Annuities, public policy and demographic change in overlapping generations models

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Chapter 4

Pensions and ageing*

* This chapter is based on Heijdra and Mierau (2011).

4.1 Introduction

The coming generational storm is arguably the strongest tempest against which current and future politicians have to sail. The policy-panacea, however, has not yet been developed and, maybe even worse, there is no consensus on what the economic consequences of an ageing population actually are. Hence, in this chapter we use a simplified version of the model developed in Chapter 2 to study the macroeconomic consequences of an increase in old age mortality. We introduce a simple Pay-As-You-Go (PAYG) pension system, which may either be run on a defined benefit (DB) or a defined contribution (DC) basis. In the former case the replacement rate acts as a policy variable whereas in the latter case the contribution rate is the policy variable. Furthermore, and in contrast to the previous chapters, we let retirement be exogenous, which allows us to consider the retirement age as a policy variable.

We find that, in principle, ageing is good for economic growth because it increases the incentive for individuals to save. However, if a defined benefit system is in place the higher contributions necessary to finance the entitlement of the additional pensioners will reduce individual savings and thereby dampen the growth increase following a longevity shock. In order to circumvent this reduction in growth the government could opt to introduce a defined contribution system in which the benefits are adjusted downward to accommodate the increased dependency ratio. Surprisingly, we find that if the government increases the retirement age such that the old age dependency ratio remains constant economic growth drops compared to both the defined benefit and the defined contribution system. This is due to an adverse savings effect following from the shortened retirement period.

The remainder of this chapter is set-up as follows. The next section introduces the model and discusses how we feed in a realistic life-cycle. Section 3 analyses the steady-state consequences of ageing and provides some policy recommendations. The final section concludes.

4.2 Model

We use the model developed in Chapter 2 and extend it to include a public pension system on the government side but simplify it by making the retirement decision exogenous on the consumer side. We refer the reader to Chapter 2 for a complete description of the model and use the remainder of this section to outline the model and its the extensions.

4.2.1 Consumers

Individual behaviour

We develop the individual's decision rules from the perspective of birth. Expected lifetime utility of an individual born at time v is given by:

$$\mathbb{E}\Lambda\left(v,v\right)\equiv\int_{v}^{v+\bar{D}}\frac{C(v,\tau)^{1-1/\sigma}-1}{1-1/\sigma}\cdot e^{-\rho(\tau-v)-M(\tau-v)}d\tau, \tag{4.1}$$

where $C\left(v,\tau\right)$ is consumption, σ is the intertemporal substitution elasticity ($\sigma>0$), ρ is the pure rate of time preference ($\rho>0$), $\bar{D}$ is the maximum attainable age for the agent, and $e^{-M(\tau-v)}$ is the probability that the agent is still alive at some future time τ ($\geq v$). Here, $M(\tau-v)\equiv\int_{0}^{\tau-v}\mu(s)ds$ stands for the cumulative mortality rate and $\mu\left(s\right)$ is the instantaneous mortality rate of an agent of age s.

The agent's budget identity is given by:

$$\dot{A}(v,\tau)=r^{A}\left(\tau-v\right)A(v,\tau)+w(v,\tau)L\left(v,\tau\right)-C(v,\tau)+PR\left(v,\tau\right)+TR\left(v,\tau\right), \tag{4.2}$$

where $A\left(v,\tau\right)$ is the stock of financial assets, $r^{A}\left(\tau-v\right)$ is the age-dependent annuity rate of interest rate, $w\left(v,\tau\right)\equiv E\left(\tau-v\right)w\left(\tau\right)$ is the age-dependent wage rate, $E\left(\tau-v\right)$ is exogenous labour productivity, $L\left(v,\tau\right)$ is labour supply, $PR(v,\tau)$ are payments received from the public pension system, and $TR\left(v,\tau\right)$ are lump-sum transfers (see below). Labour supply is exogenous and mandatory retirement takes place at age

R. Since the time endowment is unity, we thus find:

$$L(v,\tau) = \begin{cases} 1 & \text{for } 0 \leq \tau - v < R \\ 0 & \text{for } R \leq \tau - v < \bar{D} \end{cases}. \tag{4.3}$$

There is a simple PAYG pension system which taxes workers and provides benefits to retirees:

$$PR(\tau) = \begin{cases} -\theta w(v,\tau) & \text{for } 0 \leq \tau - v < R \\ \zeta w(\tau) & \text{for } R \leq \tau - v < \bar{D} \end{cases} \tag{4.4}$$

where $w(\tau)$ is the economy wide wage rate, θ ($0 < \theta < 1$) is the contribution rate and ζ is the benefit rate ($\zeta > 0$). Under a DC system, θ is exogenous and ζ adjusts to balance the budget (see below). The opposite holds under a DB system. Finally, we postulate that lump-sum transfers are age-independent:

$$TR(v,\tau) = z \cdot w(\tau), \tag{4.5}$$

where z is endogenously determined via the balanced budget requirement of the redistribution scheme (see below).

In order to replicate the salient features of the individual life-cycle we add a number of distinguishing features to our model. As these features have been discussed at length in Chapters 2-3 we suffice by stating them here and referring the reader to the previous chapters.

Imperfect annuity markets: The annuity rate of interest facing the agent is given by:

$$r^A(\tau - v) \equiv r + \lambda\mu(\tau - v), \qquad (\text{for } 0 \leq \tau - v < \bar{D}). \tag{4.6}$$

where r is the real interest rate and λ is a parameter ($0 < \lambda \leq 1$) indicating the degree of imperfection on the annuity market.

Age-dependent productivity: Labour productivity is hump-shaped over the life-cycle. A useful parameterization of the productivity profile is:

$$E(t - v) = \alpha_0 e^{-\zeta_0(t-v)} - \alpha_1 e^{-\zeta_1(t-v)}, \qquad (\text{for } 0 \leq t - v \leq \bar{D}), \tag{4.7}$$

where α_i and ζ_i are the parameters governing the curvature of the productivity profile (see Box 2.1 for details).

Age-dependent mortality: Individual mortality increases over the life-cycle. We assume that $e^{-M(t-v)}$ takes the following useful functional form:

$$e^{-M(t-v)} \equiv \frac{\eta_0 - e^{\eta_1 (t-v)}}{\eta_0 - 1}, \qquad (\text{for } 0 \leq t - v \leq \bar{D}), \tag{4.8}$$

where $\eta_0 > 1$ and $\eta_1 > 0$ are parameters (see Box 2.2 for details).

The agent chooses time profiles for $C(v, \tau)$ and $A(v, \tau)$ (for $v \leq \tau \leq v + \bar{D}$) in order to maximize (4.1), subject to (i) the budget identity (4.2), (ii) a transversality condition, $A(v, v + \bar{D}) = 0$, and (iii) the initial asset position at birth, $A(v, v) = 0$. The optimal consumption profile for a vintage-v individual of age u ($0 \leq u \leq \bar{D}$) is fully characterized by the following equations:

$$C(v, v+u) = C(v,v) \cdot e^{\sigma[(r-\rho)u - (1-\lambda)M(u)]}, \tag{4.9}$$

$$\frac{C(v,v)}{w(v)} = \frac{1}{\int_0^{\bar{D}} e^{(\sigma-1)[rs+\lambda M(s)] - \sigma[\rho s + M(s)]} ds} \cdot \frac{H(v,v)}{w(v)}, \tag{4.10}$$

$$\begin{aligned} \frac{H(v,v)}{w(v)} &= (1-\theta) \int_0^R E(s)\, e^{-(r-g)s - \lambda M(s)} ds + \zeta \int_R^{\bar{D}} e^{-(r-g)s-\lambda M(s)} ds \\ &\quad + z \int_0^{\bar{D}} e^{-(r-g)s - \lambda M(s)} ds. \end{aligned} \tag{4.11}$$

The intuition behind these expressions is as follows. Equation (4.9) is best understood by noting that the consumption Euler equation resulting from utility maximization takes the following form:

$$\frac{\dot{C}(v,\tau)}{C(v,\tau)} = \sigma \cdot [r - \rho - (1-\lambda)\,\mu(\tau - v)]\,. \tag{4.12}$$

By using this expression, future consumption can be expressed in terms of consumption at birth as in (4.9). In the absence of an annuity market imperfection ($\lambda = 1$), consumption growth only depends on the gap between the interest rate and the pure rate of time preference. In contrast, with imperfect annuities, individual consumption growth is negatively affected by the mortality rate, a result first demonstrated for the case with $\lambda = 0$ by Yaari (1965, p. 143).

Equation (4.10) shows that scaled consumption of a newborn is proportional to scaled human wealth. Finally, equation (4.11) provides the definition of human wealth at birth. The first term on the right-hand side represents the present value of the time endowment during working life, using the growth-corrected annuity rate of interest for discounting. The second term on the right-hand side denotes the present value of the pension received during retirement. Finally, the third term on the right-hand side of (4.11) is just the present value of transfers arising from the annuity market imperfection.

The asset profiles accompanying the optimal consumption plans are given for a working-age individual ($0 \leq u < R$) by:

$$\begin{aligned}\frac{A(v,v+u)}{w(v)}e^{-ru-\lambda M(u)} &= (1-\theta)\int_0^u E(s)\,e^{-(r-g)s-\lambda M(s)}ds + z\int_0^u e^{-(r-g)s-\lambda M(s)}ds \\ &\quad -\frac{C(v,v)}{w(v)}\int_0^u e^{(\sigma-1)[rs+\lambda M(s)]-\sigma[\rho s+M(s)]}ds, \qquad (4.13)\end{aligned}$$

and for a retiree ($R \leq u \leq \bar{D}$) by:

$$\begin{aligned}\frac{A(v,v+u)}{w(v)}e^{-ru-\lambda M(u)} &= \frac{C(v,v)}{w(v)}\int_u^{\bar{D}} e^{(\sigma-1)[rs+\lambda M(s)]-\sigma[\rho s+M(s)]}ds \\ &\quad -(\zeta+z)\int_u^{\bar{D}} e^{-(r-g)s-\lambda M(s)}ds. \qquad (4.14)\end{aligned}$$

Aggregate household behaviour

In general, we can define per-capita average values in general terms as:

$$x(t) \equiv \int_{t-\bar{D}}^{t} p(v,t)\,X(v,t)\,dv, \qquad (4.15)$$

where $X(v,t)$ denotes the variable in question at the individual level, $x(t)$ is the per capita average value of that same variable, and $p(v,t) \equiv \beta e^{-\pi(t-v)-M(t-v)}$ are the cohort weights (see Box 2.2 for details).

Per capita aggregate household behaviour is summarized by the following expressions:

$$\frac{c(t)}{w(t)} = \beta\frac{C(v,v)}{w(v)}\int_0^{\bar{D}} e^{\sigma[(r-\rho)s-(1-\lambda)M(s)]-(\pi+g)s-M(s)}ds, \qquad (4.16)$$

$$\begin{aligned} n(t) &= n \equiv \beta \int_0^R E(s)\, e^{-\pi s - M(s)} ds, \qquad (4.17) \\ \dot{a}(t) &= (r-\pi)\, a(t) + w(t)\, n(t) - c(t) \\ &\quad + \left[\zeta \int_R^{\bar{D}} \beta e^{-\pi s - M(s)} ds - \theta \int_0^R \beta E(s)\, e^{-\pi s - M(s)} ds \right] w(t) \\ &\quad + \left[(1-\lambda) \int_0^{\bar{D}} \beta e^{-(g+\pi)s - M(s)} \mu(s) \frac{A(v, v+s)}{w(v)} ds - z \right] w(t). \qquad (4.18) \end{aligned}$$

Equation (4.16) relates the macroeconomic consumption-wage ratio to the optimally chosen scaled consumption level by newborns. Since this ratio is time-invariant, per capita consumption grows at the macroeconomic growth rate g. Equation (4.17) shows that aggregate per capita labour supply (in efficiency units) is a time-invariant constant. Finally, the growth rate in per capita financial assets is given in equation (4.18). This expression will be discussed in more detail below.

4.2.2 Loose ends

We assume that the PAYG pension scheme is run on a balanced-budget basis. In view of (4.7), (4.4) and the demographic steady state condition this furnishes the following budget constraint:

$$\zeta w(t) \int_R^{\bar{D}} \beta e^{-\pi s - M(s)} ds = \theta w(t) \int_0^R \beta E(s)\, e^{-\pi s - M(s)} ds, \qquad (4.19)$$

where the left-hand side stands for pension payments to retirees and the right-hand side represents pension contributions by workers. The mandatory retirement age R is exogenous. Under the assumption of a DC system, θ is also exogenous and ζ adjusts to balance the budget. The opposite holds under a DB system. In view of (4.19), the PAYG system does not feature in the expression for aggregate asset accumulation, i.e. the second line of (4.18) is zero.

Excess profits of annuity firms can be written as follows:

$$EP(v,t) \equiv (1-\lambda) \int_{t-\bar{D}}^{t} p(v,t)\, \mu(t-v)\, A(v,t)\, dv. \qquad (4.20)$$

The integral on the right-hand side represents per capita annuitized assets of all individuals that die in period t. This is the total revenue of annuity firms, of which only a fraction λ is paid out to surviving annuitants. The remaining fraction, $1-\lambda$, is excess profit which is taxed away by the government and disbursed to all households in the form of lump-sum transfers, i.e. $EP(v,t) = TR(v,t)$. Using (4.5) and (4.20) we find the implied expression for z:

$$z = (1-\lambda)\int_0^{\bar{D}} \beta e^{-(g+\pi)u - M(u)}\mu(u)\,\frac{A(v,v+u)}{w(v)}du. \tag{4.21}$$

Just as for the PAYG system, the redistribution of excess profits of annuity firms also debudgets from the asset accumulation equation, i.e. the third line in (4.18) is also zero.

In the absence of government bonds, the capital market equilibrium condition is given by $A(t) = K(t)$ or, in per capita average terms, by:

$$a(t) = k(t), \tag{4.22}$$

where $k(t) \equiv K(t)/P(t)$ is the per capita stock of capital. As before we easily find:

$$y(t) = \Omega_0 k(t), \tag{4.23}$$

$$w(t)\,n(t) = (1-\varepsilon)\,y(t), \tag{4.24}$$

where $y(t) \equiv Y(t)/P(t)$ is per capita output. From (4.18)–(4.19), (4.21) and (4.22) we can derive the expression for the macroeconomic growth rate:

$$g \equiv \frac{\dot{k}(t)}{k(t)} = r - \pi + \left[n(t) - \frac{c(t)}{w(t)}\right]\frac{w(t)}{k(t)}. \tag{4.25}$$

For convenience, the key equations comprising the general equilibrium model have been gathered in Table 1. Equations (T1.1)–(T1.2), (T1.3a)–(T1.3b), (T1.4)–(T1.6), (T1.8)–(T1.9) restate, respectively, (4.10)–(4.11), (4.13)–(4.14), (4.19), (4.21), (4.25), (4.17), and (4.16). Equation (T1.7) is obtained by combining (4.23) and (4.24) and noting (4.17).

The model features a two-way interaction between the microeconomic decisions and the macroeconomic outcomes. Indeed, conditional on the macroeconomic variables, equations (T1.1)–(T1.3) determine scaled newborn consumption and human wealth, $C(v,v)/w(v)$ and $H(v,v)/w(v)$ as well as the age profile of scaled assets $A(v,v+u)/w(v)$. Conditional on these microeconomic variables, equations (T1.4)–

(T1.9) determine equilibrium pension payments and transfers, ζ and z, the macroeconomic growth rate, g, the overall wage-capital ratio, $w(t)/k(t)$, aggregate labour supply, n, and the $c(t)/w(t)$ ratio.

4.2.3 The core model

For the *core model* we postulate the existence of perfect annuities (PA, with $\lambda = 1$) and parameterized it as follows. The productivity and demographic parameters[1] are taken from the estimates outlined in Chapter 2. We assume that the rate of population growth is half of one percent per annum ($\pi = 0.005$). For the estimated demographic process, the demographic steady-state yields a birth rate equal to $\beta = 0.0204$. Since $\bar{\mu} \equiv \beta - \pi$, this implies that the average mortality rate is $\bar{\mu} = 0.0154$. The old-age dependency ratio equals 22.92%. We model an economy with a steady-state capital-output ratio of 2.5, which is obtained by setting $\Omega_0 = 0.4$. The interest rate is five percent per annum ($r = 0.05$), the capital depreciation rate is seven percent per annum ($\delta = 0.07$), and the efficiency parameter of capital is set at $\varepsilon = 0.3$. The steady-state growth rate is set equal to two percent per annum ($g = 0.02$). For the intertemporal substitution elasticity we use $\sigma = 0.7$, a value consistent with the estimates reported by Attanasio and Weber (1995). The rate of pure time preference is used as a calibration parameter, yielding a value of $\rho = 0.0112$.

Regarding the PAYG pension system we assume that the mandatory retirement age is set at $R = 47$ (corresponding with 65 in biological years) and that the pension contribution rate is seven percent of wage income, i.e. $\theta = 0.07$ which roughly corresponds with the Dutch pension system. The implied pension benefit is determined in general equilibrium.

Table 4.2(a) reports the main features of the initial steady-state growth path. With perfect annuities, there are no excess profits of annuity firms and thus no transfers, i.e. $z = 0$ in Table 4.2(a). Note also that at retirement age R a vintage-v agent receives $\zeta w(v+R)$ in the form of a pension whereas the last-received wage for this agent equals $E(R)\, w(v+R)$. The replacement rate is thus equal to $\zeta / E(R) = 0.3189$.

We visualize the life-cycle profiles for a number a key variables in Figure 1. The

[1] Remember that we consider agents from age 18 onward so that a model age of 0 corresponds to a biological age of 18.

Table 4.1. The model

(a) Microeconomic relationships:

$$\frac{C(v,v)}{w(v)} = \frac{1}{\int_0^{\bar{D}} e^{-(1-\sigma)[rs+\lambda M(s)]-\sigma[\rho s+M(s)]}ds} \cdot \frac{H(v,v)}{w(v)} \tag{T1.1}$$

$$\frac{H(v,v)}{w(v)} = (1-\theta)\int_0^R E(s)\,e^{-(r-g)s-\lambda M(s)}ds + \zeta \int_R^{\bar{D}} e^{-(r-g)s-\lambda M(s)}ds + z\int_0^{\bar{D}} e^{-(r-g)s-\lambda M(s)}ds \tag{T1.2}$$

$$\frac{A(v,v+u)}{w(v)} e^{-ru-\lambda M(u)} = (1-\theta)\int_0^u E(s)\,e^{-(r-g)s-\lambda M(s)}ds + z\int_0^u e^{-(r-g)s-\lambda M(s)}ds - \frac{C(v,v)}{w(v)}\int_0^u e^{-(1-\sigma)[rs+\lambda M(s)]-\sigma[\rho s+M(s)]}ds \tag{T1.3a}$$

$$= \frac{C(v,v)}{w(v)}\int_u^{\bar{D}} e^{-(1-\sigma)[rs+\lambda M(s)]-\sigma[\rho s+M(s)]}ds - (\zeta+z)\int_u^{\bar{D}} e^{-(r-g)s-\lambda M(s)}ds \tag{T1.3b}$$

(b) Macroeconomic relationships:

$$\zeta = \theta \cdot \frac{\int_0^R \beta E(s)\,e^{-\pi s-M(s)}ds}{\int_R^{\bar{D}} \beta e^{-\pi s-M(s)}ds} \tag{T1.4}$$

$$z = (1-\lambda)\int_0^{\bar{D}} \beta e^{-(g+\pi)u-M(u)}\mu(u)\,\frac{A(v,v+u)}{w(v)}du \tag{T1.5}$$

$$g \equiv \frac{\dot{k}(t)}{k(t)} = r-\pi+\left[n-\frac{c(t)}{w(t)}\right]\frac{w(t)}{k(t)} \tag{T1.6}$$

$$\frac{w(t)\,n}{k(t)} = (1-\varepsilon)\,\Omega_0 \tag{T1.7}$$

$$n = \beta\int_0^R E(s)\,e^{-\pi s-M(s)}ds \tag{T1.8}$$

$$\frac{c(t)}{w(t)} = \beta\frac{C(v,v)}{w(v)}\int_0^{\bar{D}} e^{\sigma[(r-\rho)s-(1-\lambda)M(s)]-(\pi+g)s-M(s)}ds \tag{T1.9}$$

Definitions: Endogenous are $C(v,v)/w(v)$, $H(v,v)/w(v)$, $A(v,v+u)/w(v)$, ζ, z, g, n, $w(t)/k(t)$, and $c(t)/w(t)$. Parameters: R retirement age, θ pension contribution rate, birth rate β, aggregate mortality rate $\bar{\mu}$, population growth rate $\pi \equiv \beta - \bar{\mu}$, imperfection annuities λ, rate of time preference ρ, capital coefficient in the technology ε, scale factor in the technology Ω_0. The interest rate is $r \equiv \varepsilon\Omega_0 - \delta$, where δ is the depreciation rate of capital.

Table 4.2. Quantitative effects

Case:	(a) PA	(b) IA	(c) PA	(d) IA	(e) PA	(f) IA	(g) PA	(h) IA
	Core cases		**DC**		**DB**		**RA**	
$\frac{C(v,v)}{w(v)}$	0.8534	0.8609	1.0784	1.0785	0.9078	0.9053	0.9329	0.9369
$\frac{H(v,v)}{w(v)}$	26.5646	27.0207	36.0229	36.2942	30.3246	30.4653	31.1617	31.5282
g (%)	2.00	1.91	3.36	3.27	2.79	2.68	2.39	2.30
n	0.9675	0.9675	0.8212	0.8212	0.8212	0.8212	0.9589	0.9589
$\frac{w(t)}{k(t)}$	0.2894	0.2894	0.3410	0.3410	0.3410	0.3410	0.2920	0.2920
$\frac{c(t)}{w(t)}$	1.0538	1.0570	0.8692	0.8720	0.8861	0.8893	1.0482	1.0514
ζ	0.3632	0.3632	0.1824	0.1824	0.3632	0.3632	0.3632	0.3632
z		0.0200		0.0142		0.0131		0.0185
θ					0.1394	0.1394		
$R+18$							75.3	75.3

Notes. PA stands for perfect annuities ($\lambda = 1$) and IA denotes imperfect annuities ($\lambda = 0.7$). Column (a) is the core model. Column (b) shows the effects of the annuity market imperfection in the core model. Columns (c)–(d) show the effects of a demographic shock under a DC pension system. Columns (e)–(f) show the effects under a DB system. In this scenario the tax rate θ adjusts to keep ζ at its pre-shock level. Columns (g)–(h) show the effects under a retirement age (RA) scenario in which θ and ζ are kept at their pre-shock levels and R is adjusted.

solid lines are associated with the core model featuring perfect annuities. For ease of interpretation, the horizontal axes report biological age, $u + 18$. Figure 1(a) shows that with perfect longevity insurance consumption rises monotonically over the life cycle. This counterfactual result follows readily from (4.12) which for $\lambda = 1$ simplifies to $\dot{C}(v,\tau)/C(v,\tau) = \sigma(r-\rho)$. Figure 1(b) depicts the age profile of scaled financial assets. At first the agent is a net borrower, i.e. a buyer of life-insured loans. Thereafter annuity purchases are positive and the profile of assets is bell-shaped. In the absence of a bequest motive, the agent plans to run out of financial assets at the maximum age $\bar{D}$. Figure 1(c) shows the profile of scaled wages over the life cycle. Despite the fact that individual labour productivity itself is bell-shaped, wages increase monotonically as a result of ongoing economic growth. Finally, in Figure 1(d) we illustrate the profile for scaled pension receipts. During the working career these payments are negative and proportional to scaled wages, whilst they are positive and proportional to the economy-wide wage rate during retirement.

Despite its simplicity, the model captures some of the main stylized facts regarding life cycles. Indeed, as is documented by *inter alia* Huggett (1996), in real life financial assets typically display a hump-shaped profile and remain non-negative in old age. The model also features a realistic age profile for labour supply. Indeed, as is pointed out by McGrattan and Rogerson (2004) (for the United States), labour supply is constant and age-invariant for most of working life and tapers off rapidly near the retirement age.

In contrast, the model does not provide a realistic profile for consumption. In the core model the age profile for consumption is monotonically increasing, whereas it is hump-shaped in reality. See, for example, Gourinchas and Parker (2002) and Fernández-Villaverde and Krueger (2007) for evidence on the US, and Alessie and de Ree (2009) for a recent study using Dutch data.

Referring to the consumption Euler equation (4.12) it is clear that an annuity market imperfection can account for a hump-shaped pattern of consumption. Indeed, with $0 < \lambda < 1$ it follows from (4.12) and Figure 1(b) that consumption growth is positive during the early phase of life because the mortality rate is low, i.e. $r - \rho > (1-\lambda)\mu(u)$. Toward the end of life, however, the instantaneous death probability rises sharply, the inequality is reversed, and the optimal consumption profile is downward sloping.[2]

In order to quantify and visualize the effects of an annuity market imperfection we recompute the general equilibrium of the model using the structural parameters mentioned above but setting $\lambda = 0.7$. This degree of annuity market imperfection is in the order of magnitude found by Friedman and Warshawsky (1988, p. 59). Table 2(b) reports the quantitative implications of the annuity market imperfection. Two features stand out. First, in the presence of imperfect annuities excess profits of annuity firms are positive and transfers are thus strictly positive ($z = 0.0200$). Each surviving agent thus receives about two percent of the macroeconomic wage rate in each period in the form of transfers. Second, the macroeconomic growth rate falls by nine basis points, from 2 percent to 1.91 percent per annum.

The ultimate effect on newborn consumption of the change in λ depends on the interplay between the human wealth effect and the propensity effect. Recall from (T1.1)–(T1.2) that $C(v,v) = \Delta \cdot H(v,v)$ where the propensity to consume is defined as:

$$\Delta \equiv \frac{1}{\int_0^{\bar{D}} e^{-(1-\sigma)[rs+\lambda M(s)]-\sigma[\rho s+M(s)]} ds}. \tag{4.26}$$

It is easy to show that with $0 < \sigma < 1$, the propensity to consume out of human wealth falls as a result of the reduction in λ:

$$\frac{d\Delta}{d\lambda} = (1-\sigma)\,\Delta^2 \cdot \int_0^{\bar{D}} M(s)\, e^{-(1-\sigma)[rs+\lambda M(s)]-\sigma[\rho s+M(s)]} ds > 0. \tag{4.27}$$

The partial derivative of scaled human wealth with respect to λ is given by:

$$\begin{aligned} \frac{\partial}{\partial\lambda}\frac{H(v,v)}{w(v)} &= -(1-\theta)\int_0^{R} M(s)\,E(s)\,e^{-(r-g)s-\lambda M(s)} ds - \zeta \int_R^{\bar{D}} M(s)\,e^{-(r-g)s-\lambda M(s)} ds \\ &\quad -z\int_0^{\bar{D}} M(s)\,e^{-(r-g)s-\lambda M(s)} ds < 0. \end{aligned} \tag{4.28}$$

A decrease in λ results in a reduction in the annuity rate of interest at all age levels and thus an increase in human wealth due to less severe discounting of non-asset in-

[2] Consumption peaks at age $\hat{u}$, which is defined implicitly by $\mu(\hat{u}) = (r-\rho)\,/\,(1-\lambda)$. Since $\mu'(u) > 0$ we find that $d\hat{\mu}/d\lambda > 0$ and $d\hat{\mu}/d(r-\rho) > 0$. Hence, the smaller is λ or $r-\rho$, the lower is the age at which consumption peaks. Note that whereas λ can help determine the location of the kink, the intertemporal substitution elasticity σ cannot do so.

come streams. Human wealth is also affected by two of the macroeconomic variables, namely transfers z and the growth rate g (note that n, ζ, and $w(t)/k(t)$ are not affected by λ). Scaled human wealth is boosted as a result of the transfers:

$$\frac{\partial}{\partial z}\frac{H(v,v)}{w(v)} = \int_0^{\bar{D}} e^{-(r-g)s-\lambda M(s)} ds > 0, \tag{4.29}$$

but it is reduced by the decrease in the growth rate:

$$\begin{aligned}\frac{\partial}{\partial g}\frac{H(v,v)}{w(v)} &= (1-\theta)\int_0^R sE(s)\, e^{-(r-g)s-\lambda M(s)} ds + \zeta \int_R^{\bar{D}} se^{-(r-g)s-\lambda M(s)} ds \\ &\quad + z\int_0^{\bar{D}} se^{-(r-g)s-\lambda M(s)} ds > 0. \end{aligned} \tag{4.30}$$

The results in Table 2(b) confirm that for our parameterization scaled consumption and human wealth both increase, i.e. the effects in (4.28) and (4.29) dominate the combined propensity effect (4.27) and growth effect (4.30).

In Figure 1 the dashed lines depict the life-cycle profiles associated with the model featuring imperfect annuities. Scaled consumption is hump-shaped but peaks at a rather high age.[3] The profiles for scaled financial assets, wages, and pension payments are all very similar to the ones for the core model.

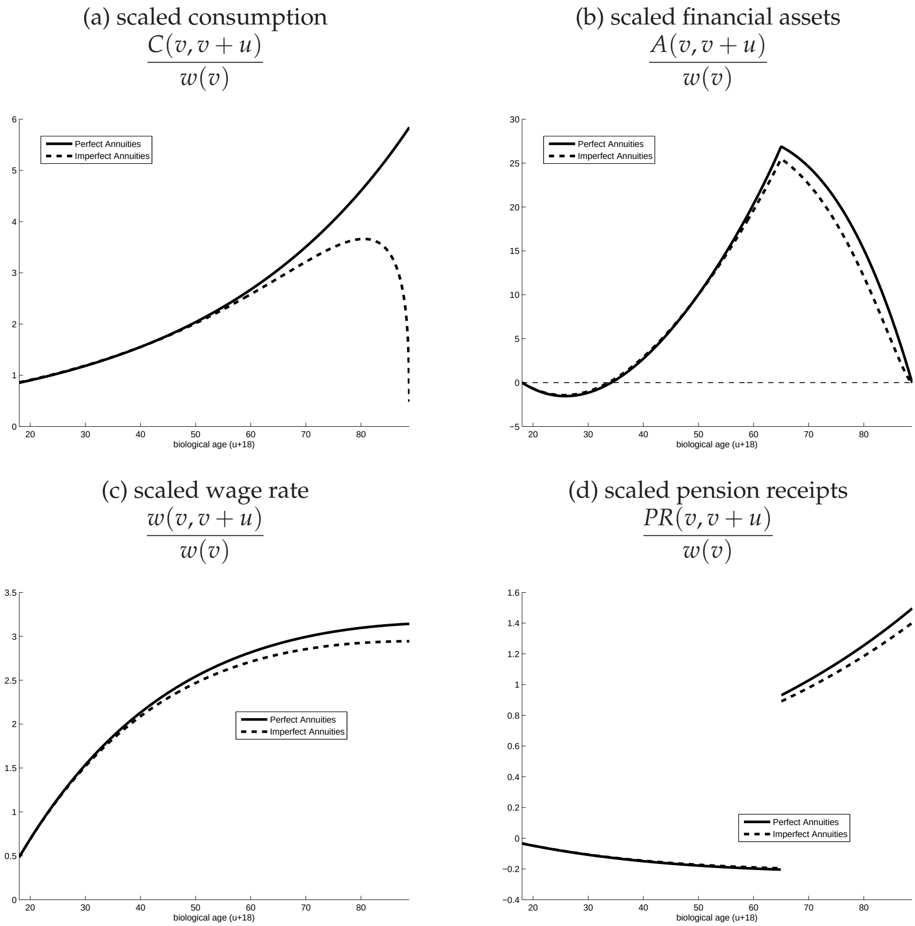
4.3 Ageing: the big picture

In this section we put our model to work on the big policy issue of demographic change. Population ageing remains one of the key issues in economic policy in the Netherlands. During the 2010 Dutch parliamentary election campaign numerous parties went so far as to call future policy on pensions and the retirement age a breaking point for the post-electoral coalition scramble. In this section we look at the big picture and study the effect of ageing and demographic change on the steady-state rate of economic growth of a country.[4]

[3] Bütler (2001) and Hansen and İmrohoroğlu (2008) also find that the hump occurs too late in life. Alessie and de Ree (2009, p. 113) decompose Dutch consumption into durables and non-durables. They find that non-durable consumption peaks at age 45 whereas durable consumption reaches its maximum at about age 41.

[4] For an accessible survey of the literature on the topic of population ageing and economic growth, see Bloom et al. (2008). Recent contributions using the endogenous growth framework include Fougère and

Figure 4.1. Life-cycle profiles and the role of annuity imperfections



We start our analysis with some stylized facts for the Netherlands.[5] In the period 2005-10 the crude birth rate is about $\beta = 1.13\%$ per annum whereas for 2035-40 it is projected to change to $\beta = 1.05\%$ per annum. The population growth rates are, respectively, $\pi = 0.41\%$ per annum for 2005-10 and $\pi = -0.01\%$ per annum 2035-40. Finally, the old-age dependency ratio is, respectively 23% in 2010 and 46% in 2040. We wish to simulate our model using a demographic shock which captures the salient features of these stylized facts. Since we restrict attention to steady-state comparisons in this paper, we make the strong assumption that the country finds itself in a demographic steady state both at present and in 2040.

4.3.1 A demographic shock

The demographic shock that we study is as follows.[6] First, we assume that the population growth rate changes from $\pi_0 = 0.5\%$ to $\pi_1 = 0\%$ per annum. Second, we use our estimated demographic process (4.8) but change the η_1 parameter in such a way that an old-age dependency ratio of 46% is obtained. Writing $e^{-M_i(u)} \equiv (\hat{\eta}_0 - e^{\eta_{1,i}u}) / (\hat{\eta}_0 - 1)$ the old-age dependency ratio can be written as:

$$dr\left(\pi_i, \eta_{1,i}\right) \equiv \frac{\int_{47}^{\bar{D}_i} e^{-\pi_i s - M_i(s)} ds}{\int_0^{47} e^{-\pi_i s - M_i(s)} ds}, \tag{4.31}$$

where $\bar{D}_i \equiv \left(1/\eta_{1,i}\right) \ln \eta_0$. Using this expression we find that η_1 changes from $\eta_{1,0} = \hat{\eta}_1 = 0.0680$ to $\eta_{1,1} = 0.0581$. The associated values for the crude birth rate are obtained by imposing the suitably modified demographic steady-state condition. We find that β changes in the model from $\beta_0 = 0.0204$ to $\beta_1 = 0.0151$. Figure 2(a) shows that the new instantaneous mortality profile shifts to the right. Figure 2(b) illustrates the change in the population composition. In the new steady state, the population distribution features less mass at lower ages and more at higher ages, i.e. the population pyramid becomes narrower and higher.

Mérette (1999), Futagami and Nakajima (2001), Heijdra and Romp (2006), and Prettner (2009).

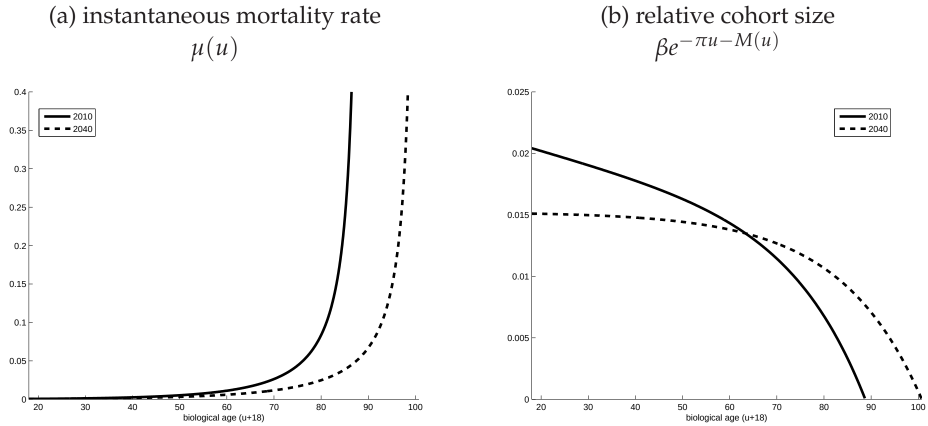
[5] These figures are taken from United Nations, *World Population Prospects: The 2008 Revision Population Data Base*, `http://esa.un.org/unpp`. We use data for the medium variant.

[6] In this chapter we focus on demographic changes induced by a change in the mortality rate. When we return to the analysis of demographic changes in Chapter 6 we study how demographic changes induced by a change in the birth rate differ from demographic changes induced by a change in the mortality rate.

The effect on the economic growth rate of the demographic shock depends critically on the type of pension system. We consider three scenarios. In the first scenario the pension system is DC, the contribution rate and retirement age are kept constant ($\theta_0 = 0.07$ and $R_0 = 47$), pension payments to the elderly are reduced to balance the budget of the PAYG system. Columns (c)–(d) in Table 2 report the results for the two cases with perfect (PA) and imperfect annuities (PA). Since the effects are qualitatively the same for PA and IA, we restrict attention to the latter case. Comparing columns (b) and (d) several features stand out. First, the ageing shock has a large effect on the supply of (efficiency units of) labour, i.e. n falls by more than fifteen percent. This is an obvious consequence of the fact that the population proportion of working-age persons declines. Second, the pension payments to retirees are almost halved. Third, notwithstanding the decrease in pensions, scaled consumption and human wealth at birth both increase dramatically. More people expect to survive into retirement and, once retired, the period of retirement is also increased. Fourth, the macroeconomic growth rate increases dramatically, from 1.91% to 3.27% per annum. The intuition behind this strong growth effect can be explained with the aid of Figure 3. The solid lines represents the core case of Table 2(b) and the dashed lines illustrate the results from Table 2(d). Following the demographic shock scaled consumption is uniformly higher and peaks at a later age. Scaled financial assets are somewhat lower during youth but much higher thereafter. As Figure 3(b) shows there is a huge savings response which explains the large increase in the macroeconomic growth rate. In conclusion, of the main growth channels identified by Bloom et al. (2008, p. 2), labour supply falls (and thus retards growth) but the capital accumulation effect is so strong as to lead to a strong positive effect of longevity on economic growth.

In the second scenario the pension system is DB, the pension payments and retirement age are kept constant ($\zeta_0 = 0.3632$ and $R_0 = 47$), and pension contributions by the young are increased to balance the budget of the PAYG system. Columns (e)–(f) in Table 2 give the results for this case. Comparing columns (b), (d) and (f) the following features stand out. First, the contribution rate increase is quite substantial, it almost doubles from $\theta_0 = 0.07$ to $\theta_1 = 0.1394$. Second, though scaled consumption, scaled human wealth, and the economic growth rate are higher than in the base case, they are lower than under the DC scenario. As Figure 3 shows, the capital accumulation effect

Figure 4.2. Demographic shock



of the longevity shock is substantially dampened under a DB system. Intuitively, by taking from the young and giving to the old the PAYG system redistributes from net savers to net dissavers.

Finally, in the third scenario both θ and ζ are kept at their pre-shock levels and the retirement age is increased to balance the budget of the PAYG system. Columns (g)–(h) in Table 2 give the results for this case. Comparing columns (b), (d), (f), and (h) the following features stand out. First, under the retirement age (RA) scenario the longevity shock necessitates an increase in the biological retirement age 65 to 75.3 years. i.e. the value of R restoring budget balance changes from $R_0 = 47$ to $R_1 = 57.3$. Second, compared to the DB and DC cases, labour supply increases strongly in the RA scenario. Third, the economic growth rate, though still higher than in the base case, is slightly lower that under DB and much lower than under DC. The intuition behind this result is clear from Figure 3(b) which shows that the savings response following the longevity shock is lower than either DB or DC.

The negative relationship between the retirement age and economic growth is surprising in light of the current (Dutch) policy debate in which an increase of the retirement age has become the paradigm for weathering the generational storm (see, for instance, Bovenberg and Gradus (2008)). This finding, however, fits well with the analysis of Bloom *et al.* (2007) who show that the recent increase in adult mortality has increased the savings rate in countries where the pension system contains a strong incentive to retire at the early eligibility age.[7]

The contrast between the findings from the literature and the policy debate is that the policy debate is predominantly occupied with the sustainability of government finances, which have come under pressure due to the additional influx of elderly into the receiving end of public pensions. The question arises whether the improvement of government finances due to an increase in the retirement age is not simply bought against a decreased incentive to save? We leave this interesting trade-off between government finances and the savings rate as an issue for future research because our current model is not equipped to give a compelling answer. However, we do emphasize that the analysis in this chapter highlights the fact that focusing solely on the sustainability of government finances may induce a policy with adverse long-run repercussions.

4.3.2 Robustness

The clear message emerging from the discussion so far is that the type of pension system in place has a quantitatively large influence on the link between longevity and macroeconomic growth. Indeed, the same longevity shock can either lead to a huge increase in growth (under DC) or only a modest increase (under RA). But how robust are these conclusions? As is pointed out by Bloom *et al.* (2008, p. 3), "population data are not sacrosanct" and UN predictions are revised substantially over time. In short, our stylized demographic facts may be more like "factoids".[8]

We study the robustness issue in Table 3. We restrict attention to the case with imperfect annuities, and column (a) in the table represents the base case. It coincides with the pre-shock steady state reported in Table 3(b). Columns (b)–(c) in Table 3 report the results under the DC scenario for alternative demographic shocks. In contrast, columns (d)-(e) show how a much more broadly defined PAYG system reacts to the original demographic shock under DC, DB, and RA.

[7] In accordance with our other findings Bloom *et al.* (2007) also find that positive impact of ageing on economic growth is mitigated if a PAYG system with high benefits is in place.

[8] De Waegenaere *et al.* (2010) provide a survey of the recent literature on longevity risk (i.e. the risk that mortality predictions turn out to be wrong). In accordance with Bloom *et al.* (2008) they show that estimates on future mortality rates differ substantially and depend on a plethora of uncertain factors.

In column (b) we assume that the old-age dependency ratio is 30% rather than 46% in 2040. As in the original shock we continue to assume that $\pi_1 = 0\%$ per annum. By using (4.31) we obtain new values for the demographic parameters, i.e. $\eta_{1,1} = 0.0662$ and $\beta_1 = \bar{\mu}_1 = 0.0172$. The alternative demographic shock causes a small increase in the economic growth rate. Whereas the original demographic shock caused growth to increase from 1.91% to 3.27% per annum (See Table 2, columns (b) and (d)), the alternative one only raises the growth rate to 2.33% per annum. The alternative ageing shock is relatively small, and pensions are reduced much less drastically than under the original demographic shock. The private savings response is quite small as a result.

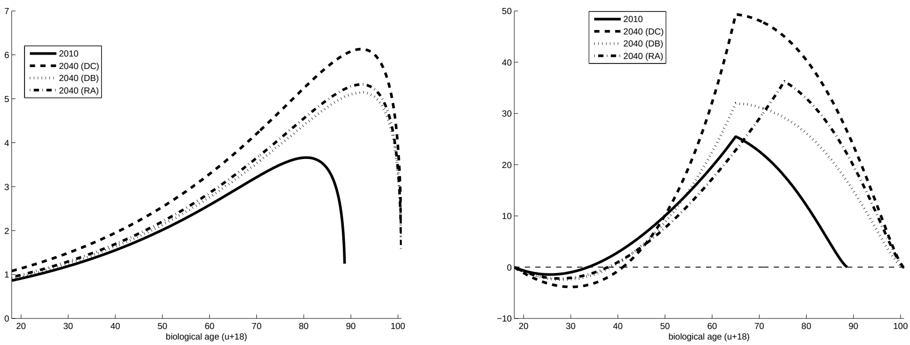
In column (c) we keep the dependency ratio at 46% but assume that the population growth rate is 0.5% rather than 0% per annum in 2040. Under this assumption the demographic parameters are equal to $\eta_{1,1} = 0.0540$, $\beta_1 = 0.0168$, and $\bar{\mu}_1 = 0.0118$. This type of demographic shock produces a huge increase in the macroeconomic growth rate. The intuition is the same as before – see the discussion relating to Table 2(d) above. The large growth effect is all the more surprising in view of the growth equation (T1.6) which directly features $-\pi$ on the right-hand side. So even though the demographic shock itself retards growth by 0.5% per annum, the huge private savings response more than compensates for this effect.

In conclusion, the two alternative demographic shocks give rise to qualitatively the same predictions as we obtained for the original shock. Under a DC system economic growth is boosted because the labour supply effect is strongly dominated by the capital accumulation effect.

As a final robustness check we investigate whether the *size* of the PAYG system influences the relationship between longevity and economic growth. We return to the original demographic shock featuring $\pi_1 = 0.05\%$ per annum and an old-age dependency ratio of 46% ($\eta_{1,1} = 0.0581$ and $\beta_1 = 0.0151$). As was pointed out by Broer (2001, p. 89), "in an ageing society, both the health insurance system and the pension system impose an increasing burden on households. ... Thus as the share of elderly in the population grows, the contribution base [of the public health insurance system, JM] shrinks at the same time when demand for health care increases." In short, it can be argued that the public health insurance system itself contains elements of a PAYG type, i.e. it taxes the young (and healthy) and provides resources to the old (and infirm).

Figure 4.3. Life-cycle profiles before and after the demographic shock

(a) scaled consumption $\frac{C(v,v+u)}{w(v)}$ (b) scaled financial assets $\frac{A(v,v+u)}{w(v)}$



Whereas it is beyond the scope of the present paper to fully model the health insurance system, we take from Broer's analysis the idea that the PAYG system may be broader than just the public pension system itself. We study the quantitative consequences of PAYG system size in columns (d)–(g) in Table 3. Column (d) shows what happens to the initial steady-state economy if the contribution rate is increased from $\theta_0 = 0.07$ to $\theta_1 = 0.15$. The comparison between columns (a) and (d) reveals that there is a huge drop in the growth rate, from $g = 1.91\%$ to $g = 1.19\%$ per annum. Intuitively the larger PAYG system takes more from the young and gives more to the old. This chokes off private savings and retards economic growth.

Columns (e)–(g) in Table 3 shows the effects of the original demographic shock under DC, DB, and RA. The growth increases under all scenarios with the largest effect occurring under the DC system. Interestingly, whereas the growth effect was smallest for the RA case in the original model with the narrowly defined PAYG system, for a large PAYG system it is smallest for the DB scenario.

4.3.3 Limitations

A few words of caution are in place when interpreting our conclusions. There are several limitations. First, our analysis consists of steady-state comparisons and space con-

Table 4.3. Alternative scenarios

	Initial PAYG system			Large PAYG system			
	(a)	(b)	(c)	(d)	(e)	(f)	(g)
		DC	DC		DC	DB	RA
$\frac{C(v,v)}{w(v)}$	0.8609	0.9268	1.0971	0.7047	0.8818	0.6109	0.7678
$\frac{H(v,v)}{w(v)}$	27.0207	29.4510	38.0243	22.1187	29.6744	20.5594	25.8384
g (%)	1.91	2.33	3.43	1.19	2.59	1.51	1.71
n	0.9675	0.9217	0.8155	0.9675	0.8212	0.8212	0.9589
$\frac{w(t)}{k(t)}$	0.2894	0.3038	0.3434	0.2894	0.3410	0.3410	0.2920
$\frac{c(t)}{w(t)}$	1.0570	1.0094	0.8465	1.0817	0.8918	0.9236	1.0715
ζ	0.3632	0.2796	0.1812	0.7783	0.3910	0.7783	0.7783
z	0.0200	0.0200	0.0111	0.0174	0.0130	0.0120	0.0148
θ						0.2986	
$R + 18$							75.3

Notes. Column (a) is the core model with imperfect annuities (column (b) in Table 2). Column (b) dependency ratio in 2040 equal to 30% instead of 46%. Column (c) population growth rate in 2040 equal to 0.5% instead of 0% per annum. Column (d) bigger PAYG system ($\theta = 0.15$). Columns (e)–(f) show the effects of the original demographic shock for the large PAYG system under DC and DB. Column (g) leaves θ and ζ unchanged and features a higher retirement age.

siderations prevent us from studying the transitional dynamics of a longevity shock. Although we find that in the steady state a longevity shock has beneficial effects on growth, it need not be the case that transition is monotonic. Second, we have merely analyzed growth but not individual welfare. However, as we assume exogenous labour supply, higher growth automatically translates into higher welfare because discounted income of individuals increases. Third, we have assumed that labour supply and the retirement age are exogenous. We have chosen this approach here in order to keep the model as simple as possible. Indeed, endogenization of both the hours decision over the life cycle and/or the retirement date is fairly straightforward – see e.g. Heijdra and Romp (2009) and Chapter 2 and 3 above. Fourth, we have ignored aggregate risk and the risk-sharing properties of pension systems. The interested reader is referred to Bovenberg and Uhlig (2008) who apply a two-period stochastic overlapping generations model featuring endogenous growth to study the consequences of particular pension systems on risk-sharing between generations. Finally, we have studied a closed economy. This is not a convincing representation of the Dutch economy which is extremely open and small in world markets. However, aging is a global phenomenon. Hence, to model a small open economy with fixed factor prices is equally unconvincing. Here we have chosen the closed economy framework to zoom in on the global consequences of ageing on capital accumulation and economic growth – the big picture.

4.4 Conclusions

In this chapter we apply the model developed in the previous chapters to study the relationship between aging and economic growth and the mediating role that government policy has on this relationship. We find that, in principle, aging increases the economic growth rate. However, if a defined benefit system is in place the growth effect weakens somewhat because of the increase in the contribution rate necessary to finance the additional pensioners. In order to circumvent this adverse effect on the growth rate, the government might consider to switch to a defined contribution system or to increase the retirement age. Surprisingly, we find that the latter policy option has adverse effects on the economy.

This chapter completes the analysis of various economic issues based on the model developed in Chapter 2. In the initial chapter we set out to analyse what the impact is of imperfect annuity markets on individual decisions and macroeconomic outcomes. In further analysis we found that the model is very versatile and can be used to study two important issues at the forefront of economic policy making. In Chapter 3 we applied to model to issues of taxation and in Chapter 4 to an analysis of pension systems. In the next chapter we return to the issue of annuities by asking whether opening up an annuity market is also welfare enhancing from the macroeconomic perspective. In Chapter 6 we close the analysis by returning to the issue of demographic change and study how different sources of demographic change affect the aggregate capital stock.