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Data-driven Loewner matrices-based modeling and model predictive control of a single machine infinite bus model

T. C. Ionescu, O. V. Iftime and I. Necoara

Abstract— **In this paper, we consider the problem of data-driven modelling and model predictive control (MPC) of a single machine infinite bus system (SMIB). When a linear state-space model of the plant is not available, a pair of Loewner matrices is constructed from a set of measured input-output data and a set of prescribed poles. We prove that the Loewner matrices are the solutions of two Sylvester equations. We compute the unique linear model of an order equal to the dimension of the data sets, that interpolates the input-output data and that has all the poles prescribed at the selected locations. As an application we perform MPC on both the original system and its approximant. The simulation show that the linear data-driven model is a good approximation of the SMIB and achieves good closed-loop MPC performance.**

I. Introduction

A power system consists of a very large number of interconnected synchronous generators rendering it as a complex nonlinear system [1]. The resulting complex model creates difficulties in the analysis simulation and control design. The mathematical model of one synchronous machine is made of eight dynamical equations with poynomial nonlinearities, see e.g [2]. The machines are interconnected through transmission lines, to a power grid modeled as an infinite bus. The power generator is called the single machine infinite bus system (SMIB), as in [3], [4]. This network model is used in the transient stability analysis, see [5]. However, the nonlinear model is difficult to use and model reduction is called for [5].

From the many methods of model reduction, moment matching techniques stand out as an efficient tool, see e.g. [6], [7], [8], [9], [6], [10], [11], [12], [13]. Employing Krylov projection methods the (reduced order) model is computed efficiently constructing a lower order rational function that approximates the rational transfer function of the given system. The approximating transfer function matches the original, higher order transfer function at various points in the complex plane. If the interpolation points are finite, then the general rational interpolation problem is solved.

In practice, transfer functions or state-space models of complex systems are hardly available. However, sets of measured data can be extracted from the real plant. For linear systems, the problem of data-driven reduced order modelling from a realization theory perspective has become popular, in the Loewner framework, pioneered in [14], [15]. In short, for a given plant, linear dynamical systems can be obtained from sets of measured input-output data. Loewner matrices are the so-called divided-difference matrices built from the data sets available. More recently, in [16], Loewner matrices have been built to include not only input-output data, but information on the desired poles, zeros and higher order moments of a given plant with a linear high order dynamical model available. Based on the Loewner matrices, a unique model that interpolates the given data and has the prescribed poles, zeros and higher order moments was obtained.

In this paper, we assume that a linear state-space model of the SMIB is not available and consider the problem of constructing a reduced order linear model from data sets containing input-output data and a set (of similar dimension) of prescribed poles. Based on the data sets, we write a pair of Loewner matrices proven to be the solutions of two Sylvester equations. With the designed Loewner matrices, we compute the unique linear model of order equal to the dimension of the data sets, that interpolates the input-output data and that has all the poles prescribed in the selected locations. Furthermore, to test the quality of the resulting approximant, we design model predictive controllers (MPC) for both the nonlinear model of the SMIB and the linear data-driven approximation. The simulations illustrate that both the input-output approximation and the closed-loop MPC trajectories of the approximant follow those of the SMIB plant.

The paper is organized as follows. In Section II, the reduced order modelling problem formulation is given, as well as the complete nonlinear physical model of the SMIB presented. In Section III, the Loewner matrices containing measured input-output data, as well as data on the set of prescribed poles, are constructed and analyzed. Furthermore, the theory on the model interpolating the given data and having the prescribed poles is developed. In Section IV, we present the nonlinear and linear MPC formulations to be used in the numerical example. Section V, illustrates the theory on the test case of the SMIB. The paper ends with conclusions.

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II. Preliminaries

In this section, we formulate the reduced order modelling problem, as well as present the complete nonlinear physical model of the SMIB.

A. Problem formulation

In practice, the dynamics of the real plant are unknown and a mathematical model is not available. Hence, we may formulate the modeling and controls problem based on data sets available for the plant. For a nonlinear system

$$\dot{x} = f(x) + g(x)u, \quad y = h(x), \tag{1}$$

where $x(t) \in \mathbb{R}^n, u(t), y(t) \in \mathbb{R}^m$, having the equilibrium point $x_e \in \mathbb{R}^n$, we measure the outputs $y(t)$ corresponding to a given set of inputs $u(t)$. The simulations are performed using numerical integration, e.g., 'ode', initialized within the set $x_0 \in [x_e - \epsilon, x_e + \epsilon]$, $\epsilon > 0$. Choosing inputs with the input data described by the set of numbers $\{s_1, \ldots, s_\nu\}$, the simulations yield output data described by the set $\{\eta_1 \ \eta_2 \ \ldots \ \eta_\nu\}, \eta_i \in \mathbb{R}^m, i = 1 : \nu$. Furthermore, based on some liniearization arguments around x_e, a set of poles may be selected. The data set resulting from the pole selection is the set $\{\eta_{\lambda_1} \ \eta_{\lambda_2} \ \ldots \eta_{\lambda_\nu}\}, \eta_{\lambda_i} \in \mathbb{R}^m, i = 1 : \nu$.

For simulation and control purposes, compute a linear model that interpolates the data set $\{\eta_1 \ \eta_2 \ \ldots \ \eta_\nu\}$ and has the poles $\{\lambda_1, \ldots \lambda_\nu\}$. Note that the resulting linear model must be a good approximation of the original system (1) in a neighbourhood of the equilibrium point x_e.

B. Model of a single machine infinite bus (SMIB)

We present the model of a single machine connected to an infinite bus, described in [17], see also, e.g., [3], [4]. Combining Kirchhoff's voltage law with Park's transformation yields:

$$\begin{bmatrix} e_d \\ e_q \end{bmatrix} = R_E \begin{bmatrix} i_d \\ i_q \end{bmatrix} + X_E \omega \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + E_b \begin{bmatrix} \sin\delta \\ \cos\delta \end{bmatrix} + X_E \frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix}. \tag{2}$$

The machine has the following constitutive equations:

$$\begin{aligned} &\dot{\Psi}_d = \omega\Psi_q + R_a i_d + e_d, \dot{\Psi}_q = -\omega\Psi_d + R_a i_q + e_q, \\ &\dot{\Psi}_{fd} = -R_{fd} i_{fd} + e_{fd}, \dot{\Psi}_r = -R_r i_r, \ r \in \{1d, 1q, 2q\}, \\ &j\dot{\omega} = -K_d(\omega - \omega_0) + T_m - (\Psi_d i_q - \Psi_q i_d), \dot{\delta} = \omega - \omega_0. \end{aligned} \tag{3}$$

Substituting (2) in (3) and denoting by $\Psi_{ds} = \Psi_d - X_E i_d$ and $\Psi_{qs} = \Psi_q - X_E i_q$, one gets:

$$\begin{aligned} &\dot{\Psi}_{ds} = \omega\Psi_q + (R_a + R_e) i_d - \omega X_e i_q + E_b \sin\delta, \\ &\dot{\Psi}_{qs} = -\omega\Psi_d + (R_a + R_e) i_q + \omega X_e i_d + E_b \cos\delta, \\ &\dot{\Psi}_{fd} = -R_{fd} i_{fd} + e_{fd}, \dot{\Psi}_r = -R_r i_r, \ r \in \{1d, 1q, 2q\}, \\ &j\dot{\omega} = -K_d(\omega - \omega_0) + T_m - (\Psi_d i_q - \Psi_q i_d), \dot{\delta} = \omega - \omega_0. \end{aligned} \tag{4}$$

The variables and parameters in (2), (3) and (4) are provided next: $\delta = \omega t + \delta_0$ is the rotor angle measured in radians, with ω_0, the synchronous speed; ω is the angular speed of the rotor; j is the inertia of the rotor; T_m is the mechanical torque; i_d, i_q are the direct axis current, quadrature axis current (of the stator), respectively; i_r are the rotor (amortisseur) currents in dq framework; i_{fd}, e_{fd} are the field current and voltage, respectively; K_d is the damping constant; R_E and X_E are transmission line resistance and reactance, respectively; Ψ_d and Ψ_q are the stator fluxes in the d-axis and q-axis, respectively; E_b is the infinite bus voltage (interconnection variable); Ψ_{fd} is the field flux; e_{fd} is the field voltage; $\Psi_i, \ i \in \{1d, 1q, 2d\}$ are the rotor fluxes due to the amortisseurs; R_a is the stator resistance; R_{fd} is the field circuit resistance; $R_i, i \in \{1d, 1q, 2d\}$ are the amortisseurs resistances. The relation between fluxes and current is:

$$\Psi = \mathcal{L} i,$$

$$\mathcal{L} = \begin{bmatrix} L_{ds} & 0 & L_{ad} & L_{ad} & 0 & 0 \\ 0 & L_{qs} & 0 & 0 & L_{aq} & L_{aq} \\ L_{ad} & 0 & L_{ffd} & L_{f1d} & 0 & 0 \\ L_{ad} & 0 & L_{f1d} & L_{11d} & 0 & 0 \\ 0 & L_{aq} & 0 & 0 & L_{11q} & L_{aq} \\ 0 & L_{aq} & 0 & 0 & L_{aq} & L_{22q} \end{bmatrix} \succ 0,$$

$$\Psi = [\Psi_{ds} \ \Psi_{qs} \Psi_{fd} \ \Psi_{1d} \ \Psi_{1q} \ \Psi_{2q}]^T, \quad i = [-i_d \ - i_q \ i_{fd} \ i_{1d} \ i_{1q} \ i_{2q}]^T,$$

with $L_{ds} = L_{ad} + L_l + X_e$ and $L_{qs} = L_{aq} + L_l + X_e$. The system can be written as

$$\dot{x} = f(x) + g(x)u,$$

where $x = [i^T \ \omega \ \delta]^T$ and $u = [E_b \ e_{fd} \ T_m]^T$. The total energy of the system is:

$$H(x) = \frac{1}{2}[i^T \ \omega] D \begin{bmatrix} i \\ \omega \end{bmatrix} > 0, \ H(0) = 0, \tag{5}$$

where $D = \mathrm{diag}(\mathcal{L}, j) > 0$. The nonlinear port Hamiltonian model of the SMIB is given by equations of the form

$$\dot{x} = (\mathcal{J}(x) - \mathcal{R})\mathcal{Q}x + Bu, \quad y = B^T \mathcal{Q} x, \tag{6}$$

with

$$\mathcal{J}(x) = \begin{bmatrix} 0 & \omega X_E & 0 & 0 & 0 & 0 & \Psi_q \\ -\omega X_E & 0 & 0 & 0 & 0 & 0 & -\Psi_d \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\Psi_q & \Psi_d & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & \sin\delta \\ 0 & 0 & \cos\delta \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\mathcal{R} = \mathrm{diag}\{R_a + R_E, R_a + R_E, R_{fd}, R_{1d}, R_{1q}, R_{2q}, K_d\},$$

and

$$\mathcal{Q} = \mathrm{diag}\left(\mathcal{L}^{-1}, \frac{1}{j}\right),$$

respectively. The state is $x = [\Psi^T \ \omega]^T \in \mathbb{R}^7$. The input is

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$$u = \begin{bmatrix} E_b \ e_{fd} \ T_m \end{bmatrix}^T \in \mathbb{R}^3 \tag{7}$$

and the measured output is

$$y = B^T \mathcal{Q} x = \begin{bmatrix} I_b \ i_{fd} \ \omega_m \end{bmatrix}^T \in \mathbb{R}^3, \tag{8}$$

where I_b is the infinite bus current and i_{fd} represents the field current.

Remark 1. Typically, in the power systems community, when the nonlinear equations of the model are available, model reduction is achieved based on physics arguments. For example, in [5], [18], under certain assumptions, a so-called third order Flux-Decay Model of nonlinear equations is obtained. Furthermore, in [17], balancing-based projections have been applied using balanced truncation arguments.

In the next section, we compute a linear model based on sets of input-output data and information about selected poles.

III. Data-driven based modeling using Loewner matrices

In this section, we present a solution for the reduced order modelling problem in Section II-A.

A. Loewner matrices from data

We now consider the situation when the models (6) or any linear counterpart are not known. In this case, we tackle the problem of data-driven reduced order modelling, using Loewner matrices, as in [14]. The Loewner matrices are constructed when the state-space matrices or the transfer function are not available, but data sets are available. Consider the input data sets, measured from a system with $m = p$ inputs and outputs, respectively, partitioned as in Table I.

Input data	Output data
directions	$\{\ell_1, \ldots, \ell_\nu\}$, $\ell_i \in \mathbb{R}^m$
$\{s_1, \ldots, s_\nu\}$, $s_i \in \mathbb{R}$	$\{\eta_1, \ldots, \eta_\nu\}$, $\eta_i \in \mathbb{R}^m$
Poles: $\{\lambda_1, \ldots, \lambda_\nu\}$, $\lambda_i \neq s_j$, $i, j = 1:\nu$	$\{\eta_{\lambda_1}, \ldots, \eta_{\lambda_\nu}\}$, $\eta_{\lambda_i} \in \mathbb{R}^m$

TABLE I

Input-output data sets for the data-driven Loewner matrices.

Then, the data-driven Loewner matrices are defined as:

$$\mathbb{L}_{ij} = \frac{\eta_{\lambda_i}^T \ell_i}{\lambda_i - s_j}, \quad i, j = 1:\nu, \tag{9a}$$

$$\sigma\mathbb{L}_{ij} = \frac{\lambda_i \eta_{\lambda_i}^T \ell_i}{\lambda_i - s_j}, \quad i, j = 1:\nu. \tag{9b}$$

Let

$$S = \operatorname{diag}(s_1, s_2, \ldots, s_\nu) \text{ and } Q = \operatorname{diag}(\lambda_1, \ldots, \lambda_\nu) \tag{10}$$

and

$$L = [\ell_1 \ \ldots \ \ell_\nu] \in \mathbb{R}^{m \times \nu}. \tag{11}$$

The next result shows that the Loewner matrices $\mathbb{L}$ and $\sigma\mathbb{L}$, defined by (9) are the solutions of two Sylvester equations.

Proposition 1. *Let S and Q be as in* (10) *and let L be as in* (11)*. Then, $\mathbb{L}$ and $\sigma\mathbb{L}$ defined in* (9) *satisfy the Sylvester equations*

$$Q\mathbb{L} - \mathbb{L}S = \mathbf{N}^T L, \tag{12a}$$

$$Q\sigma\mathbb{L} - \sigma\mathbb{L}S = Q\mathbf{N}^T L, \tag{12b}$$

where $\mathbf{N} = [\eta_{\lambda_1} \ \ldots \eta_{\lambda_\nu}]$, with η_{λ_k} from the data in Table I.

Proof: Note that (9a) yields, e.g., $\lambda_i \mathbb{L}_{ij} - \mathbb{L}_{ij} s_j = \eta_{\lambda_i}^T \ell_i, \ i, j = 1:\nu$ and (9b) yields $\lambda_i \sigma\mathbb{L}_{ij} - \sigma\mathbb{L}_{ij} s_j = \lambda_i \eta_{\lambda_i}^T \ell_i, \ i, j = 1:\nu$. In matrix form, using the definition of $\mathbf{N}$ as well as (10) yields (12). ■

B. The ν order model matching the data in Table I

In this section, we compute the linear model, with the corresponding transfer matrix and state space realization, that matches the data in Table I, in the sense that in each point s_i, along the direction ℓ_i, the output yields the data $\eta_i^T \ell_i$ and, moreover, the model has the poles in $\lambda_i, \quad i = 1:\nu$.

Theorem 1. *Let S and Q be as in* (10) *and let L be as in* (11)*. Consider $\mathbb{L}$ and $\sigma\mathbb{L}$ defined in* (9) *and let $\mathbf{M} = [\eta_1 \ \ldots \eta_\nu]$ and $\mathbf{N} = [\eta_{\lambda_1} \ \ldots \eta_{\lambda_\nu}]$, with η_k and η_λ from Table I. Then, the model described by the transfer matrix*

$$\mathbf{K} : \mathbb{C}^{m \times m} \to \mathbb{C}^{m \times m}, \ \ \mathbf{K}(s) = \mathbf{M}(\sigma\mathbb{L} - s\mathbb{L})^{-1}\mathbf{N}^T, \tag{13}$$

with a (generalized) state-space realization given by

$$\mathbb{L}\dot{\xi} = \sigma\mathbb{L}\xi + \mathbf{N}^T u, \quad \psi = \mathbf{M}\xi, \tag{14}$$

with $\xi(t) \in \mathbb{R}^\nu$ and $\psi(t) \in \mathbb{R}^m$, satisfies the following:

i) $\mathbf{K}(s_i)\ell_i = \eta_i, \quad i = 1:\nu$;
ii) $\lambda_i, \ i = 1:\nu$ *are the poles of* $\mathbf{K}$.

Proof: We prove statement i). Let

$$\epsilon_i = \eta_i - \mathbf{M}(\sigma\mathbb{L} - s_i\mathbb{L})^{-1}\mathbf{N}^T \ell_i, \tag{15}$$

for all $i = 1:\nu$, with s_i, η_i as in Table I. Then, since $\eta_i = \mathbf{M}e_i$, (15) yields

$$\begin{aligned}\epsilon_i &= \mathbf{M}e_i - \mathbf{M}(\sigma\mathbb{L} - s_i\mathbb{L})^{-1}\mathbf{N}^T \ell_i \\ &= \mathbf{M}\left[e_i - (\sigma\mathbb{L} - s_i\mathbb{L})^{-1}\mathbf{N}^T \ell_i\right] \\ &= \mathbf{M}(\sigma\mathbb{L} - s_i\mathbb{L})^{-1}\left[(\sigma\mathbb{L} - s_i\mathbb{L})e_i - \mathbf{N}^T \ell_i\right], i = 1:\nu.\end{aligned}$$

Multiplying (12a) with s to the left and subtracting it from (12b) gives $Q(\sigma\mathbb{L} - s\mathbb{L}) - (\sigma\mathbb{L} - s\mathbb{L})S = (Q - sI)\mathbf{N}^T L$. Postmultiplying now with e_i to the right, letting $s = s_i$ and taking into consideration that $Se_i = s_i e_i$, and that $\mathbf{N}^T L e_i = \mathbf{N}^T \ell_i$, yields

$$\begin{aligned}&Q(\sigma\mathbb{L} - s_i\mathbb{L})e_i - (\sigma\mathbb{L} - s_i\mathbb{L})s_i e_i = (Q - s_i I)\mathbf{N}^T \ell_i \\ &\Leftrightarrow (Q - s_i I)(\sigma\mathbb{L} - s_i\mathbb{L})e_i = (Q - s_i I)\mathbf{N}^T \ell_i.\end{aligned}$$

Hence $(\sigma\mathbb{L} - s_i\mathbb{L})e_i = \mathbf{N}^T \ell_i$ yielding $\epsilon_i = 0$.

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To prove ii), note that utilizing (10) and (12), (13) becomes

$$\begin{aligned}\mathbf{K}(s) &= \mathbf{M}S(\sigma\mathbb{L}S - s\mathbb{L}S)^{-1}\mathbf{N}^T \\ &= \mathbf{M}S(\sigma\mathbb{L}S + s\mathbf{N}^T L - sQ\mathbb{L})^{-1}\mathbf{N}^T \\ &= \mathbf{M}S(Q\sigma\mathbb{L} - Q\mathbf{N}^T L + s\mathbf{N}^T L - sQ\mathbb{L})^{-1}\mathbf{N}^T \\ &= \mathbf{M}S\left[(sI - Q)\mathbf{N}^T L + Q(\sigma\mathbb{L} - s\mathbb{L})\right]^{-1}\mathbf{N}^T \\ &= \mathbf{M}S\big[\mathbf{N}^T L + (sI - Q)^{-1}Q(\sigma\mathbb{L} - s\mathbb{L})\big]^{-1}(sI - Q)^{-1}\mathbf{N}^T.\end{aligned}$$

Hence, $\lambda_i \in \sigma(Q)$ are the poles of $\mathbf{K}$. ∎

Once a linear model (14) has been constructed, it can be used for simulations and control (e.g., model predictive control (MPC)).

IV. Data-driven model predictive control (DD-MPC)

Model predictive control (MPC) is widely used in today's process industry due to its ability to handle state and/or input constraints and to deal with multivariable control systems [19]. However, MPC is based on a model of the system in order to make future predictions. In the case of low fidelity models one can use data-driven MPC techniques. In this paper we focus on MPC design, where the characterization of system dynamics are based on models derived from measured data as presented in the previous sections. More specifically, instead of running through a parametric modelling procedure, our previous Loewner non-parametric method directly represents the system dynamics around an equilibrium point with data. For linear systems, the MPC problem is usually posed as a convex quadratic problem (QP), while for nonlinear systems one needs to solve a highly nonconvex optimization problem at regular intervals. In the next sections we formulate the MPC problem for both nonlinear and linear systems and then compare the behaviours around an equilibrium point of the closed loop MPC based on the "true" nonlinear model of the system and on its data-driven Loewner linear approximation.

A. Nonlinear DD-MPC

Given the nonlinear dynamical system (1) subject to state constraints $x \in \mathbb{X} \subseteq \mathbb{R}^n$ (e.g., given an equilibrium point $x_e \in \mathbb{R}^n$ we can enforce the behavior of the state to stay in the neighbourhood of the equilibrium point, where the linear model is a good approximation of the nonlinear system, e.g., $x \in [x_e - \epsilon, x_e + \epsilon]$ for some $\epsilon > 0$) and input constraints $u \in \mathbb{U} \subseteq \mathbb{R}^m$, we consider the following finite horizon optimal control problem with state-input constraints over a prediction horizon of length T, given the initial state x_0:

$$\begin{aligned}&\min_{x,u} \int_{t=0}^{T} \ell\left(x,u\right) + \ell_f\left(x(T)\right) \qquad (16)\\ &\text{s.t.:} \quad \dot{x} = f(x) + g(x)u, \quad x \in \mathbb{X},\ u \in \mathbb{U},\ x_0 \text{ given},\end{aligned}$$

where $\mathbb{X}$ and $\mathbb{U}$ are usually simple polyhedral sets, i.e., described by linear inequalities (e.g., $\underline{x} \leq x \leq \overline{x}$). Typically, the stage and final costs to be minimized are weighted quadratic error functions between the actual and the desirable state of the controlled variables of the system, i.e.:

$$\begin{aligned}\ell(x,u) &= (x - x_{\text{ref}})^T W(x - x_{\text{ref}}) + u^T R u,\\ \ell_f(x) &= (x - x_{\text{ref}})^T P(x - x_{\text{ref}}),\end{aligned}$$

respectively, where x_{ref} is a desired reference point (e.g., the equilibrium point x_e) and $W, P \succeq 0, R \succ 0$. After the optimization problem is solved, the first optimal control action is implemented on the system and when the next measurements are available the procedure is repeated in a receding horizon fashion. The main disadvantage of the previous MPC problem, especially when one has to control a process with nonlinear dynamics, is the computational burden associated with finding a solution of the corresponding nonconvex optimization problem (e.g., one can use ACADO to sole it [20]). Hence, in the next section, we also consider a MPC problem based on linear dynamics, which yields a simpler convex quadratic program than needs to be solved at regular intervals.

B. Linear DD-MPC

We consider a continuous time linear systems, defined by the following linear equations:

$$\dot{z} = A_c z + B_c u, \qquad (17)$$

where $z \in \mathbb{R}^\nu$ is the state and $u \in \mathbb{R}^m$ is the associated control input. We also impose state constraints $z \in \mathbb{Z}$ and input constraints $u \in \mathbb{U}$, where $\mathbb{Z}, \mathbb{U}$ are simple polyhedral sets (e.g., $\underline{z} \leq z \leq \overline{z}$). For system (17), we consider as before convex quadratic stage and final costs:

$$\begin{aligned}\ell(z,u) &= (z - z_{\text{ref}})^T W_z(z - z_{\text{ref}}) + u^T R u,\\ \ell_f(z) &= (z - z_{\text{ref}})^T P_z(z - z_{\text{ref}}),\end{aligned}$$

respectively, where z_{ref} is a reference point and $W_z, P_z \succeq 0, R \succ 0$. For a prediction horizon of length T, the finite horizon optimal control problem for system (17), given the initial state z_0, is:

$$\begin{aligned}&\min_{z,u} \int_{t=0}^{T} \ell\left(z,u\right) + \ell_f\left(z(T)\right) \qquad (18)\\ &\text{s.t.:} \quad \dot{z} = A_c z + B_c u, \quad z \in \mathbb{Z},\ u \in \mathbb{U},\ z_0 \text{ given}.\end{aligned}$$

This optimal control problem is usually much simpler to solve than (16). Indeed, one possible numerical technique to solve (18) is by discretization. More precisely, if we consider the discretized version of (17), $z_{t+1} = Az_t + Bu_t$, then (18) becomes:

$$\begin{aligned}&\min_{z_t,u_t} \sum_{t=0}^{N} \ell\left(z_t,u_t\right) + \ell_f\left(z_N\right)\\ &\text{s.t.:} \quad z_{t+1} = Az_t + Bu_t \qquad (19)\\ &\qquad\quad z_t \in \mathbb{Z},\ u_t \in \mathbb{U}\ \ \forall t = 0:N-1,\ z_0 \text{ given},\end{aligned}$$

where N is the corresponding prediction horizon. Let us show that this problem can be written equivalently as a

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convex quadratic problem. In order to present a compact reformulation of the MPC problem (19) as an optimization problem, we define the matrices:

$$\mathbf{A} = \begin{bmatrix} B & 0 & \dots & 0 \\ AB & B & \dots & 0 \\ A^2B & AB & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ A^{N-1}B & A^{N-2}B & \dots & B \end{bmatrix}, \ \mathbf{A}_N = \begin{bmatrix} A \\ A^2 \\ A^3 \\ \vdots \\ A^N \end{bmatrix}, \quad (20)$$

$$\mathbf{W} = \mathrm{diag}(\underbrace{W, \cdots, W}_{N-1 \text{ times}}, P) \quad \text{and} \quad \mathbf{R} = \mathrm{diag}(\underbrace{R, \cdots, R}_{N \text{ times}}).$$

Further, by eliminating the state variables, problem (19) is reformulated as a convex Quadratic Program (QP) [19]:

$$\begin{aligned} &\min_{\mathbf{u}} \frac{1}{2}\mathbf{u}^T\mathbf{H}\mathbf{u} + (\mathbf{q}(x_0))^T\mathbf{u}, \\ &\text{s.t.}: \ \mathbf{G}\mathbf{u} \leq \mathbf{g}(x_0) \end{aligned} \quad (21)$$

where the decision variable contains the entire sequence of commands over the prediction horizon

$$\mathbf{u} = [u_0^T, u_1^T, \cdots, u_{N-1}^T]^T \in \mathbb{R}^{Nn_u}$$

and

$$\mathbf{H} = \mathbf{R} + \mathbf{A}^T\mathbf{W}\mathbf{A} \quad \text{and} \quad \mathbf{q}(x_0) = \mathbf{A}^T\mathbf{W}(\mathbf{A}_N x_0 - e \otimes z_{\text{ref}})$$

and $\mathbf{G}, \mathbf{g}(x_0)$ are coming from the linear state and input constraints. In the next section we use the linear data-driven Loewner model for prediction in the linear MPC problem and compare its closed loop behaviour with that of the full nonlinear SMIB plant.

V. Numerical example

In this section, we illustrate the solution of the reduced order modelling problem and use it for MPC, employing the SMIB as test case.

A. Data-driven Loewner matrices-based model of SMIB

Cosider the SMIB nonlinear model (6) with the parameters taken fron, e.g., [4]. Hence we have the inductances:

$$\mathcal{L} = \begin{bmatrix} 0.22 & 0 & 0.01 & 0.01 & 0 & 0 \\ 0 & 0.219 & 0 & 0 & 0.009 & 0.009 \\ 0.01 & 0 & 1.825 & 1.660 & 0 & 0 \\ 0.01 & 0 & 1.660 & 1.8313 & 0 & 0 \\ 0 & 0.009 & 0 & 0 & 0 & 0.009 \\ 0 & 0.009 & 0 & 0 & 0.009 & 0.134 \end{bmatrix},$$

The dissipation matrix of the system is

$$\mathcal{R} = \mathrm{diag}(0.031, 0.031, 0.0006, 0.0284, 0.0061, 0.0236, 10),$$

and the inertia $j = 6$. One can observe that $x_e = 0$ is an equilibrium point, which additionally is locally asymptotically stable. To obtain the poles of the local behaviour for small variations, (6) is linearized around the equilibrium point yielding the asymptotically stable poles:

$$\begin{aligned} &\{-1.6667, -0.0109, -0.0012 \\ &-0.1132, -0.095, -0.01232, -0.0002\}. \end{aligned}$$

Using arguments in Remark 1, we select the order of the model $\nu = 3$. We compute a third order linear model (13) based on data measured from (6), described in Table II.

Input data	Output data
directions	$\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$
$0, 0.01, 0.1$	$\begin{bmatrix}0.0185\\3.49\\0.475\end{bmatrix}, \begin{bmatrix}2.255\\0.03\\60\end{bmatrix}, \begin{bmatrix}0.1685\\0.169\\0.3\end{bmatrix}$
Poles: $-1.67, -0.11, -0.095$	$\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$

TABLE II
Input-output data of (6)

Note that the first three dominant poles of the linearized model of (6) have been selected. Writing the matrices S and Q as in (10) and solving the Sylvester equations (12) yields the data-based third order linear model (13), with a state-space realization (14), with

$$\begin{aligned} \mathbb{L} &= \mathrm{diag}(0.5998, 8.333, 5.1282), \\ \sigma\mathbb{L} &= \mathrm{diag}(-1, -0.91667, -0.48718), \ \mathbf{N} = I_3, \quad (22) \\ \mathbf{M} &= \begin{bmatrix} 0.0185 & 3.49 & 0.475 \\ 2.255 & 0.03 & 60 \\ 0.1685 & 0.169 & 0.3 \end{bmatrix}. \end{aligned}$$

Note that, since $m = \nu = p = 3$, the Loewner matrices have simple canonical forms. Usually when $m \neq \nu$ and/or $p \neq \nu$, this is not the case and if the order of the data sets is large, then the Loewner matrices can be constructed based on (9) rather than solving equations (12). Figure 1 compares the step responses of the original nonlinear model of the SMIB in (6) with the step response of the linear state-space model (14), with the realization (22). The result is suitable, since (14) has been built based on data including the time-domain measurement of the DC-gain from the original model (6).

B. Data-driven MPC for SMIB

We now consider applying nonlinear MPC using ACADO [20] for the original SMIB in (6) starting from the initial state $x_0 = [0.6 \ 1 \ -0.2 \ 1 \ -0.4 \ 0 \ 1]^T$, with the state constraints $-1 \leq x \leq 1$ and cost matrices $W = I_7$ and $R = 10^{-2}I_3$. Similarly, we apply MPC for the third order approximant (22) starting from the initial state $z_0 = [0.6 \ 1 \ 0.9]^T$, with the state constraints $-1 \leq z \leq 1$ and cost matrices $Q_z = I_3$ and $R_z = 10^{-2}I_3$. Figure 2 shows the closed-loop state trajectories of the fourth state of the original model versus the second state of (22). Note that MPC is stabilizing the system in both cases. However, the nonlinear stabilization

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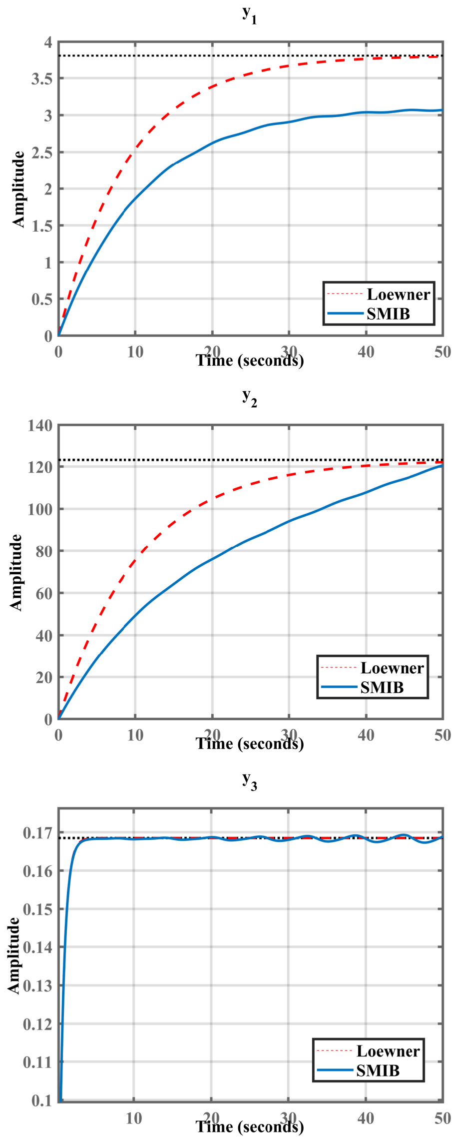

Fig. 1. Step response of (6) (solid line) v. step response of the third order data-driven model $K(s)$ as in (22) (dashed line)

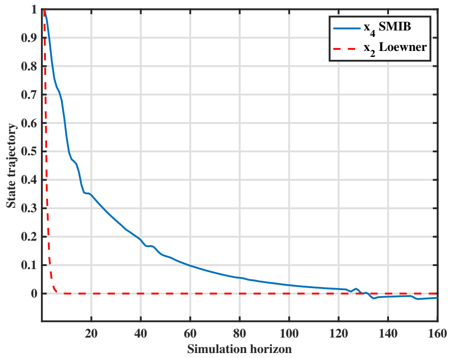

Fig. 2. Closed-loop MPC for (6) (solid line) v. the third order data-driven model $K(s)$ as in (22) (dashed line)

is slower than the approximating reduced-dimension linear counterpart.

VI. Conclusions

In this paper, we have posed the problem of data-driven modelling and model predictive control of a single machine infinite bus system. A set of Loewner matrices has been constructed from measured input-output data and a set of prescribed poles. We have computed the unique linear model of order equal to the dimension of the data sets, that matches the input-output data and that has all the poles prescribed in the desired locations. As an application we have applied MPC on both the original system and its approximant. The numerical example shows that the data-driven MPC Loewner-based controller achieves the same stabilizing goals as the nonlinear MPC applied on the original system.

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