Asset liability management for pension funds using multistage mixed-integer stochastic programming

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To Barbara

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Preface

Towards the end of my study econometrics I had the feeling that I stood at the crossroads: how would I use the acquired knowledge? The wish to do scientific research became stronger and stronger. Therefore, I discussed the idea to become a Ph.D. student with prof. dr. W.K. Klein Haneveld. During that talk, it turned out that he was the right person at the right time at the right place, because he had already the desire to supervise a Ph.D. student for research on Stochastic Linear Programming for Asset Liability Management.

As I understood from many people, a Ph.D. path is characterized by ups and downs. By now, I also belong to the group who subscribes to that viewpoint.

This Ph.D. thesis is accomplished under the supervision of prof. dr. W.K. Klein Haneveld and dr. M.H. van der Vlerk. They have learned me a lot during the four years I worked with them. Especially the structured way of thinking of prof. dr. W.K. Klein Haneveld made a profound impression on me. Furthermore, I would like to thank prof. dr. R.A.H. van der Meer, dr. H.A. Klein Haneveld, and ir. H. Stam for discussions regarding some modeling aspects.

I would also thank the people of the University of Groningen who were directly or indirectly involved with the realization of this Ph.D. thesis. Especially, I would thank dr. D.P. van Donk.

I will also thank my family and friends. They were interested in my research and supported me where possible. I am especially indebted to my wife Barbara. She gave me room to finish this dissertation, even directly after the birth of our daughter Esther. Therefore, I dedicate this thesis to Barbara.

Zuidhorn, July 2005.

Contents

List of notations

General

Time is denoted by an index t. All periods considered are periods of one year. Year t is the time span from time $t-1$ to time t. The initial decision moment is denoted by $t = 0$, and the horizon is denoted by time T. The set of all decision moments is denoted by $\mathcal{T} := \{0, \ldots, T\}$. In addition we have the subsets $\mathcal{T}_0 := \{0, \ldots, T-1\}$, and $\mathcal{T}_1 := \{1, \ldots, T\}$.

Scenarios are denoted by a superscript s. The total number of scenarios is given by S. The set of all scenarios is denoted by $\mathcal{S} := \{1, \ldots, S\}$. Moreover, i_t denotes the index of the branch at a node at time t. At time t, the state (of the world) is indicated as (t, s).

The total number of asset classes is N, and index j refers to asset class j. Its values 1,...,4 refer to stocks, bonds, real estate, and cash, respectively.
Moreover, all financial quantities are denominated in million euros.

ω_t is the vector of random parameters whose values are revealed in year t, ω_t^s is its value in scenario s. In addition, x_t^s denotes the decision vector at time t in scenario s.

In addition, we have the following:

$\mathbb{R}$	Set of real numbers.
e	$e = 2.71828\ldots$
$\log$	Natural logarithm.
$\mathbb{E}$	Expectation operator.
$P(.)$	Probability operator.
$\min$	Minimum operator.
$\sum$	Summation operator.
$\prod$	Multiplication operator.
Δ	Symbol which denotes a change.
$\lvert x \rvert$	Absolute value of x.
$(x)^+$	$\max\{0, x\}$.
$(x)^-$	$\max\{0, -x\}$.
p_t^s	Probability of scenario s at time t.
p^s	Probability of scenario s.
M	Sufficiently large number ('big M').
$\mathcal{N}(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2.

$branch_t$	Number of branches from each state at time t.
Ξ_t	Cardinality of the bundle of scenarios through any node at time t.
$\mathcal{K}_t^s$	Set which contains those $s' \in \mathcal{S}$, such that the pension fund may end up in state (T, s'), given state (t, s).
$\mathcal{K}_t^s(q)$	Set which contains all $s' \in \mathcal{S}$, such that state (q, s') can be reached with strict positive probability, and no $s'' < s'$ exists with the same history up to and including time t, given time t.
$\mathcal{S}_t$	Set which contains those $s' \in \mathcal{S}$, such that no $s'' < s'$ exists with the same history up to and including time t, given time t.
$I(1)$	Integrated process of the first order.

Decision variables

Continuous decision variables

Assets

A_t^s	Value of the assets at time t in scenario s.
X_{jt}^s	Value of investments in asset class j, at the beginning of year t in scenario s.
XI_{jt}^s	Value of assets in class j bought at time t in scenario s.
XD_{jt}^s	Value of assets in class j sold at time t in scenario s.

Liabilities

B_t^s	Benefit payments in year t in scenario s.
L_t^s	Value of the liabilities at time t in scenario s.

Underfunding and overfunding

Z_t^s	Remedial contribution by the sponsor at time t in scenario s, used to restore the level of the funding ratio α.
ZI_t^s	Remedial contribution by the sponsor at time t in scenario s, as far as it surpasses the lower bound τW_t^s.
DZ_t^s	Direct cash flow by the sponsor, because of a funding ratio below the level θ.
V_t^s	Restitution to the sponsor at time t in scenario s.
$Sur\alpha_t^s$	Surplus with respect to the level α at time t in scenario s.
$Sho\alpha_t^s$	Shortage with respect to the level α at time t in scenario s.
$Sur\Lambda_T^s$	Surplus with respect to the level Λ at time T in scenario s.
$Sho\Lambda_T^s$	Shortage with respect to the level Λ at time T in scenario s.

Contribution rate

c_t^s	Contribution rate for year $t+1$, determined in state (t,s).
ci_t^s	Increase in the contribution rate (with respect to c_{t-1}^s) greater than ρ at time t in scenario s.
cd_t^s	Decrease in the contribution rate (with respect to c_{t-1}^s) greater than η at time t in scenario s.
cdu_t^s	Deviation of the contribution rate from its upper bound.

Binary decision variables

u_t^s	Binary variable which indicates whether the funding ratio is less than α ($u_t^s = 1$) or not ($u_t^s = 0$) at time t in scenario s.
z_t^s	Binary variable which indicates whether a remedial contribution is made by the sponsor ($z_t^s = 1$) or not ($z_t^s = 0$) at time t in scenario s.
o_t^s	Binary variable which indicates whether the funding ratio is higher than β ($o_t^s = 1$) or not ($o_t^s = 0$) at time t in scenario s.
v_t^s	Binary variable which indicates whether a restitution is made to the sponsor ($v_t^s = 1$) or not ($v_t^s = 0$) at time t in scenario s.
m_t^s	Binary variable which indicates whether or not the participants of the fund receive full compensation for the increase in the general wage level in year t in scenario s.
l_t^s	Binary variable which indicates whether the participants of the fund receive full compensation for the increase in the general wage level up to and including year t in scenario s.

Parameters

Bounds with respect to funding ratios

α	Minimum required level of the funding ratio considered in mid-term risk constraints.
θ	Level of the funding ratio which is used to judge whether the sponsor has to make an immediate payment to the fund.
β	Level of the funding ratio considered for restitutions.
Λ	Minimum desired level of the funding ratio at the horizon.

Counting years

a	Number of consecutive years after which the sponsor has to make a remedial contribution if in these years the funding ratio is less than α.
b	Number of consecutive years after which the fund has to make a restitution to the sponsor if in these years the funding ratio is higher than β.

Asset allocation

A_0	Value of the assets at time 0.
X_{j0}	Initial investment in asset class j.
$\underline{f}_j$	Lower bound on the fraction of asset class j in the asset portfolio.
$\overline{f}_j$	Upper bound on the fraction of asset class j in the asset portfolio.
k_j	Proportional transaction cost for asset class j.
u_i^s	Indicator whether in year i the funding ratio was less than α ($u_i^s = 1$) or not ($u_i^s = 0$) in scenario s, $i = 1-a, 2-a, \ldots, 0$.
o_i^s	Indicator whether in year i the funding ratio was higher than β ($o_i^s = 1$) or not ($o_i^s = 0$) in scenario s, $i = 1-b, \ldots, 0$.

Contribution rate

c_{-1}	Contribution rate in year 0.
$\underline{c}$	Lower bound on the contribution rate.
$\overline{c}$	Upper bound on the contribution rate.
c^*	Minimum required contribution rate in case of a remedial contribution.
ρ	Maximum increase in the contribution rate between two consecutive years such that no penalties are incurred.
η	Maximum decrease in the contribution rate between two consecutive years such that no penalties are incurred.

Large remedial contributions

τ	Bound on a remedial contribution as a fraction of the liabilities such that no additional penalties ζ_{ZI} are incurred.

Risk

ψ	Fraction of the liabilities, such that ψL_t^s gives an upper bound on the maximum allowed expected next year's shortage.
ϕ	Prescribed probability in long-term chance constraints.
ϕ_t	Minimum required reliability corresponding to decisions at time t, used in one-year chance constraints.

Fixed costs

λ_u	Fixed costs associated with underfunding with respect to the level α.
λ_z	Fixed costs associated with a remedial contribution from the sponsor to the fund.
λ_o	Fixed benefits associated with overfunding with respect to the level β ($\lambda_0 \leq 0$).
λ_v	Fixed benefits associated with a restitution ($\lambda_v \leq 0$).
λ_m	Fixed costs associated with not giving full compensation for the increase in the general wage level in a year.

Unit costs

ζ_{ci}	Unit cost associated with an increase in the contribution rate in two consecutive years greater than ρ.
ζ_{cd}	Unit cost associated with a decrease in the contribution rate in two consecutive years greater than η.
ζ_Z	Unit cost associated with a remedial contribution Z_t^s.
ζ_{ZI}	Additional unit cost associated with a remedial contribution above the threshold value τW_t.
ζ_{DZ}	Unit cost associated with a direct remedial contribution DZ_t^s.
ζ_V	Unit benefit associated with a restitution ($\zeta_V \leq 0$).
ζ_L	Unit cost associated with a value of the liabilities below its upper bound.
$\zeta_{\Lambda d}$	Unit cost associated with a shortage with respect to the level Λ at the horizon.
$\zeta_{\Lambda i}$	Unit benefit associated with a surplus with respect to the level Λ at the horizon ($\zeta_{\Lambda i} \leq 0$).

Scenario tree

r_{jt}^s	Return (expressed as a fraction) on asset class j in year t in scenario s.
w_t^s	Change (expressed as a fraction) in the general wage level in year t in scenario s.
$\underline{L}_t^s$	Lower bound on the value of the liabilities at time t in scenario s.
$\overline{L}_t^s$	Upper bound on the value of the liabilities at time t in scenario s.
$\underline{B}_t^s$	Lower bound on the value of the benefit payments in year t in scenario s.
$\overline{B}_t^s$	Upper bound on the value of the benefit payments in year t in scenario s.

φ_t^s	Change in the liabilities from time $t-1$ to time t, not due to changes in the general wage level.
$PSC_t^s(q)$	Pension spot curve at time t in scenario s for discounting expected benefit payments which are due q years from year t.
φ_t^s	Percentage change in the liabilities in year $t+1$ in scenario s.
W_t^s	Total level of the pensionable wages of the active participants in year t in scenario s.
γ_t^s	Discount factor associated with cash flows at time t in scenario s.

Heuristic

$NCP\alpha_t^s$	Net capital position with respect to the level α at time t in scenario s.
$\Delta A_t^s(subtree)$	Level of change in payment in state (t,s), which affects the asset values in the subtree of (t,s).

Scenario generation

$\tilde{r}$	Continuously compounded return or log return.
ν_j	Autocorrelation coefficient for the returns in asset class j, $j = 1,3$.
χ	Parameter in the error-correction model, which describes the long-run relationship between r_4 and w.
ϵ_{4t}	Disturbance term in the error-correction model, associated with the returns on the bank account.
ϵ_{wt}	Disturbance term in the error-correction model, associated with the change in the general wage level.
ϑ_1	Parameter in the error-correction model which serves as a measure for the speed of adjustments.
ϑ_2	Parameter in the error-correction model which serves as a measure for the speed of adjustments.
$\sigma_{\epsilon 4}^2$	Variance of the disturbance terms of the bank account in the error-correction model.
$\sigma_{\epsilon w}^2$	Variance of the disturbance terms of the changes in the general wage level in the error-correction model.
$y_t^s(q)$	Yield corresponding to a risk-free zero-coupon bond maturing q years from time t, given the current state (t,s).
a_1	Difference between the yield on bonds with the longest and shortest term to maturity.
a_2	Parameter which controls the shape of the yield curve.
a_3	Parameter which controls the shape of the yield curve.
a_{4t}^s	Yield on bonds with the longest terms to maturity in state (t,s).
$C_t^s(q)$	Coupon payments of the bond portfolio q years from time t, given state (t,s).

$PrB_t^s(q)$	Principal payments of the bond portfolio q years from time t, given state (t, s).
PB_t^s	Value of the bond portfolio in state (t, s).
PS_t^s	Value of the stock portfolio in state (t, s).
μ_1	Mean of simple net stock return.
σ_1	Standard deviation of simple net stock return.
$\tilde{\mu}_1$	Mean of continuously compounded stock return.
$\tilde{\sigma}_1$	Standard deviation of compounded stock return.
$\varrho_{j,t+1}$	Innovation in a GARCH model for asset class j, $j = 1, 3$.
d_{j1}	Parameter in a GARCH model, which denotes a constant term in next year's volatility for asset class j, $j = 1, 3$.
h_{j1}	Measure of the extent to which a volatility shock in one year feeds through into next year's volatility in a GARCH model for asset class j, $j = 1, 3$.
h_{j2}	Parameter which serves as a measure of the rate at which previous year's volatility shocks feed through into next year's volatility in a GARCH model for asset class j, $j = 1, 3$.
D_{t+q}	Dividend payment q years ahead, given time t.
R_j	Internal rate of return on asset class j, $j = 1, 3$.
g_j	Growth rate of dividend payments for asset class j, $j = 1, 3$.
$earp_t^s$	Ex-ante risk premium at time t in scenario s.
$\underline{earp}$	Lower bound on the ex-ante risk premium.
$\overline{earp}$	Upper bound on the ex-ante risk premium.
δ_{jt}^s	Indicator which denotes whether r_j outperforms r_2 from time 0 to time t in scenario s or not for asset class j, $j = 1, 3$.
$P_t^*(r_j \geq r_2)$	Historical probability of outperformance of returns of asset class j over bond returns over a period of t years.
π_t^s	Risk neutral probability in state (t, s).
$B_t^*(q)$	Expected benefit payments q years ahead, given time t.

Output variables

F_t^s	Funding ratio at time t in scenario s.
f_{jt}^s	Fraction of asset class j in the portfolio at time t in scenario s.
I_t^s	Degree of change of indexation at time t in scenario s.
r_{pt}^s	Return on the portfolio in year t in scenario s.

Definitions of the output variables

$$F_t^s := \frac{A_t^s}{L_t^s}$$

$$f_{jt}^s := \frac{X_{jt}^s}{\sum_{i=1}^N X_{it}^s}$$

$$r_{pt}^{s} := \sum_{j=1}^{N} f_{jt}^{s} r_{jt}^{s}$$

$$I_t^s := \frac{L_t^s}{(1+\varphi_t^s)(1+w_t^s)L_{t-1}^s}.$$