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Asset liability management for pension funds using multistage mixed-integer stochastic programming

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Chapter 3

One-year risk constraints

In the model presented in Chapter 2, several flexible aspects are presented to maintain a sufficiently high level of the funding ratio: underfunding is penalized, and the sponsor has to restore the funding ratio if in too many consecutive years underfunding is recorded. In addition, also the level of such a payment is penalized. Finally, we have incorporated a target level of the funding ratio at the horizon.

The flexible aspects described above are all soft constraints in our model. However, as we will see in the next section, the supervisor of Dutch pension funds also imposes hard constraints with respect to the short-term solvency position. This is the reason why we also consider such constraints in our ALM model.

The question remains how to incorporate such short-term risk constraints. In the ALM model of Dert [24] decisions have to be made, such that the probability of underfunding in the next year is sufficiently small. However, we think that not only the probability of underfunding is important, but also the amount of a shortage.

In this chapter, these two possible ways to incorporate risk constraints which deal with underfunding in the ALM process are discussed. They are called chance constraints and integrated chance constraints respectively. They will not only be compared from an algorithmic point of view, but also their interpretations are discussed. As we will see, we prefer integrated chance constraints in our ALM model. Before we discuss these two types of risk constraints, we first describe (the developments of) the requirements pension funds have to comply with.

3.1 Solvency tests of supervisor (2002)

In Section 1.2.3 we have described how supervision is organized. We have also described which actions the board of a pension fund has to take in case of financial distress.

Currently, the financial position of Dutch pension funds are judged on the rules written in the *Actuariële Principes voor Pensioenfondsen (APP)*, which dates from 1997. According to the supervisor PVK, these principles have too many shortcomings to judge the solvency position of pension funds sufficiently. They are not dynamic enough and stress the current situation too much.

To gain more insight in a fund's financial position, the PVK has developed new rules, called *Financieel Toetsingskader (FTK)* in 2002. The central themes in the FTK are transparency, risks and results based on market values (not only for assets, but also for liabilities), and making methods explicit. In the FTK (such as formulated in 2002), three tests are described to judge the solvency position of a fund:

- A test which considers the solvency position of a pension fund in the long run, the so-called *continuïteitstoets*.
- A test of the financial position based on both the assets and the liabilities, corresponding to risks associated with the financial position in one year, called the *solvabiliteitstoets*. In this test, underfunding may occur with a prespecified acceptable, but small, probability. Both the assets and liabilities are valued using observed market prices.
- At the next balance date, the market value of the assets should at least be equal to the market value of the liabilities. This test is called the *minimumtoets*.

In formulating these three tests for pension funds, the PVK has considered developments in other sectors. Especially the regulation in the banking industry was an important reference point. The regulatory requirements for banks were introduced by the Bank of International Settlements (BIS), and started its work in 1988 (Basel Accord). Since then, it frequently updated these requirements (2000, new accord 2002). These regulations are followed by financial institutions all over the world. In the last accord, more emphasis is placed on the bank's own internal methodologies, supervisory review, and market discipline.

As we have seen in Chapter 2, funding ratios which are too low are penalized in our multiperiod model. Moreover, a remedial contribution is required if underfunding is recorded in too many consecutive years. In addition, we have incorporated a target level of the funding ratio at the planning horizon of our model. As a result, the 'continuïteitstoets' is taken into account in our model.

As will become clear in this chapter, we also incorporate one-year risk constraints in our model. However, we do not only consider the probability of underfunding in one year, but also the associated amounts. Therefore, we consider the 'solvabiliteitstoets' in an adjusted form.

In our model, the sponsor has to restore the funding ratio as soon as this ratio is less than θ . As a result, for $\theta = 1$ the requirement presented in the 'minimumtoets' would be satisfied. However, in the numerical experiments presented in Chapter 6, we have chosen to set $\theta = 0.90$, since we think that always requiring a funding ratio of at least 1 leads to solutions which are too expensive.

3.2 Chance constraints

In this section, we describe a first idea for representing risk constraints in ALM models, *chance constraints*. Incorporating chance constraints in ALM models was introduced by Dert [24].

Chance constraints serve as tools for modeling risk and risk aversion in stochastic programs. The board of a pension fund strives to satisfy the goal constraints

$$A_t^s \geq \alpha L_t^s \quad \forall t \in \mathcal{T}_1, s \in \mathcal{S} \quad (3.1)$$

for some $\alpha \geq 1$. Incorporating constraint (3.1) in our model, might lead to excessively high funding costs or to infeasibilities. Instead, the board of a pension fund may formulate the condition that the probability of a sufficiently high funding ratio in the next T years is sufficiently large. This requirement can be modeled as

$$P(A_t^s \geq \alpha L_t^s, t \in \mathcal{T}_1) \geq \phi,$$

where ϕ denotes the prescribed probability. Although such a *long-term chance constraint* makes sense, it cannot be incorporated into a linear program. In the previous section we have seen that the supervisor in The Netherlands also considers the short-term financial position of pension funds. Therefore, we restrict ourselves here to one-year chance constraints. In these chance constraints, next year's level of the funding ratio should be sufficiently large with a prescribed probability ϕ_t :

$$P(A_t^s \geq \alpha L_t^s) \geq \phi_t, \quad t \in \mathcal{T}_1. \quad (3.2)$$

Condition (3.2) acts as a constraint *on the decisions at time t* , in terms of consequences at time $t + 1$. As a matter of fact, although the representation in (3.2) does not show this explicitly, there are many chance constraints of this type at time t . In fact, there is a chance constraint for every node in the scenario tree corresponding to time $t \in \mathcal{T}_0$.

In the chance constraints (3.2), the probability distribution used is the conditional distribution of next year's random vector ω_t given the observed values of the past $\omega_1, \dots, \omega_{t-1}$. The value of the parameter ϕ_t , the *minimum required reliability* at time t , is set by the decision makers. It should not be set too low, because then it will lose its meaning of modeling a goal. On the other hand, solving models with ϕ_t too large (e.g. approximately equal to one) may lead to expensive solutions or to infeasibilities as was the case with goal constraint (3.1). Also note that ϕ_t may be time dependent: in earlier years, it may be even more undesirable to have a low funding ratio.

In formulation (3.2), $P(A_t^s \geq \alpha L_t^s)$ is called the *reliability* and $1 - P(A_t^s \geq \alpha L_t^s)$ is called the *risk of next year's insolvency* and is closely related to Surplus-at-Risk as described by H.A. Klein Haneveld [51]. Decisions that are insufficiently reliable (with respect to the next decision moment) are not accepted. This restricts the feasible region.

It is well-known that chance constraints can be represented in a linear programming framework by introducing indicator variables. We will provide the details of this representation for (3.2). For reasons to be explained afterwards, we first replace the variable L_t^s in (3.2) by the upper bound parameter \bar{L}_t^s , so that the condition becomes stronger, potentially. Using the notation introduced in Section 2.3, we now explain how inequalities (3.2) can be written in a mixed-integer programming framework. At time t , we observe the realization of ω_t , and therefore know the actual state (t, s) . Given this state, the conditional probability of each child node

is given by $(branch_t)^{-1}$, since we assume that all child nodes are equally probable. The chance constraints can now be written as:

$$\frac{1}{branch_t} \sum_{s' \in \mathcal{K}_t^s(t+1)} I_{\{A_{t+1}^{s'} < \alpha \bar{L}_{t+1}^{s'}\}}(s') \leq 1 - \phi_t, \quad t \in \mathcal{T}_0, s \in \mathcal{S}_t,$$

where $I_{\{A_{t+1}^{s'} < \alpha \bar{L}_{t+1}^{s'}\}}(s') = 1$ if $A_{t+1}^{s'} < \alpha \bar{L}_{t+1}^{s'}$ and 0 otherwise. Given the definition of the binary variable u_t^s , which was introduced in the previous chapter, the chance constraints can be written as linear inequalities for each state (t, s) , $t \in \mathcal{T}_0$, $s \in \mathcal{S}$:

$$Mu_{t+1}^{s'} \geq \alpha \bar{L}_{t+1}^{s'} - A_{t+1}^{s'}, \quad t \in \mathcal{T}_0, s \in \mathcal{S}_t, s' \in \mathcal{K}_t^s(t+1) \quad (3.3)$$

$$\frac{1}{branch_t} \sum_{s' \in \mathcal{K}_t^s(t+1)} u_{t+1}^{s'} \leq 1 - \phi_t, \quad t \in \mathcal{T}_0, s \in \mathcal{S}_t, \quad (3.4)$$

where, as before, M is a sufficiently large number.

Note that we have used the upper bound on the value of the liabilities in the chance constraints. Why not using their actual value $L_{t+1}^{s'}$? The reason is that, unlike $A_{t+1}^{s'}$, the level of these liabilities depend on decisions to be made at time $t+1$ rather than at time t for which (3.3) is formulated. At time t , the upper bound $\bar{L}_{t+1}^{s'}$ is a parameter and therefore its value is known.

If the number of child nodes is too small, the chance constraints coincide with the goal constraint (3.1). Assume for example that $\phi_t = 0.8$ and we have only two child nodes from a certain state, and the conditional probabilities associated with them are both $\frac{1}{2}$. In this case, the chance constraints lose their meaning of modeling risk, since the funding ratio is required to be greater than or equal to α in all states.

To obtain sufficiently detailed information about the probability distribution of the level of the funding ratio, one may introduce additional states, which do not have successors. As a result, a part of the scenario tree may look like the tree in Figure 3.1. In this figure, the additional states are described by the dots at the end of the dashed lines. They do not have successors. Given all the child nodes (both the ones which were already present and the new ones), sufficiently detailed information about the probability distribution of the funding ratio is present, so that the chance constraints become meaningful now. Although more subtlety is introduced, we did not succeed in working with these additional states.

We have seen that the chance constraints require that we should make decisions, such that only in a limited number of future states the funding ratio is less than α . This seems to be a nice way to model risk and it has a clear interpretation, too. And, since we already introduced the binary variables into the model to indicate whether the funding ratio is sufficiently high or not, these can be used for the chance constraints too.

Although this seems nice at first sight, we also have to deal with two less desirable properties in defining risk in this way. Chance constraints require that only in a limited number of future states the funding ratio may be less than its minimum required level α . But there are no direct restrictions on the amount of underfunding. Of course, if in a consecutive years a shortage with respect to the level α exists,

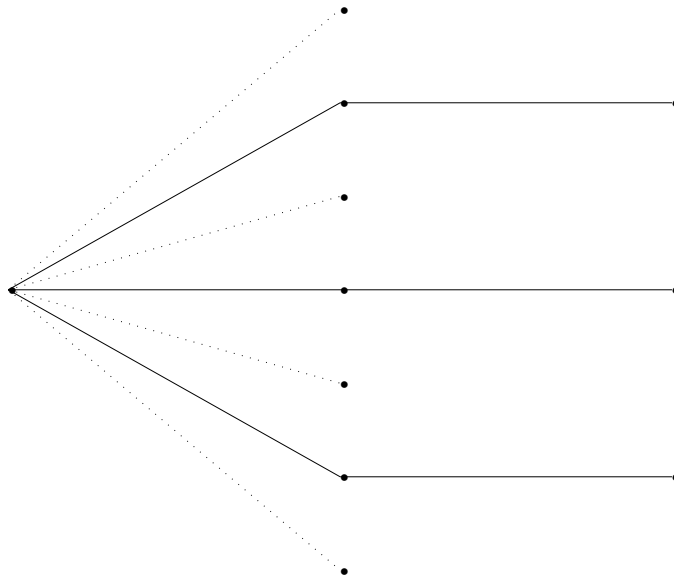


Figure 3.1: A small part of scenario tree in a model with chance constraints.

the sponsor of the fund has to make a remedial contribution, which is penalized in the objective function. But the chance constraints themselves do not impose limits on the amount of a possible shortage.

A second disadvantage of chance constraints is that for low values of $branch_t$ it is a rough way to model risk.

By means of the following example we will show that the induced feasible region may also be nonconvex in the continuous decision variables.

Example 3.1

Assume that the total asset value of a pension fund at time 0 is equal to 100, and the value of the liabilities is 90. The board of the fund requires a minimum funding ratio of 1.1. Suppose in addition that there are three states at time 1, and all conditional probabilities are $\frac{1}{3}$. In all these three states, we assume that the upper bound on the value of the liabilities equals 100.

If the minimum required reliability is set to $\frac{2}{3}$, we see that the chance constraints can be written as

$$\frac{1}{3} \sum_{s=1}^3 u_1^s \leq \frac{1}{3},$$

or,

$$\sum_{s=1}^3 u_1^s \leq 1,$$

that is, only in one of the successors underfunding is allowed.

Assume in addition that there are only two asset classes, stocks and bonds. The returns on these asset classes, which are denoted by r_1^s and r_2^s for stocks and bond respectively, are presented in Table 3.1. The investments in stocks and bonds at time 0 are denoted by X_1 and X_2 respectively. We assume that short selling is not allowed.

scenario	r_1^s	r_2^s
1	0.30	0.05
2	0.07	0.13
3	0.11	0.06

Table 3.1: Returns on stocks and bonds in the 3 scenarios of Example 3.1.

Given the description above, the pension fund has to make decisions, such that the following constraints are satisfied:

$$\begin{aligned} X_1 + X_2 &= 100 \\ Mu_1^s &\geq 110 - (1 + r_1^s)X_1 - (1 + r_2^s)X_2 \quad s = 1, 2, 3 \\ \sum_{s=1}^3 u_1^s &\leq 1 \\ X_1 &\geq 0 \\ X_2 &\geq 0 \\ u_1^s &\in \{0, 1\} \quad s = 1, 2, 3 \end{aligned}$$

The feasible portfolios, i.e. which satisfy all constraints, are depicted in Figure 3.2 by the solid line. These feasible portfolios are specified by $X_1 \in [20, 50] \cup [80, 100]$ and $X_2 = 100 - X_1$.

Note that if the minimum required reliability is set higher than $\frac{2}{3}$ in this example, the problem is infeasible. \square

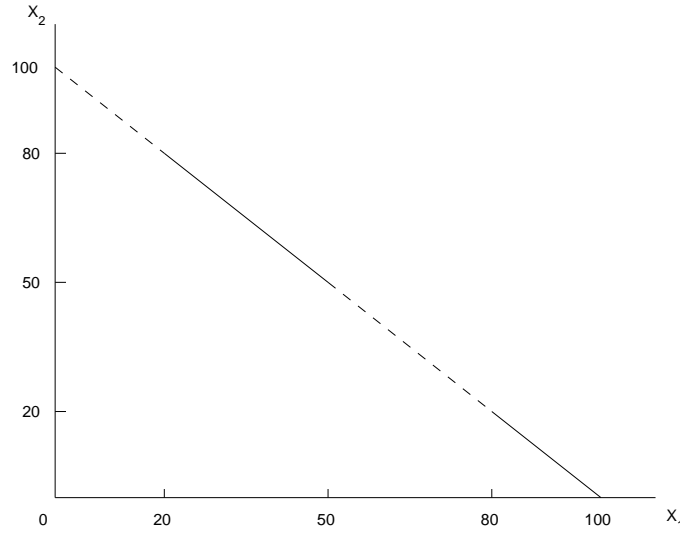


Figure 3.2: Feasible portfolios in the model with chance constraints using discrete distributions of Example 3.1.

From this example it is clear that the feasible regions of chance constraints are not convex. Of course, since we also have to deal with binary variables, we already had a nonconvex mathematical program. But also in the (X_1, X_2) -plane, we cannot expect to obtain convex feasible areas. Even here we might end-up in disjoint parts of the feasible region. This makes it very difficult to construct a feasible solution and to improve solutions. As we will see later, there is another way to model one-year risk, in which we end up with convex sets in the (X_1, X_2) -plane.

3.3 Integrated Chance Constraints

In this section, we describe a second way to incorporate one-year risk constraints into our ALM model: *integrated chance constraints* (ICCs). We formulate ICCs, give an interpretation, and describe their mathematical properties.

Integrated chance constraints are, just like the chance constraints, defined for every $t \in \mathcal{T}_0$, and $s \in \mathcal{S}_t$:

$$\mathbb{E}_{(t,s)} \left[(A_{t+1}^{s'} - \alpha \bar{L}_{t+1}^{s'})^- \right] \leq q, \quad s' \in \mathcal{K}_t^s(t+1). \quad (3.5)$$

The ICCs state that the expected next year's shortage with respect to the level α and the upper bound $\bar{L}_{t+1}^{s'}$ may not exceed q . In this formulation we have chosen to use $\bar{L}_{t+1}^{s'}$ instead of $L_{t+1}^{s'}$ to emphasize the goal of the board of the fund to strive to give full indexation in every year (so that this upper bound is the desired level), although the board may deviate from this level due to unfavorable circumstances. In

a linear programming framework, these constraints can be incorporated as follows:

$$\frac{1}{branch_t} \sum_{s' \in \mathcal{K}_t^s(t+1)} (A_{t+1}^{s'} - \alpha \bar{L}_{t+1}^{s'})^- \leq \psi L_t^s, \quad (3.6)$$

where we have replaced the right-hand side q by ψL_t^s . We refer to Klein Haneveld [53] and Klein Haneveld and Van der Vlerk [54] for mathematical details on ICCs.

Integrated chance constraints have a property which is in accordance with what financial decision makers mean by avoiding risk: not only the probability of underfunding is important, but also the *amount* of the shortage. Therefore, ICCs more closely resemble the objectives of financial risk management than chance constraints do.

The right-hand side of the ICCs of (3.6) is the maximum accepted expected shortage with respect to the funding ratio α , and is specified as a fraction ψ of the actual value of the liabilities. This is reasonable, since in this way a relative measure is found which is related to the position of the pension fund under consideration. With respect to the numerical value of ψ , we propose to relate it to the duration of the liabilities. What we mean by this, and why we propose this, will be explained now. The *duration of the liabilities* is the weighted average maturity of the stream of benefit payments. The maturity of each benefit payment (i.e. in how many years such a payment has to be made) is weighted by the fraction of L_t^s accounted for by the payment. Now, we will explain what this implies for pension funds. If the pension fund under consideration has relatively many young active participants and relatively few retired members, the duration of the liabilities is rather high. On the other hand, in case of funds with relatively many retired members, more weight is assigned to the benefit payments in the near future, and as a result, the duration is lower. For the first type of pension fund, a larger expected shortage is allowed. This makes sense, because this fund has more time to recover from a period of financial distress than the latter fund.

A nice mathematical property is that constraint (3.6) can be used in a linear programming framework without the need to introduce additional binary decision variables. This can be done by introducing additional nonnegative, continuous decision variables $Sho\alpha_t^s$. They measure the amount of shortage with respect to the level α in state (t, s) . Adding the constraints

$$A_t^s + Sho\alpha_t^s \geq \alpha \bar{L}_t^s, \quad t \in \mathcal{T}_1, s \in \mathcal{S}_t,$$

the integrated chance constraints (3.6) can be written as

$$\frac{1}{branch_t} \sum_{s' \in \mathcal{K}_t^s(t+1)} Sho\alpha_t^{s'} \leq \psi L_t^s \quad t \in \mathcal{T}_1, s \in \mathcal{S}_t.$$

The inequalities above define convex, polyhedral feasibility sets. They are very attractive from an algorithmic point of view. Since the constraints defining the integrated chance constraints are all linear, they can be used in a linear programming framework, see also Klein Haneveld and Van der Vlerk [54]. We will illustrate this by means of the following example.

Example 3.2

In this example, we will use the same data as in Example 3.1. Assume in addition that the board of the pension fund has decided that the expected next year's shortage may not exceed 1.

The feasible region is now defined by the following set of linear (in)equalities:

$$\begin{aligned}
 X_1 + X_2 &= 100 \\
 Sho\alpha_1^s &\geq 110 - (1 + r_1^s)X_1 - (1 + r_2^s)X_2 \quad s = 1, 2, 3 \\
 \frac{1}{3} \sum_{s=1}^3 Sho\alpha_1^s &\leq 1 \\
 Sho\alpha_1^s &\geq 0 \quad s = 1, 2, 3 \\
 X_1 &\geq 0 \\
 X_2 &\geq 0
 \end{aligned}$$

The resulting feasible portfolios are depicted in Figure 3.3. They are defined by $X_1 \in [20, 100]$ and $X_2 = 100 - X_1$. We see that the feasible set is convex in this case.

Note that if the maximum expected next year's shortage is less than 0.5, no feasible solution exists. \square

We have seen that chance constraints only consider probabilities of underfunding, while integrated chance constraints take into account both probabilities and amounts of underfunding. In addition, from an algorithmic point of view, the ICCs have more attractive properties than chance constraints: ICCs can be incorporated in a linear program without additional binary variables. Moreover, if the risk aversion parameter is changed, the feasible region in case of ICCs changes smoothly, while this region changes in a rough way in case of chance constraints if the number of branches is low. Because integrated chance constraints have nicer properties than chance constraints, we use ICCs as one-year risk constraints in our ALM model.

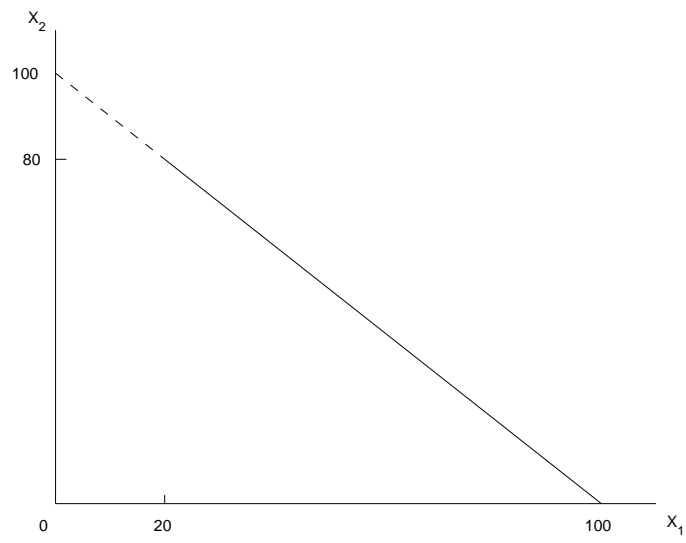


Figure 3.3: Feasible portfolios in the model with integrated chance constraints, presented in Example 3.2.