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Annuities, public policy and demographic change in overlapping generations models

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Chapter 2

Annuity market imperfections, retirement and economic growth*

* This chapter is based on Heijdra and Mierau (2009).

2.1 Introduction

One of the most robust findings in economic theory is that individuals facing an uncertain date of death derive great benefits from annuitization. In a seminal paper, Yaari (1965) showed that in the absence of a bequest motive individuals should fully annuitize all of their savings. One of the key assumptions adopted by Yaari concerns the availability of actuarially fair annuities. In a recent paper, Davidoff *et al.* (2005) have demonstrated that the full annuitization result holds in a much more general setting than the one adopted by Yaari, e.g. it obtains also when annuities are less than actuarially fair.

The objective of this chapter is to develop an overlapping generations model of finitely-lived households featuring annuity market imperfections. While the model lays the ground work for the chapters to come, we also use it to study the macroeconomic effects of annuity market imperfections. Are the optimal retirement age and the macroeconomic growth rate significantly affected by the degree of actuarial fairness of annuities or is this imperfection quantitatively unimportant? To answer this question we construct a stylized overlapping generations model of a closed economy featuring endogenous growth due to an inter-firm external effect of the “AK”-type.

Our starting point is the celebrated Blanchard (1985) model, featuring perfect annuities and age-independent mortality (perpetual youth). We extend this model in four directions. First, we endogenize the agent’s life-cycle labour supply decision. Second, we introduce an annuity imperfection parameter, which allows us to study the cases of actuarially fair and unfair annuities in one single framework. In the latter case, annuity firms make profits which are taxed away by the government and redistributed to households. Third, we introduce age-dependent labour efficiency. Fourth, we incorporate the insights of Heijdra and Romp (2008) and postulate an age-dependent mortality process.

Our main findings are as follows. First, the imperfection on the annuity market leads individuals to discount future consumption by their mortality rate as well as their pure rate of time preference. In a perpetual-youth model this leads to a flatter consumption profile. In an age-dependent mortality context this leads to a hump-shaped consumption profile. In both cases capital accumulation, and thereby economic growth, is depressed.

Second, in terms of labour supply we find that both in the perpetual-youth model and the age-dependent productivity model individuals supply less labour during their working life. However, in the perpetual-youth model individuals retire earlier whereas in the age-dependent mortality model individuals retire later. This discrepancy arises because the magnitude of profits made by annuity firms is lower in the age-dependent mortality case. Less profit means lower transfers and thus a smaller wealth effect via that channel.

Third, we show that the way in which annuity firms' profits are recycled plays a key role in the analysis. If these profits are redistributed in the form of lump-sum transfers to households, then the growth and retirement effects of even fairly substantial annuity market imperfections are quantitatively rather small. In contrast, if these profits are drained from the economy via wasteful government consumption, then growth deteriorates dramatically and the retirement age is reduced substantially.

Fourth, our analysis highlights the importance of a correctly modelled demography. Under the Blanchard (1985) assumption the impact of imperfect annuities is grossly overestimated. This is because for a properly modelled demography only the elderly are significantly affected by annuity market imperfections and of these elderly only a small portion is alive at any point in time. Hence, the individual as well as the aggregate effect is mild.

The two papers most closely associated with ours are Bütler (2001) and Hansen and İmrohoroğlu (2008). We extend the insights of Bütler (2001) to the general equilibrium case and explicitly take into account the impact of imperfect annuities on the retirement decision (as opposed to Hansen and İmrohoroğlu (2008)). Furthermore, we also study imperfect annuities in general equilibrium, not only on the individual level as Bütler (2001) and Hansen and İmrohoroğlu (2008).

Like Bütler (2001) and Hansen and İmrohoroğlu (2008) we find that imperfections on the annuity market lead to a hump-shaped profile in consumption. In addition we find that the imperfect annuity market leads to late retirement and depresses economic growth due to less capital accumulation. Furthermore, we find that labour supply during work life decreases. Finally, we show that the profits made by annuity firms play a key role in the analysis.

The remainder of the chapter is structured as follows. Section 2 sets out the core model, whilst section 3 studies the relationship between the annuity market imperfection, the retirement decision, and macroeconomic growth. Section 4 introduces the two extensions that allow our model to better resemble realistic features of life-cycle choices. Section 5 concludes.

2.2 Model

2.2.1 Firms

The production side of the model makes use of the insights of Romer (1989) and postulates the existence of sufficiently strong external effects operating between private firms in the economy. There is a large and fixed number, $\mathcal{N}$, of identical, perfectly competitive firms. The technology available to firm i is given by:

$$Y_i(t) = \Omega(t) K_i(t)^{\varepsilon_K} L_i(t)^{1-\varepsilon_K}, \qquad 0 < \varepsilon_K < 1, \tag{2.1}$$

where $Y_i(t)$ is output, $K_i(t)$ is capital use, $L_i(t)$ is the labour input, and $\Omega(t)$ represents the general level of factor productivity which is taken as given by individual firms. The competitive firm hires factors of production according to the following marginal productivity conditions:

$$w(t) = (1-\varepsilon_K)\Omega(t)\kappa_i(t)^{\varepsilon_K}, \tag{2.2}$$

$$r(t) + \delta = \varepsilon_K \Omega(t)\kappa_i(t)^{\varepsilon_K - 1}, \tag{2.3}$$

where $\kappa_i(t) \equiv K_i(t)/L_i(t)$ is the capital intensity, $w(t)$ is the wage rate, $r(t)$ is interest rate and δ is the depreciation rate. The rental rate on each factor is the same for all firms, i.e. they all choose the same capital intensity and $\kappa_i(t) = \kappa(t)$ for all $i = 1, \cdots, \mathcal{N}$. This is a very useful property of the model because it enables us to aggregate the microeconomic relations to the macroeconomic level.

Generalizing the insights of Saint-Paul (1992, p. 1247) and Romer (1989) to a growing population, we assume that the inter-firm externality takes the following form:

$$\Omega(t) = \Omega_0 \kappa(t)^{1-\varepsilon_K}, \tag{2.4}$$

where Ω_0 is a positive constant, $\kappa(t) \equiv K(t)/L(t)$ is the economy-wide capital intensity, $K(t) \equiv \sum_i K_i(t)$ is the aggregate capital stock, and $L(t) \equiv \sum_i L_i(t)$ is aggregate employment. According to (2.4), total factor productivity depends positively on the aggregate capital intensity, i.e. if an individual firm i raises its capital intensity, then *all* firms in the economy benefit somewhat because the general productivity indicator rises for all of them. Using (2.4), equations (2.1)–(2.3) can now be rewritten in aggregate terms:[1]

$$\begin{aligned} Y(t) &= \Omega_0 K(t), & (2.5)\\ w(t)L(t) &= (1-\varepsilon_K)Y(t), & (2.6)\\ r(t) &= r = \varepsilon_K \Omega_0 - \delta, & (2.7) \end{aligned}$$

where $Y(t) \equiv \sum_i Y_i(t)$ is aggregate output and we assume that capital is sufficiently productive, i.e. $\varepsilon_K \Omega_0 - \delta > 0$. The aggregate technology is linear in the capital stock and the interest rate is constant.[2]

2.2.2 Consumers

Individual behaviour

We generalize the Blanchard (1985) model of consumer behaviour by including an endogenous labour-leisure decision and by assuming potentially imperfect annuity markets. At time t, expected remaining-lifetime utility of an individual born at time v

[1] All firms use the same capital intensity ($\kappa_i(t) = \kappa(t)$), so that $Y_i(t) = \Omega(t) L_i(t) \kappa(t)^{\varepsilon_K}$ and $Y(t) = L(t)\Omega(t)\kappa(t)^{\varepsilon_K}$. By using (2.4) in this expression, we find (2.5). For the wage we find $w(t) = (1-\varepsilon_K)\Omega(t)\kappa(t)^{\varepsilon_K} = (1-\varepsilon_K)\Omega_0\kappa(t)$, which can be rewritten to get (2.6). Finally, for the rental rate on capital we find $r(t)+\delta = \varepsilon_K \Omega(t)\kappa(t)^{\varepsilon_K-1} = \varepsilon_K\Omega_0$.

[2] Romer (1989, p. 90) makes $\Omega(t)$ dependent on the stock of capital $K(t)$, an approach also adopted by Saint-Paul (1992, p. 1247). Romer rationalizes his formulation by appealing to the public good character of knowledge and by assuming that physical capital and knowledge are produced in constant proportions. Both Romer and Saint-Paul assume a constant labour force. In order to accommodate population growth, we make the knowledge spillover dependent on the capital intensity. Note that the original Romer specification would result in $r(t)+\delta = \varepsilon_K\Omega_0 L(t)^{\varepsilon_K-1}$, i.e. a downward trend in the real interest rate *contra* Kaldor's stylized facts.

$(v \leq t)$ is given by:

$$\mathbb{E}\Lambda\left(v,t\right) \equiv \int_{t}^{\infty} \ln\left[C(v,\tau)^{\varepsilon_C} \cdot [1-L(v,\tau)]^{(1-\varepsilon_C)}\right] \cdot e^{(\rho+\mu)(t-\tau)} d\tau, \tag{2.8}$$

where $C\left(v,\tau\right)$ is consumption, $L\left(v,\tau\right)$ is labour supply (the time endowment is equal to unity), ρ is the pure rate of time preference, and μ is the instantaneous mortality rate.[3]

The agent's budget identity is given by:

$$\dot{A}(v,\tau) = r^A A(v,\tau) + w(\tau) L\left(v,\tau\right) - C(v,\tau) + TR\left(v,\tau\right), \tag{2.9}$$

where $A\left(v,\tau\right)$ is the stock of financial assets, r^A is the annuity rate of interest, $w\left(\tau\right)$ is the wage rate, and $TR\left(v,\tau\right)$ are lump-sum transfers from the government (see below), all defined in real terms. Following Yaari (1965), we postulate the existence of annuity markets, but unlike Yaari we allow the annuities to be less than actuarially fair. Since the agent is subject to lifetime uncertainty and has no bequest motive, he/she will fully annuitize so that the annuity rate of interest facing the agent is given by:

$$r^A \equiv r + \theta\mu, \tag{2.10}$$

where r is the real interest rate (see (2.7)), and θ is a parameter $(0 \leq \theta \leq 1)$. Our specification for the annuity rate can be rationalized in three ways. First, $1-\theta$ may be interpreted as a load factor needed to cover the administrative costs of organizing the annuity firm – see Horneff *et al.* (2008, p. 3595). Second, as Hansen and İmrohoroğlu (2008, p. 569) suggest, θ may represent the fraction of assets that are annuitized. Provided θ is strictly less that unity, there will be unintended bequests under this interpre-

[3] As is well known from the Real Business Cycle (RBC) literature, in the presence of technological change, certain restrictions must be imposed on preferences in order to allow for a meaningful steady state to exist. King *et al.* (2002, p. 94-95) show that the only admissible felicity functions take the following form:

$$U \equiv \frac{1}{1-\sigma} C^{1-\sigma} \cdot v\left(1-L\right), \qquad \sigma \neq 1, \quad \sigma > 0 \tag{A}$$

$$\equiv \ln C + v\left(1-L\right) \tag{B}$$

With specification (A), $v\left(1-L\right)$ must be increasing and concave if $\sigma < 1$ and decreasing and convex if $\sigma > 1$. Further restrictions are needed to ensure overall concavity. Under specification (B), all we need is that $v\left(1-L\right)$ is increasing and concave. Our function is a special case of (B) with $v\left(1-L\right)$ log-linear in leisure.

tation. Third, annuity firms may possess some market power, allowing them to make a profit by offering a less than actuarially fair annuity rate. In this chapter, we adopt the market-power interpretation. We shall refer to $1-\theta$ as the degree of imperfection in the annuity market.[4]

Our specification is quite general and incorporates three important cases:

- *Perfect annuities* (PA). The case of perfect (actuarially fair) annuities is obtained by setting $\theta = 1$. Life insurance companies break even, and $TR(v,\tau) = 0$.
- *Imperfect annuities* (IA). The case of imperfect (less than actuarially fair) annuities is obtained by assuming $0 < \theta < 1$. Life insurance companies make excess profits, $\mu(1-\theta)A(\tau)$, which are taxed away by the government and distributed in a lump-sum fashion to surviving agents.
- *No annuities* (NA). For $\theta = 0$ there are no annuity markets. The agent can save at the interest rate r, but borrowing is impossible because, with lifetime uncertainty, he/she faces a probabilistic time-of-death wealth constraint of the form, $\text{prob}\{A(v,\tau) \geq 0\} = 1$ (Yaari, 1965, p. 139). By definition, $TR(v,\tau) = 0$.

In the remainder of this chapter we restrict attention to the PA and IA cases.

The agent chooses time profiles for $C(v,\tau)$, $A(v,\tau)$, and $L(v,\tau)$ (for $\tau \geq t$) in order to maximize (2.1), subject to (i) the budget identity (2.2), (ii) a No Ponzi Game (NPG) condition, $\lim_{\tau\to\infty} A(v,\tau)\, e^{(r+\theta\mu)(t-\tau)} = 0$, (iii) the initial asset position in the planning period, $A(v,\tau)$, and (iv) a non-negativity condition, $L(v,\tau) \geq 0$. We restrict attention to the optimal individual life-cycle decisions in the context of an economy moving along a steady-state balanced growth path.

Along the balanced growth path, labour productivity grows at a constant exponential rate, γ (see below), and as a result individual agents face an upward sloping path

[4] Another explanation for the overpricing of annuities is adverse selection (Finkelstein and Poterba, 2002). That is, agents with a low mortality rate are more likely to buy annuities than agents with high mortality rates. However, because mortality is private information annuity firms "mis-price" annuities for low-mortality agents, thus creating a load factor. Abel (1986) and Heijdra and Reijnders (2009) study this adverse selection mechanism in a general equilibrium model featuring healthy and unhealthy people and with health status constituting private information. The unhealthy get a less than actuarially fair annuity rate whilst the healthy get a better than actuarially fair rate for part of life. An alternative source of imperfection may arise from the way that the annuity market is structured. Yaari (1965) assumes that there is a continuous spot market for annuities. In reality, however, investments in annuities are much lumpier. See Pissarides (1980) for an early analysis of this issue.

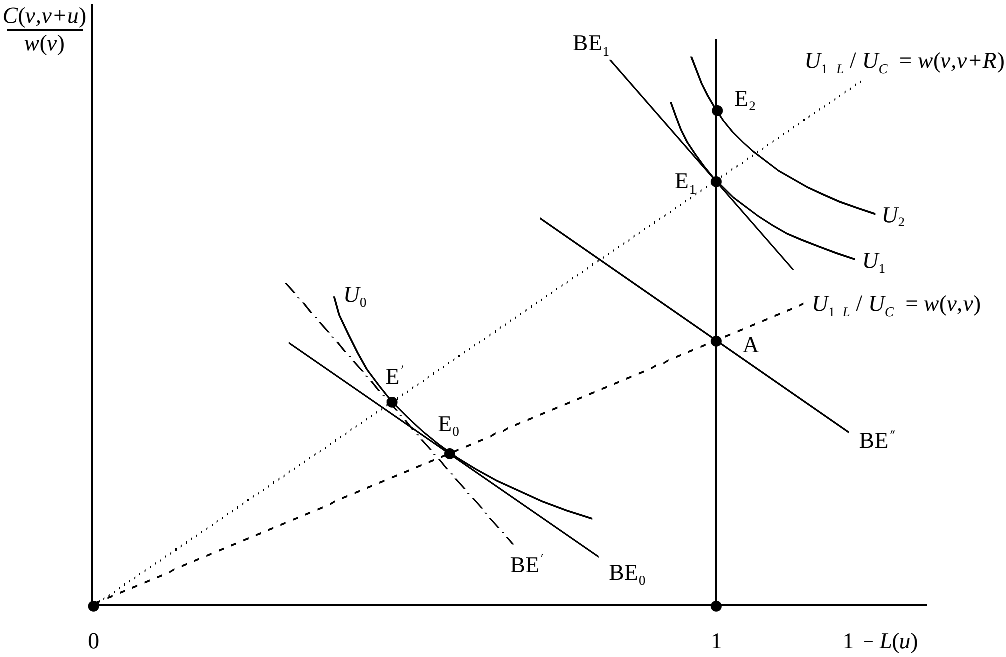

Figure 2.1. Life-cycle consumption, labour supply, and retirement

for real wages over their lifetimes:

$$w(\tau) = w(v)\, e^{\gamma(\tau - v)}. \tag{2.11}$$

The individual consumption Euler equation is given by:

$$\frac{\dot{C}(v,\tau)}{C(v,\tau)} = r - \rho - (1-\theta)\,\mu > 0. \tag{2.12}$$

With imperfect annuities, individual consumption growth is affected by the mortality rate, a result first demonstrated for the case with $\theta = 0$ by Yaari (1965, p. 143). During the working period, the agent equates the marginal rate of substitution between leisure and consumption to the wage rate at all times:

$$\frac{(1-\varepsilon_C)\,/\,(1-L(v,\tau))}{\varepsilon_C / C(v,\tau)} = w(\tau). \tag{2.13}$$

The consumption-leisure choice is illustrated in Figure 1, where $C(v, v+u)\,/w(v)$ and

$L(u)$ stand for, respectively, consumption (scaled by the wage rate at birth) and labour supply of the agent at age u. The initial choice at age $u = 0$ is at point E_0 where there is a tangency between an indifference curve (labeled U_0) and a "budget line" (labeled BE_0).[5] If there were no economic growth, the wage rate would be constant over the agent's lifetime and the optimum would gradually move along the dashed line from E_0 to A at which point it is optimal to retire. This move reflects the positive wealth effect on the demands for consumption and leisure. After retirement, the agent would move along the vertical leisure constraint in the direction of points E_1 and E_2.

Matters are slightly more complicated in the presence of economic growth and an upward sloping wage profile (2.11). Over the agent's life the utility-expansion path rotates in a counter-clockwise fashion inducing substitution effects. In terms of Figure 1, the agent retires at point E_1 where the marginal rate of substitution between leisure and consumption is equal to $w(R)$, where R stands for this agent's *age* at retirement. Using the dotted utility-expansion line through point E_1 we find that the total effect on consumption and leisure during working life is given by the move from E_0 to E_1. The pure substitution effect is given by the move from E_0 to E', and the wealth effect is the move from E' to E_1.

Armed with this graphical apparatus we can explain the following analytical expressions. Consumption of a newborn is given by:

$$C(v,v) = \frac{\varepsilon_C(\rho+\mu)}{\varepsilon_C + (1-\varepsilon_C)\left[1-e^{-(\rho+\mu)R(v)}\right]} \cdot LI(v,v), \qquad (2.14)$$

where $R(v)$ is the retirement age chosen by an agent born at time v, and $LI(v,v)$ is lifetime income of the agent:

$$LI(v,v) = w(v) \cdot \frac{1-e^{-(r-\gamma+\theta\mu)R(v)}}{r-\gamma+\theta\mu} + LT(v,v), \qquad (2.15)$$

[5] During the working period, the budget line is given by:

$$X(v,\tau) = w(\tau)\left[1-L(v,\tau)\right] + C(v,\tau),$$

where $X(v,\tau)$ is full consumption. The line BE_0 is obtained by substituting $X(v,v)$.

where $LT(v,v)$ are lifetime transfers received from the government:

$$LT(v,v) \equiv \int_v^\infty TR(v,\tau)\, e^{(r+\theta\mu)(v-\tau)} d\tau. \tag{2.16}$$

Equation (2.14) shows that consumption of a newborn is proportional to lifetime income. The marginal propensity to consume out of lifetime income is decreasing in the retirement age. Equation (2.15) provides the definition of lifetime income. The first term on the right-hand side represents the present value of the time endowment during working life, using the growth-corrected annuity rate of interest $(r-\gamma+\theta\mu)$ for discounting. The later one retires, the larger is this term. The second term on the right-hand side of (2.15) is just the present value of transfers, defined in (2.16).

Point E_1 in Figure 1 is attained at the point where consumption satisfies:

$$C(v, v+R(v)) = \frac{\varepsilon_C}{1-\varepsilon_C} w(v)\, e^{\gamma R(v)}. \tag{2.17}$$

By using (2.12) we find that $C(v,\tau) = C(v,v)\, e^{(r-\rho-(1-\theta)\mu)(\tau-v)}$ so that (2.17) can be rewritten as:

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} e^{-[r-\gamma-\rho-(1-\theta)\mu]R(v)}. \tag{2.18}$$

Equations (2.14) (with (2.15) substituted), and (2.18) represent a simultaneous system implicitly determining $C(v,v)/w(v)$ and $R(v)$ as a function of the structural parameters (ε_C, ρ, μ, r, and θ), the macroeconomic growth rate (γ), and scaled lifetime transfers ($LT(v,v)/w(v)$).

We illustrate the optimal retirement choice in Figure 2. This figure is based on the following parameter settings. The interest rate is set at six percent per annum ($r = 0.06$) whilst the rate of time preference is three and a half percent ($\rho = 0.035$). These values imply that in the presence of perfect annuities, individual consumption grows at 2.5 percent per annum (see (2.12)). The instantaneous mortality rate is estimated with Dutch mortality data for the cohort born in 1960 (see below for details). This yields a value of 1.26 percent per annum ($\mu = 0.0126$), implying an expected remaining lifetime of 79.4 years. We assume that labour productivity growth equals two percent per annum ($\gamma = 0.02$), and set the utility parameter for consumption at such a value that the optimal retirement age with perfect annuities is $R = 42$ years. This yields a

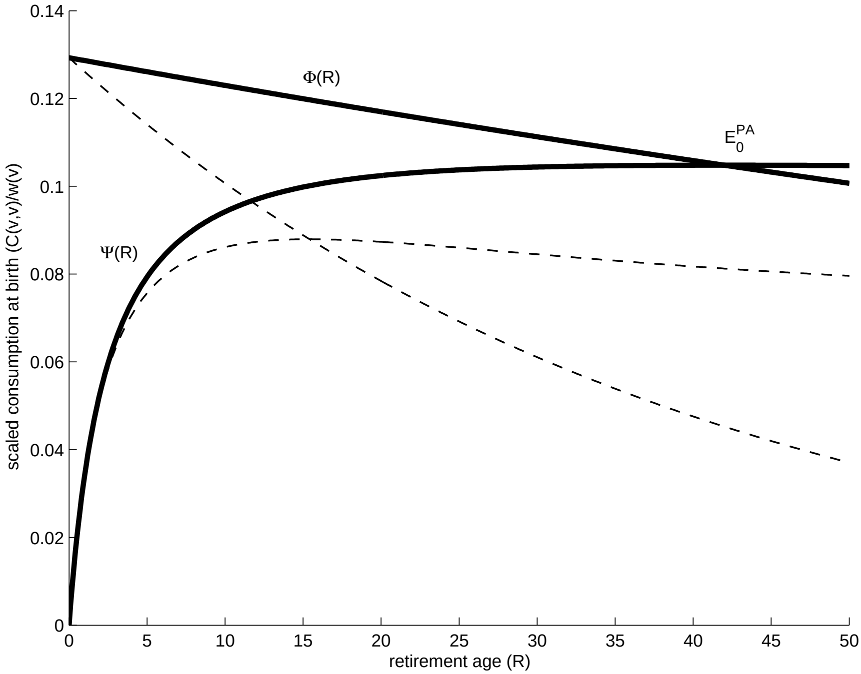

Figure 2.2. Optimal retirement age

value of $\varepsilon_C = 0.1145$. Finally, we assume that annuities are perfect, i.e. $\theta = 1$ in Figure 2. This simplifies matters somewhat because $LT\,(v, v) = 0$ for this case.

In Figure 2, the $\Psi\,(R)$ function plots the combinations between $C\,(v, v)\,/w\,(v)$ and $R\,(v)$ implied by equations (2.14)–(2.15) (with $LT\,(v, v) = 0$ imposed). Despite the fact that the marginal propensity to consume is a downward sloping function of the retirement age, lifetime income is sharply increasing in the retirement age and $\Psi\,(R)$ is upward sloping as a result. The downward sloping $\Phi\,(R)$ function plots equation (2.18) and intersects $\Psi\,(R)$ at point E_0^{PA}. There is a unique optimal retirement age which, for the parameters used here, equals $R = 42$.

Figure 2 also illustrates the partial equilibrium effects of a change in the macroeconomic growth rate, γ. Indeed, the thin dashed lines depict the $\Phi\,(R)$ and $\Psi\,(R)$ functions for the zero-growth case ($\gamma = 0$), for which the optimal retirement age is $R = 15.6$ years. In terms of Figure 1, this is the case where the agent moves from E_0

to A along the dashed utility-expansion curve. With a flat wage profile, equilibrium consumption at birth (and at all ages) and the retirement age are both lower than with an upward sloping wage profile. Finally, we note that an increase in lifetime transfers leads to an upward shift in $\Psi(R)$, higher consumption at birth and a lower retirement age. The transfers thus cause a negative wealth effect on the optimal retirement age.

With imperfect annuities ($0 < \theta < 1$) we must confront the issue of redistribution of excess profits and recognize the fact that $LT(v, v)$ will be positive in general. To keep things as simple as possible, we assume that the lump-sum transfers are set according to:

$$TR(v, \tau) = z \cdot w(\tau), \tag{2.19}$$

where z is a positive parameter, that is taken as given by individual agents but determined endogenously in general equilibrium via the balanced budget requirement of the redistribution scheme (see below). By using (2.19) in (2.16) we find:

$$\frac{LT(v, v)}{w(v)} \equiv \frac{z}{r - \gamma + \theta\mu}. \tag{2.20}$$

It is easy to show that z is constant along the balanced growth path.[6] Equations (2.14)–(2.15), (2.18), and (2.20) in combination imply that the retirement age is independent of v, i.e. $R(v) = R$ for all v. We summarize this important result in the following proposition.

Proposition 2.1. *Consider lump-sum redistribution of excess profits of life-insurance companies, of the form $TR(v, \tau) = z \cdot w(\tau)$. In that case: (i) the optimal retirement age is independent of v, i.e. $R(v) = R$ for all v; (ii) the optimal ratio between consumption at birth and the wage rate at birth is independent of v, i.e. $C(v, v) / w(v) = \frac{\varepsilon_C}{1-\varepsilon_C} e^{-(r-\gamma-\rho-(1-\theta)\mu)R}$ for all v.*

[6] An alternative feasible redistribution scheme would set $TR(v, \tau) = z \cdot w(v)$, implying that $LT(v, v) / w(v) = z / (r + \theta\mu)$ along the balanced growth path. Interestingly, "actuarially fair" lump-sum redistribution, setting transfers according to $TR(v, \tau) = \mu(1 - \theta) A(v, \tau)$, is infeasible. Under such a scheme, $LT(v, v)$ becomes unbounded which is clearly infeasible.

Aggregate household behaviour

In this subsection we derive expressions for per-capita average consumption, saving, and labour supply. We allow for constant population growth π and distinguish between the birth rate, β, and the mortality rate rate, μ, so that $\pi \equiv \beta - \mu$. The relative cohort weights evolve according to:

$$p(v,t) \equiv \frac{P(v,t)}{P(t)} = \beta e^{\beta(v-t)}, \qquad t \geq v, \tag{2.21}$$

where $P(v,t)$ is the size of cohort v at time t and $P(t)$ is the total population. Using (2.21), we can define per-capita average values in general terms as:

$$x(t) \equiv \int_{-\infty}^{t} p(v,t)\, X(v,t)\, dv, \tag{2.22}$$

where $X(v,t)$ denotes the variable in question at the individual level and $x(t)$ is the per capita average value of the same variable.

Off the steady-state growth path, exact analytical aggregation of the individual behavioural decision rules is impossible. To see why this is the case, note, for example, that consumption of workers features an age-dependent propensity to consume out of age-dependent wealth making aggregation impossible. We, therefore, focus on steady-state relationships. We know that $R(v) = R$ for all v, so for consumption we find:

$$C(v,v) = \frac{\varepsilon_C}{1-\varepsilon_C} w(v)\, e^{-[r-\gamma-\rho-(1-\theta)\mu]R}, \tag{2.23}$$

$$C(v,t) = C(v,v)\, e^{[r-\rho-(1-\theta)\mu](t-v)}, \tag{2.24}$$

whilst the wage rate satisfies equation (2.11). Using (2.22), per capita average consumption is thus given by:

$$c(t) \equiv \int_{-\infty}^{t} p(v,t)\, C(v,t)\, dv \equiv \frac{C(v,v)}{w(v)} \cdot \frac{\beta w(t)}{\gamma+\beta+\rho+(1-\theta)\mu - r}. \tag{2.25}$$

It follows from (2.13) and (2.23)-(2.24) that labour supply of workers in period t

$(t - v \leq R)$ can be written as:

$$L(v, t) = 1 - e^{-[r-\gamma-\rho-(1-\theta)\mu](R+v-t)}. \tag{2.26}$$

Since $L(v, t) = 0$ for retirees $(t - v > R)$, per capita average labour supply is equal to:

$$\begin{aligned} l(t) &\equiv \int_{t-R}^{t} p(v, t) L(v, t) \, dv \\ &= \left[1 - e^{-\beta R}\right] - \beta e^{-\beta R} \cdot \frac{e^{[\gamma+\beta+\rho+(1-\theta)\mu-r]R} - 1}{\gamma + \beta + \rho + (1-\theta)\mu - r} \equiv l, \end{aligned} \tag{2.27}$$

with $0 < l < 1$. The term in square brackets on the right-hand of (2.27) provides the first mechanism by which l falls short of unity: agents retire and their unit time endowment is consumed in full in the form of leisure. The second composite term on the right-hand side of (2.27) represents the other mechanism by which l falls short of unity: as workers age they reduce their labour supply.

At the individual level, financial assets are accumulated according to:

$$\dot{A}(v, t) = (r + \theta\mu) A(v, t) + w(t) L(v, t) + zw(t) - C(v, t), \tag{2.28}$$

where $L(v, t) = 0$ for retirees (for $t - v > R$). Per capita aggregate assets are defined as $a(t) \equiv \int_{-\infty}^{t} p(v, t) A(v, t) \, dv$ so that:

$$\dot{a}(t) = \int_{-\infty}^{t} p(v, t) \dot{A}(v, t) \, dv - \beta a(t), \tag{2.29}$$

where we have incorporated the fact that individual agents are born bare of financial assets $(A(v, v) = 0)$ and that cohort shares evolve over time according to $\dot{p}(v, t) = -\beta p(v, t)$. Substituting (2.28) into (2.29) and noting (2.27) we obtain:

$$\dot{a}(t) = (r + \theta\mu - \beta) a(t) + w(t) l(t) + zw(t) - c(t). \tag{2.30}$$

The balanced-budget requirement for the lump-sum redistribution scheme is given in per capita terms by:

$$\mu(1 - \theta) a(t) = zw(t). \tag{2.31}$$

Table 2.1. Balanced growth and retirement in the core model

(a) Microeconomic relationships:

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C(\rho+\mu)}{\varepsilon_C+(1-\varepsilon_C)\left[1-e^{-(\rho+\mu)R}\right]}\cdot\frac{1-e^{-(r-\gamma+\theta\mu)R}+z}{r-\gamma+\theta\mu} \quad \text{(T1.1)}$$

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C}e^{-(r-\gamma-\rho-(1-\theta)\mu)R} \quad \text{(T1.2)}$$

(b) Macroeconomic relationships:

$$z = \mu(1-\theta)\frac{k(t)}{w(t)} \quad \text{(T1.3)}$$

$$\gamma \equiv \frac{\dot{k}(t)}{k(t)} = r-\pi+\left[l-\frac{c(t)}{w(t)}\right]\cdot\frac{w(t)}{k(t)} \quad \text{(T1.4)}$$

$$\frac{w(t)\,l}{k(t)} = (1-\varepsilon_K)\,\Omega_0 \quad \text{(T1.5)}$$

$$l \equiv 1-e^{-\beta R}-\beta e^{-\beta R}\frac{e^{[\gamma+\beta+\rho+(1-\theta)\mu-r]R}-1}{\gamma+\beta+\rho+(1-\theta)\mu-r} \quad \text{(T1.6)}$$

$$\frac{c(t)}{w(t)} \equiv \frac{\beta}{\gamma+\beta+\rho+(1-\theta)\mu-r}\cdot\frac{C(v,v)}{w(v)} \quad \text{(T1.7)}$$

Definitions: Endogenous are $C(v,v)/w(v)$, R, z, γ, l, $w(t)/k(t)$, and $c(t)/k(t)$. Parameters: birth rate β, mortality rate μ, population growth rate $\pi \equiv \beta-\mu$, imperfection annuities θ, rate of time preference ρ, capital coefficient in the technology ε_K, consumption coefficient in tastes ε_C, scale factor in the technology Ω_0. The interest rate is $r \equiv \varepsilon_K\Omega_0-\delta$, where δ is the depreciation rate of capital.

Finally, by substituting (2.31) into (2.30) we obtain:

$$\dot{a}(t) = (r+\mu-\beta)\,a(t)+w(t)\,l(t)-c(t). \quad (2.32)$$

Like in the standard case with perfect annuities, the aggregate per capita annuity receipts, $\theta\mu a(t)$, do not feature directly in (2.32) because they constitute pure transfers from the dead to the living. In each period, life insurance companies receive $\mu a(t)$ from the estates of the deceased and pay $\theta\mu a(t)$ to their surviving customers. The resulting profit, $(1-\theta)\,\mu a(t)$, is taxed away by the government and redistributed to the surviving agents. The transfers are eliminated from the per capita average asset accumulation equation.

2.2.3 Balanced growth path

The capital market equilibrium condition is given by $A(t) = K(t)$. In per capita average terms we thus find:

$$a(t) = k(t), \tag{2.33}$$

where $k(t) \equiv K(t)/P(t)$ is the per capita stock of capital. From (2.5)-(2.6) we easily find:

$$\begin{aligned} y(t) &= \Omega_0 k(t), && (2.34)\\ w(t)\,l(t) &= (1-\varepsilon_K)\,y(t), && (2.35)\end{aligned}$$

where $y(t) \equiv Y(t)/P(t)$ is per capita output.

The macroeconomic growth model has been written in a compact format in Table 1. Equation (T1.1) is obtained by substituting (2.15) and (2.20) into (2.14). Equation (T1.2) is the same as (2.23). Equation (T1.3) is (2.31) with (2.33) substituted. Equation (T1.4) is obtained by substituting (2.33) into (2.32). Equation (T1.5) is obtained by combining (2.34)-(2.35) and noting (2.27). Equation (T1.6) is the same as (2.27). Finally, (T1.7) is the same as (2.25).

The model features a two-way interaction between the microeconomic decisions and the macroeconomic outcomes. Equations (T1.1)-(T1.2) determine scaled newborn consumption, $C(v,v)/w(v)$, and the optimal retirement age, R, as a function of the key macroeconomic variables. Equations (T1.3)-(T1.7) determine equilibrium transfers, z, the macroeconomic growth rate, γ, the overall wage-capital ratio, $w(t)/k(t)$, aggregate labour supply, l, and the $c(t)/w(t)$ ratio as a function of the optimal retirement age and scaled newborn consumption.

2.3 Retirement, growth and annuities

In this section we compute and visualize the comparative static general equilibrium effects for the core model of Table 1. To compute the initial general equilibrium we assume that annuities are perfect ($\theta = 1$) and use the coefficient values mentioned above (in the paragraph below equation (2.18)). We assume that rate of population growth is one percent per annum ($\pi = 0.01$). Since $\pi \equiv \beta - \mu$, this implies that, for the mortality rate that was postulated above, the birth rate is $\beta = 0.0226$. The capital depreciation rate is ten percent per annum ($\delta = 0.10$). We use the efficiency parameter of capital as a calibration parameter and find $\varepsilon_K = 0.8348$.[7] It follows that the constant in the production function is equal to $\Omega_0 = (r + \delta) / \varepsilon_K = 0.1917$. The initial steady-state growth path has the following features: $C(v, v) / w(v) = 0.1048$, $R = 42$, $z = 0$, $\gamma = 0.02$, $l = 0.0691$, $c(t) / w(t) = 0.1346$, and $w(t) / k(t) = 0.4583$. For convenience these values are restated in the first column in Table 2(a).

Figure 3 visualizes some of the key features of the calibration. Figure 3(a) depicts the *general equilibrium* determination of the retirement age and the macroeconomic growth rate. The solid line represents the microeconomic equilibrium condition, i.e. it depicts (γ, R) combinations for which (T1.1) and (T1.2) are equated (recall that $z = 0$ in the base case, so the microeconomic equilibrium can be computed conditional on the macroeconomic growth rate only). In Figure 3(a), the dashed line depicts the macroeconomic equilibrium conditions, i.e. it depicts (γ, R) combinations for which (T1.3)–(T1.7) are satisfied. The equilibrium is at point E_0, where the two lines intersect.

Figure 3 also illustrates the steady-state age profiles for the key variables (solid lines). Figure 3(b) shows that scaled consumption is exponential in the agent's age. Figure 3(c) shows that the agent gradually reduces the number of hours supplied to the labour market, and retires permanently at age $R = 42$. Finally, Figure 3(d) shows that the path of financial assets is monotonically increasing in age, and features a slight kink at the retirement age.

Next we consider the equilibrium under imperfect annuities. Instead of setting $\theta = 1$, we simulate the model with a value of $\theta = 0.70$ and keep all other parameters the same.[8] The new equilibrium values for the different variables are reported in the second column in Table 2(a). Obviously, with imperfect annuities lump-sum transfers become positive. Interestingly, agents reduce lifetime labour supply slightly but retire at about the same age as under perfect annuities.

[7] This is, of course, an implausibly high value, signalling that it is hard to obtain a calibration for the core model that yields plausible values for all parameters. Below we introduce some model extensions that allow us to substantially improve the quality of the calibration in this respect.

[8] Friedman and Warshawsky (1988, p. 59) estimate a load factor of 48 cents per dollar of expected present value. They suggest that 15 cents of this amount may be due to adverse selection and the remaining 33 cents due to costs, taxes, and profit.

The new growth rate is about thirty five basis points lower than under perfect annuities. In Figure 4 we visualize the general equilibrium effects of θ on the retirement decision and scaled consumption of a newborn. The solid lines depict the case with perfect annuities ($\theta = 1$). The equilibrium is at point E^{PA}. The thick dashed lines illustrate the case with imperfect annuities ($\theta = 0.70$), taking into account the general equilibrium effects on γ and z. The equilibrium with imperfect annuities is at point E^{IA}, which lies north-east of point E^{PA}. Agents retire slightly later on in life and consume more at birth. The thinly dashed line in Figure 4 depicts the $\Psi(R)$-line for imperfect annuities, but assuming that the transfers are zero. The total effect of the move from E^{PA} to E^{IA} can thus be decomposed into a part that is caused by the effect of the growth rate, and a part that is caused by lump-sum transfers.

In order to better understand these growth effects, we use (2.34), (T1.5) in (T1.4) to obtain:

$$\gamma = r - \pi + \Omega_0 \cdot \left[1 - \varepsilon_K - \frac{c(t)}{y(t)}\right]. \tag{2.36}$$

The model features an inverse relationship between the growth rate and the macroeconomic consumption-output ratio. In the bottom row of Table 2(a) we find that the decrease in the growth rate is accompanied by an increase in the consumption-output ratio from 0.3217 to 0.3398.

The new steady-state age profiles for the imperfect annuity case have been illustrated in Figures 3(b)–(d) (see the dashed lines). The growth rate in individual consumption is reduced somewhat because $-(1-\theta)\mu$ features in equation (2.12). Figure 3(c) shows that the agent reduces labour supply especially at early age levels. Finally, Figure 3(d) shows that the age profile for scaled financial assets continues to be upward sloping, though it is lower than under perfect annuities.

2.4 Extensions

In the previous section we used a calibrated version of the core model to show that an imperfection in the annuity market leads to a slight increase in the optimal retirement age and a decrease in the macroeconomic growth rate. The core model, though useful for analytical purposes, suffers from a number of empirical deficiencies. These are:

Table 2.2. Growth and retirement: quantitative effects

	(a) *Core case*		(b) *Productivity*		(c) *Mortality*		(d) *Combined*		
	$\theta = 1.0$	$\theta = 0.7$	$\theta = 1.0$	$\theta = 0.7$	$\theta = 1.0$	$\theta = 0.7$	$\theta = 1.0$	$\theta = 0.7$	
								(i)	(ii)
$\frac{C(v,v)}{w(v)}$	0.1048	0.1062	0.0926	0.0934	0.1028	0.1057	0.1017	0.1007	0.0963
S (years)	0	0	7.47	7.65	0	0	7.47	7.40	6.75
R (years)	42	42.02	42	41.83	42	48.78	42	42.40	39.97
z	0	0.0078	0	0.0044	0	0.0031	0	0.0025	0
γ (%)	2.00	1.65	2.00	1.64	2.00	1.92	2.00	1.89	1.69
l (or n)	0.0691	0.0651	0.0831	0.0802	0.0825	0.0804	0.1100	0.1078	0.1060
$\frac{c(t)}{w(t)}$	0.1346	0.1340	0.1188	0.1189	0.1179	0.1157	0.1166	0.1144	0.1158
$\frac{w(t)}{k(t)}$	0.4583	0.4863	0.8401	0.8701	0.8482	0.8712	4.6038	4.6995	4.7792
$\frac{c(t)}{y(t)}$	0.3217	0.3398	0.4343	0.4501	0.4348	0.4384	0.8050	0.8067	0.8300

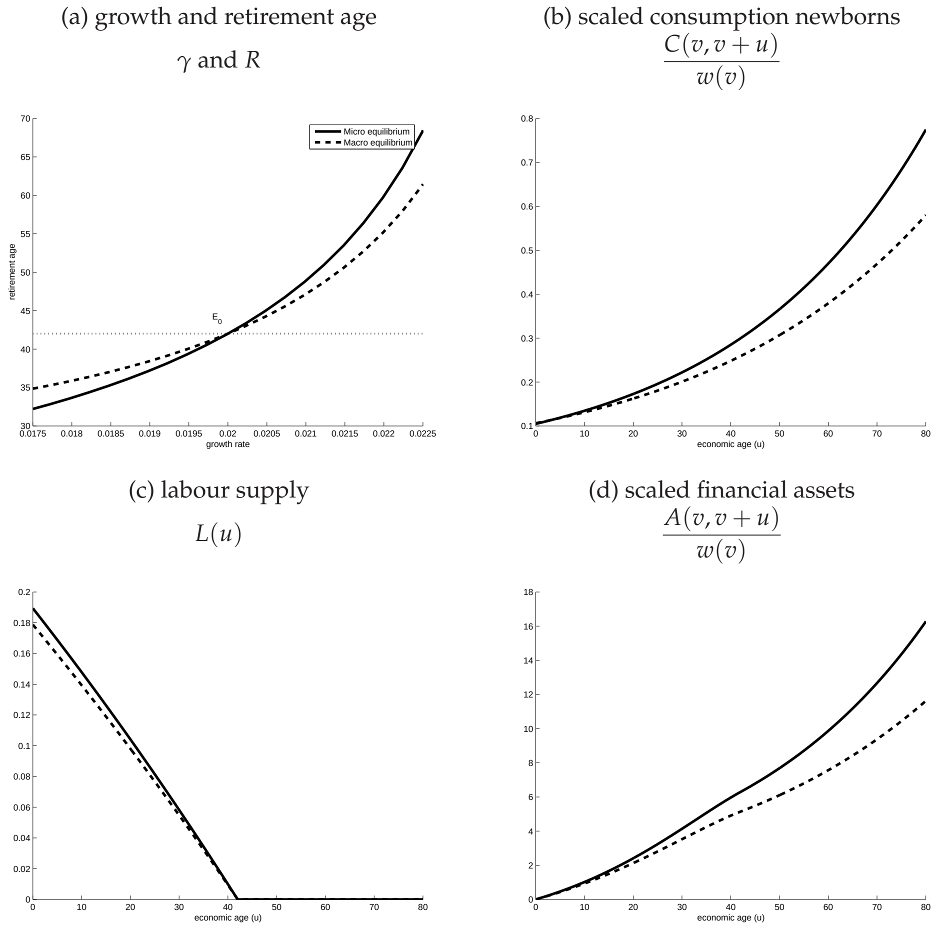

Figure 2.3. General equilibrium in the core model

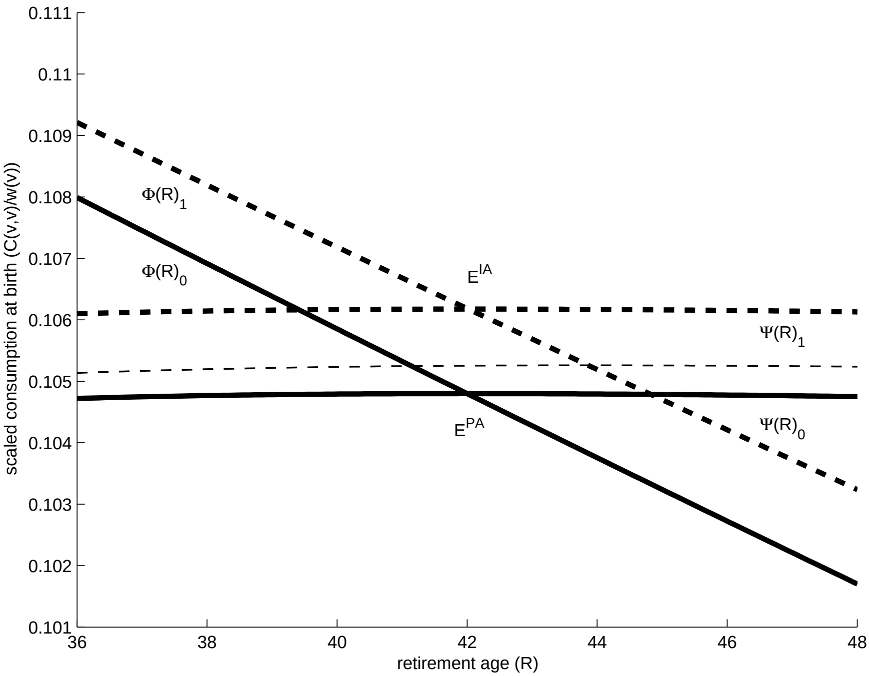

Figure 2.4. Imperfect annuities and the retirement date

(ED1). The age profile for consumption is monotonically increasing, whereas it is hump-shaped in reality (Gourinchas and Parker, 2002, and Fernández-Villaverde and Krueger, 2007).

(ED2). The age profile for labour supply is monotonically decreasing. In reality, labour supply is constant and age-invariant for most of working life and tapers off rapidly near the optimal retirement age (see, for example, McGrattan and Rogerson (2004) for the United States).

(ED3). Labour productivity is age-independent, whereas in reality it appears to be hump-shaped (cf. Hansen, 1993 and Rios-Rull, 1996).

(ED4). Under perfect annuities, the age profile for financial assets is monotonically rising. In reality, financial assets (a) display a hump-shaped profile, and (b) remain non-negative in old age (Huggett, 1996).

(ED5). To calibrate the model for a realistic retirement age and macroeconomic growth rate, an implausibly high efficiency parameter for capital must be postulated.

In this section we consider two important model extensions, namely age-dependent labour productivity and age-dependent mortality. In each case we study whether, and to what extent, the model extension under consideration can solve the empirical deficiencies of the core model. Both individual decisions and (simulated) general equilibrium effects are studied.

2.4.1 Hump-shaped productivity

In this section we directly address empirical deficiency (ED3) and assume that labour productivity of individuals is hump-shaped. That is, labour productivity is non-negative throughout life, starts out positive, is rising during the first life phase, and declines thereafter. For ease of exposition and future reference we collect the results concerning the individual labour productivity profile in Box 2.1 below.

The production side of the model is affected as follows. The total stock of efficiency

(i) Labour productivity of an u-year old is given by:

$$E(u) = \alpha_0 e^{-\zeta_0 u} - \alpha_1 e^{-\zeta_1 u}. \tag{B1.1}$$

we assume that $\alpha_0 > \alpha_1 > 0$, $\zeta_1 > \zeta_0 > 0$, and $\alpha_1\zeta_1 > \alpha_0\zeta_0$.

(ii) We easily find that:

$$E(0) = \alpha_0 - \alpha_1 > 0, \qquad \lim_{u\to\infty} E(u) = 0, \tag{B1.2}$$

$$E'(u) = -\zeta_0\alpha_0 e^{-\zeta_0 u} + \beta_1\alpha_1 e^{-\zeta_1 u} \begin{cases} > 0 & \text{for } 0 \le u < \bar{u} \\ < 0 & \text{for } u \ge \bar{u} \end{cases} \tag{B1.3}$$

where $\bar{u}$ is the age at which labour productivity is at its maximum:

$$\bar{u} = \frac{1}{\zeta_1 - \zeta_0} \ln\left(\frac{\alpha_1\zeta_1}{\alpha_0\zeta_0}\right). \tag{B1.4}$$

(iii) Along the balanced growth path the wage of an u-year old is given by (see below):

$$w(u) = w(0)\, e^{\gamma u}\left[\alpha_0 e^{-\zeta_0 u} - \alpha_1 e^{-\zeta_1 u}\right], \tag{B1.5}$$

Box 2.1: Labour Productivity Profile

units of labour at time t is denoted by $N(t)$ and is defined in the usual way:

$$N(t) \equiv \int_{-\infty}^{t} P(v,t)\, E(t-v)\, L(v,t)\, dv, \tag{2.37}$$

where $L(v,t)$ stands for raw labour supply in hours, and $P(v,t)$ is the size of cohort v at time t. Replacing L_i by N_i in equation (2.1), and redefining $\kappa_i \equiv K_i/N_i$ and $\kappa \equiv K/N$, we find that (2.5) and (2.7) are still satisfied but (2.6) must be changed to:

$$w(t)\, N(t) = (1 - \varepsilon_K)\, Y(t), \tag{2.38}$$

where $w(t)$ stands for the rental rate on efficiency units of labour. The wage faced at time t by a worker born at time v is thus given by:

$$w(v,t) \equiv E(t-v)\, w(t). \tag{2.39}$$

The household side of the model is affected as follows. In the household budget identity (2.9), $w(\tau)$ is replaced by $w(v,\tau)$. Along the balanced growth path, $w(v,\tau)$ can be written as:

$$w(v,\tau) = w(v)\, e^{\gamma(\tau-v)} \left[\alpha_0 e^{-\zeta_0(\tau-v)} - \alpha_1 e^{-\zeta_1(\tau-v)}\right], \tag{2.40}$$

where we have used (B1.1) and (2.39).[9] The consumption Euler equation is still given by (2.12). Interestingly, with a hump-shaped wage profile, it may be optimal for the agent to delay labour market entry somewhat. Indeed, we now have two relevant dates for the working decision of an agent, namely the optimal labour market entry date, S, and the optimal retirement date, R.[10] Obviously, we must have that $R > S \geq 0$. During working life ($S \leq \tau - v \leq R$) the condition (2.13) still holds but with $w(v,\tau)$ replacing $w(\tau)$.

Scaled consumption of a newborn agent is given by:

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C(\rho+\mu)}{\varepsilon_C + (1-\varepsilon_C)\left[e^{-(\rho+\mu)S} - e^{-(\rho+\mu)R}\right]} \cdot \frac{LI(v,v)}{w(v)}, \tag{2.41}$$

where $LI(v,v)/w(v)$ is defined as:

$$\frac{LI(v,v)}{w(v)} \equiv \int_S^R E(s)\, e^{-(r-\gamma+\theta\mu)s} ds + \frac{z}{r-\gamma+\theta\mu}. \tag{2.42}$$

For an interior solution (with $S > 0$), the labour market entry condition is given by:[11]

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} E(S)\, e^{-[r-\gamma-\rho-(1-\theta)\mu]S}, \tag{2.43}$$

[9] Equation (2.40) shows that it is *in principle* possible for the individual's wage to fall after a certain age, namely if the fall in labour productivity exceeds the macroeconomic growth rate ($\dot{E}(u)/E(u) < -\gamma$). This effect does not occur in our calibrated model so the wage path is monotonically increasing in age.

[10] As was the case in the core model of the previous section, household preferences and the redistribution scheme are such that S and R are generation independent, i.e. $S(v) = S$ and $R(v) = R$ for all v.

[11] It is not difficult to show that an interior solution for S is obtained if the following condition is satisfied:

$$\frac{C(v,v)}{w(v)} > \frac{\varepsilon_C}{1-\varepsilon_C} E(0).$$

If this condition is violated, then $L(v,v)$ attains an interior solution satisfying:

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} E(0)\left[1 - L(v,v)\right].$$

whereas the retirement condition is given by:

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} E(R) e^{-[r-\gamma-\rho-(1-\theta)\mu]R}. \tag{2.44}$$

Equations (2.41) (with (2.42) substituted), (2.43), and (2.44) form a three-equation system with three unknowns, viz. $C(v,v)/w(v)$, S, and R (see Table 3(a)). This system can be solved conditional on the macroeconomic variables, γ and z.

Using cross-section efficiency data for male workers aged between 18 and 70 from Hansen (1993, p. 74) we find the solid pattern in Figure 5(a). We interpolate these data by fitting equation (B1.1) using non-linear least squares. We find the following estimates (t-statistics in brackets): $\alpha_0 = 4.494$ (fixed), $\hat{\alpha}_1 = 4.010$ (71.04), $\hat{\zeta}_0 = 0.0231$ (24.20), $\hat{\zeta}_1 = 0.050$ (17.81) and the $R^2 = 0.80$ The fitted productivity profile is illustrated with dashed lines in Figure 5(a).

We have collected the key equations of the macroeconomic growth model in Table 3. Effectively this table provides the hump-shaped productivity analogue to Table 1. Compared to Table 1, the main changes are as follows. First, there is an additional equation governing the entry decision of households. Second, total labour supply is measured in efficiency units (i.e. n rather than l features in (T3.5)–(T3.7)). Third, the labour productivity age profile features prominently in (T3.2)–(T3.3) and (T3.7). The key features of the initial steady-state growth path have been reported in the first column of Table 2(b).

Figures 5(b)–(d) provide a visualization of the extended model. The key panel to consider is 5(b), which shows that with a hump-shaped productivity profile, the labour supply profile also features a hump-shaped pattern. This model extension thus somewhat alleviates empirical deficiency (ED2) of the core model. That is, we now have a labour supply profile that increases rapidly in young age, briefly touches a plateau and then drops to zero (i.e. retirement) quickly. Interestingly, the remaining empirical deficiencies (ED1) and (ED4)–(ED5) are not solved by the introduction of age dependent labour productivity. Consumption and assets are not hump shaped, and the required capital efficiency parameter, though lower than for the core model, is still too high ($\varepsilon_K = 0.6963$).

As before, the dashed lines in Figures 5(b)–(d) visualize the implications of an im-

Table 2.3. Balanced growth and retirement with age-dependent productivity

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(a) Microeconomic relationships:

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C(\rho+\mu)}{\varepsilon_C + (1-\varepsilon_C)\left[e^{-(\rho+\mu)S} - e^{-(\rho+\mu)R}\right]} \cdot \left[\int_S^R E(s)\, e^{-(r-\gamma+\theta\mu)s} ds + \frac{z}{r-\gamma+\theta\mu}\right] \quad \text{(T3.1)}$$

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} E(S)\, e^{-(r-\gamma-\rho-(1-\theta)\mu)S} \quad \text{(T3.2)}$$

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} E(R)\, e^{-(r-\gamma-\rho-(1-\theta)\mu)R} \quad \text{(T3.3)}$$

(b) Macroeconomic relationships:

$$z = \mu(1-\theta)\frac{k(t)}{w(t)} \quad \text{(T3.4)}$$

$$\gamma \equiv \frac{\dot{k}(t)}{k(t)} = r - \pi + \left[n - \frac{c(t)}{w(t)}\right] \cdot \frac{w(t)}{k(t)} \quad \text{(T3.5)}$$

$$\frac{w(t)\, n}{k(t)} = (1-\varepsilon_K)\,\Omega_0 \quad \text{(T3.6)}$$

$$n \equiv \int_S^R \beta E(s)\, e^{-\beta s} ds - \beta e^{-\beta R} E(R)\, \frac{e^{[\gamma+\beta+\rho+(1-\theta)\mu-r](R-S)} - 1}{\gamma+\beta+\rho+(1-\theta)\mu - r} \quad \text{(T3.7)}$$

$$\frac{c(t)}{w(t)} \equiv \frac{\beta}{\gamma+\beta+\rho+(1-\theta)\mu-r} \cdot \frac{C(v,v)}{w(v)} \quad \text{(T3.8)}$$

Definitions: Endogenous are: $C(v,v)/w(v)$, S, R, z, γ, n, $w(t)/k(t)$, and $c(t)/w(t)$. Parameters: birth rate β, mortality rate μ, population growth rate $\pi \equiv \beta - \mu$, imperfection annuities θ, rate of time preference ρ, capital coefficient in the technology ε_K, consumption coefficient in tastes ε_C, scale factor in the technology Ω_0. The interest rate is $r \equiv \varepsilon_K \Omega_0 - \delta$, where δ is the depreciation rate of capital.

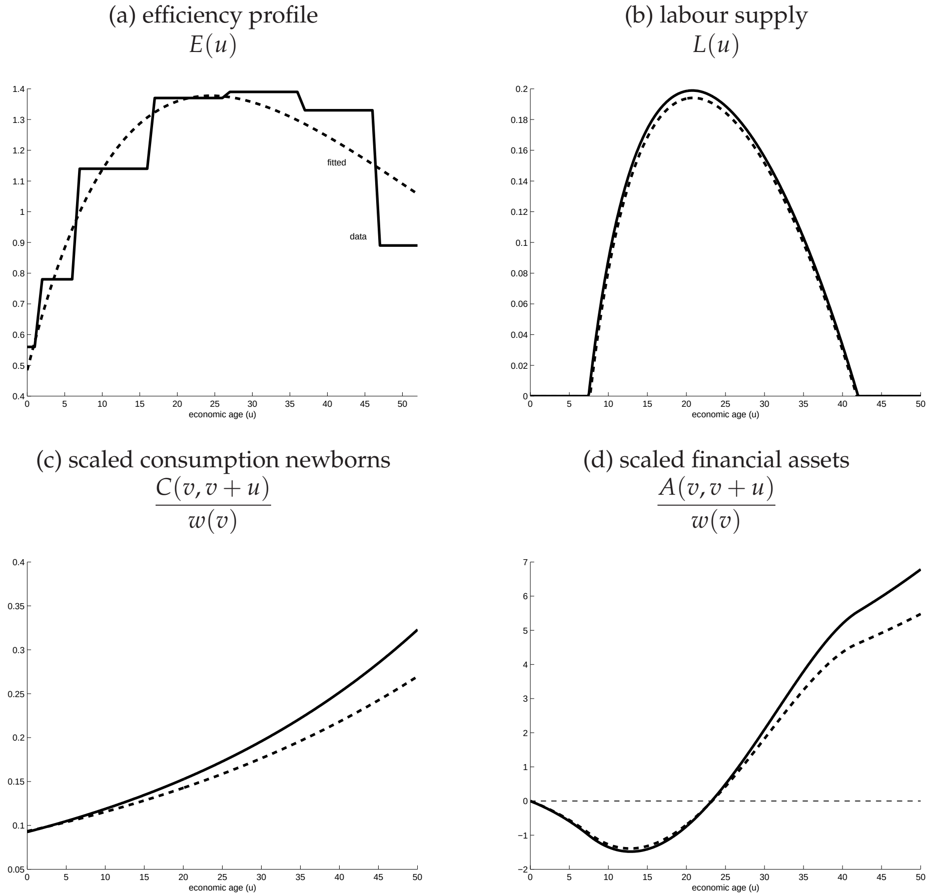

Figure 2.5. General equilibrium with age-dependent labour productivity

perfect annuity market (captured by $\theta = 0.7$). The key features of the new steady-state growth path have been reported in the second column of Table 2(b). The composite impact of an imperfect annuity market on individual decisions is that agents delay labour market entry, work less during working life and retire early. In general equilibrium this leads to a substantial reduction in economic growth. Interestingly, the effect on economic growth is very similar for the core model and the extended model.

Note that because the profits made by annuity firms are taxed away by the government, the annuity market imperfection acts as an implicit tax on the annuity premium. Furthermore, in the initial phase of the life-cycle the annuity market imperfection may act as a subsidy on loans. Although this is a somewhat troubling feature of the model, the effect of this feature is negligible because the absolute magnitude of loans as well as $\mu(t-v)$ are low for the young.

2.4.2 Age-dependent mortality

In this section we assume $E(u) = 1$ for all u and instead augment the core model by assuming age-dependent mortality. For ease of exposition, we use a demographic process which incorporates a finite maximum age; the Boucekkine, de la Croix, and Licandro (BCL) model suggested by Boucekkine *et al.* (2002). As with the labour productivity profile we collect the results concerning the mortality structure in Box 2.2 below.

We use data from age 18 onward for the Dutch cohort that was born in 1960. Following Heijdra and Romp (2008), we denote the actual surviving fraction up until model age u_i by S_i, and estimate the parameters of the parametric distribution function by means of non-linear least squares. The model to be estimated is thus:

$$S_i = 1 - \Phi(u_i) + \varepsilon_i = d(u_i \le D) \cdot \frac{\eta_0 - e^{\eta_1 u_i}}{\eta_0 - 1} + \varepsilon_i, \tag{2.45}$$

where $d(u_i \le \bar{D}) = 1$ for $u_i \le \bar{D}$, and $d(u_i \le \bar{D}) = 0$ for $u_i > \bar{D}$, and ε_i is the stochastic error term. We find the following estimates (with t-statistics in brackets): $\hat{\eta}_0 = 122.643$ (11.14), $\hat{\eta}_1 = 0.0680$ (48.51). The standard error of the regression is $\hat{\sigma} = 0.02241$, and the implied estimate for $\bar{D}$ is 70.75 model years (i.e., the maximum age in biological years

(i) The surviving fraction up to age u (from the perspective of birth) is given by:

$$1-\Phi(u) \equiv \frac{\eta_0 - e^{\eta_1 u}}{\eta_0 - 1}, \tag{B2.1}$$

with $\eta_0 > 1$, $\eta_1 > 0$ and $\bar{D} = (1/\eta_1) \ln \eta_0$ is the maximum attainable age.

(ii) For $0 < s < \bar{D}$, the cumulative mortality rate is:

$$M(u) \equiv -\ln[1-\Phi(u)], \tag{B2.2}$$

so that the exponential discounting factors are given by:

$$e^{-M(s)} \equiv \frac{\eta_0 - e^{\eta_1 s}}{\eta_0 - 1}, \qquad e^{M(s)} \equiv \frac{\eta_0 - 1}{\eta_0 - e^{\eta_1 s}}. \tag{B2.3}$$

(iii) The instantaneous mortality (or hazard) rate at age u is given by:

$$\mu(u) \equiv \frac{\Phi'(u)}{1-\Phi(u)} = \frac{\eta_1 e^{\eta_1 u}}{\eta_0 - e^{\eta_1 u}}. \tag{B2.4}$$

The mortality rate is increasing in age and becomes infinite at $u = \bar{D}$.

(iv) The relative cohort size is:

$$p(v,t) \equiv \frac{P(v,t)}{P(t)} \equiv \beta e^{-\pi(t-v)-M(t-v)} = \beta \frac{\eta_0 - e^{\eta_1 (t-v)}}{\eta_0 - 1} e^{-n(t-v)} \tag{B2.5}$$

where β is the crude birth rate and π is the population growth rate.

(v) The demographic steady-state is given by (see d'Albis (2007, p.416) and Heijdra and Romp (2008, p.94)):

$$\frac{1}{\beta} = \frac{1}{\eta_0 - 1} \left[\eta_0 \frac{1-e^{-n\bar{D}}}{n} + \frac{1-e^{(\eta_1 - n)\bar{D}}}{\eta_1 - n} \right] \tag{B2.6}$$

For a given birth rate, equation (B2.6) determines the unique population growth rate consistent with the demographic steady state. The average population-wide mortality rate, $\bar{\mu}$, follows residually from the fact that $\pi \equiv \beta - \bar{\mu}$.

Box 2.2: Boucekkine, de la Croix and Licandro (2002) Mortality Structure

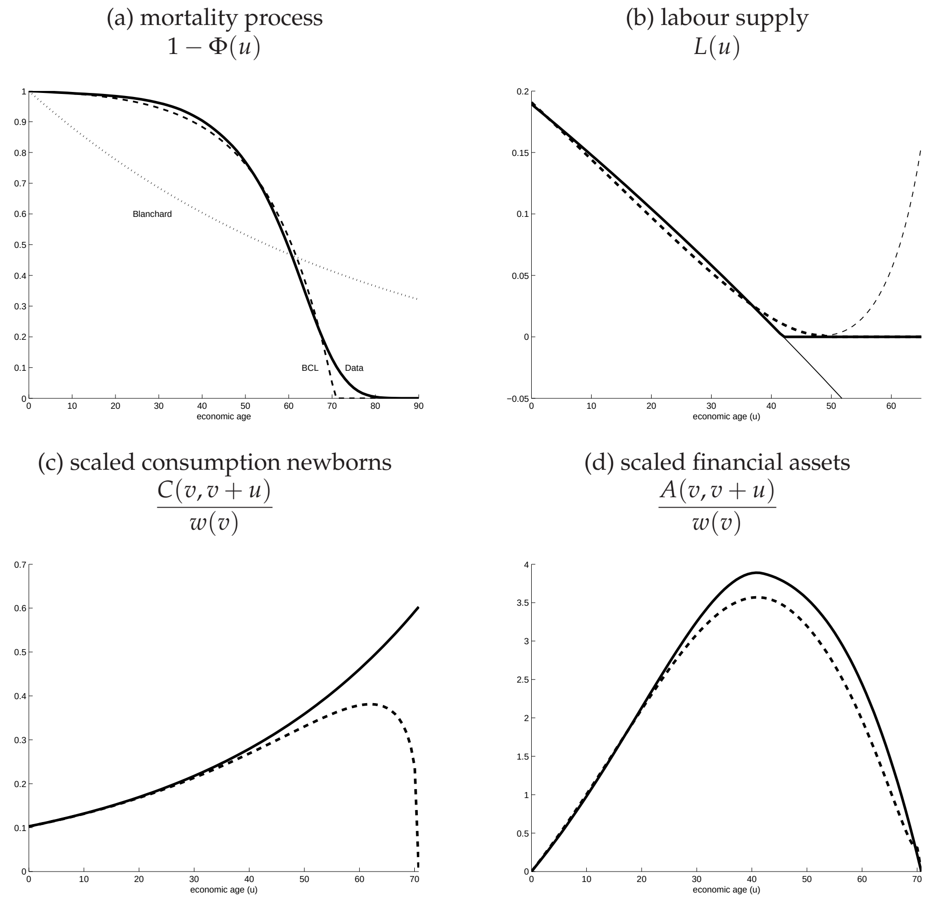

Figure 2.6. General equilibrium with age-dependent mortality

is 88.75). Figure 6(a) depicts the actual and fitted survival rates with, respectively, solid and dashed lines. Up to age 69, the BCL model fits the data rather well. For higher ages the fit deteriorates as the BCL model fails to capture the fact that some people are expected to live to very ripe old ages in reality.

Using the same data, we also estimate the parameter of the Blanchard demography, by running the following regression by means of non-linear least squares: $S_i = e^{-\mu u_i} + \varepsilon_i$. We find $\hat{\mu} = 0.0126$ (11.41), and $\hat{\sigma} = 0.2466$. The dotted line in Figure 6a depicts the fitted survival rates implied by the Blanchard demography. The fit is much worse than that of the BCL model. Relative to the data, the Blanchard model "kills off" the young too quickly and the old too slowly.

In the presence of age-dependent mortality, the core model is changed as follows. First, as is explained in Heijdra and Romp (2008, p. 92), the lifetime utility function (2.8) is now given by:

$$\mathbb{E}\Lambda\left(v,t\right) \equiv e^{M(t-v)} \cdot \int_t^{v+\bar{D}} \ln\left[C(v,\tau)^{\varepsilon_C}\cdot[1-L(v,\tau)]^{(1-\varepsilon_C)}\right]\cdot e^{-\rho(\tau-t)-M(\tau-v)}d\tau, \tag{2.46}$$

where (a) the maximum possible age is incorporated in the upper limit of the integral, and (b) the discounting factor due to lifetime uncertainty, $e^{-M(\tau-v)}$, depends on the agent's age at some future time τ.

Second, the annuity rate given in (2.10) above is modified to reflect the fact that the mortality rate depends on age:

$$r^A\left(\tau-v\right) \equiv r+\theta\mu\left(\tau-v\right), \qquad (\text{for } 0\le\tau-v<\bar{D}). \tag{2.47}$$

Older agents attract a higher annuity rate than younger agents do because they feature a higher mortality rate (note that at age $\tau-v=\bar{D}$ no life insurance is available). Utility maximization gives rise to an individual consumption Euler equation that is different from the one given in (2.12) above:

$$\frac{\dot{C}\left(v,\tau\right)}{C\left(v,\tau\right)} = r-\rho-\left(1-\theta\right)\mu\left(\tau-v\right). \tag{2.48}$$

Provided annuities are imperfect ($\theta<1$), optimal consumption growth is age dependent.

The key expressions characterizing individual behaviour are given in equations (T4.1)–(T4.3) in Table 2.4. Equation (T4.1) gives the expression for scaled consumption at birth. It contains specific values for a general demography-dependent function that is defined as follows:

$$\Xi\left(\lambda_1,\lambda_2\right)_{u_0}^{u_1} = \int_{u_0}^{u_1} e^{-\lambda_1 s}\cdot\left[\frac{\eta_0-e^{\eta_1 s}}{\eta_0-1}\right]^{\lambda_2} ds, \tag{2.49}$$

with $0\le u_0<u_1\le\bar{D}$ and $\lambda_2\ge0$. Provided λ_1 is finite, the integral exists and is

strictly positive. It follows that $\Xi\left(r-\gamma,\theta\right)_S^R>0$, $\Xi\left(r-\gamma,\theta\right)_0^{\bar{D}}>0$, $\Xi\left(\rho,1\right)_S^R>0$, and $\Xi\left(\rho,1\right)_0^{\bar{D}}>0$, so scaled newborn consumption is positive and depends positively on the amount of transfers.[12]

Interestingly, despite the fact that productivity is age-independent, equation (T4.2) shows that with imperfect annuities it is *in principle* possible for the individual agent to postpone labour market entry somewhat, i.e. to choose $S>0$. With a realistic demography, however, this scenario does not materialize, i.e. *in practice* labour market entry is immediate and $S=0$. Intuitively, this results from the fact that the mortality process only cuts in toward the end of the agent's life.

The macroeconomic part of the model is given by equations (T4.5)–(T4.9) in Table 2.4. Compared to the core model, the main changes are found in (T4.5) and (T4.8)–(T4.9). In (T4.5), transfers can no longer be related to a single aggregate variable but must be computed (numerically) by using the scaled wealth paths of existing cohorts. Expressions (T4.8)–(T4.9) generalize (T1.6)–(T1.7), making use of the $\Xi\left(\lambda_1,\lambda_2\right)_{u_0}^{u_1}$ function defined in (2.49) above.

Just as for the previous two models, we calibrate the model for an initial steady state with perfect annuities ($\theta=1$), a growth rate of two percent ($\gamma=0.02$), and an optimal retirement age of 42 years ($R=42$). The key features of the initial steady-state growth path have been reported in the first column of Table 2(c). As was mentioned above, labour market entry is immediate for the cases considered in Table 2(c).

Figures 6(b)–(d) provide a visualization of the extended model. The key panels to consider are 6(c) and 6(d). With imperfect annuities, consumption features a hump-shaped pattern thus addressing empirical deficiency (ED1)–see the dashed lines in Figure 6(c). This finding is in line with Yaari (1965), Abel (1985), Bütler (2001), and Hansen and İmrohoroğlu (2008): with imperfect annuities the mortality rate features in the individual Euler equation. Hence, if the mortality rate is age-dependent, agents will discount consumption later on in life more heavily, thus creating a hump-shaped profile. From an empirical point of view it should be noted that we–like Bütler (2001) and Hansen and İmrohoroğlu (2008)–also find that the hump occurs too late in life. Also, as is illustrated in Figure 6(d), financial assets feature a hump-shaped pattern

[12] Using the Ξ-function we can define the demographic steady-state as $\frac{1}{\beta}=\Xi\left(\pi,1\right)_0^{\bar{D}}$ which simply generalizes (2.21) to the case with age-dependent mortality.

Table 2.4. Balanced growth and retirement with age-dependent mortality

(a) Microeconomic relationships:

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C \Xi(r-\gamma,\theta)_S^R + \varepsilon_C \Xi(r-\gamma,\theta)_0^{\bar{D}} \cdot z}{(1-\varepsilon_C)\,\Xi(\rho,1)_S^R + \varepsilon_C \Xi(\rho,1)_0^{\bar{D}}} \tag{T4.1}$$

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} e^{-(r-\gamma-\rho)S+(1-\theta)M(S)} \tag{T4.2}$$

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} e^{-(r-\gamma-\rho)R+(1-\theta)M(R)} \tag{T4.3}$$

$$\frac{A(v,v+u)}{w(v)} \cdot e^{-ru-\theta M(u)} = -\frac{C(v,v)}{w(v)} \cdot \Xi(\rho,1)_0^u + z \cdot \Xi(r-\gamma,\theta)_0^u \tag{T4.4a}$$

$$= -\frac{C(v,v)}{w(v)} \cdot \left[\Xi(\rho,1)_0^S + \frac{1}{\varepsilon_C}\Xi(\rho,1)_S^u\right] + z \cdot \Xi(r-\gamma,\theta)_0^u + \Xi(r-\gamma,\theta)_S^u \tag{T4.4b}$$

$$= \frac{C(v,v)}{w(v)} \cdot \Xi(\rho,1)_u^{\bar{D}} - z \cdot \Xi(r-\gamma,\theta)_u^{\bar{D}} \tag{T4.4c}$$

(b) Macroeconomic relationships:

$$z = (1-\theta) \cdot \int_0^{\bar{D}} \beta e^{-(\pi+\gamma)u-M(u)} \mu(u) \frac{A(v,v+u)}{w(v)} du \tag{T4.5}$$

$$\gamma \equiv \frac{\dot{k}(t)}{k(t)} = r - \pi + \left[l - \frac{c(t)}{w(t)}\right] \cdot \frac{w(t)}{k(t)} \tag{T4.6}$$

$$\frac{w(t)\,l}{k(t)} = (1-\varepsilon_K)\,\Omega_0 \tag{T4.7}$$

$$l = \beta \cdot \left[\Xi(\pi,1)_S^R - \frac{1-\varepsilon_C}{\varepsilon_C}\frac{C(v,v)}{w(v)} \cdot \Xi(\pi+\rho+\gamma-r,2-\theta)_S^R\right] \tag{T4.8}$$

$$\frac{c(t)}{w(t)} \equiv \frac{C(v,v)}{w(v)} \cdot \beta \Xi(\pi+\rho+\gamma-r,2-\theta)_0^{\bar{D}} \tag{T4.9}$$

Definitions: Endogenous are $C(v,v)/w(v)$, S, R, z, γ, l, $w(t)/k(t)$, and $c(t)/w(t)$. Parameters: birth rate β, aggregate mortality rate $\bar{\mu}$, population growth rate $\pi \equiv \beta - \bar{\mu}$, imperfection annuities θ, rate of time preference ρ, capital coefficient in the technology ε_K, consumption coefficient in tastes ε_C, scale factor in the technology Ω_0. The interest rate is $r \equiv \varepsilon_K \Omega_0 - \delta$, where δ is the depreciation rate of capital.

both with perfect and with imperfect annuities.[13] The model extension thus fixes empirical deficiency (ED4) to a large extent. Finally, empirical deficiency (ED5) is reduced somewhat in this extension as the required efficiency parameter for capital is equal to $\varepsilon_K = 0.6956$ (rather than 0.8348 in the core model).

As before, the dashed lines in Figures 6(b)–(d) visualize the implications of an imperfect annuity market (captured by $\theta = 0.7$). The key features of the new steady-state growth path have been reported in the second column of Table 2(c). Just as in the core model, individual and aggregate saving and thus the macroeconomic growth rate are all lower when annuity markets are imperfect rather than perfect.[14] Furthermore, and in contrast to both the core model and the model with age-dependent productivity, we now find that agents also delay labour market exit by almost seven years. Hence, the composite impact of an imperfect annuity market on individual decisions is that agents work slightly fewer hours during most of their working life, but retire much later thus limiting the fall in the aggregate supply of labour. In general equilibrium, this retirement effect explains why the reduction in economic growth is smaller than for the previous two models.

Compared to the core model, the annuity market imperfection operates quite differently in the model with a realistic demographic structure – compare panels (a) and (c) in Table 2. First, instead of finding a near-zero retirement effect, in the extended model agents *delay* retirement by almost 7 years. Intuitively, with age-dependent mortality and imperfect annuities, agents discount future felicity by their ever increasing mortality rate. This ensures that both consumption and leisure are hump-shaped and hence labour supply is U-shaped. Because retirement is an absorbing state, however, the upward sloping part of the labour supply path is not attained – see Figure 6(b).[15] In the calibration underlying Table 2(c), labour supply bottoms out at zero thus explaining the large shift in the retirement age.

Second, instead of experiencing a reduction in the economic growth rate of 35 basis

[13] In contrast to the core model, with age-dependent mortality an actuarially fair redistribution scheme of the form $TR(v,\tau) = (1-\theta)\,\mu(\tau - v)\,A(v,\tau)$ is feasible. See also footnote 6 on this issue.

[14] This finding regarding growth has previously been highlighted by Abel (1985) and Fuster (1999) who suggest that capital accumulation decreases with imperfect annuities provided (i) the elasticity of intertemporal substitution is no less than unity and (ii) there is steady-state growth.

[15] In Figure 6(b) the thin dotted line after retirement gives the labour supply path if retirement were not an absorbing state. Similar, the thin solid line gives labour supply if negative labour supply were allowed.

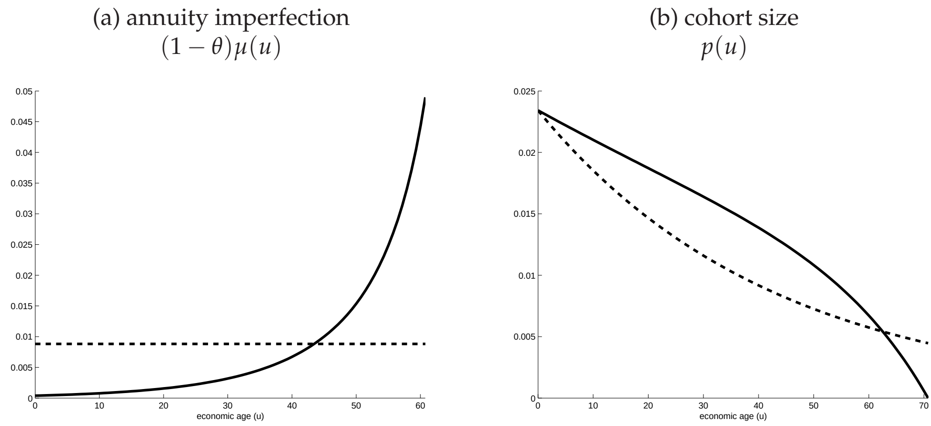

Figure 2.7. The annuity market imperfection

points, in the extended model we see a much smaller reduction of only 8 basis points. To appreciate the origin of these differences, note Figures 7(a)–(b). Figure 7(a) visualizes the annuity market imperfection faced by the agent over the life-cycle. The dashed line shows the imperfection for the Blanchard mortality process whilst the solid line depicts the imperfection for the realistic-mortality case. From here it is immediately clear that the Blanchard mortality process overstates the magnitude of the annuity market imperfection for a substantial part of the life-cycle. In contrast, for a realistic demography the annuity market imperfection only becomes an issue later on in life. Furthermore, as can be seen in Figure 7(b), the relative size of cohorts that are actually affected is quite small. In the core model assets grow indefinitely with age, thus overstating the effect of older agents on the growth rate. A given change in θ thus has a large effect on growth because it most strongly affects the asset-rich older agents of whom there are too many. In the extended model, however, these agents are not only relatively few in numbers but are also decumulating assets. As a result, they have a much smaller effect on the growth rate.

2.4.3 Full model

In this section we visualize the full model, simultaneously incorporating age-dependent labour productivity and mortality. The key equations for the full model have been collected in Table 2.5, whilst Figure 8 visualizes some of its salient life-cycle features. Finally, the quantitative effect of imperfect annuities are reported in Table 2(d).

Figure 8(a) plots the right-hand sides of (T5.2) and (T5.3) as a function of age. For $\theta = 1$, there is a unique entry age ($S = 7.47$, at point A) and a unique retirement age ($R = 42$, at point B). In contrast, for $\theta = 0.7$, there appears to be a second labour market entry point located to the right of point B. This point is not feasible, however, because we assume that labour market exit is an absorbing state. Hence, also for $\theta = 0.7$, there are unique entry and exit ages, i.e. $S = 7.40$ and $R = 42.40$ – see Table 2(d).

Figure 8(b) shows the age profile for labour supply. It is hump-shaped because labour productivity is, i.e. Figure 8(b) looks very much like Figure 5(b) above.

Figure 8(d) shows the age profile for financial assets. This figures captures the main features of Figure 6(d), but adds a borrowing period at the start of life. Agents delay labour market entry and –upon entry– face rather low wages and supply few hours early on in life. They finance their rising consumption profile by borrowing during that first life phase.

Interestingly, the quantitative effects of θ on the optimal retirement age and growth are rather small, as is revealed in column (i) of Table 2(d). The full model is a hybrid of the two extended models. With respect to the optimal retirement decision, the effects explained by age-dependent productivity outweigh the effects of age-dependent mortality. In contrast, the impact of θ on the growth rate is predominantly driven by the effects of age-dependent mortality. Furthermore, empirical deficiency (ED5) is eliminated in the full model as the required efficiency parameter for capital is equal to $\varepsilon_K = 0.2402$.

2.4.4 The role of transfers in the full model

Up until now we have focused on the situation where the profits made by the annuity firms are redistributed toward the agents in the form of a lump-sum transfer. These transfers have allowed us to focus solely on the substitution effect of the annuity market imperfection. However, in order to study the full (i.e. income and substitution) effect of the imperfection we need to consider an alternative general equilibrium mechanism by which the profits of the annuity firms are spent. In this subsection we assume

Table 2.5. Balanced growth and retirement with age-dependent productivity and mortality

(a) Microeconomic relationships:

$$\frac{C(v,v)}{w(v)} = \frac{\alpha_0 \varepsilon_C \Xi(r+\zeta_0-\gamma,\theta)_S^R - \alpha_1 \varepsilon_C \Xi(r+\zeta_1-\gamma,\theta)_S^R}{(1-\varepsilon_C)\Xi(\rho,1)_S^R + \varepsilon_C \Xi(\rho,1)_0^{\bar{D}}} + \frac{\varepsilon_C \Xi(r-\gamma,\theta)_0^{\bar{D}} \cdot z}{(1-\varepsilon_C)\Xi(\rho,1)_S^R + \varepsilon_C \Xi(\rho,1)_0^{\bar{D}}} \tag{T5.1}$$

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} E(S)\, e^{-(r-\gamma-\rho)S+(1-\theta)M(S)} \tag{T5.2}$$

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} E(R)\, e^{-(r-\gamma-\rho)R+(1-\theta)M(R)} \tag{T5.3}$$

$$\frac{A(v,v+u)}{w(v)} \cdot e^{-ru-\theta M(u)} = -\frac{C(v,v)}{w(v)} \cdot \Xi(\rho,1)_0^u + z \cdot \Xi(r-\gamma,\theta)_0^u \tag{T5.4a}$$

$$= -\frac{C(v,v)}{w(v)} \cdot \left[\Xi(\rho,1)_0^S + \frac{1}{\varepsilon_C}\Xi(\rho,1)_S^u\right] + z \cdot \Xi(r-\gamma,\theta)_0^u + \left[\alpha_0 \Xi(r+\zeta_0-\gamma,\theta)_S^u - \alpha_1 \Xi(r+\zeta_1-\gamma,\theta)_S^u\right] \tag{T5.4b}$$

$$= \frac{C(v,v)}{w(v)} \cdot \Xi(\rho,1)_u^{\bar{D}} - z \cdot \Xi(r-\gamma,\theta)_u^{\bar{D}} \tag{T5.4c}$$

(b) Macroeconomic relationships:

$$z = (1-\theta) \cdot \int_0^{\bar{D}} \beta e^{-(\pi+\gamma)u-M(u)} \mu(u) \frac{A(v,v+u)}{w(v)} du \tag{T5.5}$$

$$\gamma \equiv \frac{\dot{k}(t)}{k(t)} = r - \pi + \left[n - \frac{c(t)}{w(t)}\right] \cdot \frac{w(t)}{k(t)} \tag{T5.6}$$

$$\frac{w(t)\,n}{k(t)} = (1-\varepsilon_K)\,\Omega_0 \tag{T5.7}$$

$$n = \beta \cdot \left[\alpha_0 \Xi(\pi+\zeta_0,1)_S^R - \alpha_1 \Xi(\pi+\zeta_1,1)_S^R - \frac{1-\varepsilon_C}{\varepsilon_C}\frac{C(v,v)}{w(v)} \cdot \Xi(\pi+\rho+\gamma-r,2-\theta)_S^R\right] \tag{T5.8}$$

$$\frac{c(t)}{w(t)} \equiv \frac{C(v,v)}{w(v)} \cdot \beta \Xi(\pi+\rho+\gamma-r,2-\theta)_0^{\bar{D}} \tag{T5.9}$$

Definitions: Endogenous are $C(v,v)/w(v)$, S, R, z, γ, n, $w(t)/k(t)$, and $c(t)/w(t)$. Parameters: birth rate β, aggregate mortality rate $\bar{\mu}$, population growth rate $\pi \equiv \beta - \bar{\mu}$, imperfection annuities θ, rate of time preference ρ, capital coefficient in the technology ε_K, consumption coefficient in tastes ε_C, scale factor in the technology Ω_0. The interest rate is $r \equiv \varepsilon_K \Omega_0 - \delta$, where δ is the depreciation rate of capital.

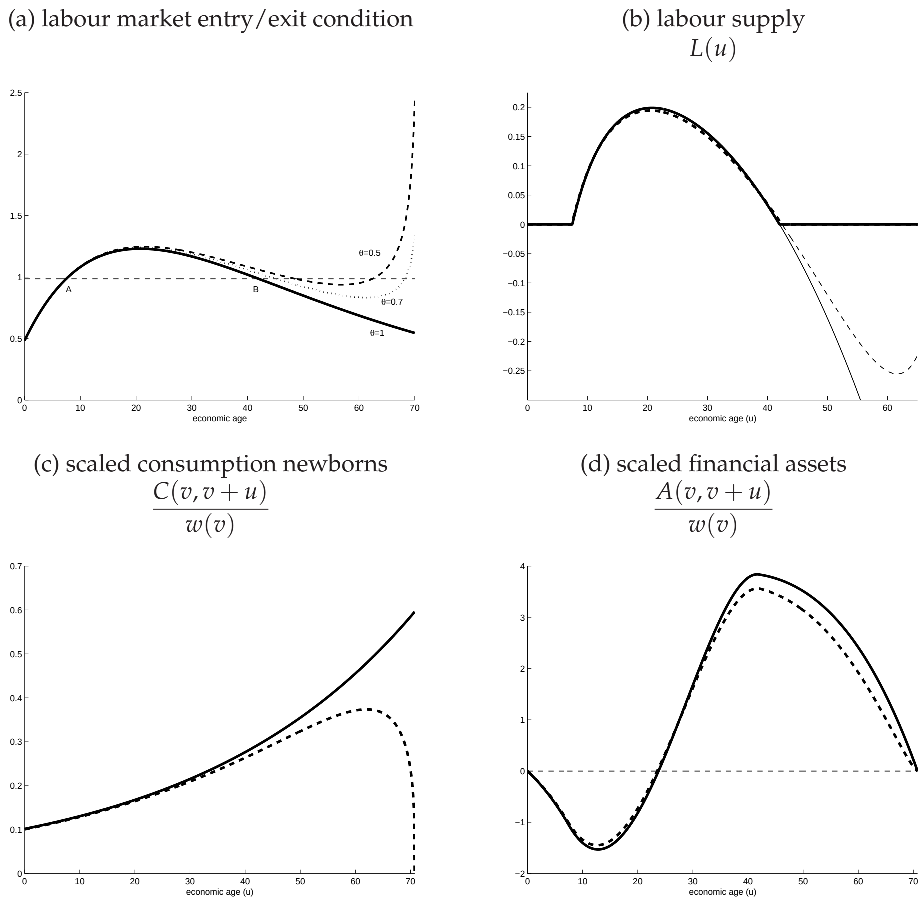

Figure 2.8. General equilibrium with age-dependent productivity and mortality

that the government uses the funds for non-productive spending.

Compared to Table 5, there are two major changes. First, transfers are zero both with perfect and imperfect annuities. Second, the imperfection surfaces directly in the relationship for the growth rate. Indeed, equation (T5.5) is replaced by:

$$\gamma = \frac{\dot{k}(t)}{k(t)} = r - \pi - (1-\theta) \cdot \frac{w(t)\,\Gamma}{k(t)} + \left[n - \frac{c(t)}{w(t)} \right] \frac{w(t)}{k(t)}, \qquad \text{(T5.5}'\text{)}$$

where $w(t)\,\Gamma$ is given by:

$$w(t)\,\Gamma \equiv \int_{t-\bar{D}}^{t} p(v,t)\,\mu(t-v)\,A(v,t)\,dv.$$

Figure 9 visualizes the impact of the annuity market imperfection on labour supply and financial assets (as in the full model with transfers, consumption is hump shaped). Figure 9(b) shows that assets accumulation is increased slightly for younger agents and reduced substantially for older agents.

Comparing columns (i) and (ii) two main features stand out. First, the growth rate drops substantially under non-productive government spending. This is a direct consequence of draining productive resources from the economy. Second, in the model with unproductive spending agents enter and retire at an earlier age and shorten the length of their working career. The retirement effect is a direct consequence of the fall in the growth rate – see for example the discussion surrounding Figure 2.

2.5 Conclusions

We study the impact of imperfect annuity markets on individual decisions and macroeconomic outcomes. We develop a concise overlapping generations model of a closed economy featuring endogenous growth. We demonstrate that this model replicates the most salient life-cycle features of asset holdings, labour supply, and consumption. For this, annuities must be imperfect and both the mortality process and labour productivity must be age dependent. The annuity imperfection accounts for a hump-shaped consumption profile, age-dependent mortality gives rise to a life-cycle pattern of saving, and age-dependent productivity captures the life-cycle pattern of labour supply.

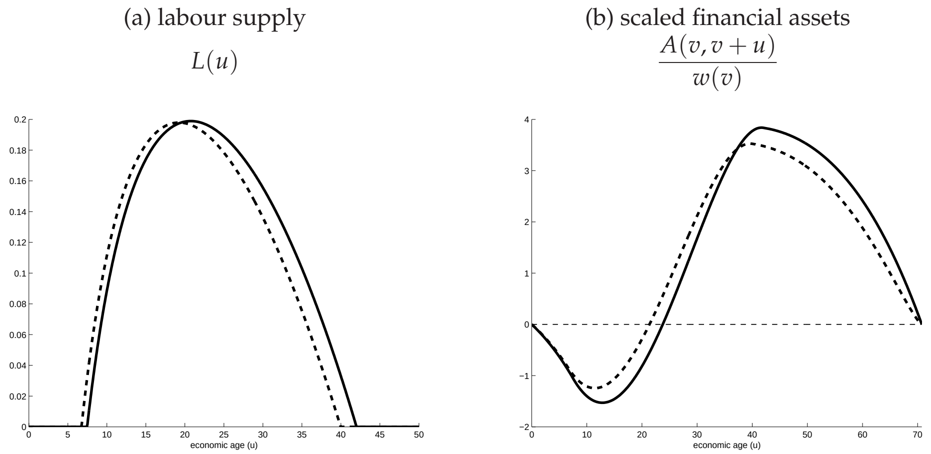

Figure 2.9. Full model with useless government spending

The empirical evidence suggests that the annuity imperfection parameter employed in this chapter may be quite substantial. Our model shows that the microeconomic effects of such an imperfection are rather large if the lump-sum transfers (arising from annuity firms' profits) are not taken into account. But this partial equilibrium result, though commonly stressed in the literature, is rather misleading. Indeed, in the presence of transfers, i.e. in a general equilibrium setting, both microeconomic and macroeconomic effects of quantitatively significant annuity imperfections are small.

The model developed in this chapter can be amended and extended in a straightforward fashion to study a variety of public policy issues. In the next chapter we use the model to study the implications of labour income and consumption taxation. In Chapter 4 we amend the model slightly and study how different public pension arrangements affect a decrease in adult mortality.