t^s

Also in favorable circumstances like overfunding, we still have
to check whether the definitions of the binary decision variables are satisfied. We do this as follows:

- **$A_t^s \leq \beta L_t^s$ and $V_t^s > 0$.**
  Set $o_t^s = v_t^s = 0$, $\Delta A_t^s(subtree) = -V_t^s$, and $V_t^s = 0$ and consider the effects in the subtree of (t, s).

- **$A_t^s > \beta L_t^s$ and $V_t^s = 0$.**
  Set $o_t^s = 1$. If in the last b years overfunding is registered, set $v_t^s = 1$, $\Delta A_t^s(subtree) =$

$-(A_t^s - \beta L_t^s)$, and $V_t^s = \beta L_t^s - A_t^s$ and consider the effects in the subtree of (t, s). However, if a restitution is forced, but $L_t^s < \overline{L}_t^s$, the value of the liabilities are set equal to its upper bound in this state. It is checked again whether a restitution has to be made.

Remark

In Section 4.1.2 we have noted that changes in the level of a remedial payment or restitution may result in changed asset values in all states in its subtree. Therefore, as soon as an asset value is changed in Step 2.2 or 2.3, we consider its subtree. Also in this subtree, the states are considered in an increasing order of time. Moreover, Steps 2.2 and 2.3 are applied again. Note that in this way recursion arises.

4.3.2 Step 4: Instruments

As we have noted in the Section 4.1.1, decisions have to be changed to obtain a new feasible solution with a specific binary variable changed from 1 to 0. The details with respect to the possible instruments to reach a local target will be discussed in this section. Before we do that, we first introduce the *net capital position* with respect to the level α of the fund. This net capital position is defined as $A_t^s - \alpha L_t^s$, and is abbreviated as $NCP\alpha_t^s$. Note that this net capital position may be positive, zero, or negative. A positive (negative) $NCP\alpha_t^s$ is also called a surplus (shortage) with respect to the level α, and is denoted by $Sur\alpha_t^s$ ($Sho\alpha_t^s$) for state (t, s).

$NCP\alpha_t^s$ has to be increased if $u_t^s = 1$ is considered to change into $u_t^s = 0$. To do so, several instruments are at the disposal of the board of a pension fund. Possible instruments are an increase in a contribution rate or a remedial contribution at times $0, 1, \ldots, t-1$. Moreover, L_t^s may be decreased, and/or the composition of the asset portfolios may be changed. If $m_t^s = 1$, L_t^s has to be increased.

In the remainder of this section we describe the consequences of an increase in the contribution rate and remedial contribution, a decrease in the value of the liabilities and changed compositions of the asset portfolios. These are all instruments to avoid underfunding. Moreover, an increase in L_t^s is considered if $m_t^s = 1$. We assume that fixed costs are present in state (t, s).

An increase in a contribution rate

If there exists a state (q, s), $0 \le q \le t-1$ in which $c_q^s < \overline{c}$, we have found a possibility to increase the net capital position of the fund in state (t, s): an increase in the level of the contribution rate.

Of course, such an increase has certain consequences. The first consequence associated with an increase in c_q^s are the direct costs: the increase in c_q^s leads to larger contributions of the active participants of the fund in states $(q+1, s')$, $s' \in \mathcal{K}_t^s(q+1)$.

Other effects have to do with penalties associated with large increases and decreases in the contribution rate. First of all, if an additional penalty is incurred due to a larger increase in the contribution rate at time q, compared to the level of c_{q-1}^s. The second one has to do with changes in c_{q+1}^s, compared to the level of c_q^s. If penalty costs are incurred, because of a large decrease in the contribution rate at time $q + 1$, these penalty costs are increased now.

On the other hand, an increase in c_q^s may also lead to a reduction in penalty costs. Due to $\Delta c_q^s > 0$, we observe a more moderate decrease in c_q^s (compared to the level of c_{q-1}^s), where Δc_q^s denotes the change in the contribution rate in state (q, s). The last effect which occurs, is a more moderate increase in states $(q+1, s')$, $s' \in \mathcal{K}_t^s(q+1)$. If a large increase in the contribution rate in year $q+1$ resulted in penalty costs, these are lowered due to $\Delta c_t^s > 0$.

An increase in a remedial contribution

If there exists a state (q, s), $0 \leq q \leq t-1$, in which $u_q^s = z_q^s = 1$ a remedial contribution is allowed in this state. According to the decision rules in our ALM model, such a remedial contribution may be increased. Although such a payment is also allowed if only $u_q^s = 1$, we only consider increases in an existing remedial contribution. This choice is made to avoid additional fixed penalty costs. Indeed, the focus of the heuristic is on avoiding these fixed costs.

If a remedial contribution is increased, the associated costs are also increased. An increase in Z_q^s only leads to additional variable costs. The current level of Z_q^s is important, however. If $Z_q^s < \tau W_q$, the marginal costs are ζ_Z. On the other hand, if $Z_q^s \geq \tau W_q$, the marginal costs are $\zeta_Z + \zeta_{ZI}$.

A decrease in the value of the liabilities

A third instrument to improve the net capital position of the fund is a decrease in L_t^s. This instrument can be used to improve the financial position if L_t^s is strictly larger than its lower bound, $\underline{L}_t^s$.

The marginal costs associated with a decrease in L_t^s are ζ_L, since deviations of L_t^s from its upper bound are penalized by this penalty parameter. In addition, if $L_t^s = (1+\varphi_t^s)(1+w_t^s)L_{t-1}^s$, a decrease in the value of the liabilities also leads to additional fixed penalty costs λ_m.

Changed composition in the asset portfolios

By changing the compositions of the asset portfolios at times 0,…,$t-1$, underfunding may be prevented in state (t, s). However, contrary to the three instruments discussed above, a changed composition of an asset portfolio in a certain state not necessarily leads to higher asset values in all its child nodes. As a result, unfavorable events may be shifted from one state to another at the same decision moment. Because we do not want that, we do not consider this instrument to avoid underfunding in a state.

An increase in the value of the liabilities

All the instruments discussed above can be used to avoid underfunding. In case of fixed penalty costs due to a deterioration of indexation, L_t^s has to be increased. This increase should be equal to $(1+\varphi_t^s)(1+w_t^s)L_{t-1}^s - L_t^s$, where L_{t-1}^s and L_t^s are the values of the liabilities in states $(t-1, s)$ and (t, s) in the current feasible solution respectively. This increase also has consequences for the states in the subtree of state

(t, s), since a higher value of the liabilities implies higher future benefit payments in states $(t+1, s')$, $s' \in \mathcal{K}_t^s(t+1)$, as can be seen in equation (2.18).

4.3.3 Consequences in scenario tree

It is straightforward that an increase in c_q^s or Z_q^s leads to a higher asset value in states $(q+1, s')$, $s' \in \mathcal{K}_t^s(q+1)$. In case of $\Delta c_q^s > 0$, active participants (and the sponsor) pay a larger fraction of the pensionable salaries to the fund, resulting in larger cash inflows. If $Z_q^s > 0$, the fund immediately has more money at its disposal.

In case of a decrease in L_t^s, the asset values increase in the child nodes of (t, s). The reason is the relationship between the value of the liabilities and the level of the benefit payments given by (2.18): if the fund does not index pension rights fully, less money has to be paid to retired people.

However, it is not necessarily true that an increase in A_t^s always results in larger values for $A_q^{s'}$, $t+1 \leq q \leq T$, $s' \in \mathcal{K}_t^s(q)$. This can be seen as follows. If a state (q, s') exists, $t+1 \leq q \leq T$, $s' \in \mathcal{K}_t^s(q)$, in which $Z_q^{s'} > \alpha L_q^{s'} - A_q^{s'} > 0$, and of course $u_q^{s'} = 1$, an increase in $A_q^{s'}$ may result in $u_q^{s'} = 0$. Because of the decision rules of our ALM model, a remedial contribution is not allowed anymore. Therefore, the remedial contribution has to be removed, to obtain a feasible solution again.

The heuristic described in this chapter is used in the computational results which are presented in Chapter 6. Before these results are presented, we first describe how the realizations of the stochastic parameters are found. This is the subject of the next chapter.

Chapter 5

Scenario generation

In this chapter we describe how the scenarios for the multistage stochastic program are generated. Before we do that, we first recall which stochastic parameters appear in our ALM model. We compare these parameters with those which appear in some other ALM models for pension funds in the literature. Then, we will indicate some properties that scenarios for multistage stochastic programs should satisfy. It will be argued that both consistency with historical data, and consistency with financial theory is important. After a discussion on the interdependencies between the stochastic parameters, we devote the remainder of this chapter to describe how we actually find realizations for all stochastic parameters.

5.1 Probabilities and stochastic parameters

Before we describe the probabilities and the stochastic parameters, we first recall some notation which is used regularly in this chapter. In Chapter 2, we introduced the definitions of the time sets $\mathcal{T}_0 := \{0, \ldots, T-1\}$ and $\mathcal{T}_1 := \{1, \ldots, T\}$.

As we have seen in Chapter 2, a scenario tree is needed to define multistage stochastic programs. Moreover, *probabilities* have to be assigned to each node of the tree. We assume that the branching structure is specified in advance and that each node at time $t \in \mathcal{T}_0$ has the same number of child nodes. This number is denoted by $branch_t$. In addition, we assume that given a certain state, the conditional probabilities associated with every child node are the same, i.e. they are given by $(branch_t)^{-1}$. As a result of this choice, all scenarios have equal probability S^{-1}, and the probability of a state (t, s) is given by

$$p_t^s = \left(\Pi_{q=1}^{t-1} branch_q\right)^{-1} \quad t \in \mathcal{T}_1, s \in \mathcal{S}_t.$$

As we will see in Section 5.3, the choice of assigning equal probabilities to all child nodes is convenient in our method to match returns on stocks and real estate with returns on bonds.

Given the specification of the scenario tree and their corresponding probabilities, we have to find numerical values for the vector of stochastic parameters in each state (t, s) in the scenario tree. In Chapter 2, we have denoted the vector of stochastic parameters by ω_t^s, defined as:

$$\omega_t^s = (r_{1t}^s, r_{2t}^s, \ldots, r_{Nt}^s, w_t^s, \underline{L}_t^s, \overline{L}_t^s, \underline{B}_t^s, \overline{B}_t^s, \gamma_t^s, W_t^s).$$

In this chapter, we focus on the first five elements of ω_t^s, and call the new vector

$$\varpi_t^s := (r_{1t}^s, r_{2t}^s, r_{3t}^s, r_{4t}^s, w_t^s).$$

The stochastic parameters which appeared in ω_t^s, but not in ϖ_t^s, are (deterministic) functions of the elements of ϖ_t^s and other data.

In our ALM model, more stochastic parameters (which are present in ω_t^s) appear than in Consigli and Dempster [17], Dert [24], and Kouwenberg [55]. The main differences are due to the fact that we use indexation as a decision in the stochastic program, whereas the other authors use fixed values for the level of the liabilities. In addition, Consigli and Dempster consider exchange rates and borrowing rates, while we do not use these parameters; we assume that the exchange rates are considered in the asset returns. Borrowing money, which is also considered by Kouwenberg, is not a possibility in our model. We assume that the fund only invests money.

Pflug [77] describes a method to generate realizations of a stochastic process for multiperiod financial problems, using optimal discretization. However, he considers a one-dimensional stochastic process, while we deal with a 5-dimensional vector ϖ_t^s.

Dupačová et al. [28] describe properties which scenarios for multistage stochastic programs should have. In building representative scenario trees, one should keep in mind underlying probability assumptions, the existing data, and the purpose of the application. On the one hand, trees must represent the underlying distribution, on the other hand they should be such that the model produces good decisions. Explicitly formulated additional requirements concerning properties of the probability distribution can help. The statistical properties can be made specific through a suitable manipulation of the data to obtain the prescribed moments, given a fixed tree structure. According to Dupačová et al. [28], building a scenario tree should be done such that some statistical properties of the data process are retained. For instance, one should take into account specified expectations, other moments, and correlations between the stochastic variables. Moreover, Dupačová et al. [28] argue that one should also consider the purpose of the model under consideration. For example in financial optimization problems, one should build trees which are arbitrage free (see Section 5.3.4).

The difficulty in generating scenarios for ϖ_t^s, $t \in \mathcal{T}_1$, $s \in \mathcal{S}$, are the dependencies between the components. These dependencies may be necessary from a theoretical point of view, or may be the result of wishes of the model user. These issues will be discussed now briefly.

Consistency with historical data

We would like that the sample we use to represent returns on stocks, real estate, and the bank account in the scenario tree are consistent with empirical data. Therefore, we specify the stochastic processes for these parameters, and estimate the parameters of such models using empirical data. To obtain numerical values for ϖ_t^s in the scenario tree, we sample (in a deterministic or stochastic way) from the specified stochastic processes.

We use econometric models to describe these stochastic processes. In these econometric models, special attention is paid to empirically based autocorrelation and lower and upper bounds. Because the return on the bank account and the change in the general wage level both depend on the level of the inflation, and are cointegrated of the first order as we shall see, we implement these processes in an error-correction model. In addition, the variances of the historical returns on stocks and real estate vary over time and are modeled as a GARCH(1,1) process. These returns are assumed to be lognormal. Finally, we consider excess returns of stocks over bonds and excess returns of real estate over bonds to obtain dependencies between these stochastic parameters which are in accordance with historical data.

Consistency with financial theory

Because some stochastic parameters which appear in our ALM model are interrelated, we have to be aware that consistency is obtained. Given the parameters of ϖ_t^s, the following wishes arise with respect to the specification of realistic values of other stochastic parameters in the scenarios.

- The numerical values of the lower and upper bounds on the value of the liabilities and benefit payments are based on discounting future streams of cash flows. In order to be able to compare the asset value with the value of the liabilities, one should use appropriate discount factors to find market values of the liabilities.
- Discount rates, which are used to compare cash flows over time, should be consistent with the ones found in valuing the liabilities. Moreover, they should be consistent with expected returns on the bank account.
- Bond returns should be consistent with the yield curve, and should be generated taking into account coupon and principal payments.
- Returns on stocks, bonds, real estate, and the bank account should be assigned to the nodes of the scenario tree, such that no arbitrage opportunities are present.

Vector Autoregressive Models

In the financial literature, Vector Autoregressive (VAR) models are often used to find realizations for stochastic parameters, see for example Boender et al. [7], Dert [24], and Kouwenberg [55]. VAR models were popularized by Sims [89]. However, VAR models have some drawbacks, see Maddala and Kim [63]. In practice it has been found that the unrestricted VAR model gives very erratic estimates, because of high multicollinearity among the explanatory variables (which are parameters in the ALM model). In addition, if the variables in the VAR model are cointegrated, this imposes restrictions on the parameters of this model. In this case, standard estimation techniques do not lead to a good description of the stochastic process. Moreover, using a VAR model may even lead to inconsistencies, for example in generating bond returns. This will be explained in Section 5.3.1.

Because of these disadvantages of VAR models, we propose an alternative method to find numerical values for the stochastic parameters of the ALM model. This method is based on dependencies between some of the parameters. Now, we will describe these dependencies in more detail.

Dependencies between the stochastic parameters in ω_t^s

As we have argued, there are good reasons to pay attention to dependencies between various stochastic parameters, when one is generating possible realizations of them. In Table 5.1 we give an overview of the relationships we have modeled in our scenario generation algorithm. Each row in this table gives a relationship between two or more stochastic parameters, by placing a $\bullet$ on the corresponding position. Moreover, a brief comment is given, indicating why these parameters should not be specified independently. We describe these relationships in the next subsections. They are indicated here to show which numerical values have to be generated simultaneously.

r_4	w	r_2	r_1	r_3	$\underline{L}$	$\overline{L}$	$\underline{B}$	$\overline{B}$	γ	W	Relationship
$\bullet$	$\bullet$										Cointegration
$\bullet$		$\bullet$									Yield curve
$\bullet$			$\bullet$								Bounds on stock returns
		$\bullet$	$\bullet$								Probability of outperformance (stocks/bonds)
$\bullet$				$\bullet$							Bound real estate returns
		$\bullet$		$\bullet$							Probability of outperformance (real estate/bonds)
$\bullet$		$\bullet$	$\bullet$	$\bullet$							No arbitrage
$\bullet$									$\bullet$		Consistent discount rates
$\bullet$					$\bullet$					$\bullet$	Definition $\underline{L}$
$\bullet$	$\bullet$					$\bullet$				$\bullet$	Definition $\overline{L}$
					$\bullet$		$\bullet$				Definition $\underline{B}$
	$\bullet$							$\bullet$			Definition $\overline{B}$

Table 5.1: Relationships between the stochastic parameters which are present in ω_t^s. The first five elements are also part of ϖ_t^s.

The relationships also determine the order in which numerical values have to be found, since for some parameters the values of other parameters are needed. Given the relationships between the parameters, as presented in Table 5.1, the following order is used to find numerical values for the stochastic parameters for a fixed state (t, s), assuming that all parameters of all predecessors already have numerical values:

1. Because of lack of data, we use $W_t^s = W_t$.
2. Find return on the bank account r_{4t}^s and the development in the general wage level W_t^s.
3. Generate bond return r_{2t}^s.
4. Generate stock return r_{1t}^s and returns on the real estate portfolio r_{3t}^s.
5. Find lower and upper bounds on the value of the liabilities, $\underline{L}_t^s$ and $\overline{L}_t^s$, and of the benefit payments, $\underline{B}_t^s$ and $\overline{B}_t^s$, and find discount rates γ_t^s.

5.2 Returns on the bank account and changes in the general wage level

It is to be expected that r_{4t} and w_t, which specify the return on the bank account and the change in the general wage level respectively, are both integrated processes of the first order, denoted by I(1). Moreover, they are correlated. These expectations will be discussed in the next paragraph.

The developments in the general wage level in the next few years may be the result of negotiations in a given year. Moreover, if there are negotiations about the wage level in one sector, the outcomes of these negotiations tend to be used in other sectors. We think that the stochastic process r_{4t} is also an integrated process of the first order, because it is to be expected that information regarding the short-term interest rate in a given year also has predictive power regarding this rate one year later. This follows for example from the fact that this level depends on the phase of the economy in the business cycle. These cycles generally last more than one year, see for example Mankiw [65]. As soon as the changes in the general wage level are high (low), the inflation rate tends to be high (low), which results in a high (low) nominal interest rate, since the nominal return on a bank account is equal to the sum of the real return and the inflation. On the other hand, if the interest rate is high, the inflation is likely to be high, and it is reasonable to expect that employees want to be compensated for the high price level, so they ask for higher wages.

As a result, we expect that the stochastic processes r_{4t} and w_t are not only I(1)-processes, but that they are even cointegrated. In order to validate this assumption, we studied the returns on the bank account and the changes in the general wage level in The Netherlands from 1983 to 2002. These data were derived from the Dutch central bureau of statistics, CBS [16]. After applying Dickey-Fuller tests [25], we conclude with 95 percent confidence that the returns on a bank account and the changes in the general wage level are indeed integrated processes of the first order.

Because of the arguments given above, we believe that these two stochastic processes can be described by an error-correction model. Error-correction models were first introduced into the econometric literature by Sargan [84], and were popularized by Davidson et al. [21]. The main characteristics of error-correction models are the notion of an equilibrium long-run relationship and the introduction of past disequilibriums as explanatory variables in the dynamic behavior of current variables. Granger and Weiss [37], have demonstrated that if two variables are integrated of order 1, and are cointegrated, they can be modeled as having been generated by an error-correction model. The error-correction model, used to describe the dependencies between r^s_{4t} and w^s_t, is presented in Appendix 5.A.1.

In order to test the hypothesis that the returns on the bank account and the changes in the general wage level are cointegrated indeed, we applied the Johansen cointegration test [46] on the same data set as described above. With 99 percent confidence, we conclude that the returns on a bank account and the change in the general wage level are cointegrated.

Generating r^s_{4t} and w^s_t with the error-correction model

We have seen that autoregressive terms are important in modeling the stochastic processes for the short-term risk-free interest rate and the change in the general wage level. This is the reason why we generate r^s_{4t} and w^s_t in a forward manner. That is, we generate the values for these stochastic processes from time 1 to time T. Note that at time 0 the risk-free interest and the (change in last year's) general wage level are known.

Basically, the error-correction model generates numerical values for r^s_{4t} and w^s_t by first estimating the parameters and then generating values for the error terms. Given state (t, s), we apply a stratified sampling procedure to find normally distributed error terms associated with $r^{s'}_{4,t+1}$ and $w^{s'}_{t+1}$, denoted by $\epsilon^{s'}_{4,t+1}$ and $\epsilon^{s'}_{w,t+1}$, $s' \in \mathcal{K}^s_t(t+1)$. Stratification is suitable, since if one samples randomly from a normal distribution, and the number of realizations is relatively low, the sample estimation approximates the underlying distribution relatively poorly.

To find numerical values for $\epsilon^{s'}_{4,t+1}$ and $\epsilon^{s'}_{w,t+1}$, such that they are stratified sampled from a normal distribution, we first find $branch_t$ points, which are all different, have equal probabilities, and approximate a uniform $[0, 1]$-distribution. These $branch_t$ points describe a discrete uniform distribution with equidistant values and leads to correct values for the mean and variance. They are given by

$$e_i = (i - \frac{branch_t + 1}{2})\sqrt{\frac{1}{branch_t^2 - 1}} + \frac{1}{2}, \quad i = 1, \ldots, branch_t. \tag{5.1}$$

The inverse transform method is used to transform the uniform error terms (5.1) into normally distributed ones.

Because $\epsilon^{s'}_{4,t+1}$ and $\epsilon^{s'}_{w,t+1}$ are assumed to be independent in the error-correction model, we generate numerical values of these error terms also independently. This is done by allocating the error terms $\epsilon^{s'}_{4,t+1}$ and $\epsilon^{s'}_{w,t+1}$ randomly to the nodes $(t+1, s')$, $s' \in \mathcal{K}^s_t(t+1)$. Given the error terms, we use the error-correction model to generate numerical values for $r^{s'}_{4,t+1}$ and $w^{s'}_{t+1}$.

To obtain scenarios which are consistent with financial theory, we also enforce lower and upper bounds on r^s_{4t} and w^s_t. Moreover, we also enforce a lower bound on the increase in the general wage level. We use historical data to find numerical values for these bounds.

5.3 Returns on bonds, stocks, and real estate

In this section, we describe how returns are generated for bonds, stocks and real estate. Before the procedure is described to generate returns for each of these three asset classes, we first give a list of properties we would like the returns to satisfy:

- Returns on the bond portfolio are consistent with observed yield curves.
- If developments of variances of historical data of returns on stocks and real estate are best described by means of a GARCH specification, then the same specification is also used when generating future returns.
- The probability of excess returns of stock returns over bond returns and returns on real estate over bond returns are consistent with historically observed values, to obtain consistency with the method to value liabilities that will be described in Section 5.4.
- In our model we make the assumption that returns on stocks and real estate follow lognormal distributions, because this assumption is also made frequently in financial theory (see for example the Black-Scholes option pricing model [5]).
- If autoregressive terms are observed in historical time series for the returns, these are also taken into account.
- Lower and upper bounds on the returns are considered, which are consistent with observed market prices.
- The scenario tree is arbitrage free.

Given these wishes, we now describe how the returns are generated for bonds, stocks, and real estate, respectively.

5.3.1 Bond returns

In many ALM models in the financial literature, bond returns are generated by simply drawing from a return distribution. This is for example the case if a VAR model is used. Next to the disadvantages of using VAR models mentioned in Section 5.1, the use of these models may also lead to implied very low (or even negative) yields. This is made clear by means of the following example.

Example 5.1

Consider a zero-coupon, non-callable bond with maturity ten years. If the current price of the bond, with principal €$1,000$, is €675.56, the implied 10-year yield equals 4%.

Assume that in one scenario the return on this bond is 9% per year. This implies that after 5 years, the price of the bond equals $(1.09)^5$€675.56 =€$1,039.44$. This means that one is willing to pay €$1,039.44$ to receive €$1,000$ five years from then! In other words, a negative implied yield is observed. □

To avoid such unrealistic returns, we would like to find bond returns, which are consistent with observed yields. For that reason we will specify a yield curve in every state of the scenario tree. These yield curves are used to discount future coupon and principal payments in order to obtain market values of the bond portfolio. These market values are used in the specification of bond returns. The advantages of this approach are twofold: one can specify any current bond portfolio, and find consistent future returns, using observed market prices. Moreover, these curves are used in Section 5.4 to value the liabilities, using observed market prices. We use the following equation, which is derived from Haugen [39], to define a yield curve in state (t, s):

$$y_t^s(q) = (a_1 + a_2 q)e^{-a_3 q} + a_{4t}^s, \quad q = 0, 1, 2, \dots \tag{5.2}$$

In equation (5.2), $y_t^s(q)$ denotes the yield corresponding to a risk-free zero-coupon bond maturing q years from time t, given the current state (t, s). Moreover, coefficient a_{4t}^s is the yield on bonds with the longest terms to maturity, and a_1 is the difference between the yield on bonds with the longest and shortest terms to maturity. This can easily be seen by considering $q = \infty$ and $q = 0$, respectively. The other two coefficients, a_2 and a_3, control the shape of the curve between the shortest en longest maturities.

For simplicity, we assume only parallel shifts in the yield curve. As a result, the coefficients a_1, a_2 and a_3 in (5.2) do not depend on t and s; they have to be estimated only once (at time 0). In other states, only a_{4t}^s will be adjusted, so that the new yield curve is consistent with expected one-year returns on the bank account.

At time 0, the yield curve (5.2) is estimated. To do so, we used data on March 1, 2002, of yields implied by Dutch government bonds with maturity 10 years. These data were derived from Datastream [20]. Numerical values for the coefficients in (5.2) are found by using a nonlinear regression routine. In this routine, numerical values for the four coefficients in (5.2) are found which minimizes the sum of the squared vertical distances from the curve. As a result, the yield curve at time 0 is specified.

Now we explain how yield curves are found in all other states (t, s), $t \in \mathcal{T}_1$, $s \in \mathcal{S}$. Since we assume that only parallel shifts in the yield curve occur, the estimates $\hat{a}_1$, $\hat{a}_2$, and $\hat{a}_3$ are used in every state to specify the shape of the yield curve. As a result, only the values of $\hat{a}_{4t}^s$ have to be found in all states (t, s), $t \in \mathcal{T}_1$, $s \in \mathcal{S}$. This can be accomplished by asking for consistency with the values of the risk-free interest rates. Recall that these are already present in the scenario tree. This consistency can be obtained by considering the following definition:

$$\mathbb{E}_{(t,s)}[r_{4,t+1}^{s'}] = y_t^s(1), \quad t \in \mathcal{T}_1, s \in \mathcal{S}, s' \in \mathcal{K}_t^s(t+1). \tag{5.3}$$

Equation (5.3) has to be satisfied, since both the left-hand side and the right-hand side define the expected next year's risk-free rate. Given equation (5.3), it is relatively easy to find updated values of $\hat{a}_{4t}^s$, given that its value in its predecessor is already known:

$$\hat{a}_{4t}^s = \hat{a}_{4,t-1}^s - y_{t-1}^s(1) + \mathbb{E}_{(t,s)}[r_{4,t+1}^{s'}], \quad t \in \mathcal{T}_1, s' \in \mathcal{K}_t^s(t+1).$$

Therefore, we consider the scenario tree in an increasing order of time. As a result, a parallel shift in the yield curve arises, and all term structures in the scenario tree are consistent with the expected returns on the bank account.

Generating bond returns

We make the following assumptions in generating bond returns:

- The pension fund under consideration only invests in risk-free, non-callable bonds.
- The coupon and principal payments of the current bond portfolio are known (and therefore, also the duration of the portfolio).
- Each year, the board of the fund adjusts the bond portfolio, so that the duration at the beginning of each year is the same.

We would stress here that each of these assumptions may be relaxed, although relaxing some of these assumptions may lead to additionally required data.

Given the yield curve in a state, we use the implied discount rates to find the current price of the bond portfolio. The payoffs of the bond portfolio, i.e. coupon and principal payments, together with the change in the price of the bond portfolio, define the return on this portfolio. The mathematical details of these calculations are given in Appendix 5.A.2.

5.3.2 Stock returns

Before we describe how stock returns are generated, we first discuss mean returns, variances, autoregressive terms and bounds on returns. These issues are considered, because we would like to obtain scenarios which are in accordance with historically observed characteristics.

Mean stock returns

The considered period of observation is crucial in estimating mean stock returns, and can lead to large differences. In Table 5.2 mean yearly stock returns, used in some financial models in the literature, are presented.

Author(s)	Mean return (in %)
Boender et al. [7]	10.7
Cariño and Turner [15]	11.0
Dert [24]	8.6
Ibbotson and Sinquefield [43]	10.5
Kouwenberg [55]	10.2
Rudolf and Zimmermann [83]	7.1

Table 5.2: Mean yearly stock returns used in financial models.

For most of the mean returns presented in Table 5.2, the considered period started either in 1956 or in 1926 and ended in the late 1990s. To obtain an estimate of mean stock returns based on many observations, we use the results presented in Siegel [88]. He found a yearly return of 6.7 percent in the period 1802-1992. This figure is based on research by Schwert [87], Schiller [85], and data from the Center for Research in Stock Prices, CRSP [19].

We use the returns on the broadly diversified MSCI World-index to include even more recent data in the estimate of mean stock returns. In the period 1993-2002, the mean return on the MSCI World-index was 9.0%. Taking into account both periods, we decided to specify the mean return as 6.8%.

Variances

Variances of stock returns are not stable over time, see for example Bollerslev et al. [11] and French et al. [34]. They conclude that the stochastic processes for the variances of the returns, are best described by a GARCH(1,1)-model. We have tested this condition (against the alternative to include higher moments) using historical values of the returns on the MSCI World-index in the period 1970-2002. We concluded that a GARCH(1,1) representation is indeed the best one.

ARCH models were introduced in the econometric literature by Engle [31] and generalized by Bollerslev [9], which led to the introduction of GARCH models. These models are widely used in various branches of econometrics, especially in financial time series analysis. Details about the GARCH(1,1) model, used to describe time varying variances, are presented in Appendix 5.A.3.

Autoregressive terms

As already mentioned before, in the literature it is customary to model the relationships between the stochastic parameters as a first-order autoregressive model. In these models, the autoregressive components for the returns on stocks are omitted. This is done to avoid predictability of asset returns. However, Lo and MacKinlay [61] found positive correlation in daily, weekly, and monthly index returns. If yearly returns are considered, as we do in our ALM model, Fama and Frech [33] and Poterba and Summers [79] found negative serial correlation in index returns.

To find an appropriate description for the development of the stock returns, we follow the analysis made by Campbell et al. [13] to test the presence of autocorrelation in the lognormal stock returns. Assume that the returns on stocks are described by the following model:

$$\tilde{r}_{1t} = \nu_1 \tilde{r}_{1,t-1} + \epsilon_{1t}, \tag{5.4}$$

where the disturbance terms ϵ_{1t} are assumed to be independent and identically distributed (IID), and follow a normal distribution with mean 0.

We tested the null hypothesis that yearly stock returns are IID (which implies $\nu_1 = 0$). The estimate of ν_1 is 0.056. We used again yearly returns on the MSCI World-index from 1970 to 2002. The Ljung-Box Q-statistic [60], and the corresponding probability are 0.2186 and 0.607 respectively. We conclude that the first order autoregressive term is not needed in the description of stock returns.

Bounds on stock returns

Based on historical observations, one may expect a higher return on a broadly diversified stock portfolio than on a corresponding bond portfolio. As a result, the *ex-ante risk premium*, the difference between the returns on a stock and bond portfolio, is strictly positive. This risk premium may vary through time, and may for example be influenced by the level of the interest rate. If this rate is very low, one may not expect large positive returns. On the other hand, a low interest rate may lead to many investment opportunities by companies, resulting in more economic activity. Therefore, one may expect a relatively high return on stocks. However, even though the ex-ante risk premium may vary, it is reasonable to assume that an upper bound on this premium exists.

We want to generate stock returns in such a way that the implied internal rate of return on the stock portfolio, based on the growth model developed by Gordon [36], does not violate lower and upper bounds on the ex-ante risk premium. This implies lower and upper bounds on stock returns. The mathematical details about the Gordon growth model and the derivation of the bounds on stock returns are presented in Appendix 5.A.4.

Given time t, the lower and upper bounds on the stock returns imply that a mean reverting component is introduced, taking into account the whole history from time 0 to time t. This mean reverting effect depends on the lower and upper bound on the ex-ante risk premium. Depending on these values, these bounds may prevent that scenarios are present in which stock returns are extremely high or low every year.

Generating stock returns

We assume that the stock return in each state (t, s), denoted by r^s_{1t}, follows a lognormal distribution. This is consistent with the assumption underlying many pricing models of derivatives based on stock prices. This is for example the case in the Black-Scholes option pricing model, see Black and Scholes [5]. In the description below, we distinguish r_{1t} and $\tilde{r}_{1t}$. The first is the so-called *simple net return*, while $\tilde{r}_{1t}$ denotes the *continuously compounded return*, or the *log return*. The values of r_{1t} and $\tilde{r}_{1t}$ are related by means of the following equation:

$$\tilde{r}_{1t} := \log(1 + r_{1t}).$$

Given this assumption, we need numerical values for next year's mean return and variance to generate returns. However, we cannot directly use the numerical values for the mean and variance of the simple net returns, denoted by μ_1 and σ_1^2 respectively. It is well known that given the estimates of μ_1 and σ_1^2, the mean and variance of the continuously compounded returns are given by

$$\tilde{\mu}_1 = \log\Big(\frac{\mu_1 + 1}{\sqrt{1 + \frac{\sigma_1}{\mu_1+1}}}\Big), \tag{5.5}$$

$$\tilde{\sigma}_1 = \log\Big(1 + \big(\frac{\sigma_1}{\mu_1 + 1}\big)^2\Big), \tag{5.6}$$

see for example Campbell et al. [13].

We assume that the mean return is the same for every year (although this may later be adjusted if returns are truncated), and the variances are given by the GARCH(1,1) specification. Given the expected mean return and variance for each state (t, s),

$t \in \mathcal{T}_0$, $s \in \mathcal{S}_t$ in the scenario tree, given by (5.5) and (5.6), we apply the same procedure as described in Section 5.2 to find numerical values which approximate a normal distribution. Given state (t, s), this gives us $branch_t$ values for $\tilde{r}^{s'}_{1,t+1}$, $t \in \mathcal{T}_0$, $s' \in \mathcal{K}^s_t(t+1)$. These $branch_t$ values are transformed to obtain simple net returns by means of the formula

$$r^{s'}_{1,t+1} = e^{\tilde{r}^{s'}_{1,t+1}} - 1. \tag{5.7}$$

Finally, we check whether the returns (5.7) satisfy their lower and upper bounds. See for the formulas of these bounds Appendix 5.A.4. If these bounds are not satisfied, the returns are truncated (and therefore the probability distribution is adjusted), so that these bounds are satisfied.

Probability of outperformance of stocks over bonds

As a result of the calculations above, we have, for any state (t, s), a set of precisely $branch_t$ values for the simple net return of stocks in the next year. Together with the (equal) conditional probabilities they represent the marginal distribution of $r_{1,t+1}$ given (t, s). Moreover, already in Section 5.3.1 a similar marginal distribution of the bond returns $r_{2,t+1}$ was derived. The question comes up: how to join these marginal distributions to a joint distribution? An easy way to do so is by assuming independence: then random assignments will be appropriate. But there are good reasons to assume that interdependencies between the returns of stocks and bonds are realistic. In the literature, special attention is paid to the probability that stock returns outperform bond returns, depending on the length t of the time period, see for example Bernstein [2], who concludes that in the long run, stocks are fundamentally less risky than bonds. It is argued, that this probability of outperformance increases with t. Indeed, according to H.A. Klein Haneveld [51] this probability is even equal to 1 if broadly diversified stock and bond portfolios are considered (as we do) over periods longer than 20 years.

We use a heuristic way to assign stock returns to nodes in the tree, taking into account the probability of outperformance. We would like to minimize the value of

$$\sum_{t=1}^{T} |P_t(r_1 \geq r_2) - P^*_t(r_1 \geq r_2)|, \tag{5.8}$$

where, for any t, $P_t(r_1 \geq r_2)$ denotes the probability of outperformance. It is calculated as

$$P_t(r_1 \geq r_2) = \sum_{s \in \mathcal{K}^s_0(t)} p^s_t \delta^s_{1t}$$

with

$$\delta^s_{1t} := \begin{cases} 1 & \text{if } \Pi^t_{q=1}(1 + r^s_{1q}) \geq \Pi^t_{q=1}(1 + r^s_{2q}) \\ 0 & \text{otherwise.} \end{cases} \tag{5.9}$$

Moreover, $P^*_t(r_1 \geq r_2)$ denotes the historical probability of outperformance of stock returns over bond returns over a period of t years.

We do not want to violate the following constraints in the minimization of 5.8, since these are more important in our opinion:

- The mean values and the variances of the stock returns over all sets of successors may not be changed.

- Bond returns may not be altered, because of the relationship with the yields.

Now we will describe the heuristic we use to assign stock returns to the nodes of the tree. We start with a random assignment. Then, we try to find a lower value of (5.8), given the current realizations of r^s_{1t} and r^s_{2t}. We do this by considering an interchange of two stock returns in two nodes with the same predecessor. Here comes the choice of the equal conditional probabilities into play. This choice for the conditional probabilities allows for an interchange of the returns on the stock portfolio, such that the conditions listed above are still satisfied: the marginal distributions of the stock returns remain unaltered in this case.

We consider the scenario tree from $t = 0$ to time $T - 1$. If $|P_{t+1}(r_1 \geq r_2) - P^*_{t+1}(r_1 \geq r_2)|$ is larger than $\frac{1}{|\mathcal{S}_t|}$, we consider the interchange of the stock returns in two states $(t+1, s^{'})$, $(t+1, s^{''})$, with $s^{'}, s^{''} \in \mathcal{K}^s_t(t+1)$. If it leads to a lower value of (5.8), this interchange is made. Note that given an interchange of two returns, all implications for future years have to be considered in the definition of δ^s_{1t} in (5.9).

In this way, all states before the horizon are considered. If an improvement is found, i.e. a lower value of (5.8) is obtained, the corresponding adjustments in the scenario tree are made, otherwise not.

5.3.3 Returns on real estate

Returns on the real estate portfolio are generated in the same way as returns on the stock portfolio. They are also assumed to be lognormally distributed. The variance of the continuously compounded returns are described by a GARCH(1,1)-process. Numerical values for the GARCH(1,1) specification for a world index for real estate can be found in Appendix 5.A.3.

We also tested whether an autoregressive term is present in the historical yearly returns on real estate. If one assumes that (5.4) describes the returns (with index 3 instead of 1), the estimate for ν_3 gets the value 0.076 for the given data. The corresponding Q-statistic and probability are given by 0.3744 and 0.541, respectively. We conclude that the first-order autoregressive term is not needed in generating returns on the real estate portfolio.

Given the specification of the mean and variance for the returns, we truncate the returns if the lower or upper bounds, as found by using the Gordon growth model [36], are not satisfied. Moreover, these returns are adjusted, so that they represent a lognormal distribution. Finally, the interdependencies with the bond returns are specified in a heuristic way to obtain realized probabilities of outperformance which are close to empirically observed ones.

5.3.4 No arbitrage

A very important concept in financial models, is the *no arbitrage condition*, see e.g. Pliska [78]. The existence of arbitrage in the data of portfolio models means that, without risk, money can be made from nothing (so that so-called *money machines* or *free lunches* are possible). That is, if arbitrage is present, it is possible to construct portfolios that finance themselves and give a sure win.

Arbitrage is a theoretical concept, which should be considered in financial optimization models in order to avoid spurious outcomes, because optimization models exploit these opportunities if they exist. Indeed, as Klaassen [50] shows, if scenarios are not arbitrage free, stochastic programming makes use of the arbitrage opportunities, leading to portfolios that are biased to spurious profit opportunities in a nonrealistic way. It is obvious that realistic scenarios for returns on investments should be arbitrage free. In the remainder of this section, we describe the arbitrage concept. This concept is not especially related to ALM. Therefore, we discuss arbitrage in a more general context. In particular we may consider an investment of zero.

We first consider arbitrage opportunities in a one-period sense. Given state (t, s), arbitrage opportunities are present if:

- the total amount invested in state (t, s) is equal to zero,
- in each state $(t+1, s')$, $s' \in \mathcal{K}_t^s(t+1)$ the value of the portfolio is nonnegative, and
- the expected next year's value of the portfolio is strictly positive. Therefore, in at least one state at time $t+1$ with (t, s) as parent, the value of the portfolio is strictly positive.

Dert [24] formulated a linear program to test for arbitrage opportunities. Another way to check for arbitrage opportunities is introduced by Harrison and Kreps [38]. There are no arbitrage opportunities if and only if numbers $\pi_{t+1}^{s'} > 0$ exist for the set of child nodes for each state (t, s), $t \in \mathcal{T}_0$ and $s \in \mathcal{S}_t$, such that the following system has a solution:

$$\sum_{s' \in \mathcal{K}_t^s(t+1)} (1 + r_{j,t+1}^{s'}) \pi_{t+1}^{s'} = \sum_{s' \in \mathcal{K}_t^s(t+1)} (1 + r_{1,t+1}^{s'}) \pi_{t+1}^{s'} \qquad j = 2, \ldots, N, \tag{5.10}$$

$$\sum_{s' \in \mathcal{K}_t^s(t+1)} \pi_{t+1}^{s'} = 1, \tag{5.11}$$

$$\pi_{t+1}^{s'} > 0. \tag{5.12}$$

Such $\pi_{t+1}^{s'}$, $s' \in \mathcal{K}_t^s(t+1)$ are called *risk neutral probabilities*. If the returns $r_{j,t+1}^{s'}$, $j = 1, \ldots, N$, $s' \in \mathcal{K}_t^s(t+1)$ are given, one can test whether arbitrage opportunities exist by solving a linear program that maximizes the value of ϵ while satisfying constraints (5.10) and (5.11), and replacing constraint (5.12) by

$$\pi_{t+1}^{s'} \geq \epsilon.$$

Given the values of $r_{j,t+1}^{s'}$, there are N linear equalities in $branch_t$ unknown probabilities and $branch_t$ positivity constraints in order to rule out arbitrage opportunities in year $t+1$.

In the context of our ALM model, arbitrage implies that current investments could be extended for free. To be sure that no arbitrage opportunities exist in the multiperiod sense, one has to consider the whole scenario tree at once. Given a scenario tree which is not arbitrage free, it is very difficult to improve the realizations of the stochastic variables, so that the tree becomes arbitrage free. Because we do not expect that we could implement testing for arbitrage opportunities in reasonable time, we ignored this item in the numerical realization of scenario trees.

Fortunately, if the number of branches increases, whereas the number of asset classes remains the same, the probability that arbitrage opportunities are present decreases. This can be seen in model (5.10), (5.11), and (5.12), because more decision variables $\pi_{t+1}^{s'}$ are present in this case. Dert [24] also shows numerically that the probability that arbitrage opportunities are present decreases rapidly if the number of scenarios increases.

5.4 Liabilities, benefit payments, discount rates, and wages

In this section we describe how we obtain numerical values for $\underline{L}_t^s$, $\overline{L}_t^s$, $\underline{B}_t^s$, $\overline{B}_t^s$, γ_t^s, and W_t^s. First, we start with answering the question how we get values for the lower and upper bounds on the initial value of the liabilities, based on observed market prices. Then we will describe how we get values for the lower bounds $\underline{L}_t^s$ for all remaining states (t, s), $t \in \mathcal{T}_1$, $s \in \mathcal{S}$.

After it is made clear how the values for $\underline{L}_t^s$ are generated, we describe how values for $\overline{L}_t^s$ are found. Finally, we concentrate on the stochastic parameters $\underline{B}_t^s$ and $\overline{B}_t^s$, the lower and upper bounds on the benefit payments, the discount rates γ_t^s, and the level of the wages W_t^s.

Initial value of the liabilities

The initial value of the liabilities is by definition the present value of the nominal expected future benefit payments of the current built-up rights. A large Dutch pension fund provided us data with respect to these nominal expected benefit payments, and also data with respect to developments of the built-up rights in the next years. An example of these expected nominal future benefit payments is given in Figure 5.1. From this figure we see that it is expected that in the near future more people will retire, and as a result, expected future benefit payments will increase. In the years thereafter, only a fraction of the people survive that year and therefore, we get the (long) tail to the right. As will become clear in this chapter, the data we obtained will be transformed.

The major issue to determine the current value of the liabilities, is the choice of the discount factors to be used. Of course, the board of a pension fund cannot determine these discount factors themselves; instead, prescriptions with respect to these rates by the supervisor have to be satisfied. In The Netherlands, pension funds used to value their liabilities using a (prescribed) fixed discount rate, that is, one rate for all future years. However, as already mentioned in Section 1.2, the Dutch supervisor of pension funds wants that the liabilities are valued, such that a good judgement can be make about the financial position of a fund, see the discussion

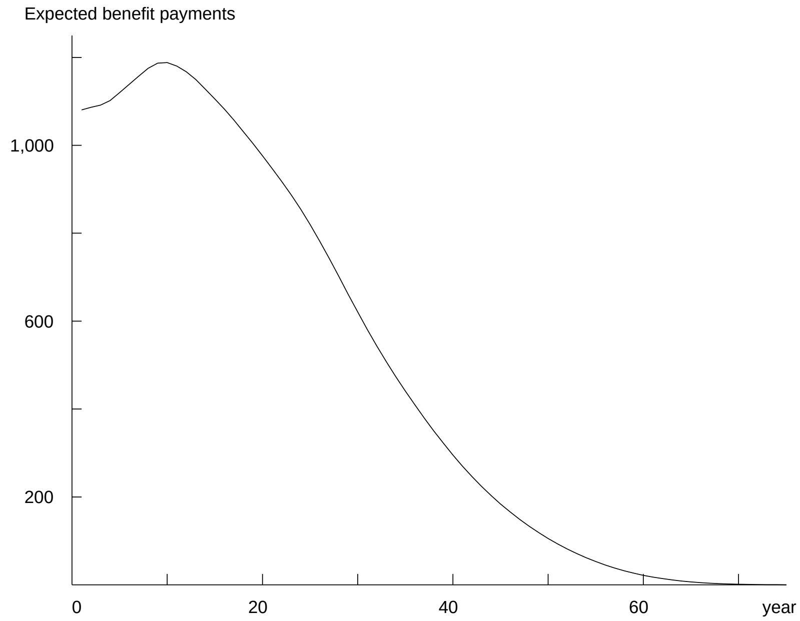

Figure 5.1: Expected future benefit payments, based on the current built-up rights (of a large Dutch pension fund), contained in the liabilities of year 0.

paper [75]. Since the assets are valued based on observed market prices, it is therefore better to use market values to value the liabilities too. In the discussion paper [75], the supervisor of the pension funds in The Netherlands writes that a market value for the liabilities should be found.

Although the concept of finding a market value of the liabilities seems easy (the fund 'only' has to discount future cash flows), it is far from trivial to answer the question which discount factors should be used to value the liabilities. It is important that these discount factors are based on the prices in the capital markets at the moment one wants to find the market value. One possibility to discount future benefit payments is to use risk free rates. However, H.A. Klein Haneveld [51] suggests not only to use risk free rates, but also to use internal rates of returns on stocks and bonds in the discount factors. He gives two reasons why to use the internal rates:

- Time diversification. This means that above average returns tend to offset below average returns over long horizons, see Kritzman [56].

- Returns on stocks and bonds. As we have seen in Section 5.3.2, Bernstein [2] concluded that in the long run, stocks are fundamentally less risky than bonds.

Therefore, H.A. Klein Haneveld [51] suggests to take into account the maturity of the expected future benefit payments to decide if returns on stocks should be considered into the discount factors. We also used this approach to discount future benefit payments. Now, we will describe how we implemented this approach.

In the discussion how returns on bonds are generated, we have already introduced yield curves. Moreover, we used the *internal rate of return* of the stock portfolio in generating stock returns by considering the Gordon growth model (see Appendix 5.3.2). Next to the yield curve, we introduce two additional curves: the *stock spot curve*, and the *pension spot curve*. These two curves are based on H.A. Klein Haneveld [51]. One point on the stock spot curve is the internal rate of return of stocks. We assume that this curve and the yield curve run parallel to each other. The difference between these two curves is the so-called *equity risk premium*.

The third curve is the pension spot curve. This is a combination of the previously defined two spot curves. The pension spot curve is used to discount expected future benefit payments. Because returns of a broadly diversified stock portfolio always outperformed a broadly diversified bond portfolio if the considered period was longer than 20 years (see H.A. Klein Haneveld [51]), we use the stock spot curve to discount cash flows with a maturity of 20 years or more. Because investments in stocks with shorter maturities are exposed to more risk, a combination between stock and bond investments is considered to discount cash flows with maturities less than 20 years. For simplicity, we let the fraction of stocks increase linearly in the first 20 years to obtain discount rates. In state (t, s), the discount rate q years from year t on the pension spot curve, denoted by $PSC_t^s(q)$, is defined as follows:

$$PSC_t^s(q) := \begin{cases} y_t^s(q) + \frac{q}{20} earp_t^s & \text{if } 0 \leq q \leq 20 \\ y_t^s(q) + earp_t^s & \text{if } q \geq 20, \end{cases}$$

where $earp_t^s$ denotes the *ex-ante equity risk premium*. The numerical values of the ex-ante risk premiums vary from state to state. These values depend on previously realized returns, the growth rate and the levels of the dividends, and the yield curve. In Appendix 5.A.4, the formulas of these ex-ante risk premiums are given. A typical example of these three curves, is given in Figure 5.2.

The discount rates used in our model, which are denoted by γ_t^s, are the values on the pension spot curve as determined at time 0. We use these discount rates to get consistency in computing financial cash flows at all times, just as in valuing liabilities.

Generating lower bounds on the value of the liabilities

At time 0, the lower bound on the value of the liabilities, denoted by $\underline{L}_0^1$, is given by

$$\underline{L}_0^1 := \sum_{q=0}^{\infty} \frac{B_0^*(q)}{\Pi_{q'=1}^{q}(1 + PSC_0^1(q'))},$$

where $B_0^*(q)$ is the current expected benefit payment over q years. In Figure 5.1 the values of $B_0^*(q)$ are presented. Moreover, $PSC_0^1(q)$ denotes the pension spot curve at time 0, for all maturities $q \geq 1$.

Numerical values for $\underline{L}_t^s$ for all states (t, s) with $t \geq 1$, are found in a similar way. Future values $B_t^*(q)$, which denote the conditional expected benefit payments in year $t+q$, given time t, were obtained from a large Dutch pension fund for all future

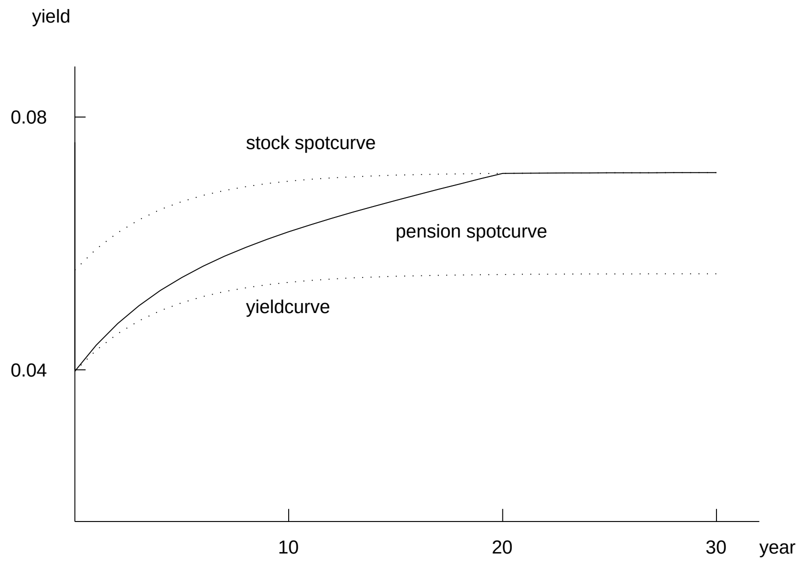

Figure 5.2: The pension spot curve, the curve used to discount future cash flows, is composed of the yield curve and the stock spot curve.

times $t \in \mathcal{T}_1$. In addition, we have calculated the pension spot curves $PSC_t^s(q)$ in every state of the scenario tree. Appropriate discounting gives numerical values for $\underline{L}_t^s$ for all (t, s):

$$\underline{L}_t^s := \sum_{q=t+1}^{\infty} \frac{B_t^*(q)}{\Pi_{q'=1}^{q}(1 + PSC_0^1(q'))}.$$

Upper bound on the value of the liabilities

The upper bound on the value of the liabilities in state (t, s) is found by increasing the expected future benefit payments with observed accumulated increases in the general wage level:

$$\overline{L}_t^s = \underline{L}_t^s \Pi_{q=0}^t (1 + w_q^s) \qquad t \in \mathcal{T}, s \in \mathcal{S}.$$

We see that the upper bound on the value of the liabilities is found by always giving full indexation to the participants of the fund.

Lower and upper bound on the benefit payments

The lower bound $\underline{B}_t^s$ on the benefit payments in state (t, s), is equal to $B_t^*(0)$, the nominal benefit payment. The corresponding upper bounds, $\overline{B}_t^s$, are equal to their lower bound, increased with the observed cumulative increase in the general wage level.

Wages

We obtained data for the expected level of the pensionable wages of the active participants for the next years. In our ALM model, it is assumed that this level is the same for all scenarios at time t. Ideally, one would like these levels to be state dependent, so that the interactions between the values of the assets and liabilities are reflected even better. However, obtaining appropriate values for W_t^s would require information with respect tot the development of the pensionable wages in each state of the scenario tree. These developments require the introduction of business scenarios, because the total pensionable wages depends for example on the number of employees of the company. This number is correlated with other realizations in the scenario tree, because they all depend on the state of the economy. In the absence of the additionally required data to calculate W_t^s we let the values of W_t^s only depend on t and not on s, so that for each state at time t the same value is used.

5.5 Future research

In this chapter, we have seen that scenarios are generated by considering dependencies between stochastic parameters. Moreover, both empirical and theoretical reasons were given to justify the used approaches. These have a number of desirable properties. For example, returns on the bond portfolio were generated, while keeping in mind that the implied yields do not become negative.

Although this seems nice at first sight, we will give here some suggestions for improvements. Because of a time constraint, we were not able to implement these refined ideas yet.

First of all, relationships between returns on stocks, bonds, and real estate should be considered in more detail. Although these relationships are considered by means of probabilities of outperformance (and this approach is consistent with the way the liabilities are valued), one may explicitly use empirically observed correlations between the returns. An alternative way to consider relationships between returns, which also makes use of historical data, is to apply principal component analysis. Another alternative is to consult an expert, who may choose ‘good’ combinations from a number of alternatives. One can also use a heuristic to match moments, so that also for example the skewness and kurtosis of the marginal distributions are taken into account, see Høyland et al. [42].

In the generation of bond returns, some assumptions were made for simplicity. In order to find possible future bond returns for pension funds, some of these assumptions may be relaxed. For example, one should include the possibility to invest not only in risk-free non-callable bonds. Moreover, nonparallel shifts of the yield curve should be considered.

Another important issue is the no arbitrage concept. Although the probability of arbitrage opportunities decreases if the tree becomes more bushy, it would be nice to generate returns such that it is guaranteed that these opportunities are excluded.

5.A Appendix: Mathematical details

In this appendix, we discuss mathematical backgrounds regarding concepts underlying our scenario generator. We first describe the error-correction model which we use to model the relationship between the returns on the bank account and changes in the general wage level. Then, we will consider bond returns. In Section 5.A.3, we discuss GARCH(1,1)-models. Finally, we describe how we find bounds on stock returns and the implications on the ex-ante risk premium.

5.A.1 Error-correction model

In this section, we describe the error-correction model which is used to model the interdependency between the returns on the bank account and the change in the general wage level (see Section 5.2).

The cointegrating equation is described by

$$w_t = \chi r_{4t}, \tag{5.13}$$

and can be interpreted as a long-run equilibrium relationship between the short-term risk-free interest rate and the change in the general wage level. The parameter χ gives the proportionality between r_{4t} and w_t in the long run. The error-correction model is given by

$$\Delta r_{4t} = \vartheta_1 (w_{t-1} - \chi r_{4,t-1}) + \epsilon_{4t} \tag{5.14}$$

$$\Delta w_t = \vartheta_2 (w_{t-1} - \chi r_{4,t-1}) + \epsilon_{wt}, \tag{5.15}$$

with $\epsilon_{4t} \sim \mathcal{N}(0, \sigma^2_{\epsilon 4})$ and $\epsilon_{wt} \sim \mathcal{N}(0, \sigma^2_{\epsilon w})$, and $\epsilon_{4t}, \epsilon_{wt}$ are all mutually independent. In equalities (5.14) and (5.15), Δ denotes a change, that is $\Delta r_{4t} := r_{4t} - r_{4,t-1}$, and $\Delta w^s_t := w^s_t - w^s_{t-1}$. In addition, the parameters ϑ_1 and ϑ_2 are measures for the speed of adjustment.

The error-correction model described above forces r_{4t} and w_t to converge in the long run to their cointegrating relationship, while allowing a wide range of short-run (and randomly) dynamics. Therefore, deviation from the long-run equilibrium is corrected gradually through a series of partial short-term adjustments.

We use risk free interest rates from 1981 to 2001 from Datastream [20], and data of changes in the general wage level of the same years from the website of the Dutch central bureau of statistics, CBS [16] to estimate the equations (5.13), (5.14) and (5.15). To do so, we use the quasi-maximum likelihood method, as described by Bollerslev et al. [10]. The numerical values are presented in Table 5.3. These parameter estimates imply the following. The estimated value of χ means that there is a positive relationship between the short-term risk-free interest rate and the change in the general wage level. This is also what we expected, because both r_{4t} and w_t are influenced by the inflation rate.

The negative signs of ϑ_1 and ϑ_2 are also consistent with economic theory. If r_{4t} is above the long-run relationship (5.13), relatively high wage increases lead to a relatively high value of the inflation. As a result, the interest rate will increase, since this rate consists of the inflation and a real part. Therefore, it is to be expected that r_{4t} will increase. If r_{4t} is above the long-run relationship, w_t is of course below

parameter	estimation	t-value
χ	1.320	6.38
ϑ_1	-0.368	-2.83
ϑ_2	-0.390	-2.49
σ_4^2	0.01323	
σ_w^2	0.0204	

Table 5.3: Estimated parameter values of the error-correction model.

this relationship. In this case, the government may call for a wage restraint, since further high wage increases will harm the economy.

On the other hand, if r_{4t} is below the long-run relationship (5.13), low inflation may result in a decreasing risk-free interest rate. This may for example be the case if economic growth is very low (or even negative): the central bank will decrease the risk-free interest rate to stimulate the economy. Moreover, w_t is below the long-run relationship with r_{4t}. In this case, it is to be expected that unions claim higher wages.

5.A.2 Bond returns

In this section we describe the details about generating bond returns (see Section 5.3.1).

Price of a bond portfolio

We assume that future payoffs, i.e. coupon payments and principal payments of the bond portfolio at time 0 are known. Moreover, we assume that these payments are made with certainty in the future. Therefore, we assume that these are risk-free and that non-callable bonds are considered.

Given state (t, s), portfolio coupon and principal payments, which are made q years ahead, are denoted by $C_t^s(q)$ and $PrB_t^s(q)$, respectively. The initial values, i.e. at time zero, have to be specified by the user. These values describe the initial bond portfolio of the pension fund under consideration.

The value of the bond portfolio at time t in scenario s, denoted by PB_t^s, is (by definition) given by

$$PB_t^s = \sum_{q=1}^{\infty} \frac{C_t^s(q) + PrB_t^s(q)}{\Pi_{q'=1}^{q}(1 + y_t^s(q'))}.$$

This gives us the price of the bond portfolio in each state (t, s), given the future stream of coupon and principal payments.

As time passes, the duration (the expected mean term) of the bond portfolio decreases. To avoid this, new zero-coupon bonds with maturity strictly greater than the desired duration are bought. The number of newly acquired bonds is chosen, so that the duration is exactly equal to its desired level.

Generating bond returns

We have explained how the price of the bond portfolio changes, given the desired level of the duration and the movements in the yield curve. Now, we are able to generate bond returns. By definition, the return on the bond portfolio in state (t, s) is given by

$$r_{2t}^s = \frac{C_{t-1}^s(1) + PrB_{t-1}^s(1) + PB_t^s}{PB_{t-1}^s} - 1.$$

That is, the return is based on the coupon and principal payments made in state (t, s), and on the current and previous (market) values of the bond portfolio.

If one would like to add also more risky bonds to the bond portfolio, one may change the corresponding payments in expected payments and increase the discount rates appropriately.

5.A.3 GARCH(1,1) models

A GARCH(1,1) model for the continuously compounded returns on stocks, as discussed in Section 5.3.2, are defined as follows:

$$\tilde{r}_{1t} = \tilde{\mu}_1 + \varrho_{1,t+1} \tag{5.16}$$

$$\varrho_{1,t+1} = \tilde{\sigma}_{1t}\varsigma_{1,t+1}, \tag{5.17}$$

$$\tilde{\sigma}_{1t}^2 = d_{11} + h_{12}\tilde{\sigma}_{1,t-1}^2 + h_{11}\varrho_{1t}^2 \tag{5.18}$$

where $\varsigma_{1,t+1} \sim \mathcal{N}(0, 1)$ is assumed to be standard normally distributed. As a result, $\varrho_{1,t+1} \sim \mathcal{N}(0, \tilde{\sigma}_{1t}^2)$. Moreover, the $\tilde{}$-sign denotes that continuously compounded returns are used.

In this GARCH(1,1) model, $\varrho_{1,t+1}$ can be interpreted as an innovation: it has mean zero, conditional on time t information. In addition, $\tilde{\sigma}_{1t}^2$ is the time t conditional variance of $\varrho_{1,t+1}$ or, equivalently, the conditional expectation of $\varrho_{1,t+1}^2$. This means that (5.18) can be rewritten as follows:

$$\tilde{\sigma}_{1t}^2 = d_{11} + (h_{11} + h_{12})\tilde{\sigma}_{1,t-1}^2 + h_{11}(\varrho_{1t}^2 - \tilde{\sigma}_{1,t-1}^2). \tag{5.19}$$

In equality (5.19), $(\varrho_{1t}^2 - \tilde{\sigma}_{1,t-1}^2)$ has mean zero, conditional on time $t-1$ information, and can be thought of as the shock to volatility. The extent to which a volatility shock this year feeds through into next year's volatility is given by h_{11}, while $(h_{11} + h_{12})$ measures the rate at which this effect dies out over time.

Estimating the parameters of the GARCH(1,1) model for the returns on the MSCI World-index in the period 1967-2002 using the maximum likelihood method, gives the following results. $\tilde{\mu}_1 = 0.075$, $d_{11} = 0.013$, $h_{11} = 0.251$, $h_{12} = 0.533$.

We also estimated the parameters of the GARCH(1,1)-model for returns on real estate. We used the returns of a world index for real estate from 1970 to 2002. These data were derived from Datastream [20]. The estimates of the parameter values are $\tilde{\mu}_3 = 0.068$, $d_{31} = 0.012$, $h_{31} = 0.267$, and $h_{32} = 0.487$.

5.A.4 Bounds on stock returns

The Gordon growth model [36] mentioned in Section 5.3.2 gives the relation between the price of a stock(portfolio), the dividend payments of this stock(portfolio), and a growth rate of the dividend payments. It is assumed that the discount rates used to discount future dividend payments, is the same for all time periods. The price of a stock(portfolio) at time t, denoted by PS_t, is determined by its future stream of dividends:

$$PS_t = \mathbb{E}_t[\sum_{q=1}^{\infty} \frac{D_{t+q}}{(1+R_1)^q}], \tag{5.20}$$

where $\mathbb{E}_t$ denotes the conditional expectation given the state at time t (e.g. given D_t), and D_{t+q} denotes the dividend payment q years ahead, and $R_1 = \mathbb{E}_t[r_{1q}]$, $q \geq t+1$ so R_1 denotes the assumed constant expected stock return, also called *internal rate of return*.

Since the dividends are assumed to grow at a constant rate g_1 (which is less than R_1, a prerequisite to keep the stock price finite), the following relationship holds:

$$\mathbb{E}_t[D_{t+q}] = (1+g_1)\mathbb{E}_t[D_{t+q-1}] = (1+g_1)^q D_t. \tag{5.21}$$

Substituting (5.21) into (5.20), the well-known Gordon growth model is obtained:

$$PS_t = \frac{(1+g_1)D_t}{R_1 - g_1}. \tag{5.22}$$

Although the growth rate of stock dividends may fluctuate much for individual stocks, this figure is rather stable for stock portfolios, see Jagannathan et al. [45]. This growth rate was approximately 5 percent per year in the period 1927-1999. The reason why many companies prefer to pay stable, but increasing dividends (even though profits fluctuate significantly), is that the company seems financially sound, see Smith [90].

We will explain now what implication the Gordon growth model has on the stock returns in the scenario tree. For that reason, (5.22) is reformulated as follows:

$$R_1 = \frac{(1+g_1)D_t}{PS_t^s} + g_1. \tag{5.23}$$

Given the numerical values for PS_0, D_0 and g_1, we can find R_1. As explained in the beginning of this section, the ex-ante risk premium is assumed to be positive, and finite. The lower and upper bounds on this ex-ante risk premium are denoted by $\underline{earp}$ and $\overline{earp}$, respectively, and are assumed to be time independent. We require

$$r_{4t}^s + \underline{earp} \leq R_1 \leq r_{4t}^s + \overline{earp}. \tag{5.24}$$

Therefore, the implied internal rate of return of the stock portfolio is related to the one-year risk-free interest rate, r_{4t}^s, and bounds on the ex-ante risk premium.

Now, we explain how bounds on the returns on the stock portfolio are derived. The price of the stock portfolio in state (t,s) is by definition given by

$$PS_t^s = (1+r_{1t}^s)PS_{t-1}^s. \tag{5.25}$$

Because of (5.23) and (5.24), we obtain the upper bound on PS_t^s as follows:

$$\frac{(1+g_1)D_t}{PS_t^s} \geq r_{4t}^s + \underline{earp} - g_1,$$

or

$$PS_t^s \leq \frac{(1+g_1)D_t}{r_{4t}^s + \underline{earp}} - g_1.$$

Given definition (5.25), we obtain the upper bound on r_{1t}^s:

$$(1+r_{1t}^s)PS_{t-1}^s \leq \frac{(1+g_1)D_t}{r_{4t}^s + \underline{earp} - g_1},$$

$$r_{1t}^s \leq \frac{(1+g_1)D_t}{(r_{4t}^s + \underline{earp} - g_1)PS_{t-1}^s} - 1, \tag{5.26}$$

where PS_{t-1}^s is known at time t.

Analogously, one can find the lower bound on the stock returns:

$$r_{1t}^s \geq \frac{(1+g)D_t}{(r_{4t}^s + \overline{earp} - g)PS_{t-1}^s} - 1. \tag{5.27}$$

If a return on the stock portfolio does not satisfy (5.26) or (5.27), in our scenario generator we truncate it, so that these bounds are forced. This means that the distribution of the stock returns is adjusted.

Ex-ante risk premium

Given the Gordon growth model, we would like to find its implications on the ex-ante risk premium in state (t, s), denoted by $earp_t^s$. By definition, we have

$$earp_t^s := R_{1t}^s - y_t^s(1), \tag{5.28}$$

where R_{1t}^s denotes the internal rate of return of the stock portfolio in state (t, s). Using the definition of R_{1t}^s, as given by (5.23), we immediately obtain the formula for $earp_t^s$:

$$earp_t^s = \frac{(1+g_1)D_t}{PS_t^s} + g_1 - y_t^s(1).$$

From this equation we see that its value depends on PS_t^s and, as a result, on returns r_{1q}^s, $q = 1, \ldots, t-1$. In addition, $earp_t^s$ also depends on g_1, D_t and the expected next year's return on the bank account.

Chapter 6

Numerical experiments

In this chapter we report the first impressions we gained from numerical experiments. In Section 6.1 we consider an *illustrative case*. It is an ALM model for a fictitious pension fund. The values of the deterministic parameters are discussed in relation to the position of the interested parties. The numerical specification of the parameters in the scenario tree are described in the previous chapter and Appendix 5.A. The illustrative case is used to test the heuristic. The results are interpreted in some detail.

The illustrative case is also used as the starting point for *sensitivity analyses*. We consider sensitivity analyses with respect to modeling choices, model justification, and scenario trees. The results of these experiments are presented in Section 6.2.

In order to find the numerical results, we used a personal computer with a Pentium IV 2 GHz processor and 512 MB memory. The multistage stochastic programs with only continuous decision variables (obtained by fixing the values for the binary decision variables) are solved by OSL [71], using the callable library OSL Stochastic Extensions [72]. The heuristic is programmed in Microsoft Visual C++ [66].

6.1 Illustrative case

The illustrative case deals with a fictitious pension fund. Whereas data with respect to nominal liabilities of a Dutch pension fund are used in our scenario generator, as have been described in Chapter 5, all other parameters have been chosen by ourselves, without any relation to this fund. Of course, we have tried to provide realistic specifications. In fact, we aimed to specify the following situation. The sponsor is far from wealthy: he has to borrow money to be able to make a remedial contribution. The retired people are assumed to have relatively much influence, so that a deterioration of indexation is considered as harmful. Finally, the active participants do not have much influence in the decision making process.

As a result of the supposed positions of the supervisor, the sponsor and the retired people, the fixed penalty costs associated with underfunding, a remedial payment and a deterioration of indexation are all high. Are these positions realistic?

In our opinion, the assumed positions of the supervisor and the sponsor adequately represent current practice. Although currently the active participants may have more influence than the retired people, in the (near) future this may change. This more powerful role of the retirees is reflected in the illustrative case.

Before we present the numerical results, we first describe the parameter settings.

6.1.1 Parameter settings

In this illustrative case, the planning horizon is 4 years. The number of branches per year are 6, 6, 5 and 5 respectively. This implies that we consider 900 scenarios and 1,123 states in the scenario tree. The number of continuous and binary decision variables are respectively 28,075 and 6,738, and in the illustrative case the model has 45,147 constraints.

The tolerance, i.e. the upper bound on the required relative gap between the primal and dual solutions, is set equal to 0.0001.

In the description of parameter settings, we make a distinction between parameters which reflect the positions of the parties involved, and other parameters. The various settings of the first group of parameters, corresponding to the illustrative case and subsequent cases, allow for an interpretation in terms of the relative weights that are given to the interests of the parties involved.

Positions interested parties

In Chapter 1 we have seen that there are various interested parties in the ALM decision process. We will consider these parties and indicate what they have agreed on during the negotiations.

- **Active participants**
  Active participants like both a low and stable contribution rate. Moreover, they would like to receive full indexation with respect to increases in the general wage level.

- **Deferred members and retired people**
  Deferred members and retired people have stipulated that a large decrease in the level of the contribution rate is not desirable.

- **Sponsor**
  The sponsor of the fund does not like large increases of the contribution rate between two consecutive years. Moreover, the sponsor is not financially sound: the sponsor has to borrow money to be able to make a remedial payment. As a result, the associated fixed penalty costs will be set high, and large remedial contributions are very expensive.

- **Supervisor**
  Both underfunding and not indexing benefit rights are considered to be undesirable by the supervisor. As a result, the corresponding fixed penalty costs are relatively high. Moreover, the supervisor imposes a restriction on one-year expected shortages.

The specific parameter values for the fixed and variable costs, which are used to penalize undesirable events, together with the maximum decrease and increase in the contribution rate such that no penalty costs are incurred, are presented in Table 6.1. We think that the numerical values of these parameters correspond to the above described characteristics of the interested parties. In particular, the numerical value of ζ_{DZ} is chosen sufficiently high, as we will see in the output of this case in Section 6.1.2. In the remainder of this subsection we will comment on our choices underlying the specification in Table 6.1.

In Table 6.1, the fixed penalty costs are related to the level of the regular contributions $c_0 W_0$ paid to the fund in the previous year (i.e. in year 0). This makes it possible to compare the fixed costs with cash flows. Moreover, one can get a feeling how much the level of the contribution rate should be increased to avoid unfavorable events.

Other parameter settings

Next to the parameter settings which describe the relative position of the interested parties, there are some other parameters in our ALM model. The numerical values of these parameters which are used in the illustrative case are also presented in Table 6.1. We will discuss these values now.

- **Initial positions assets and liabilities**
  The current value of the assets, i.e. the value of the assets just before the portfolio may be changed at time 0, is €18,000 million. This amount is currently divided as follows: 47% is invested in stocks, 25% in bonds, 21% in real estate, and 7% in cash.

  The lower and upper bounds on the value of the liabilities at time 0 are €14,993 million and €15,517 million respectively. This implies that the funding ratio at time 0 is at least 1.16 and at most 1.20.

- **Contribution rate**
  Last year, the contribution rate was set equal to 11% of the pensionable salaries. The minimum level for this rate is 0%, so that a non-contributory pension is not excluded. The maximum level of the contribution rate is set to 21%. Moreover, the interested parties agreed that if the sponsor makes a remedial contribution, the contribution rate should be at least equal to 12.5%.

- **Underfunding and remedial contributions**
  The minimum required level of the funding ratio is 105%. This is similar to the requirements of the Dutch supervisor, described in the FTK [73]. Moreover, if in 2 consecutive years the funding ratio is less than 105%, the sponsor is forced to make a remedial payment. After this payment, the funding ratio should be at least equal to the minimum required level.

  In Chapter 2 we introduced the level θ. If the funding ratio falls below this level, the sponsor is forced to restore the funding ratio with respect to this level immediately. In the illustrative case, we have chosen to set $\theta = 0.90$. As

Initial position			
$A_0/\overline{L}_0$	1.16		
Contribution rate			
$\underline{c}$	0	ρ	0.010
$\overline{c}$	0.210	η	0.015
c^*	0.125	ζ_{ci}	0.750
c_0	0.110	ζ_{cd}	0.500
Underfunding and remedial contributions			
α	1.05	λ_u/c_0W_0	2.20
θ	0.90	λ_z/c_0W_0	3.40
a	2	ζ_Z	1.50
u_{-1}	0	ζ_{ZI}	5.00
u_{-2}	0	ζ_{DZ}	20.00
τ	0.25		
One-year risk constraints			
ψ	0.09		
Indexation			
λ_m/c_0W_0	1.75	ζ_L	0.50
Overfunding and restitutions			
β	2.50	λ_o/c_0W_0	-0.00
b	2	λ_v/c_0W_0	-0.00
o_{-1}	0	ζ_V	-0.02
o_{-2}	0		
Horizon			
Λ	1.25	$\zeta_{\Lambda d}$	0.10
$\zeta_{\Lambda i}$	-0.01		
Portfolio			
$\underline{f}_1$	30	$\%X_{10}/A_0$	47
$\underline{f}_2$	30	$\%X_{20}/A_0$	25
$\underline{f}_3$	10	$\%X_{30}/A_0$	21
$\underline{f}_4$	0	$\%X_{40}/A_0$	7
$\overline{f}_1$	60	$k_1(\times 100)$	0.43
$\overline{f}_2$	60	$k_2(\times 100)$	0.25
$\overline{f}_3$	25	$k_3(\times 100)$	0.43
$\overline{f}_4$	20	$k_4(\times 100)$	0.05

Table 6.1: Data for the illustrative case.

can be seen in Table 6.1, the variable costs associated with a remedial contribution with respect to the level θ, denoted by ζ_{DZ} are very high. This reflects the current financial position of the sponsor of the fund.

On the balance dates of the last two years, the funding ratio was at least equal to 105%, i.e. $u_{-2} = u_{-1} = 0$. However, if underfunding is recorded and the sponsor of the fund makes a remedial contribution, the level of the remedial contribution is also important. This follows from the modeling of remedial contributions in Chapter 2. The marginal costs associated with large remedial payments are much higher than those associated with low ones. In the illustrative case, all remedial payments which are larger than a quarter of the total pensionable salaries are indicated as large.

- **One-year risk constraints**
  In the illustrative case, the maximum allowed expected next year's shortage is set equal to 9 percent of the market value of the liabilities.

- **Overfunding and restitutions**
  In this case, overfunding is present if the funding ratio is greater than 2.5. If overfunding is present in two consecutive years, the pension fund is obliged to restitute the excess wealth to the sponsor. On the last two balance dates, overfunding did not occur.

- **Horizon**
  The board of the pension fund strives for a funding ratio of at least 1.25 in 4 years. If the funding ratio is less than this level, a penalty will be incurred. On the other hand, surplusses with respect to the level 1.25 in 4 years are rewarded. As can be deduced from Table 6.1, the penalty associated with not reaching the target is 10 times higher than the reward associated with reaching it.

- **Portfolios**
  We have already discussed the market value and the composition of the initial portfolio. In our ALM model, more parameters associated with the asset portfolios appear. To be specific, lower and upper bounds on the fraction of assets invested in each asset class are present. Moreover, transaction costs associated with buying and selling assets are taken into account. For the numerical values of these parameters, we refer to Table 6.1.

6.1.2 Output

In this section, we first consider CPU times and computation statistics. Then, we discuss the decisions of the ALM model in detail.

CPU times and computation statistics

The heuristic needed 38 minutes and 29 seconds. The LP-relaxation was solved in 20 seconds. The total number of MSLPs solved is 708. In 654 of these, or in 92%, an improvement, i.e a lower value of the objective function, was found. An overview of the computation statistics is presented in Table 6.2.

In Table 6.2 the values of the objective function associated with the LP-relaxation, the first feasible solution and the heuristic solution are presented under A, B, and C, respectively. In addition, we solved the LP-relaxation with the heuristic decisions at time 0 fixed. This gives a lower bound on the value of the objective function of the heuristic solution, see D. Next to the levels of the objective function A-D, two gaps are presented. The first gap, between the first feasible solution and the heuristic solution, expresses how much the value of the objective function is improved by the execution of the heuristic. We not only present the absolute level, but also the relative improvement, which is calculated as (B-C)/C. The second gap presented in

Table 6.2 is the one between the heuristic solution and the solution obtained by fixing the decisions found in the heuristic and solving the LP-relaxation. The smaller the difference between the values of the objective function of the heuristic solution and this one, the more confidence we have that good recourse actions are found by the heuristic. Also for this gap we not only present an absolute number, but also calculate its relative value (C-D)/C.

Note that from the results presented in Table 6.2 we see the effects of the hot starts. The average time needed to solve a MSLP is 3.26 seconds, while it took 20 seconds to solve the LP-relaxation.

In addition to Table 6.2, in Figure 6.1 the development of the value of the objective function is presented. In this figure it is also shown at which decision moment (time $t = 0, \ldots, 4$) an improvement is searched for. We see that the heuristic performs 4 iterations. In each iteration at time 1 few, but large, improvements are found. On the other hand, at time 4 many small improvements are found. Note that no improvement is found at time 0. This is not surprising, since in the first feasible solution (B) all binary decision variables at $t = 0$ have the value 0.

Comparing the iterations, we see that the total improvement during an iteration is less than that of the previous one. Actually, the very first improvement gave the largest decrease of the objective function. The initial value (B) is not represented in Figure 6.1 because the vertical scaling would then obscure subsequent improvements.

Solution times	
Total solution time (h:mm:ss)	0:38:29
Solution time LP-relaxation (h:mm:ss)	0:00:20
Development value of objective function	
(A) LP-relaxation	1,746
(B) First feasible solution	51,512
(C) Heuristic solution	7,350
(D) Decisions $t = 0$ fixed, LP-relaxation	3,232
Gap (B)-(C) (absolute/relative)	44,164 (86%)
Gap (C)-(D) (absolute/relative)	4,118 (56%)
Number of MSLPs solved and improvements found	
Number of MSLPs solved	708
Number of improvements found (absolute/relative)	653 (92%)

Table 6.2: Computation statistics for the illustrative case.

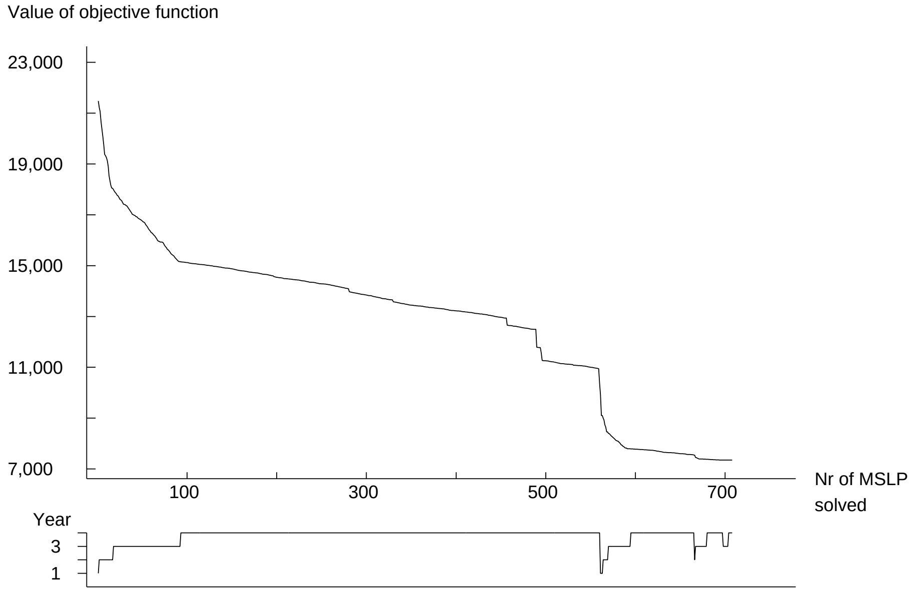

Figure 6.1: Value of the objective function after each MSLP which is solved in the illustrative case. Also the decision moment at which an improvement is searched for is presented.

Decisions

Now, we discuss in detail the decisions generated by the heuristic. A summary of some important characteristics can be found in Table 6.3. Numbers in **bold** face are used to indicate that the corresponding number coincides with a lower or upper bound.

In the last rows of column 't=-1' we have presented the total expected discounted funding costs. Moreover, these costs are split-up in contributions and remedial payments. All these costs are printed in *italics*. These numbers are found by discounting the costs of times $t = 0, \ldots, 4$ by the pension spot curve at time 0.

We will discuss the results in Table 6.3 in more detail now.

- **Development of funding ratios**
  The development of the funding ratios is one of the most important outcomes of the ALM model. Their minimum, mean and maximum values are contained in Table 6.3. More detailed information is given in Figures 6.2 and 6.3.

  At times 0 and 1, the funding ratio is always sufficiently high, i.e. greater than or equal to $\alpha = 105\%$. At later decision moments (t = 2, 3, and 4), underfunding does appear. In 17 of the 1,123 states (1.5%) in the scenario tree, the funding ratio is below this critical level. In only 7 states of these 17 (that is, in 41%), the sponsor makes a remedial contribution. Here we see the effect of the flexible modeling of risk. At time 4, the sponsor is forced to make a remedial payment in 2 states, because the funding ratio is less than 105% in

	$t=-1$	$t=0$	$t=1$	$t=2$	$t=3$	$t=T=4$
Funding ratio						
$\mathbb{E}[F]$		1.16	1.16	1.15	1.12	1.14
$\min F$			1.05	0.91	0.90	0.89
$\max F$			1.28	1.48	1.44	1.73
Contributions						
$\mathbb{E}[c]$	0.110	0.120	0.122	0.097	0.059	
$\min c$			0.068	**0**	**0**	
$\max c$			0.164	0.202	**0.210**	
Underfunding and remedial contributions						
$\mathbb{E}[u] \times 100$	0	0	0	5.56	3.33	1.00
$\mathbb{E}[z] \times 100$	0	0	0	2.78	1.67	0.33
$\mathbb{E}[Z/A_0]$	0	0	0	0.00	0.00	0.00
$\min[Z/A_0]$			0	0	0	0
$\max[Z/A_0]$			0	0.15	0.15	0.14
$\mathbb{E}[DZ/A_0]$	0	0	0	0	0	0
$\mathbb{E}[ZI/A_0]$	0	0	0	0	0	0
One-year risk constraints						
P(ICCs binding)		0	0	0	0.01	
Indexation and liabilities						
$\mathbb{E}[m] \times 100$	0	0	0	0	0.56	0
$\mathbb{E}$[degree of indexation]		1	1	1	0.99	1
Horizon						
$\mathbb{E}[Sho\Lambda/A_0 \mid A_T < \Lambda L_T]$						0.14
$\mathbb{E}[Sur\Lambda/A_0 \mid A_T \geq \Lambda L_T]$						0.02
$P(A_T < \Lambda L_T)$						0.81
Portfolio						
$\mathbb{E}$[% stocks]	47	42	53	44	42	
$\mathbb{E}$[% real estate]	21	**25**	17	15	15	
$\mathbb{E}$[% bonds]	25	**30**	30	32	32	
$\mathbb{E}$[% cash]	7	3	0	9	11	
Expected funding costs in million €	*4,014*	0	1,270	1,372	1,079	652
$\mathbb{E}[cW]$	*3,903*	-	1,270	1,298	1,039	645
$\mathbb{E}[Z]$	*110*	0	0	74	39	7
$\mathbb{E}[DZ]$	*0*	0	0	0	0	0

Table 6.3: Output and decisions for the illustrative case.

two consecutive years.

In Figure 6.2 the probability distributions of the funding ratios are presented for times 1 to 4. The dashed lines represent the minimum required level set by the supervisor. Moreover, the second dashed line at time 4 denotes the target level of the funding ratio at the horizon. These two important levels of the funding ratio are also depicted in Figure 6.3.

From Figure 6.2 we conclude that at times 2, 3 and 4 in many states the funding ratio is only slightly above 105%.

- **Decisions at time 0**

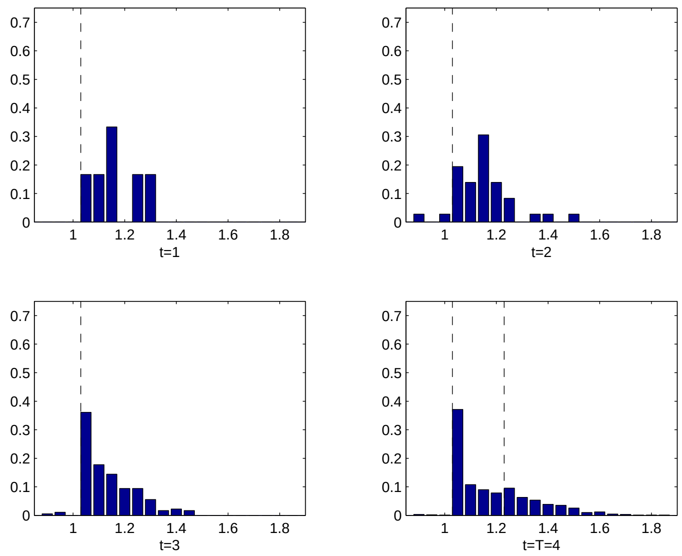

Figure 6.2: Probability distributions of the funding ratios at times 1 to 4 in the illustrative case. Dashed lines indicate levels $\alpha = 1.05$ and $\Lambda = 1.25$.

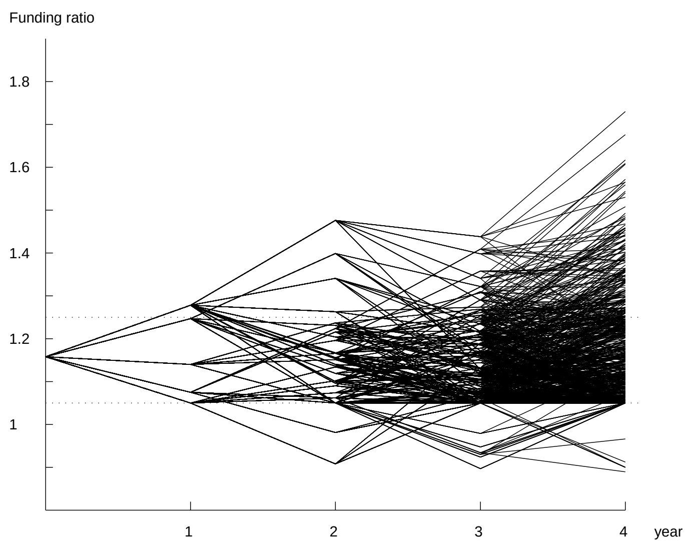

Figure 6.3: Development of the funding ratios in all scenarios in the illustrative case. Dashed lines indicate levels $\alpha = 1.05$ and $\Lambda = 1.25$.

The pension fund under consideration decreases its investments in stocks with 5 percent points. Also the position in cash is reduced (with 4 percent points). These resources are invested in bonds (plus 5 percent points) and real estate (plus 4 percent points).

The contribution rate increases with 1 percent point to 12%. This is the maximum increase, such that no additional penalty costs are incurred.

- **Contribution rate**
At time 1, the expected level of the contribution rate is marginally higher than at time 0. From time 2 on, we see a decrease in the expected level of the contribution rate. Moreover, we see a relatively large range of values of the contribution rate at time 3. At that decision moment, the contribution rate is 0 in some states, and equal to its upper bound (21%) in some other states.

- **Asset portfolios**
We have already discussed the asset portfolios at time 0. At later decision moments, especially the developments of the expected fraction invested in stocks is striking. This expected fraction is 53% at time 1, and slightly above 40% at other times. At the same time, the amount invested in real estate is reduced. This implies that the pension fund takes more risk. At times 3 and 4 we see that the fund invests more conservative: a larger fraction of the assets is invested in cash.

- **Indexation**
In only 1 state (at time 3) the future benefit rights are not indexed fully. In all other states, the rights of the participants are adjusted fully for inflation.

6.2 Sensitivity analysis

In the previous section, we have described an illustrative case in detail. Now, we describe the first impressions we gained by performing sensitivity analyses. We focus on modeling choices, model justification, and scenario trees, respectively.

6.2.1 Sensitivity analysis with respect to modeling choices

From the description of the ALM model in chapters 2 and 3 it follows that this model is rather flexible: a user has to make many choices. In the description of the illustrative case we have presented one such setting. In this section we sketch the outlines of our experiences we obtained by considering some alternative modeling choices.

In the illustrative case, the sponsor of the fund had to make a remedial payment if the funding ratio was too low in two consecutive years. We also examined the effect if this requirement with respect to mid-term risks are strengthened by the supervisor: as soon as the funding ratio is below the minimum required level, the sponsor should restore the financial position of the fund. As a result of these strengthened requirements, the levels of the contribution rate increased considerably in many states of the scenario tree. Moreover, the total level of the expected discounted remedial contributions increased.

We also considered a case in which the supervisor of pension funds imposes more stringent demands with respect to the expected level of underfunding after one year. From numerical experiments we infer that both the number of states in which underfunding is observed, and the number of remedial payments decreased.

On the other hand, the contribution of the active participants increased, and a larger fraction of the asset portfolio was invested less risky. The total cost of funding increased.

To obtain an even better impression of the effects of the integrated chance constraints, we considered a case in which these constraints were omitted. This may be reasonable if the supervisor thinks that requirements with respect to mid-term risks are sufficient. In this case, underfunding and remedial contributions were registered more often. Moreover, the contribution of the active participants increased. As a result, the total funding costs increased.

The last case described briefly here is one in which the interested parties agreed about a higher penalty if the target level of the funding ratio at the horizon is not reached. This higher penalty may be the result of a relative large power of the supervisor and the retired people. Although both the number of states in which underfunding and remedial contributions was observed decreased, the total level of the funding costs increased. Moreover, a larger fraction of the asset portfolio was invested in risky assets.

Summarizing, the first impression of the numerical experiments is that the changes in the outcomes (compared with the illustrative case) are mainly in line with our expectations. However, much more research is needed to make more definite statements about the influences presented in this subsection.

6.2.2 Model justification

In this section we focus on the effects of some characteristics of our model. Recall that the ALM model presented in this thesis is a mixed-integer stochastic program. In this section, we analyze the effects of the introduced fixed penalty costs and the flexible modeling of mid-term risk constraints (by using binary variables) and dynamic rather than static decision strategies. To do so, we first consider an ALM model without binary decision variables. In Section 6.2.4, a static model is considered. In this static model, the fund is only allowed to make decisions with respect to the composition of the asset portfolio and the level of the contribution rate at time 0.

6.2.3 Model without binary decision variables

In order to consider the effect of the binary decision variables, we simplify the model by removing them. That is, in the objective function all fixed-costs terms are deleted, just as all constraints that contain binary decisions. Instead, the inflexible policy rule is implemented, that the sponsor has to restore the funding ratio to the level α =105% as soon as it is too low. Similarly, as soon as the level β =205% is reached, the overflow is restituted. We did not add new constraints on indexation when removing the binaries.

In this case, the active participants get relatively more power in the decision making process. Indeed, by not assigning fixed penalty costs to unfavorable events, these events may occur more often now.

The role of the supervisor is also changed in this case. On the one hand, underfunding is not penalized directly anymore. On the other hand, as soon as the funding ratio is too low, the sponsor has to make a remedial payment. Indeed, the sponsor lost much power, due to these new requirements. Also the position of the current old aged deteriorates: the costs associated with not indexing benefit rights decreased.

In the numerical results, the more powerful role of the active participants can be seen: on average, a lower fraction of the pensionable salaries are transferred to the fund. However, even though this leads to lower total funding costs, one may question whether the supervisor is satisfied. Indeed, underfunding occurs more often in this case. Due to the fact that the sponsor has to restore the funding ratio in those situations immediately, also the weaker role of the sponsor is visible.

Although the fixed costs associated with not indexing benefit rights are removed, we do not see that this instrument is used more often. This may be explained by the relatively high variable costs associated with them.

The board tries to obtain higher returns on its asset portfolio. This is done by investing more in stocks and real estate.

To find numerical results of this model, the computer needed 21 seconds. This is a reduction of 99% compared with the CPU time needed to finish the heuristic for the illustrative case. This is an indicator that larger problems can be solved without binary decision variables, although much flexibility is lost.

6.2.4 Static model

Now we will consider the effect of the dynamic structure in our stochastic program. Due to the multistage character of the model, the fund can react on observed realizations of the stochastic parameters by adjusting the asset portfolio and the contribution rate, for instance. To analyze the effect of static decisions with respect to these decisions, we have set the penalty parameter for a change in contribution rate extremely high. In addition, the transaction costs associated with buying and selling assets also are set at an unrealistic high value. As a consequence, in case of underfunding at a stage after $t = 0$ the only recovery actions that are left for the board are reduction of indexation and a remedial contribution of the sponsor. Obviously, the positions of the active participants and the sponsor deteriorates in the adjusted model.

The total solution time the heuristic needed for this static model, 18 minutes and 36 seconds, is 52% less than in the basic case. This can be explained by the fact that in this case the total number of MSLP problems solved is reduced by (23%).

Numerical results indicate that the active participants have to pay a larger fraction of their pensionable salaries to the fund. However, this is not sufficient to prevent that the funding ratio is too low more often. Indeed, the expected levels of the funding ratios are lower. As a result, it is to be expected that the supervisor will not be satisfied.

Due to the fact that underfunding is observed more often, the sponsor has to make a remedial contribution in more states in the scenario tree. The higher regular and remedial payments resulted in a considerable increase of the funding costs.

Surprisingly, only the retired people would not complain in this case. Their benefit rights are indexed as often as in the illustrative case. This may be the result of the relatively high variable costs associated with not indexing benefit rights

Tree description			
Case	Structure	# Scenarios	# Nodes
Illustrative	6-6-5-5	900	1,124
1	6-6	36	43
2	10-10	100	111
3	15-15	225	241
4	6-6-6	216	259
5	10-10-10	1,000	1,111
6	6-6-6-6	1,296	1,555
7	3-3-3-3-3	243	364
8	4-4-4-4-4	1,024	1,365
9	5-5-5-5-5	3,125	3,906

	CPU time and characteristics					Discounted funding costs		
Case	Obj. fun.	LP-rel.	CPU time (h:mm:ss)	# MSLPs	# Impr. (%)	$\mathbb{E}$[tot costs]	$\mathbb{E}$[tot c]	$\mathbb{E}$[tot Z]
Illustrative	7,350	1,746	0:38:29	708	653 (92)	4,014	3,903	110
1	2,184	-542	0:00:01	27	27 (100)	1,255	1,255	0
2	3,361	-413	0:00:05	60	43 (72)	1,233	1,057	176
3	3,284	-129	0:00:39	154	126 (82)	1,639	1,618	21
4	4,018	231	0:01:05	161	149 (93)	2,506	2,438	68
5	5,258	582	0:45:13	810	625 (77)	2,853	2,792	61
6	7,344	1,243	1:31:05	1,239	1,071 (86)	4,141	4,013	128
7	9,768	2,296	0:01:55	210	202 (96)	4,824	4,536	288
8	11,809	1,938	0:44:47	930	889 (96)	4,952	4,040	912
9	9,920	2,541	7:33:09	2,852	2,678 (94)	5,818	5,526	292

	Initial decisions (i.e. at time $t = 0$)					Performance (total #)		
Case	$\%X_1$	$\%X_3$	$\%X_2$	$\%X_4$	$\%c$	$\#\{u = 1\}$ (%)	$\#\{z = 1\}$ (%)	$\#\{m = 1\}$ (%)
Illustrative	42	25	30	3	12.00	17 (2)	7 (1)	1 (0)
1	30	25	30	15	9.00	2 (5)	0 (0)	0 (0)
2	40	10	30	20	7.36	11 (10)	2 (2)	0 (0)
3	31	19	30	20	9.50	18 (7)	5 (2)	0 (0)
4	56	13	31	0	10.78	2 (1)	1 (0)	0 (0)
5	45	25	30	0	12.00	38 (3)	13 (1)	0 (0)
6	49	21	30	0	12.00	31 (2)	12 (1)	0 (0)
7	38	25	30	7	12.84	4 (1)	2 (1)	0 (0)
8	52	18	30	0	12.00	13 (1)	10 (1)	0 (0)
9	52	18	30	0	18.00	27 (1)	23 (1)	0 (0)

Table 6.4: Summary statistics for different tree structures: CPU times and characteristics, discounted funding costs, initial decisions, and performance.

completely.

6.2.5 Sensitivity analysis with respect to scenario trees

In this section, we present the results of the experiences we obtained by using different shapes of the scenario tree, other seeds in the scenario generator, and random sampling instead of stratified sampling.

Tree structures

Recall that the tree structure of the model in Section 6.1 is 6-6-5-5; that is, the horizon $T = 4$, the number of branches in nodes of time t is 6 for $t = 0, 1$ and 5 for $t = 2, 3$. Now we formulate nine variants of the illustrative case, that have different tree structures: the horizon varies from 2 to 5, and the number of branches per node varies from 3 to 10. See the top of Table 6.4 for details.

We applied the heuristic to all nine cases, with the same parameter values. Of course, for new scenario trees new scenarios had to be generated. In Table 6.4 the results are presented.

The numerical results indicate that the longer the horizon, the higher the initial contribution rate. For example, in the tree with structure 10-10, this rate is 7.36%, while this is 18% in the tree in which we have 5 time periods and 5 branches per node each time. Also the composition of the asset portfolios differs much. The longer the horizon, the larger the fraction of the assets invested in stocks. The fund invests less in cash if the length of the horizon increases.

These results can be explained by the fact that for longer horizons and more scenarios, the range of possible funding ratios also increases. Because unfavorable circumstances are penalized relatively hard in our ALM model, the fund tries to avoid this. The instruments to do this are the contribution rate, the composition of the asset portfolio and not indexing benefit payments. In case of more time periods, the fund tries to benefit from the higher expected return on investments in stocks. To prevent underfunding in some states, the level of the contribution rate should be increased. Finally, not indexing benefit payments is (very) expensive, so that this instrument is not used. Of course, these results are also influenced by the fact that for different tree structures, new scenarios had to be generated.

The required solution time grows exponentially with the number of scenarios in the ALM model. This can be seen in the CPU times needed to solve models with different tree structures in Table 6.4. We solved one case with 3,125 scenarios and 3,906 nodes. The required CPU time to finish the heuristic was slightly more than seven and a half hours.

Other seeds

In this section, the focus is on the question whether the solutions differ much if the seeds in the scenario generator are adjusted. In these numerical experiments, we still use the stratified sampling procedure. These seeds are for example used in generating returns for stocks, real estate and cash. The closer the solutions of the different cases, the more confidence we have in the way the scenarios are generated. So we repeated the calculations of Section 6.1 with 25 different sets of randomly chosen seeds. A summary of the calculations performed to answer this question can be found in Table 6.5.

Considering the results of Table 6.5, we see that the solutions are very different for different values of the seed. Although the required amount of time needed to finish the heuristic is fairly constant, the range of decisions is very large. For example, the level of the contribution rate at time 0 varies from 12% to 18%. Also the composition of the asset portfolios (and the corresponding risk profiles) differs much from case to case. Only the fraction of assets invested in bonds remains rather constant. Moreover, the fund almost always gives the participants full compensation for increases in the general wage level. This may be the result of the relatively high costs associated with not indexing benefit rights.

The total expected discounted funding costs differ also much from case to case. Also the range of the expected discounted regular contributions and remedial payments are very large. The same conclusion can be drawn if we consider the total number of states in which the funding ratio is less than 105% and the total number of times the sponsor makes a remedial contribution.

The large discrepancies in the outcomes of the different cases are not only the result of the heuristic approach, but also due to the scenarios. This can be concluded from the values of the objective function of the LP-relaxations. The difference between the minimum and maximum value is 51%. The difference between the minimum and maximum value of the objective function of the heuristic solutions is 23%.

Random sampling

Up to now, we used the stratified sampling procedure described in Chapter 5 in generating realizations for the stochastic parameters. We would like to know what the effect of this stratification strategy is. Therefore, we have generated 25 scenario trees, in which the scenarios are generated without stratification. A summary of the numerical results can be found in Table 6.6.

Just as for the cases presented in Table 6.4 (in which 25 cases were considered with other seeds), we conclude that the range of outcomes is very large. This conclusion holds for the level of the contribution rate at time 0, and the composition of the asset portfolios.

Moreover, also the total number of times underfunding is registered (20-181 times) and the number of times the sponsor makes a remedial contribution (6-142 times) are striking. Contrary to the cases discussed in the previous section, there are many cases in which the sponsor makes a remedial contribution as soon as the funding ratio is below its minimum required level.

Also the results of these cases can (at least partially) be assigned to the way the scenarios are generated. The difference between the minimum and maximum value of the objective function associated with the LP-relaxations and the heuristic solutions are 56% and 45% respectively.

	CPU time and characteristics					Discounted funding costs		
Case	Obj. fun.	LP-rel.	CPU time (h:mm:ss)	# MSLPs	# Impr. (%)	$\mathbb{E}$[tot costs]	$\mathbb{E}$[tot c]	$\mathbb{E}$[tot Z]
Illustrative	7,350	1,746	0:38:29	708	653 (92)	4,014	3,903	110
1	7,652	1,734	1:08:02	1,125	964 (86)	4,398	4,267	131
2	7,475	2,105	0:47:08	791	714 (90)	4,593	4,445	148
3	7,176	2,078	0:31:52	756	700 (93)	12,953	4,596	8,357
4	8,457	1,924	0:43:47	809	731 (90)	16,133	3,759	12,374
5	6,624	1,308	0:50:46	809	713 (88)	10,170	4,030	6,140
6	6,512	1,544	0:42:49	835	713 (85)	4,365	4,297	68
7	6,959	1,627	0:50:28	713	655 (92)	6,488	4,422	2,066
8	7,387	1,892	0:41:23	736	634 (86)	11,981	4,205	7,776
9	7,159	1,979	0:39:53	725	646 (89)	6,932	4,354	2,578
10	8,458	1,714	0:34:28	731	623 (85)	12,787	3,412	9,375
11	9,058	1,873	0:29:09	798	661 (83)	11,995	3,640	8,355
12	8,217	1,916	0:38:15	761	663 (87)	8,075	3,880	4,195
13	8,131	2,021	0:21:25	719	635 (88)	3,719	3,455	264
14	9,409	2,274	0:24:39	666	566 (85)	11,224	3,370	7,854
15	9,288	1,974	0:26:44	720	623 (87)	6,682	3,616	3,066
16	7,756	1,843	0:20:33	738	645 (87)	3,967	3,851	116
17	8,883	1,997	0:21:14	689	604 (88)	12,885	3,475	9,410
18	7,299	1,562	0:25:43	688	599 (87)	3,806	3,533	273
19	7,619	1,687	0:26:26	777	667 (86)	3,954	3,841	113
20	7,088	1,224	0:32:26	810	718 (89)	9,046	3,841	5,205
21	7,724	1,610	0:24:13	677	581 (86)	3,832	3,553	279
22	7,185	1,831	0:25:16	662	570 (86)	4,106	3,861	245
23	8,484	1,614	0:28:24	808	691 (86)	8,347	3,702	4,645
24	9,222	1,843	0:33:54	768	679 (88)	20,200	3,555	16,645
25	8,769	2,039	0:26:57	747	636 (85)	6,110	3,678	2,432
$\mathbb{E}$	7,920	1,809	0:50:53	762	666	8,350	3,866	4,484
min	6,512	1,224	0:40:37	662	566	3,719	3,370	68
max	9,409	2,274	1:14:18	1,125	964	20,200	4,596	16,645

	Initial decisions (i.e. at time $t = 0$)					Performance (total #)		
Case	$\%X_1$	$\%X_3$	$\%X_2$	$\%X_4$	$\%c$	$\#\{u = 1\}$ (%)	$\#\{z = 1\}$ (%)	$\#\{m = 1\}$ (%)
Illustrative	42	25	30	3	12.00	17 (2)	7 (1)	1 (0)
1	60	10	30	0	18.00	24 (2)	11 (1)	0 (0)
2	46	22	32	0	16.64	14 (1)	9 (1)	0 (0)
3	51	10	30	9	18.00	5 (0)	2 (0)	0 (0)
4	46	21	33	0	15.50	14 (1)	7 (1)	0 (0)
5	49	21	30	0	15.50	15 (1)	6 (1)	0 (0)
6	45	23	31	1	15.50	19 (2)	11 (1)	4 (0)
7	52	17	31	0	16.55	14 (1)	8 (1)	2 (0)
8	48	22	30	0	16.46	22 (2)	14 (1)	1 (0)
9	56	10	30	4	18.00	17 (2)	10 (1)	1 (0)
10	45	24	31	0	12.00	25 (2)	25 (2)	4 (0)
11	44	25	31	0	12.00	25 (2)	25 (2)	1 (0)
12	49	21	30	0	15.50	22 (2)	10 (1)	1 (0)
13	56	10	30	4	12.00	22 (2)	22 (2)	3 (0)
14	31	23	35	11	12.00	23 (2)	23 (2)	9 (1)
15	50	10	30	10	12.00	27 (2)	27 (2)	1 (0)
16	36	14	32	18	13.73	22 (2)	22 (2)	4 (0)
17	45	25	30	0	12.00	19 (2)	19 (2)	1 (0)
18	59	10	31	0	12.00	20 (2)	20 (2)	3 (0)
19	58	12	30	0	13.94	19 (2)	19 (2)	6 (1)
20	59	11	30	0	12.00	14 (1)	14 (1)	6 (1)
21	60	10	30	0	12.00	26 (2)	26 (2)	0 (0)
22	48	18	34	0	12.00	23 (2)	23 (2)	0 (0)
23	57	10	33	0	12.00	20 (2)	20 (2)	1 (0)
24	49	21	30	0	12.00	21 (2)	21 (2)	2 (0)
25	55	10	34	1	12.00	19 (2)	19 (2)	2 (0)
$\mathbb{E}$	50	16	31	2	13.49	19.64	16.52	2.08
min	31	10	30	0	12.00	5	2	0
max	60	25	35	18	18.00	27	27	9

Table 6.5: Summary statistics for cases in which other seeds are used in the scenario generator: CPU times and characteristics, discounted funding costs, initial decisions, and performance.

	CPU time and characteristics					Discounted funding costs		
Case	Obj. fun.	LP-rel.	CPU time (h:mm:ss)	# MSLPs	# Impr. (%)	$\mathbb{E}$[tot costs]	$\mathbb{E}$[tot c]	$\mathbb{E}$[tot Z]
Illustrative	7,350	1,746	0:38:29	708	653 (92)	4,014	3,903	110
1	6,670	1,415	0:42:43	786	648 (82)	3,518	3,444	74
2	5,913	1,457	0:41:14	770	667 (87)	4,016	4,002	14
3	7,217	1,292	0:44:21	855	730 (85)	3,896	3,650	246
4	6,033	1,485	0:52:17	817	712 (87)	3,902	3,854	48
5	6,219	1,267	0:57:49	789	660 (84)	3,622	3,550	72
6	5,234	1,158	1:02:28	1,137	477 (42)	2,791	2,256	535
7	7,355	1,363	0:45:59	845	703 (83)	3,943	3,726	217
8	5,922	1,244	0:52:17	878	719 (82)	3,840	3,816	24
9	6,568	1,346	0:49:49	829	689 (83)	3,602	3,526	76
10	6,077	1,268	1:14:18	961	817 (85)	4,037	3,992	45
11	6,184	1,362	0:47:43	784	685 (87)	3,427	3,396	31
12	5,995	1,277	0:57:17	925	736 (80)	3,756	3,714	42
13	5,589	1,020	0:45:03	876	714 (82)	3,840	3,784	56
14	6,228	1,291	0:40:37	781	681 (87)	3,846	3,793	53
15	6,288	1,157	0:55:48	851	692 (81)	3,645	3,626	19
16	6,268	1,311	0:42:12	832	689 (83)	3,566	3,533	33
17	6,797	1,148	0:53:34	816	698 (86)	3,556	3,522	34
18	6,090	1,207	1:00:56	947	747 (79)	3,829	3,802	27
19	5,903	1,126	0:42:26	761	672 (88)	3,951	3,917	34
20	6,540	1,439	0:50:48	777	619 (80)	3,674	3,633	41
21	6,168	1,135	0:50:24	819	674 (82)	3,946	3,882	64
22	5,055	1,039	0:56:38	1,140	489 (43)	2,739	2,196	543
23	6,095	1,100	0:49:47	785	648 (83)	3,619	3,523	96
24	6,197	1,187	0:41:48	905	720 (80)	3,655	3,612	43
25	5,558	954	0:53:48	932	763 (82)	4,029	4,029	20
$\mathbb{E}$	6,167	1,241	0:50:53	864	682	3,691	3,591	99
$\min$	5,055	954	0:40:37	761	477	2,739	2,196	14
$\max$	7,355	1,485	1:14:18	1,140	817	4,029	4,029	543

	Initial decisions (i.e. at time $t = 0$)					Performance (total #)		
Case	$\%X_1$	$\%X_3$	$\%X_2$	$\%X_4$	$\%c$	$\#\{u = 1\}$ (%)	$\#\{z = 1\}$ (%)	$\#\{m = 1\}$ (%)
Illustrative	42	25	30	3	12.00	17 (2)	7 (1)	1 (0)
1	51	19	30	0	12.00	68 (6)	19 (2)	1 (0)
2	49	21	30	0	17.00	21 (2)	7 (1)	0 (0)
3	47	21	32	0	12.00	22 (2)	10 (1)	0 (0)
4	48	21	30	1	16.20	27 (2)	6 (1)	0 (0)
5	45	25	30	0	12.00	40 (4)	11 (1)	0 (0)
6	45	25	30	0	9.50	181 (16)	141 (13)	3 (0)
7	43	25	31	1	12.00	22 (2)	10 (1)	0 (0)
8	52	18	30	0	12.00	20 (2)	11 (1)	2 (0)
9	45	25	30	0	12.00	36 (3)	14 (1)	0 (0)
10	49	21	30	0	16.30	25 (2)	8 (1)	7 (1)
11	51	19	30	0	12.00	32 (3)	11 (1)	2 (0)
12	55	15	30	0	12.00	37 (3)	13 (1)	0 (0)
13	49	19	32	0	15.50	26 (2)	9 (1)	3 (0)
14	57	10	33	0	12.00	27 (2)	11 (1)	5 (0)
15	49	21	30	0	12.00	24 (2)	10 (1)	0 (0)
16	51	21	31	0	12.00	37 (3)	12 (1)	6 (1)
17	45	25	30	0	12.00	30 (3)	6 (1)	4 (0)
18	49	21	30	0	12.00	31 (3)	13 (1)	0 (0)
19	42	22	36	0	16.80	23 (2)	11 (1)	4 (0)
20	40	23	30	7	12.00	31 (3)	9 (1)	0 (0)
21	48	22	30	0	17.00	31 (3)	9 (1)	0 (0)
22	41	25	34	0	9.50	179 (16)	142 (13)	5 (0)
23	45	25	30	0	12.00	33 (3)	13 (1)	6 (1)
24	45	25	30	0	12.00	28 (2)	13 (1)	2 (0)
25	48	22	30	0	15.70	30 (3)	12 (1)	0 (0)
$\mathbb{E}$	48	31	21	0	13.02	42 (4)	21 (2)	2 (0)
$\min$	40	30	10	0	9.50	20 (2)	6 (1)	0 (0)
$\max$	57	36	25	7	17.00	181 (16)	142 (13)	7 (1)

Table 6.6: Summary statistics for cases in which random sampling is used in the scenario generator: CPU times and characteristics, discounted funding costs, initial decisions, and performance.

Chapter 7

Conclusions

In this thesis, we have presented an optimization model to tackle ALM problems for pension funds. In this model, special attention is paid to the incorporation of risk constraints, so that they fit into the framework of the requirements of the supervisor in The Netherlands. Because the model is formulated as a multistage stochastic program, we need scenarios in order to find numerical results. These scenarios, which describe future developments of uncertain parameters like returns on stocks and bonds, are the outcome of the scenario generator presented in Chapter 5.

Given the ALM model and the numerical values which describe future uncertainties, we apply a heuristic to find numerical results for the decision variables in our model. A heuristic is needed, since, due to the introduced binary decision variables, which are needed to incorporate the realistic flexible risk measures and to penalize unfavorable events, optimization is not possible in reasonable time for realistically sized instances. However, given a setting for the binaries, optimal decisions for the continuous decision variables of the multistage stochastic program could be found by the optimization software OSL [71], using the callable library OSL Stochastic Extensions [72].

The ALM model described in this thesis closely fits the developments and interests in society. Indeed, the relative positions of the interested parties in the ALM decision process can be represented by choosing appropriate parameter values. The fixed penalty costs play an important role in describing the relative positions of the interested parties.

However, it is not easy to find a suitable setting for the parameter values. Moreover, fine tuning of these values is very time consuming. Therefore, an expert is needed to find a good setting to represent the characteristics and interests of a specific pension fund. Moreover, this expert is needed, since the outcomes of the model are sensitive to the choice of some parameter values on certain intervals. Therefore, computational experience is indispensable to work with such models in real world practice.

From the numerical experiments of the presented illustrative case, in which future uncertainties were represented by 900 scenarios and 1,123 decision nodes, we conclude that a heuristic solution can be found in reasonable time. Moreover, the first impressions of the performed sensitivity analyses with respect to modeling choices and model justification, are not unsatisfactory: the changes in the decisions are almost always in line with our expectations. The numerical results show that it is possible to find numerical solutions for mixed-integer stochastic programs in spite of a large number of binary decision variables. However, we also found that the outcomes are (extremely) sensitive with respect to the scenarios. Indeed, for the computational experiences presented in Section 6.2.5, one sees that if one set of scenarios is replaced by another, the decisions may be changed considerably. There are two potential sources why this happens. The first one is that the scenarios may be too sensitive with respect to some small adjustments. In Section 5.5 we have listed some elements which may improve the quality of the scenarios. Moreover, this may influence the stability of the outcomes. A second source of the unstable outcomes may be the heuristic approach to find results for the mixed-integer stochastic program. To make more definite statements about these two sources of instability, more research is needed.

Appendix A

Mathematical formulation ALM model

In this Appendix, the constraints and the objective function of our ALM model are presented.

Accounting and policy constraints

$$A_t^s = \sum_{j=1}^{N}(1 + r_{jt}^s)X_{jt}^s + c_t^s W_t^s - B_t^s,$$

$$X_{j,t+1}^s = (1 + r_{jt}^s)X_{jt}^s + XI_{jt}^s - XD_{jt}^s,$$

$$X_{j0}^s = X_{j0} + XI_{j0}^s - XD_{j0}^s - k_j(XI_{j0}^s + XD_{j0}^s)$$

$$\sum_{j=1}^{N} X_{j,t+1}^s = A_t^s + Z_t^s + DZ_t^s - V_t^s - \sum_{j=1}^{N} k_j(XI_{jt}^s + XD_{jt}^s),$$

$$\underline{f}_j \sum_{i=1}^{N} X_{it}^s \le X_{jt}^s \le \overline{f}_j \sum_{i=1}^{N} X_{it}^s,$$

$$XI_{jt}^s \ge 0, \quad XD_{jt}^s \ge 0,$$

$$Z_t^s \ge 0, \quad DZ_t^s \ge 0, \quad V_t^s \ge 0.$$

Initial position

$$X_{j0}^s = X_{j0} + XI_{j0}^s - XD_{j0}^s - k_j(XI_{j0}^s + XD_{j0}^s).$$

Cash flows from the sponsor in case of financial distress

$$A_t^s - \alpha L_t^s \geq -M u_t^s,$$

$$A_t^s - \alpha L_t^s \leq M(1 - u_t^s) - \frac{1}{M},$$

$$Z_t^s \geq M(z_t^s - 1) - (A_t^s - \alpha L_t^s),$$

$$Z_t^s \leq M z_t^s,$$

$$z_t^s \leq u_t^s,$$

$$z_t^s \geq \sum_{i=t-a+1}^{t} u_i^s - (a - 1),$$

$$ZI_t^s \geq Z_t^s - \tau W_t^s,$$

$$ZI_t^s \geq 0,$$

$$DZ_t^s \geq \theta L_t^s - A_t^s - M z_t^s$$

$$u_t^s \in \{0, 1\}, \quad z_t^s \in \{0, 1\}.$$

Contribution rate

$$\underline{c} \leq c_t^s \leq \overline{c},$$

$$ci_t^s \geq c_t^s - c_{t-1}^s - \rho,$$

$$cd_t^s \geq c_{t-1}^s - c_t^s - \eta,$$

$$c_t^s - c^* \geq M(z_t^s - 1),$$

$$ci_t^s \geq 0, \quad cd_t^s \geq 0.$$

Indexation

$$\underline{L}_t^s \leq L_t^s \leq \overline{L}_t^s,$$

$$L_t^s - (1 + \varphi_t^s)(1 + w_t^s) L_{t-1}^s \geq -M m_t^s,$$

$$L_t^s - (1 + \varphi_t^s)(1 + w_t^s) L_{t-1}^s \leq M(1 - m_t^s) - \frac{1}{M},$$

$$m_t^s \in \{0, 1\}.$$

Benefit payments

$$B_t^s = \underline{B}_t^s + \frac{L_{t-1}^s - \underline{L}_{t-1}^s}{\overline{L}_{t-1}^s - \underline{L}_{t-1}^s} \left(\overline{B}_t^s - \underline{B}_t^s\right).$$

Restitutions

$$\beta L_t^s - A_t^s \geq -M o_t^s,$$

$$\beta L_t^s - A_t^s \leq M(1 - o_t^s) - \frac{1}{M},$$

$$V_t^s \geq M(v_t^s - 1) - (\beta L_t^s - A_t^s),$$

$$V_t^s \leq M v_t^s,$$

$$v_t^s \leq o_t^s,$$

$$v_t^s \geq \sum_{i=t-b+1}^{t} o_i^s - (b-1),$$

$$\overline{L}_t^s - L_t^s \leq M l_t^s,$$

$$\overline{L}_t^s - L_t^s \geq M l_t^s - \frac{1}{M},$$

$$V_t^s \leq M(1 - l_t^s),$$

$$o_t^s \in \{0,1\}, \quad v_t^s \in \{0,1\} \quad l_t^s \in \{0,1\}.$$

One-stage risk constraints

$$\frac{1}{branch_t} \sum_{s' \in \mathcal{K}_t^s(t+1)} \left(A_{t+1}^{s'} - \alpha L_{t+1}^{s'}\right)^- \leq \psi L_t^s.$$

Objective function

$$\sum_{s=1}^{S} \Big[\sum_{t=0}^{T} p_t^s \gamma_t^s (c_t^s W_t^s + Z_t^s)$$

$$+ \sum_{t=1}^{T} p_t^s \gamma_t^s (\zeta_{ci} ci_t^s + \zeta_{cd} cd_t^s) W_t^s$$

$$+ \sum_{t=0}^{T} p_t^s \gamma_t^s (\lambda_u u_t^s)$$

$$+ \sum_{t=0}^{T} p_t^s \gamma_t^s (\lambda_z z_t^s + (\zeta_Z - 1) Z_t^s + \zeta_{ZI} ZI_t^s)$$

$$+ \sum_{t=0}^{T} p_t^s \gamma_t^s (\zeta_{DZ} DZ_t^s)$$

$$+\sum_{t=0}^{T} p_t^s \gamma_t^s (\lambda_m m_t^s)$$

$$+\sum_{t=0}^{T} p_t^s \gamma_t^s (\zeta_L (L_t^s - \overline{L}_t^s)^-)$$

$$+\sum_{t=0}^{T} p_t^s \gamma_t^s (\lambda_o o_t^s)$$

$$+\sum_{t=0}^{T} p_t^s \gamma_t^s (\lambda_v v_t^s + \zeta_V V_t^s)$$

$$+ p_T^s \gamma_T^s (\zeta_{\Lambda d} (A_T^s - \Lambda L_T^s)^- + \zeta_{\Lambda i} (A_T^s - \Lambda L_T^s)^+)\Big].$$

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Index

Samenvatting

Pensioenfondsen worden geconfronteerd met investeringsbeslissingen. Zij hebben in het algemeen de komende tientallen jaren verplichtingen. De ontwikkelingen van rendementen en verplichtingen zijn in deze toekomstige jaren uiteraard onzeker. In de financiële planning zijn institutionele beperkingen en strategieën belangrijke aspecten. Ook overheidsregulering, in Nederland uitgevoerd door de Pensioen- & Verzekeringskamer (PVK, recentelijk gefuseerd met De Nederlandsche Bank), speelt een essentiële rol. Het vakgebied dat zich met deze strategische financiële planning bezighoudt heet *Asset Liability Management* (ALM).

Wanneer een specifiek ALM systeem in een aantal overlegronden tussen belanghebbende partijen en modelbouwers adequaat is gemodelleerd, komt de vraag op welke strategische beslissingen in deze context optimaal zijn. Voor dit doel geeft de *Stochastische Lineaire Programmering* (SLP) een passend framework. De *lineaire programmering* (LP) is heel geschikt, omdat cash flow ontwikkeling in lineaire relaties kunnen worden weergegeven, en omdat de lineaire structuur het toelaat om institutionele, wettelijke en beleidsbeperkingen, evenals transactiekosten mee te nemen. Deze onzekere exogene ontwikkelingen kunnen adequaat in *stochastische* LP modellen worden gepresenteerd. Bovendien kan risico-aversie worden opgenomen in een wijze die past in de belevingswereld van de investeerders, bijvoorbeeld door strafkosten op te nemen in geval bepaalde doelstellingen niet gerealiseerd worden. Essentieel voor *multistadia* SLP is de ingebouwde dynamische beslissingsstructuur: de ontwikkeling van de onzekere parameters wordt gemodelleerd als een scenarioboom, met een beperkt aantal tijdstippen (stadia), en voor iedere beslissing wordt een aantal conditionele beslissingsvariabelen geïntroduceerd, die weergeven welke beslissingen worden genomen in afhankelijkheid van de actuele 'state of the world'. Zo kunnen bijvoorbeeld in ALM systemen voor pensioenfondsen toekomstige premie-aanpassingen in afhankelijkheid van de feitelijke ontwikkelingen als recourse variabelen in het SLP model worden opgenomen.

In Hoofdstuk 1 wordt de achtergrond van de probleemstelling nader beschouwd. Daarbij wordt onder andere gekeken naar typen pensioenen en pensioenfondsen, indexatie van verplichtingen en naar recente ontwikkelingen. Daarbij wordt niet alleen de situatie in Nederland geschetst, maar wordt ook de internationale context niet vergeten. Daarna gaan we verder in op ALM voor pensioenfondsen: welke belanghebbende partijen zijn er, welke instrumenten heeft het bestuur van een pensioenfonds tot z'n beschikking, hoe is het toezicht geregeld en aan welke risico's staat het fonds bloot. Tenslotte wordt een historische ontwikkeling van de aanpak van ALM problemen geschetst.

Hoofdstukken 2 en 3 staan in het teken van de model formulering. Om het model in de context van SLP te presenteren worden daartoe eerst scenario's en de beslissingsstructuur geïntroduceerd. In hoofdstuk 2 wordt vervolgens het grootste deel van het ALM model gebouwd. In het bijzonder wordt daarbij aandacht besteed aan (het modelleren van) indexaties en flexibele risicomaatstaven. Deze risicomaatstaven vereisen dat indien de dekkingsgraad (de verhouding tussen de bezittingen en de verplichtingen) in een aantal achtereenvolgende jaren te laag is, de sponsor gedwongen wordt om het tekort aan te zuiveren. In hoofdstuk 3 wordt ingegaan op nieuwe risicocriteria die de PVK geïntroduceerd heeft en hoe deze aansluiten bij risico-restricties die wij in ons ALM model beschouwen. In het bijzonder wordt daarbij het risico van onderdekking over één jaar bekeken.

Door het opnemen van flexibele risicomaatstaven en vaste strafkosten in geval van ongewenste gebeurtenissen, zijn binaire variabelen (die de waarde 0 of 1 aannemen) onvermijdelijk. Het aldus verkregen gemengd geheeltallige multistadia SLP model behoort tot de moeilijkst oplosbare optimaliseringsproblemen. Het is dan ook niet te verwachten dat optimale oplossingen in een redelijke tijd gevonden kunnen worden voor realistisch grote gevallen. Vandaar dat in hoofdstuk 4 een heuristiek beschreven wordt om iteratief oplossingen te verbeteren die aan alle beperkingen voldoen. Daarbij wordt gebruik gemaakt van inzichten in de problematiek.

Zoals hierboven reeds is aangegeven, wordt gebruik gemaakt van scenariobomen om de onzekere toekomst te modelleren. In hoofdstuk 5 wordt de aanpak beschreven hoe rendementen en ontwikkelingen in de veranderingen van de loonsom gemodelleerd worden. Ook toekomstige veranderingen van de (marktwaarde van de) verplichtingen en discontofactoren worden behandeld.

Hoofdstuk 6 staat in het teken van de ervaringen die zijn opgedaan met numerieke experimenten. In dat hoofdstuk wordt eerst een illustratieve case uitvoerig behandeld. Vervolgens wordt kort ingegaan op gevoeligheidsanalyses. We rapporteren de eerste indrukken die we hebben opgedaan met modelleringskeuzes, modelrechtvaardiging en scenariobomen. Daarbij is aangetoond dat gerekend kan worden met een grootschalig multistadia stochastisch programmeringsprobleem met vele binairen. Tenslotte worden enige voorzichtige conclusies getrokken. Deze staan beschreven in hoofdstuk 7.