Asset liability management for pension funds using multistage mixed-integer stochastic programming

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Chapter 4

Heuristic

In the previous two chapters, we have described our ALM model for pension funds in detail. We have seen that binary decision variables play an important role in this model. They are needed to model flexible risk measures and to penalize unfavorable events. As a result, our ALM model is a multistage stochastic program (MSLP) with both continuous and binary decision variables. It is well-known that in general mixed-integer problems are extremely difficult to solve, and that for very large problems (like realistically sized ALM problems), we may not expect to find an optimal solution in reasonable time, see for example Schrijver [86]. Because we still want to find good feasible solutions, we construct a heuristic.

This chapter is organized as follows. In Section 4.1 the background of the heuristic is presented: the conceptual ideas are discussed, the terminology is introduced, the order in which states are visited is clarified, and flowcharts of the heuristic are presented. In Section 4.2, the central section of this chapter, the heuristic is described. Finally, the details of some parts of the heuristic are given in Section 4.3. The reader who is only interested in the main ideas of the heuristic may skip this last section.

4.1 Background

Although the multistage mixed-integer stochastic program is extremely difficult to solve, from computational experiences with advanced SLP software OSL [71] we found that the MSLP (at least of the size we will calculate with, see Section 6.1) can be solved over the continuous decision variables. Thus, given a feasible solution, we can re-solve the problem relatively fast for changed values of a few binary decision variables. This is the result of so-called *hot starts*: the previous solution is used as a starting point to solve the problem again.

In the heuristic presented in this chapter it is tried to avoid (large) fixed penalty costs in case of unfavorable events: underfunding, a remedial contribution, or not indexing built-up rights with respect to increases in last year's general wage level. As a result, we do not consider the binary decision variables l_t^s, o_t^s, and v_t^s directly, although their definitions are taken into account appropriately in the heuristic.

In Section 4.1.1 we discuss the conceptual ideas of the heuristic. Section 4.1.2 focuses on the order of visiting nodes. In Section 4.1.3 flowcharts of the heuristic are presented. Finally, in Section 4.1.4 we discuss a more refined heuristic.

4.1.1 Conceptual ideas

In Chapter 2 we have seen that in three unfavorable events (large) fixed penalty costs are incurred: in case of underfunding, in case of a remedial contribution, and in case of a deterioration of indexation. The heuristic aims at avoiding these fixed penalty costs. Given a feasible solution, we try to improve this solution by considering changes of the value of some binary decision variables. Such potential improvements are inspired by insight in the model. To be specific, the following two steps are considered to improve a feasible solution.

1. Change the values of the binary decision variables, guided by *local targets*, using available *instruments* (based on insight of the problem under consideration). If a target is reachable, the corresponding node is called a *candidate* node (for improvement).

2. Given a candidate, update the binary variables according to the instruments used. With these updated fixed values of the binaries, resolve the MSLP. This gives us *optimal values for the continuous decision variables, given the values of the binary variables*. If this results in a *global improvement*, i.e. a lower value of the objective function, the candidate node is called a *suitable* one, and we keep the new values of the binaries. Otherwise the candidate is rejected and the values of the binaries remain the same.

These two steps are repeated until no suitable candidate is found anymore. How to find a suitable candidate is explained in detail in Section 4.2.

The *local targets*, mentioned in the first step, are to avoid one (or more) unfavorable event mentioned above: to avoid underfunding, to avoid a remedial contribution or to restore full indexation. The possible *instruments* to reach these targets are an increase of a contribution rate and/or a remedial contribution, and changed compositions of the asset portfolios in predecessor nodes of a candidate. Moreover, the value of the liabilities may be decreased in the state under consideration to avoid underfunding in that state.

Which instrument(s) are used to reach a target in a candidate node will be discussed in the next section. Moreover, in Section 4.3 the corresponding details are discussed.

4.1.2 Order of visiting nodes

In the search for candidates, we consider the nodes of the scenario tree in an increasing order of time. At each time, the nodes are considered according to the lexicographical ordering of the scenarios, as described in Section 2.3. This particular order of visiting nodes to search for candidates is chosen because of the following reasons.

First, as soon as a suitable candidate is selected, the corresponding effects in other states in the scenario tree have to be calculated in order to update the values of the binary decision variables according to their definitions. In this way, unfavorable events further down the tree may already be eliminated by considering the scenario tree in an increasing order of time. For example, if the contribution rate at time 0 is increased to avoid underfunding at time 1, usually all asset values in the scenario tree will increase, too (but not necessarily as we will see in Section 4.3.3).

Another reason why we consider the scenario tree in the order described above, is that the probabilities associated with states at early decision moments are larger than those corresponding to later decision moments, and cash flows are discounted in our ALM model. As a result, payments at early decision moments have a larger impact on the objective function value. Thus, one may expect that larger decreases in the value of the objective function can be found if decisions are adjusted early in the scenario tree.

4.1.3 Flowcharts

In this section, we present two flowcharts. The first one, which is shown in Figure 4.1, gives the main steps of the heuristic. The second one shows how the search for suitable candidates is organized. This second flowchart is presented in Figure 4.2. The steps presented in these two flowcharts are described in detail in Section 4.2.

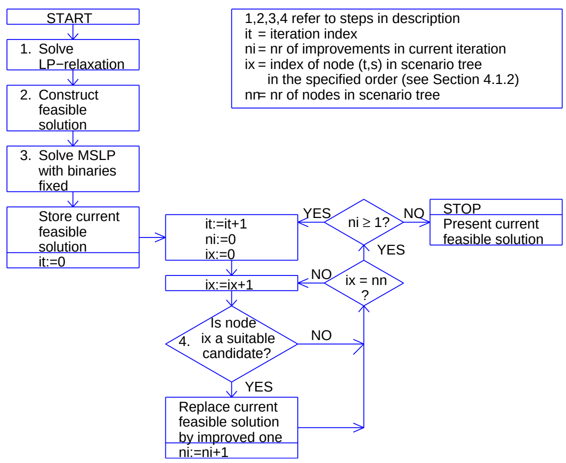

Figure 4.1: Flowchart of the main steps of the heuristic.

4.1.4 Refined heuristic

The heuristic presented in this chapter can be seen as a *greedy heuristic*: as soon as an improvement is found, it is implemented. The main advantage of such a greedy

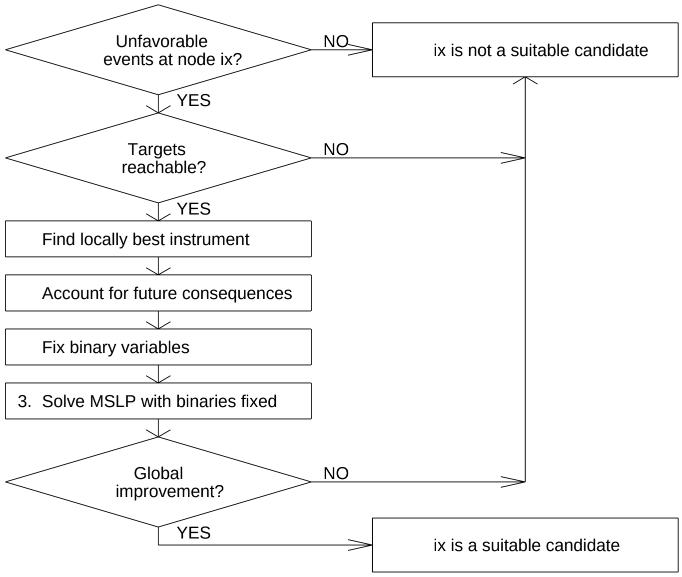

Figure 4.2: Flowchart of Step 4: identification of suitable candidates.

heuristic is that it is relatively fast.

Of course, it is possible to study the performance of a more refined heuristic. Such a refined heuristic could for example consider all binary variables with value 1 in the current feasible solution simultaneously. The changes associated with the one which leads to the largest reduction in the value of the objective function (if a reduction is found at all) may be implemented. Then, such a search is performed again, until no improvements can be found anymore in this way.

Although such a refined heuristic possibly will need more time, computational results might lead to the conclusion that this additional time is worthwhile. Recall that the ALM model is a model to support strategic decisions. From that point of view, a small increase in CPU time may not be disastrous. Unfortunately, the implementation of the refined heuristic described above cannot be presented in this thesis because of a time constraint.

4.2 Steps 1, 2, 3, and 4

In this section we will discuss the main ideas which lie behind the heuristic, and which are presented in the flowcharts in Figure 4.1 and Figure 4.2, in more detail.

Before we describe how the heuristic tries to avoid fixed penalty costs if a suitable candidate is found, an overview of the decisions which are kept fixed in the main steps of the heuristic, as presented in the flowcharts in Figure 4.1 and Figure 4.2, is given in Table 4.1. The fixed decisions are marked with a $\bullet$. Note that if the fractions of the investment in asset class j in the asset portfolio in state (t, s), denoted by f_{jt}^{s}, are fixed, the returns on the asset portfolios are also fixed. The assumption of fixed portfolio returns in Steps 2 and 4 is a simplification, which allows to carry out the necessary calculations described in this chapter.

Step	c_t^s	f_{jt}^s	L_t^s	u_t^s	z_t^s	m_t^s	l_t^s	o_t^s	v_t^s
1									
2	•	•	•						
3				•	•	•	•	•	•
4		•							

Table 4.1: Fixed components in each state (t, s) in the main steps of the heuristic.

4.2.1 Step 1: Initialization

In the first step, the linear programming (LP) relaxation is solved. This is done to find a good starting point to construct a feasible solution in Step 2.

The LP-relaxation is defined as the LP model that arises from the mixed integer model by replacing the binary variables u_t^s, z_t^s, m_t^s, l_t^s, o_t^s, and v_t^s by corresponding continuous decision variables with lower bound 0 and upper bound 1. The multistage stochastic linear program is now solved with only continuous decision variables, including the relaxed indicators.

4.2.2 Step 2: Construct a feasible solution

In the second step a feasible solution is constructed. We need a feasible solution as a starting point for improvements. However, the result of Step 1 is not feasible in general: its binary indicators may have fractional values.

In this step we fix the contribution rates c_t^s, the fraction of the amounts invested in the N asset classes in the portfolio ($f_{jt}^s, j = 1, \ldots, N$), and the value of the liabilities (L_t^s) in all states (t, s) in the scenario tree.

Given the values of the liabilities in each state of the scenario tree, the numerical values for m_t^s and l_t^s follow immediately from their definitions. Given the fixed levels of the contribution rates, the composition of the asset portfolios and the values of the liabilities, we apply the decision rules and the definition of the binary decision variables to find appropriate values for u_t^s, z_t^s, o_t^s and v_t^s. In the construction of a feasible solution, the states in the tree are considered in the order described in Section 4.1.2. The central issue in constructing a feasible solution is therefore to find appropriate values for DZ, Z and V in each state and adjust the asset values in the subtree correctly.

In Section 4.3.1 the details are described which are taken into account in the construction of a feasible solution.

4.2.3 Step 3: Continuous improvement

We assume that after the execution of Step 2 a feasible solution has been constructed. In Step 3, the multistage stochastic program is solved with the values of all binary decision variables fixed, as found in the previous step. Therefore, the multistage stochastic program is solved with respect to the continuous decision variables.

The result of Step 3, a feasible solution that is optimal for fixed values of the binary variables, is used as a starting point to search for improvements, in Step 4. The same Step 3 is to be executed in Step 4, each time a suitable candidate is found.

4.2.4 Step 4: Search for suitable candidates

When this step is executed, there is a current feasible solution, and a specific node (t, s) of the scenario tree has been selected. If the current feasible solution in this node does not describe unfavorable events, we are done. Otherwise, we check whether the state in which the unfavorable event (i.e. underfunding and/or no full indexation is given) is observed has the following properties.

- It is possible to avoid underfunding and/or it is possible to increase the value of the liabilities sufficiently, so that no fixed penalty costs are present any more.
- The resulting new feasible solution has a lower value of the objective function.

Given that we focus on u_t^s, m_t^s, and z_t^s in this heuristic (although the values of l_t^s, o_t^s, and v_t^s are taken into account appropriately), there are four possible combinations we have to consider to check if a different combination of the values of the binary variables can be found, such that the value of the objective function is decreased. These four cases are:

1. $u_t^s = 1, m_t^s = 0$ (and $z_t^s = 0$),
2. $u_t^s = 0, m_t^s = 1$ (and $z_t^s = 0$),
3. $u_t^s = 1, m_t^s = 1$ (and $z_t^s = 0$),
4. $z_t^s = 1$

Note that the possible combination, $u_t^s = 0$, $m_t^s = 0$ (and $z_t^s = 0$) does not need to be considered, since no fixed penalty costs can be removed in this case. Note also that states with $z_t^s = 1$ are considered separately, because we need to check whether a remedial payment is forced by the decision rules or not. We come back to this issue below.

All four possible combinations are considered now in detail.

Case 1. $u_t^s = 1$, $m_t^s = 0$ (and $z_t^s = 0$)

In scenario s at time t the funding ratio is below the minimum required level α. However, the participants of the fund do receive full compensation for last year's increases in the general wage level. In this case, the 'target' of Step 4 is to realize $u_t^s = 0$ by adjusting previous decisions.

To do so, we consider increases in the levels of the contribution rates, and/or increases in a remedial contribution if such a payment was already made before time

t in scenario s. If underfunding can not be avoided by means of these instruments, a decrease in L_t^s is considered.

If $u_t^s = 0$ is still not possible, the target $u_t^s = 0$ is not reachable. However, if the target is reachable by using one or more instruments, the best instruments to do so will be found. This is done by fixing the values of the binary decision variables and executing Step 3, i.e. solving the MSLP. If a global improvement is possible, state (t, s) is indeed a suitable candidate.

In order to determine if u_t^s is reachable by considering only increases in the levels of the contribution rate and an increase in a remedial contribution, we consider scenario s *backwards*, starting at time $t-1$. If the contribution rate in state $t-1$ is strictly below $\overline{c}$, this level is increased to

$$\min\{\frac{\alpha L_t^s - A_t^s}{W_t^s}, \overline{c}\}. \tag{4.1}$$

This implies that c_{t-1}^s is increased until either the shortage with respect to the level α disappears, or till the level of the contribution rate is set equal to its upper bound. If c_{t-1}^s cannot be increased sufficiently to avoid $u_t^s = 1$, we go to time $t-2$ (if $t-2 \geq 0$). This procedure is continued, where the shortage (the numerator in (4.1)) is adjusted appropriately each time. Moreover, the denominator is replaced by $W_q^s \Pi_{q'=q+2}^t (1 + rp_{q'}^s)$ if state (q, s), $0 \leq q \leq t-2$, is considered. This is necessary to find the appropriate increase in A_t^s. As soon as in the backward procedure we observe $z_q^s = 1$, the level of the corresponding remedial contribution is increased such that $u_t^s = 0$ is obtained:

$$\Delta Z_q^s = \frac{\alpha L_t^s - A_t^s}{\Pi_{q'=q+1}^t (1 + rp_{q'}^s)}.$$

This increase is always possible, since no (hard) upper bound on such a remedial payment exists.

As already mentioned above, if the target $u_t^s = 0$ is not reachable by considering increases in (remedial) contributions, a decrease in L_t^s is considered. First, L_t^s will be decreased such that its value equals $(1 + \varphi_t^s)(1 + w_t^s)L_{t-1}^s$ (if the current value is higher than this threshold value). This value is chosen, so that $m_t^s = 0$ still holds. If this decrease is insufficient to reach the target, L_t^s is set equal to its lower bound, $\underline{L}_t^s$. Indeed, by a deterioration of indexation, it may be possible to satisfy the minimum requirements with respect to the level of the funding ratio. Also in this case, the previous feasible solution is replaced by the new one if after optimization with respect to the continuous decision variables, the value of the objective function is decreased.

Remark

A backward search is performed because of the following reasons. First of all, the factor time is considered in our ALM model. This is done by discounting cash flows. As a result, the later payments have to be made, the cheaper it is. Moreover, in the stochastic program probabilities are taken into account. This implies also that the later certain payments can be made in scenario s to prevent underfunding in state (t, s), the smaller the corresponding probability. A third argument why we consider scenario s backwards, is that in this way increases in a regular contribution cannot lead to vanished opportunities to increase a remedial contribution. This would be the case if an increase in c^s_{q1} leads to $u^s_{q2} = z^s_{q2} = 0$, $q1 < q2 < t$, while before this increase a remedial contribution was made in state $(q2, s)$. Due to the higher regular contribution in state $(q1, s)$, $u^s_t = 0$ is not possible anymore if $Z^s_{q2} > 0$ is necessary to reach the local target.

Case 2. $u^s_t = 0, m^s_t = 1$ (and $z^s_t = 0$)

In the current feasible solution, the funding ratio is at least equal to α in state (t, s). However, in this state, the participants of the fund are not fully compensated for increases in the general wage level of the last year. The only instrument we have to consider in this case is an increase in L^s_t, such that $L^s_t = (1 + \varphi^s_t)(1 + w^s_t)L^s_{t-1}$. This increase is the minimum level of L^s_t such that $m^s_t = 0$. Then, all the consequences in the subtree are considered appropriately.

If u^s_t remains 0, we solve the MSLP with the values of the binary decision variables fixed and continue with the heuristic. If u^s_t becomes 1 due to the increase in L^s_t, we check whether $u^s_t = 0$ is possible. If this is possible, regular and remedial contributions are adjusted in scenario s before time t to prevent underfunding in state (t, s). This is done in the same way as described in the previous case. Again, we solve the multistage stochastic program.

If $u^s_t = 1$ cannot be avoided due to the fact that benefit rights are indexed, or the case $u^s_t = m^s_t = 0$ mentioned before did not lead to a better feasible solution, we check whether the combination $m^s_t = 0$, $u^s_t = 1$ leads to a lower value of the objective function after the MSLP is solved.

Case 3. $u^s_t = 1, m^s_t = 1$ (and $z^s_t = 0$)

If in the current feasible solution $u^s_t = 1$ and $m^s_t = 1$ in state (t, s), we first set $L^s_t = (1 + \varphi^s_t)(1 + w^s_t)L^s_{t-1}$, to obtain the target $m^s_t = 0$. Given this adjustment, the remainder of this case consists of the following two procedures:

(**A**) If the target $u^s_t = 0$ is reached by appropriate adjustments in regular and remedial payments, we check whether $u^s_t = m^s_t = 0$ leads to an improved solution. If this is indeed the case, B is not considered anymore.

If $u^s_t = 0$ is not possible due to the increase in the value of the liabilities, or if $u^s_t = 0$ is possible, but the value of the objective function is not decreased, go to B.

(**B**) We consider two possible targets: $u^s_t = 1$, $m^s_t = 0$ and $u^s_t = 0$, $m^s_t = 1$. The adjustments associated with both possible targets are

made as described under Case 1 and Case 2. Both combinations are considered, since we do not know which of these two cases leads to the best solution.

Case 4. $z^s_t = 1$ (hence $u^s_t = 1, m^s_t = 0$ or 1)

If in the current feasible solution a remedial contribution is made in state (t, s), it is checked whether this payment is forced by the modelling assumptions. This is the case if at the last a decision moments underfunding is recorded. Since such a payment is forced by the constraints of our ALM model in this case, we do not consider to remove such a payment.

However, if $Z_t^s > 0$ is not forced, it is considered whether it is attractive to remove such a payment. This may lead to an improved solution, since the fixed penalty costs λ_z are avoided now.

4.3 Details

In this section we describe some details of how we constructed a feasible solution. Moreover, we also consider the possible instruments to reach a target in more detail. Finally, Section 4.3.3 is devoted to the consequences of a change in an asset value for the states in its subtree.

4.3.1 Step 2: construction of a feasible solution

As already mentioned above, before we construct a feasible solution, the LP-relaxation is solved. This gives us compositions of the asset portfolios in each state before the horizon. As a result, we can also find the portfolio returns in each state (t, s), denoted by rp_t^s:

$$rp_t^s = \sum_{j=1}^{N} f_{jt}^s r_{jp}^s,$$

where

$$f_{jt}^s := \frac{X_{jt}^s}{\sum_{j=1}^{N} X_{jt}^s}$$

denotes the fraction of assets invested in asset class j at time t in scenario s, $j = 1, \ldots, N$.

In the construction of a feasible solution we use the portfolio returns found in the LP-relaxation. We use these returns, because the LP-relaxation gives us a good starting point and we don't know how to find better ones.

The construction of a feasible solution consists of the following three steps.

Step 2.1: Find numerical values for m_t^s and l_t^s

The values of the binary decision variables m_t^s and l_t^s only depend on the values of the liabilities in state (t, s). Therefore, given the values of the liabilities in each state in the scenario tree (as found in the LP-relaxation), we apply their definitions:

$$m_t^s = \begin{cases} 1 & \text{if } L_t^s < (1 + \varphi_t^s)(1 + w_t^s)L_{t-1}^s \\ 0 & \text{otherwise,} \end{cases}$$

$$l_t^s = \begin{cases} 1 & \text{if } L_t^s < \overline{L}_t^s \\ 0 & \text{otherwise.} \end{cases}$$

Step 2.2: Apply policy rules to find numerical values for u_t^s and z_t^s

In this second step, the nodes in the scenario tree are considered in the order described in Section 4.1.2. The reason of this order is that a change in the asset value in one state may have effects in all other states of the subtree of that state.

Recall from Section 2.3 that if a remedial payment is made, it is only made after the evaluation of the financial position of a fund. Therefore, if a remedial payment is observed in state (t, s), this payment does not affect A_t^s. However, such a payment does affect the asset value in all states in the subtree of (t, s). To indicate that asset values in the subtree of (t, s) are adjusted, because of a change in a payment in state (t, s) by an amount $x \in \mathbb{R}$, we use the notation $\Delta A_t^s(subtree) = x$. To be specific, $\Delta A_t^s(subtree) \neq 0$ means that A_t^s is not changed, but in all states in the subtree of (t, s), the asset values may be changed.

In the description of the actions to undertake, we assume that we are in state (t, s) of the scenario tree. We distinguish the following situations:

- **$A_t^s < \theta L_t^s$ and $0 < DZ_t^s < \theta L_t^s - A_t^s$.**
  Set $u_t^s = 1$, $z_t^s = 0$, $\Delta A_t^s(subtree) = \theta L_t^s - A_t^s - DZ_t^s$, and $DZ_t^s = \theta L_t^s - A_t^s$, and consider the effects in the subtree of (t, s).

- **$A_t^s < \theta L_t^s$, $DZ_t^s = 0$, and $0 \leq Z_t^s < \alpha L_t^s - A_t^s$.**
  Set $u_t^s = z_t^s = 1$, $\Delta A_t^s(subtree) = \alpha L_t^s - A_t^s - Z_t^s$, and $Z_t^s = \alpha L_t^s - A_t^s$ and consider the effects in the subtree of (t, s).

- **$\theta L_t^s \leq A_t^s < \alpha L_t^s$ and $DZ_t^s > 0$.**
  Set $u_t^s = 1$, $z_t^s = 0$, $\Delta A_t^s(subtree) = -DZ_t^s$, and $DZ_t^s = 0$ and and consider the effects in the subtree of (t, s). If in the last a years underfunding is recorded, set $z_t^s = 1$ $\Delta A_t^s(subtree) = \alpha L_t^s - A_t^s$, and $Z_t^s = \alpha L_t^s - A_t^s$ and consider the effects in the subtree of (t, s).

- **$\theta L_t^s \leq A_t^s < \alpha L_t^s$, $DZ_t^s = 0$ and $0 \leq Z_t^s < \alpha L_t^s - A_t^s$.**
  Set $u_t^s = z_t^s = 1$ $\Delta A_t^s(subtree) = \alpha L_t^s - A_t^s - Z_t^s$, and $Z_t^s = \alpha L_t^s - A_t^s$ and consider the effects in the subtree of (t, s).

- **$A_t^s \geq \alpha L_t^s$ and $DZ_t^s > 0$ and/or $Z_t^s > 0$.**
  Set $u_t^s = z_t^s = 0$, $\Delta A_t^s(subtree) = -Z_t^s - DZ_t^s$, and $Z_t^s = DZ_t^s = 0$ and consider the effects in the subtree of (t, s).

Step 2.3: Apply policy rules to find numerical values for o_t^s and v_t^s

Also in favorable circumstances like overfunding, we still have
to check whether the definitions of the binary decision variables are satisfied. We do this as follows:

- **$A_t^s \leq \beta L_t^s$ and $V_t^s > 0$.**
  Set $o_t^s = v_t^s = 0$, $\Delta A_t^s(subtree) = -V_t^s$, and $V_t^s = 0$ and consider the effects in the subtree of (t, s).

- **$A_t^s > \beta L_t^s$ and $V_t^s = 0$.**
  Set $o_t^s = 1$. If in the last b years overfunding is registered, set $v_t^s = 1$, $\Delta A_t^s(subtree) =$

$-(A_t^s - \beta L_t^s)$, and $V_t^s = \beta L_t^s - A_t^s$ and consider the effects in the subtree of (t, s). However, if a restitution is forced, but $L_t^s < \overline{L}_t^s$, the value of the liabilities are set equal to its upper bound in this state. It is checked again whether a restitution has to be made.

Remark

In Section 4.1.2 we have noted that changes in the level of a remedial payment or restitution may result in changed asset values in all states in its subtree. Therefore, as soon as an asset value is changed in Step 2.2 or 2.3, we consider its subtree. Also in this subtree, the states are considered in an increasing order of time. Moreover, Steps 2.2 and 2.3 are applied again. Note that in this way recursion arises.

4.3.2 Step 4: Instruments

As we have noted in the Section 4.1.1, decisions have to be changed to obtain a new feasible solution with a specific binary variable changed from 1 to 0. The details with respect to the possible instruments to reach a local target will be discussed in this section. Before we do that, we first introduce the *net capital position* with respect to the level α of the fund. This net capital position is defined as $A_t^s - \alpha L_t^s$, and is abbreviated as $NCP\alpha_t^s$. Note that this net capital position may be positive, zero, or negative. A positive (negative) $NCP\alpha_t^s$ is also called a surplus (shortage) with respect to the level α, and is denoted by $Sur\alpha_t^s$ $(Sho\alpha_t^s)$ for state (t, s).

$NCP\alpha_t^s$ has to be increased if $u_t^s = 1$ is considered to change into $u_t^s = 0$. To do so, several instruments are at the disposal of the board of a pension fund. Possible instruments are an increase in a contribution rate or a remedial contribution at times $0, 1, \ldots, t-1$. Moreover, L_t^s may be decreased, and/or the composition of the asset portfolios may be changed. If $m_t^s = 1$, L_t^s has to be increased.

In the remainder of this section we describe the consequences of an increase in the contribution rate and remedial contribution, a decrease in the value of the liabilities and changed compositions of the asset portfolios. These are all instruments to avoid underfunding. Moreover, an increase in L_t^s is considered if $m_t^s = 1$. We assume that fixed costs are present in state (t, s).

An increase in a contribution rate

If there exists a state (q, s), $0 \leq q \leq t-1$ in which $c_q^s < \overline{c}$, we have found a possibility to increase the net capital position of the fund in state (t, s): an increase in the level of the contribution rate.

Of course, such an increase has certain consequences. The first consequence associated with an increase in c_q^s are the direct costs: the increase in c_q^s leads to larger contributions of the active participants of the fund in states $(q+1, s')$, $s' \in \mathcal{K}_t^s(q+1)$.

Other effects have to do with penalties associated with large increases and decreases in the contribution rate. First of all, if an additional penalty is incurred due to a larger increase in the contribution rate at time q, compared to the level of c_{q-1}^s. The second one has to do with changes in c_{q+1}^s, compared to the level of c_q^s. If penalty costs are incurred, because of a large decrease in the contribution rate at time $q+1$, these penalty costs are increased now.

On the other hand, an increase in c_q^s may also lead to a reduction in penalty costs. Due to $\Delta c_q^s > 0$, we observe a more moderate decrease in c_q^s (compared to the level of c_{q-1}^s), where Δc_q^s denotes the change in the contribution rate in state (q, s). The last effect which occurs, is a more moderate increase in states $(q+1, s')$, $s' \in \mathcal{K}_t^s(q+1)$. If a large increase in the contribution rate in year $q+1$ resulted in penalty costs, these are lowered due to $\Delta c_t^s > 0$.

An increase in a remedial contribution

If there exists a state (q, s), $0 \leq q \leq t-1$, in which $u_q^s = z_q^s = 1$ a remedial contribution is allowed in this state. According to the decision rules in our ALM model, such a remedial contribution may be increased. Although such a payment is also allowed if only $u_q^s = 1$, we only consider increases in an existing remedial contribution. This choice is made to avoid additional fixed penalty costs. Indeed, the focus of the heuristic is on avoiding these fixed costs.

If a remedial contribution is increased, the associated costs are also increased. An increase in Z_q^s only leads to additional variable costs. The current level of Z_q^s is important, however. If $Z_q^s < \tau W_q$, the marginal costs are ζ_Z. On the other hand, if $Z_q^s \geq \tau W_q$, the marginal costs are $\zeta_Z + \zeta_{ZI}$.

A decrease in the value of the liabilities

A third instrument to improve the net capital position of the fund is a decrease in L_t^s. This instrument can be used to improve the financial position if L_t^s is strictly larger than its lower bound, $\underline{L}_t^s$.

The marginal costs associated with a decrease in L_t^s are ζ_L, since deviations of L_t^s from its upper bound are penalized by this penalty parameter. In addition, if $L_t^s = (1+\varphi_t^s)(1+w_t^s)L_{t-1}^s$, a decrease in the value of the liabilities also leads to additional fixed penalty costs λ_m.

Changed composition in the asset portfolios

By changing the compositions of the asset portfolios at times $0,\ldots,t-1$, underfunding may be prevented in state (t, s). However, contrary to the three instruments discussed above, a changed composition of an asset portfolio in a certain state not necessarily leads to higher asset values in all its child nodes. As a result, unfavorable events may be shifted from one state to another at the same decision moment. Because we do not want that, we do not consider this instrument to avoid underfunding in a state.

An increase in the value of the liabilities

All the instruments discussed above can be used to avoid underfunding. In case of fixed penalty costs due to a deterioration of indexation, L_t^s has to be increased. This increase should be equal to $(1+\varphi_t^s)(1+w_t^s)L_{t-1}^s - L_t^s$, where L_{t-1}^s and L_t^s are the values of the liabilities in states $(t-1, s)$ and (t, s) in the current feasible solution respectively. This increase also has consequences for the states in the subtree of state

(t, s), since a higher value of the liabilities implies higher future benefit payments in states $(t+1, s')$, $s' \in \mathcal{K}_t^s(t+1)$, as can be seen in equation (2.18).

4.3.3 Consequences in scenario tree

It is straightforward that an increase in c_q^s or Z_q^s leads to a higher asset value in states $(q+1, s')$, $s' \in \mathcal{K}_t^s(q+1)$. In case of $\Delta c_q^s > 0$, active participants (and the sponsor) pay a larger fraction of the pensionable salaries to the fund, resulting in larger cash inflows. If $Z_q^s > 0$, the fund immediately has more money at its disposal.

In case of a decrease in L_t^s, the asset values increase in the child nodes of (t, s). The reason is the relationship between the value of the liabilities and the level of the benefit payments given by (2.18): if the fund does not index pension rights fully, less money has to be paid to retired people.

However, it is not necessarily true that an increase in A_t^s always results in larger values for $A_q^{s'}$, $t+1 \leq q \leq T$, $s' \in \mathcal{K}_t^s(q)$. This can be seen as follows. If a state (q, s') exists, $t+1 \leq q \leq T$, $s' \in \mathcal{K}_t^s(q)$, in which $Z_q^{s'} > \alpha L_q^{s'} - A_q^{s'} > 0$, and of course $u_q^{s'} = 1$, an increase in $A_q^{s'}$ may result in $u_q^{s'} = 0$. Because of the decision rules of our ALM model, a remedial contribution is not allowed anymore. Therefore, the remedial contribution has to be removed, to obtain a feasible solution again.

The heuristic described in this chapter is used in the computational results which are presented in Chapter 6. Before these results are presented, we first describe how the realizations of the stochastic parameters are found. This is the subject of the next chapter.