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Published in:
Nuclear Physics B

DOI:
10.1016/S0920-5632(97)00520-3

ELSEVIER

Nuclear Physics B (Proc. Suppl.) 61A (1998) 75–85

NUCLEAR PHYSICS B
PROCEEDINGS
SUPPLEMENTS

Properties of p–Branes, D–Branes and M–Branes

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In these lectures I will discuss some basic properties of p–brane, D-brane and M-brane solutions corresponding to supergravity theories in ten and eleven dimensions. In the first lecture I will discuss the p-brane solutions, in the second the ten–dimensional D-brane solutions and in the third and last lecture the eleven–dimensional M-brane solutions.

1. LECTURE I: p-Branes

Strings are one–dimensional spacelike structures that generalize the notion of a particle. The different vibration modes of the string correspond to (massless as well as massive) elementary particles. It is natural to extend this idea and also consider membranes which are two-dimensional extended structures. In that case the different vibration modes of the membrane correspond to elementary particles. Similarly, one may consider p-dimensional extended objects, or simply p-branes, which are the topic of this lecture. In the next two lectures I will discuss two special types of p-branes, which have appeared in the recent literature. These are the so-called Dp-branes and Mp-branes or, shortly, D-branes [1] and M-branes [2]. The work on intersections which I will describe in these three lectures is based upon work done in collaboration with E. Eyras, B. Janssen, M. de Roo and J.P. van der Schaar.

I will discuss the following three aspects of extended objects:

1. Worldvolume p-brane actions
2. Target space effective actions
3. Extended object solutions

For a review of extended object solutions, see [3]. In the case of strings, i.e. $p = 1$, there exist several techniques to relate the world-sheet action of the string (in a curved background) to a target space effective action which, at low energies and weak coupling, is given by a supergravity theory. These techniques do not generalize to the case $p > 1$ except for one which is based on the so-called κ symmetry. I discuss κ symmetry first for the case $p = 1$.

Consider the Nambu-Goto action for a string $(i = 0, 1; \mu = 0, 1, \cdots, d-1)$:

$$S_{string} = \frac{1}{\alpha'}\int d\sigma d\tau\sqrt{|det\partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu}|}\,, \tag{1}$$

with embedding coordinates X^μ. This action is invariant under worldsheet reparametrizations (with parameter ξ^i) and target space translations (with parameter a^μ):

$$\delta X^\mu = \xi^i\partial_i X^\mu\,, \tag{2}$$
$$\delta X^\mu = a^\mu\,. \tag{3}$$

To make the string into a superstring we must construct an action that is not only invariant under the translations (3) but also under under target space supersymmetry transformations (with parameter ϵ):

$$\delta X^\mu = a^\mu + \bar\epsilon\gamma^\mu\theta\,, \tag{4}$$
$$\delta\theta = \epsilon\,. \tag{5}$$

Such an action necessarily involves an additional fermionic variable θ. The invariant line-elements corresponding to (4), (5) are given by

*This work is supported by the European Commission TMR programme ERBFMRX-CT96-0045, in which E.B. is associated to the university of Utrecht.

0920-5632/98/$19.00
PII S0920-5632(97)00520-3

$$\Pi_i^\mu = \partial_i X^\mu - \bar\theta\gamma^\mu\partial_i\theta\,, \tag{6}$$
$$\Pi_i^\alpha = \partial_i\theta^\alpha\,, \tag{7}$$

or

$$\Pi_i^A = \partial_i Z^M E_M{}^A\,, \tag{8}$$

with $Z^M = (X^\mu, \theta^{\underline{\alpha}})$ $(M = (\mu, \underline{\alpha}),\ A = (a, \alpha))$.

It turns out that replacing the bosonic line-element $\partial_i X^\mu$ in the action (1) by the supersymmetric line-elements (6), (7) is not enough to obtain the action for a superstring. For instance, Bose-Fermi matching would not work:

$$d = 10: \quad 10 - 2 \neq 16\,. \tag{9}$$

One must add an additional Wess-Zumino (WZ) term such that the so-called Green-Schwarz (GS) superstring action becomes

$$S_{GS} = \frac{1}{\alpha'}\int d\sigma d\tau\Big\{\sqrt{|det\Pi_i^a\Pi_j^b\eta_{ab}|} + \epsilon^{ij}\partial_i Z^M\partial_j Z^N B_{NM}\Big\}\,, \tag{10}$$

where B_{MN} is a 2-form superfield. The first (kinetic) and second (WZ) term together turn out to be invariant under worldsheet κ transformations. The variation of the kinetic and WZ term gives rise to terms proportional to the torsion $T^A_{MN}(X,\theta)$ and curvature 3-form $H_{ABC}(X,\theta)$, respectively. Requiring worldsheet κ symmetry requires constraints on T and H which turn out to be the ones describing a supergravity theory in superspace. This establishes the relation between the world-sheet superstring action and the corresponding target space effective action.

Bose-Fermi matching works in the presence of κ symmetry for target space dimensions $d = 3, 4, 6, 10$:

$$d - 2 = \frac{n_\theta}{2}\,, \tag{11}$$

where n_θ is the number of independent components of θ. We are now faced with a dilemma. On the one hand side we know that from the supergravity point of view $d = 11$ supergravity is special. On the other hand side, superstrings seem to lead, via the above Bose-Fermi matching, to $d \leq 10$ supergravity theories only. A way out of this is to consider super p-branes with $p > 1$ in which case the Bose-Fermi matching rule becomes:

$$d - (p+1) = \frac{n_\theta}{4}\,. \tag{12}$$

Alternatively, since κ symmetry and target space supersymmetry, after gauge fixing, become worldsheet supersymmetry, one may consider all scalar multiplets in $p+1$ dimensions (see Table 1):

Table 1
Scalar multiplets with T scalars in $p+1$ dimensions.

$p+1$	T	T	T	T
1	1	2	4	8
2	1	2	4	8
3	1	2	4	8
4		2	4	
5			4	
6			4	

Both the Bose-Fermi matching rule (12) as well as Table 1 lead to 16 possible super p-branes with $p \leq 5$ and $d \leq 11$ which can be summarized by a so-called "p-brane scan".

We next consider the target space effective actions that correspond, via κ symmetry, to the 16 possible super p-branes. These actions contain a metric $g_{\mu\nu}$, a dilaton ϕ and a p+1-form gauge field $A_{(p+1)}$ with curvature $F_{(p+2)}$. In the Einstein frame the relevant part of the different actions are given by

$$\mathcal{L}_{E,d} = \sqrt{|g|}\Big[R + \tfrac{1}{2}(\partial\phi)^2 + \frac{(-)^{p+1}}{2(p+2)!}e^{a\phi}F^2_{(p+2)}\Big] \tag{13}$$

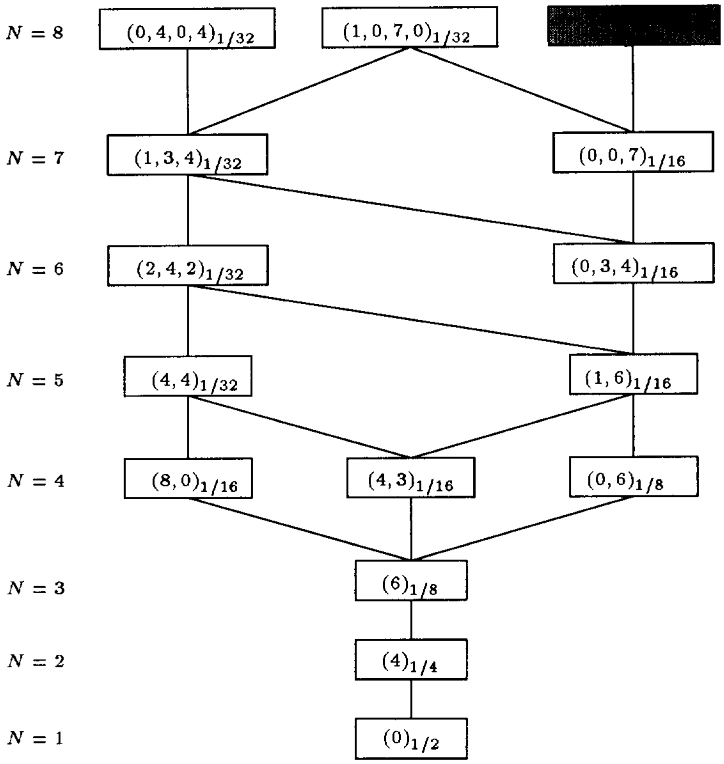

Figure 1. **p-brane intersections with $n = 4$ in 10 dimensions:** *The configuration $(5^k)[l,m]$ indicates k intersecting S5–solutions with l common worldvolume and m overall transverse directions. The number N indicates the number of independent harmonics. The subscripts indicate the amount of residual supersymmetry. To each configuration with one spacelike isometry direction one can add a single F1–solution in that direction.*

in terms of the three parameters (d, p, a). It is natural to consider extended object solutions of the above actions. Making the Ansatz

$$\begin{aligned} ds^2 &= H^\alpha dx^2_{(p+1)} - H^\beta dx^2_{(d-p-1)}\,, \\ e^{2\phi} &= H^\gamma\,, \\ F_{0\cdots pI} &= \delta\partial_I H^\epsilon\,, \end{aligned} \tag{14}$$

with $H = H(x^I)$ a harmonic function of x^I ($I = 1,\cdots,d-p-1$) and $\alpha,\cdots,\epsilon$ constant parameters, one finds that for $D > 2$ there is a unique solution given by[2]

$$\begin{aligned} \alpha &= -\frac{4(d-p-3)}{(d-2)\Delta}\,, \\ \beta &= \frac{4(p+1)}{(d-2)\Delta}\,, \\ \gamma &= \frac{4a}{\Delta}\,, \\ \delta^2 &= \frac{4}{\Delta}\,, \qquad \epsilon = -1\,, \end{aligned} \tag{15}$$

with

$$\Delta = a^2 + 2\frac{(p+1)(d-p-3)}{d-2}\,. \tag{16}$$

[2] We use here a form of the solution as given in [4].

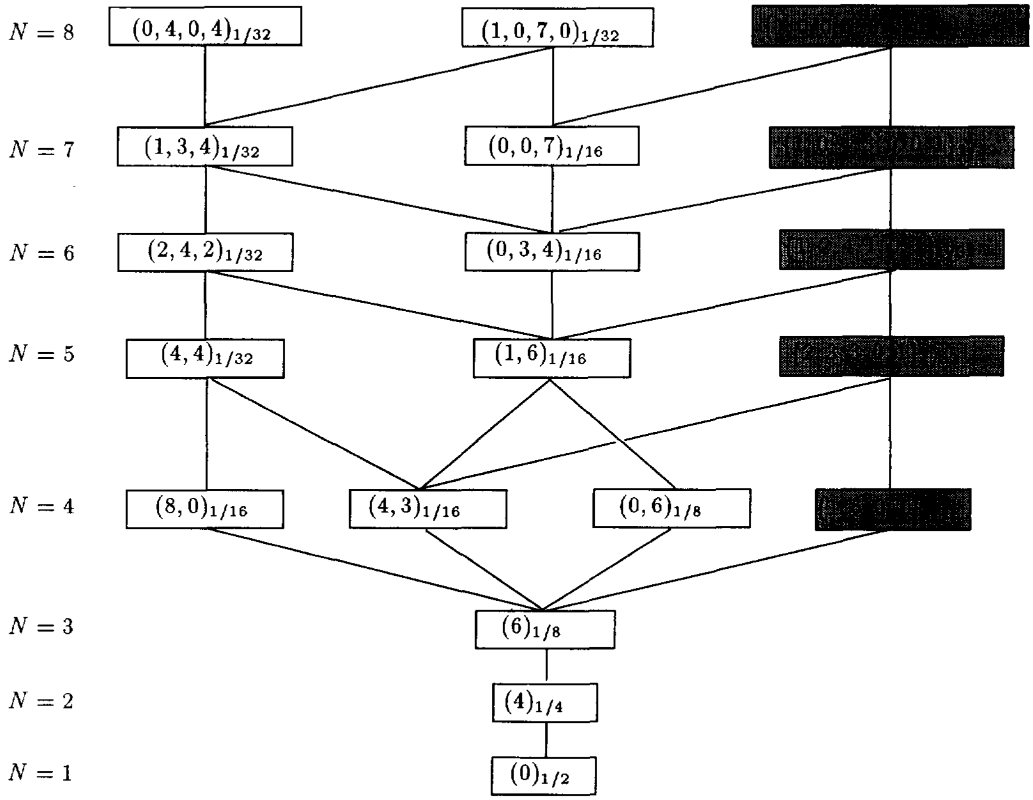

Figure 2. **D-brane intersections with $n = 4$ in 10 dimensions:** *the numbers $(n_1, n_2, \ldots)$ label the number of times a building block with $(1, 2, \ldots)$ worldvolume directions is used. The subscript in the Figure indicates the amount of supersymmetry preserved in each solution. The number N indicates the number of independent harmonics. The lines between solutions indicate how one configuration follows from another by adding (or deleting) a harmonic function. The grey box corresponds to an intersection that does not correspond to a M-brane intersection in eleven dimensions (see lecture III).*

Similarly, one finds that in the string frame (with dilaton coupling a) the parameters $\alpha, \cdots, \epsilon$ are given by

$$\begin{aligned} \alpha &= \frac{1}{N}(2-a)\,, \\ \beta &= -\frac{1}{N}(2+a)\,, \\ \gamma &= \frac{1}{N}\left[2(p+1) + (2+a)(1-\frac{1}{2}d)\right]\,, \\ \delta^2 &= -\frac{4}{N}\,, \qquad \epsilon = -1\,, \end{aligned} \tag{17}$$

with

$$N = (p+1)a + (1-\frac{1}{2}d)(1+\frac{1}{2}a)^2\,. \tag{18}$$

For instance, the string effective action (take $(d,p,a) = (10,1,-2)$) has a fundamental string or F1-solution [5] which in string frame is given by:

$$\begin{aligned} ds^2_{S,10} &= H^{-1}dx^2_{(2)} - dx^2_{(8)}\,, \\ e^{2\phi} &= H^{-1}\,, \\ F_{01I} &= \partial_I H^{-1}\,. \end{aligned} \tag{19}$$

There is also a dual five-brane or S5-solution [6,7] (take $(d,p,a) = (10,5,2)$) given by

$$\begin{aligned} ds^2_{S,10} &= dx^2_{(6)} - H dx^2_{(4)}\,, \\ e^{2\phi} &= H\,, \\ F_{012345I} &= \partial_I H^{-1}\,. \end{aligned} \qquad (20)$$

In the above solutions the isometry directions are called worldvolume directions and the remaining ones are called transverse directions.

We finally discuss solutions that are compositions or intersections of the F1- and S5-solutions. First, it is convenient to introduce a notation where for a given solution every worldvolume (transverse) direction is indicated by a $\times(-)$ [8]. Using this notation the F1- and S5-solutions are given by

$$F1 \;:\; \times | \times - - - - - - - - \,, \qquad (21)$$

$$S5 \;:\; \times | \times \times \times \times \times - - - - \,. \qquad (22)$$

The coordinates $t = x^0, x^1, \ldots x^9$ are indicated from left to right.

Intersections of a pair of brane-solutions are at the basis of the construction of multiple intersections. Determining all possible pairs is equivalent to calculating the "no-force" condition between two branes. Substituting an appropriate Ansatz describing a pair configuration into the field equations one finds two possibilities: a F1 embedded into a S5 and two S5-solutions intersecting over a 3-brane. In an obvious notation, using the $-/x$-notation, we indicate these two possibilities by

$$(1|F1, S5) = \left\{ \begin{array}{c|c} \times & \times - - - - - - - - \\ \times & \times \times \times \times \times - - - - \end{array} \right.$$

$$(3|S5, S5) = \left\{ \begin{array}{c|c} \times & \times \times \times \times \times - - - - \\ \times & \times \times \times - - \times \times - - \end{array} \right.$$

The common worldvolume of the first intersection is two-dimensional (x^0, x^1), the overall transverse space four-dimensional $(x^6, \ldots, x^9)$, and there are four relative transverse directions $(x^2, \ldots, x^5)$. The space-like direction x^1 corresponds to an isometry of the intersection. Note that in both intersections given above the number n of relative transverse directions is four.

Multiple intersections can be obtained by playing the following "$\times/-$ game". One imposes the consistency condition that in a multiple intersection each pair obtained by setting all but two of the independent harmonic functions equal to one, must be one of the two pairs given above. A simplifying feature is that there are no pairs involving two F1-solutions. We first consider intersections involving only S5-solutions. They are given in Figure 1[3].

It turns out that to each configuration of Figure 1 with one spacelike isometry direction one can add a single F1-solution in that direction. This concludes our description of the basic p-brane solutions in ten dimensions and their intersections.

2. LECTURE II: Dp-Branes

It turns out that a given supergravity theory allows much more extended object solutions then the ones occurring in the p-brane scan discussed in the first lecture. In this lecture we consider a particular class of these solutions: the ten-dimensional ones whose charge is carried by a Ramond-Ramond (R-R) gauge field. These solutions are the Dp-branes of [1]. In ten-dimensional IIA and IIB supergravity the following potentials occur:

$$IIA \;:\; A_{(1)}\,, A_{(3)}\,, A_{(5)}\,, A_{(7)}\,, A_{(9)}\,, \qquad (23)$$

$$IIB \;:\; A_{(0)}\,, A_{(2)}\,, A^{+}_{(4)}\,, A_{(6)}\,, A_{(8)}\,. \qquad (24)$$

Note that the 5-form curvature of the 4-form potential is self-dual. The 9-form potential in the IIA theory describes a cosmological constant. The above potentials lead to the following Dp-brane solutions:

$$\begin{aligned} ds^2_{S,10} &= H^{-1/2} dx^2_{(p+1)} - H^{1/2} dx^2_{(9-p)}\,, \\ e^{2\phi} &= H^{-1/2(p-3)}\,, \\ F_{0\cdots pI} &= \partial_I H^{-1}\,, \end{aligned} \qquad (25)$$

with $H = H(x^I)$. For different p the solutions are related to each other via the T-duality rules

[3]This figure was obtained together with E. Eyras, B. Janssen, M. de Roo and J.P. van der Schaar.

$$
\begin{aligned}
g'_{\mu\nu} &= g_{\mu\nu}\,, \quad g'_{xx} = \frac{1}{g_{xx}}\,, \\
e^{2\phi'} &= \frac{e^{2\phi}}{(-g_{xx})}\,, \\
A'_{\mu_1\cdots\mu_p} &= A_{x\mu_1\cdots\mu_p}\,, \quad A'_{x\mu_1\cdots\mu_p} = A_{\mu_1\cdots\mu_p}\,.
\end{aligned} \tag{26}
$$

We deduce that all Dp-branes are T-dual to each other. The extreme cases $p = -1$ and $p = 9$ are special. The D9-solution describes flat Minkowski space (the open superstring can move anywhere in the worldvolume of this 9-brane). The D-1-solution, or D-instanton corresponds to a flat Euclidean space with a non-zero dilaton and $A_{(0)}$ [9].

Why are the Dp-branes missing on the p-brane scan discussed in the first lecture? The answer is that the assumption that the dynamics of the bosonic brane is only described by the embedding coordinates X^μ is too restrictive. Adding an extra worldvolume vector changes the Bose-Fermi matching into

$$d - (p+1) + (p-1) = d - 2 = \frac{n_\theta}{4}\,. \tag{27}$$

This rule leads to new possibilities corresponding to the Dp-brane solutions discussed above. Alternatively, we may scan the vector multiplets in $p+1$ dimensions (see Table 2).

Table 2
Vector multiplets with T scalars in $p+1$ dimensions.

$p+1$	T	T	T	T
1	2	3	5	9
2	1	2	4	8
3	0	1	3	7
4		0	2	6
5			1	5
6			0	4
7				3
8				2
9				1
10				0

We see that there are 23 possible Dp-branes with $p \le 9$ and $d \le 10$. They can be summarized in a so-called "D-brane scan".

What are the worldvolume actions corresponding to these Dp-brane solutions? They must involve the extra worldvolume vector V_i. We first consider the bosonic case. It turns out that the action is given by the sum of a kinetic and WZ term:

$$S^{(p)} = \int d^{p+1}\xi \left(\mathcal{L}^{(p)}_{kin} + \mathcal{L}^{(p)}_{WZ} \right)\,. \tag{28}$$

The kinetic term is of the following Dirac-Born-Infeld form [10]:

$$\mathcal{L}^{(p)}_{kin} = e^{-\phi}\sqrt{|det(g_{ij} + \mathcal{F}_{ij}|}\,, \tag{29}$$

with

$$
\begin{aligned}
g_{ij} &= \partial_i X^\mu \partial_j X^\nu g_{\mu\nu}(X)\,, & (30) \\
\mathcal{F}_{ij} &= \partial_i V_j - \partial_j V_i - \partial_i X^\mu \partial_j X^\nu B_{\mu\nu}(X)\,. & (31)
\end{aligned}
$$

Like the corresponding solutions, the different Dp-actions are T-dual to each other. An interesting feature is that under each T-duality a scalar embedding coordinate gets converted into a component of the Born-Infeld (BI) vector or vice-versa. This can be seen from the fact that the physical degrees of freedom described by a Dp-brane action are $9-p$ scalars and $p-1$ vectors whereas the same degrees of freedom are described via a D(p+1)-action as 8-p scalars and p vectors.

The WZ term in (28) is given by [11]

$$\mathcal{L}^{(p)}_{WZ} = Ae^{\mathcal{F}} + mI_{CS}\,. \tag{32}$$

In the first term

$$A = \sum_{p=0}^{10} A_{(p)} \tag{33}$$

is a formal sum of the RR potentials $A_{(p)}$. It is understood that after expanding the potential only the (p+1)-form is retained. The last term is only present for odd p (the IIA case) [12,13]. Its coefficient m is the IIA mass parameter.

Again the extreme cases, $p = -1$ and $p = 9$, are special. The D9-brane action is the original BI action (it contains in a physical gauge only a single BI vector). Note that the leading term of the WZ term vanishes since there is no 10-form potential $A_{(10)}$ in the IIB supergravity theory[4]. On the other hand, the D-1–brane action is given by (it contains only constant scalar embedding coordinates)[5]

$$S^{(-1)} = e^{-\phi} + A_{(0)} \,. \tag{34}$$

In the supersymmetric case the Dp-brane actions look the same with the understanding that the coordinates X^μ have been replaced by supercoordinates Z^M and the fields have been replaced by corresponding superfields, whose leading component in a θ-expansion is the original (bosonic) field. The supersymmetric case, including κ symmetry, has been discussed recently in [15,16,14,17].

Finally, we discuss intersections of D-brane solutions. Assuming that the harmonic functions depend on the overall transverse directions only, it turns out that the no-force condition is given by the restriction that the number n of relative transverse directions is four, like in the first lecture. All pairs with $n = 4$ are T-dual to each other. More generally, T-dual multiple intersections are characterized by the number of basic building blocks that make up the relative transverse space. For instance, for intersections involving $N = 2, 3$ or 4 branes the basic building blocks are given by

$$N = 2 \;:\; \begin{pmatrix} \times \\ - \end{pmatrix} , \quad N = 3 : \begin{pmatrix} \times \\ - \\ - \end{pmatrix} , \tag{35}$$

$$N = 4 \;:\; \begin{pmatrix} \times \\ - \\ - \\ - \end{pmatrix} \; and \; \begin{pmatrix} \times \\ \times \\ - \\ - \end{pmatrix} . \tag{36}$$

Using the no-force condition given above one finds the D-brane intersections given in Figure 2 [8].

[4] It turns out that in the supersymmetric case this is no longer the case [14].

[5] E. Bergshoeff and T. Ortín, work in progress.

3. LECTURE III: Mp-Branes

In this third and last lecture I will discuss another class of extended objects, the so-called Mp-branes or, shortly, M-branes [2]. They correspond to solutions of the eleven-dimensional Lagrangian

$$\begin{aligned} \mathcal{L}(d = 11) &= \sqrt{|g|}\Big(R - \tfrac{1}{2\cdot 4!}F_{(4)}^2\Big) \\ &\quad - \tfrac{1}{(3!\cdot 4!)^2}\epsilon A_{(3)}F_{(4)}F_{(4)} \,. \end{aligned} \tag{37}$$

This Lagrangian allows a M2-solution [18]

$$\begin{aligned} ds^2_{E,11} &= H^{-2/3}dx^2_{(3)} - H^{1/3}dx^2_{(8)} \,, \\ F_{012I} &= \partial_I H^{-1} \,, \end{aligned} \tag{38}$$

and a M5-solution [19]

$$\begin{aligned} ds^2_{E,11} &= H^{-1/3}dx^2_{(6)} - H^{2/3}dx^2_{(5)} \,, \\ F_{012345I} &= \partial_I H^{-1} \,. \end{aligned} \tag{39}$$

Strictly speaking, the M2 solution was already encountered in the first lecture. From now on this solution will be called a M2 solution.

The M5 solution is new. It turns out that the dynamics of this solution is not only described by the scalar embedding coordinates X^μ but also by a selfdual 2-form worldvolume gauge field V_{ij}. There are two multiplets containing such a selfdual tensor (see Table 3).

Table 3
Tensor multiplets with T scalars in $p + 1$ dimensions.

$p+1$	T	T
6	1	5

The corresponding "M-brane scan" involves a 5-brane in seven and eleven dimensions.

Only recently, the structure of the M5-brane action and/or equations of motion has been unravelled (see [20–22] and references therein.). Again the action consists of a kinetic and WZ term:

$$S^{(5)} = \int d^6\xi \left(\mathcal{L}^{(5)}_{kin} + \mathcal{L}^{(5)}_{WZ}\right). \qquad (40)$$

In lowest (nontrivial) order of the 2-form field the kinetic term is given by

$$\mathcal{L}^{(5)}_{kin} = \sqrt{|g|}\left[1 + \tfrac{1}{2}\mathcal{H}^2 + O(\mathcal{H}^4)\right], \qquad (41)$$

with

$$\begin{aligned} g_{ij} &= \partial_i X^\mu \partial_j X^\nu g_{\mu\nu}(X), & (42)\\ \mathcal{H}_{ijk} &= \partial_i V_{jk} + \partial_k V_{ij} + \partial_j V_{ki} \\ &- \partial_i X^\mu \partial_j X^\nu \partial_k X^\rho A_{\mu\nu\rho}, & (43) \end{aligned}$$

and

$$\mathcal{H} = {}^*\mathcal{H} + O(\mathcal{H}^3). \qquad (44)$$

As is well known, the latter selfduality relation prohibits the construction of a covariant action (see, however, [21] where, using an auxiliary scalar, a covariant action is constructed).

To describe the complete, nonlinear in $\mathcal{H}$, field equations, it is convenient, following [20], to introduce an auxiliary variable $h_{\mu\nu\rho}$ with $h = {}^*h$. Note that h is not the curl of a potential. The most general equations of motion and Bianchi identity in terms of h are given by

$$\begin{aligned} \partial_i\left(\alpha h_{ijk} + \beta h^2_{[im} h^m{}_{jk]}\right) &= 0, & (45)\\ \partial_{[i}\left(\gamma h_{jkl]} + \delta h^2_{jm} h^m{}_{kl]}\right) &= 0, & (46) \end{aligned}$$

for constant $\alpha, \cdots, \delta$. Note that $\mathcal{H}$ is related to h, using the Bianchi identity, via

$$\mathcal{H}_{ijk} = \gamma h_{ijk} + \delta h^2_{[im} h^m{}_{jk]}. \qquad (47)$$

Due to the selfduality relation of h the equations of motion (45) and the Bianchi identities (46) are related and consistency requires that $\alpha = \gamma$ but $\delta = -\beta$. A special choice of the independent parameters α, β leads, after reduction, to the BI action and hence this choice is the proper one for the M5-brane action. This M5-brane action establishes an interesting generalization of the BI action to higher rank potentials.

Finally, the WZ term is given by

$$\mathcal{L}^{(5)}_{WZ} = A_{(6)} + A_{(3)}\mathcal{H}_{(3)}, \qquad (48)$$

with

$$dA_{(6)} - A_{(3)} dA_{(3)} = {}^*(dA_{(3)}). \qquad (49)$$

We next discuss intersections of M-branes [23]. Using the $-/\times$ notation the M2 and M5 solutions are given by

$$\begin{aligned} M2 &: \times | \times \times - - - - - - - - , & (50)\\ M5 &: \times | \times \times \times \times \times - - - - - . & (51) \end{aligned}$$

The coordinates $t = x^0, x^1, \ldots x^{10}$ are indicated from left to right. Assuming that the harmonics depend on the overall transverse directions only, there are three possible intersections of a pair of M2/M5 solutions. These three possibilities are given by

$$\begin{aligned} (0|M2, M2) &= \begin{cases} \times \,|\, \times \times - - - - - - - - \\ \times \,|\, - - \times \times - - - - - - \end{cases} \\ (1|M2, M5) &= \begin{cases} \times \,|\, \times \times - - - - - - - - \\ \times \,|\, \times - \times \times \times \times - - - - \end{cases} \\ (3|M5, M5) &= \begin{cases} \times \,|\, \times \times \times \times \times - - - - - \\ \times \,|\, \times \times \times - - \times \times - - - \end{cases} \end{aligned}$$

It is now a relatively straightforward manner to determine all possible intersections based on the above three basic pairs. The result is summarized in Figure 3 [8].

The M-brane intersections discussed in this lecture are not, via dimensional reduction, in one-to-one correspondence to the D-brane intersections discussed in the previous lecture. To obtain such a correspondence we must extend our class of M-brane and D-brane solutions as follows:

$$M - branes \quad \rightarrow \quad M - branes$$

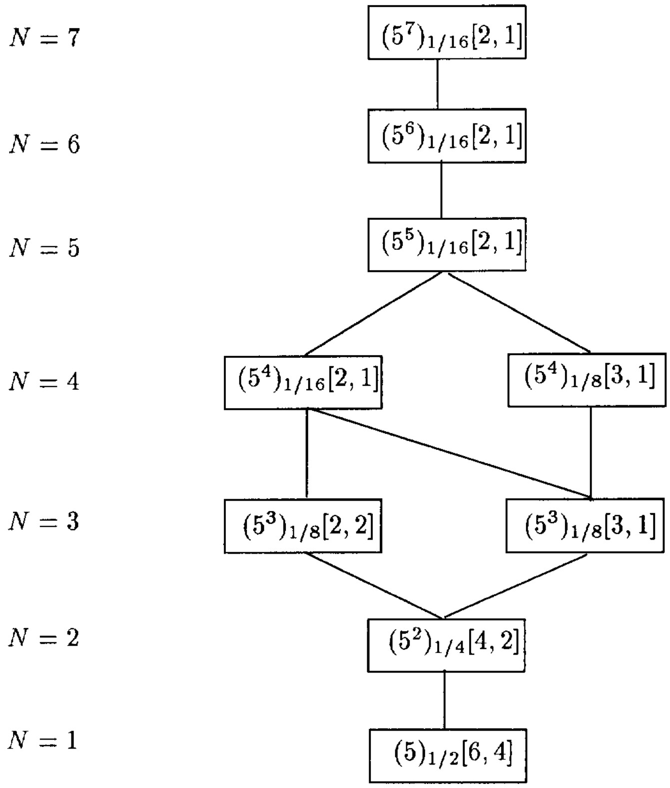

Figure 3. **M-brane intersections with $n = 4, 5$ in 11 dimensions:** *the numbers $(n_1, \cdots, n_{[N/2]})$ are the same labels used in $D = 10$, and indicate to which D-brane intersection the $D = 11$ solution reduces. The configurations in gray rectangles only reduce to $D = 10$ intersections involving F1 and/or S5 solutions. The subscripts indicate the amount of residual supersymmetry.*

$$\begin{aligned} & + \quad waves + monopoles\,, & (52) \\ D-branes & \rightarrow \quad D-branes \\ & + \quad waves + monopoles\,, \\ & + \quad F1 + S5\,. & (53) \end{aligned}$$

The Brinkmann wave in D dimensions is given by the metric [24]

$$\begin{aligned} ds^2 &= (2-H)dt^2 - Hdz^2 + 2(1-H)dtdz \\ &- (dx_2^2 + \ldots + dx_{(D-1)}^2), \end{aligned} \tag{54}$$

where H is a harmonic function in the variables $t+z, x_2, \ldots, x_{(D-1)}$. In ten dimensions the wave solution is T-dual to the fundamental string F1.

The metric for the Kaluza-Klein monopole reads $(i = 1, 2, 3)$ [25]

$$\begin{aligned} ds^2 &= dt^2 - dx_1^2 - \ldots - dx_{(D-5)}^2 \\ &- H^{-1}(dz + A_i dy_i)^2 - H dy_i^2\,, \end{aligned} \tag{55}$$

where H and A_i depend on y_i, and the relation between H and A_i is

$$F_{ij} \equiv \partial_i A_j - \partial_j A_i = \epsilon_{ijk}\partial_k H\,. \tag{56}$$

Here the directions t, x_μ $(\mu = 1, \cdots, (D-5))$ and z are isometry directions.

All solutions of Type IIA theory now have an eleven dimensional interpretation [2]. Indeed, the fundamental string (F1) [5] and the solitonic five-brane (S5) [6,7] are the double dimensional reduction of the eleven dimensional M2-brane [18]

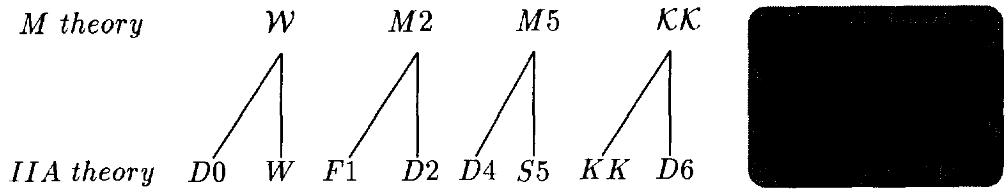

Figure 4. **The relation between $D = 10$ IIA and $D = 11$ solutions:** *Vertical lines imply direct dimensional reduction, diagonal lines double dimensional reduction. The shadowed area indicates the relationship between known ten-dimensional solutions and a conjectured 9-brane in $D = 11$.*

and the direct dimensional reduction of the eleven dimensional M5-brane [19], respectively. The Dirichlet D2- and D4-branes can be obtained from M2 and M5 via direct and double dimensional reduction, respectively. The D0- and D6-branes in the IIA theory are related to the purely gravitational Brinkmann wave [24] ($\mathcal{W}$) and the Kaluza-Klein monopole [25] ($\mathcal{KK}$) in eleven dimensions. These eleven dimensional solutions also have their counterparts in $D = 10$, which we denote by W and KK. Each of these solutions preserves 1/2 of the $D = 11$ (or $D = 10\,, N = 2$) supersymmetry. In Figure 4 (taken from [26]) we summarize the relationship between these $D = 10$ IIA and $D = 11$ solutions. The eleven dimensional interpretation of the Type IIA 8-brane [27,28] is still a mystery. Presumably, it is related to a 9-brane [6] in $D = 11$. The direct reduction of such a 9-brane is expected to lead to $D = 10$ Minkowski space.

The basic pair intersections of the solutions in Figure 4, assuming that the harmonics involved depend on the overall transverse directions, and the multiple intersections based upon these pairs have been discussed recently in [26].

[6] The conjectured 9-brane is also discussed in [28,17,29–31].

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