The microeconomics of strategic activism

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Chapter 6

Endogenous Group Formation and Lobbying

6.1 Introduction

Lobbying is a widespread phenomenon with considerable impact on political processes. Concerns regarding lobbyists' influence on policy outcomes are even used during elections to attract votes. For instance, during the 2008 United States presidential election both Barack Obama ("I'm in this race to tell the lobbyists in Washington that their days of setting the agenda are over."[1]) and John McCain ("I've fought lobbyists who stole from Indian tribes."[2]) promised the electorate that they would reduce the clout of lobbies as soon as they took office. The pervasiveness of lobbying is illustrated by the observation that in 1999 3,835 organizations had registered with the U.S. government as political action committees, i.e. as organizations which are allowed to contribute to political candidates.[3]

It is therefore not surprising that lobbying has garnered generous attention in the political economy literature over the past few decades. A substantial literature, pioneered by Becker (1983), studies how exogenously given special interest groups can affect policy outcomes by lobbying governments (cf. Grossman and Helpman, 1994, Dixit et al., 1997, Aidt, 1998). More recent work (Grossman and Helpman, 1996, Besley and Coate, 2001, Prat, 2002, amongst others) takes into account that such lobbying impacts political competition. A few papers investigate endogenous lobbying or endogenous lobby formation. Mitra (1999) endogenizes lobby formation

[1] Excerpt from a speech held in Las Vegas, 15 November, 2007.

[2] Excerpt from a speech held in Minneapolis, 4 September, 2008.

[3] The introductory chapter of Grossman and Helpman (2001), from which this figure is taken, contains ample qualitative and quantitative evidence regarding lobbying.

in the realm of trade policy, using communication-based refinements to arrive at equilibria with endogenous lobby formation. Felli and Merlo (2006) study endogenous lobbying, taking the groups capable of engaging in lobbying activities as exogenously given. Laussel (2006) and Zudenkova (2008) allow lobbies to be endogenously formed. Yet, just like Mitra (1999), both papers presuppose a high degree of communication among those agents who could possibly benefit from lobbying. Thus, in these papers it is implicitly assumed that groups already exist. The question is whether these become active as lobby groups. However, the question as to why and when these groups are formed in the first place, is left unanswered. This chapter addresses these issues.

We present a theory that explains lobby group formation. In our model special interest groups engaging in lobbying activities, simply called *lobbies*, have to be built up from scratch by citizens who oppose the socially optimal policy. Some citizens prefer a more left-wing policy to the socially optimal policy and some prefer a more right-wing policy to the socially optimal policy.[4] Each citizen can initiate a special interest group, but it is costly to set up such an organization. A special interest group subsequently has to raise funds from citizens to finance its lobbying activities. In the last stage of the game the actual lobbying takes place.[5] The policy maker, who has both welfarist and rent-seeking motives, need not be swayed by a lobby's efforts. This source of uncertainty and the fact that a lobby could face competition from another lobby with opposing interests imply that starting a lobby is a risky endeavour. Importantly, because citizens cannot communicate with each other during the lobby formation stage, coordination failures and free-riding might impede the formation of a, from an individual citizen's perspective, desirable lobby.

Both the formation of lobbies and the probability that lobbying is successful, i.e. the probability that the government caters to a lobby's demands, is endogenous in our model. Lobby formation, lobby success, and the impact lobbying has on expected welfare depends on the level of political polarization, the affinity for contributions of the incumbent policy maker, the cost of getting organized, and the numbers of agents at each end of the political spectrum. We compare lobbying with another way to influence political outcomes: trying to get elected as a policy maker. This comparison sheds light on the question why some opt to lobby rather than participate in elections.

We model lobbying as a rent-seeking contest (Tullock, 1980, Konrad, 2009). We

[4]The left-wing/right-wing nomenclature is only used for expositional purposes.

[5]Lobbies could also have an incentive to gather information and provide it to the policy maker. See also Grossman and Helpman (2001). We abstract from this possibility and focus on activities which can directly affect the policy maker's payoff.

assume however that with some probability the policy maker ignores the lobbying efforts and simply implements the socially optimal policy. If the policy maker does give in to the efforts of a lobby, then the policy advocated by that lobby is implemented. This generalization of the standard contest specification, which is due to Dasgupta and Nti (1998), allows us to endogenize the design of the contest by letting the policy maker decide on the rules of the game. If the policy maker only cares about policy issues, then he completely ignores any lobbying activities. By contrast, a policy maker who derives utility from lobbying expenditures - for example, a corrupt policy maker - optimally trades off expected lobbying expenditures with social welfare when designing the contest. Only in the latter case does lobbying occur and need the socially optimal policy not be implemented. Our approach assigns a more active role to the policy maker than the common agency framework (Douglas Bernheim and Whinston, 1986) that is predominantly used to study lobbying.[6]

From the group perspective initiating a special interest group is certainly worthwile as long as the benefit of such an action outweighs its cost. However, since individual citizens cannot coordinate their plans and have an incentive to free-ride on the efforts borne by others, forming a lobby boils down to a collective action problem (Olson, 1965). In equilibrium each citizen initiates a lobby with the probability that balances the expenditures saved by free-riding with the cost of forgoing the posssibility to lobby the policy maker. The equilibrium of our model is therefore characterized by a *random* number of lobbies. Consequently, both excessive, purely wasteful lobby formation (several lobbies trying to accomplish the same) and a total lack of lobby formation can occur in equilibrium. Our model thus explains the multitude of special interest groups trying to acccomplish similar policy changes. For instance, Wikipedia lists ten interest groups advocating the right to own and bear firearms in the United States and eight interest groups which oppose such rights.[7]

A citizen who forms a lobby provides a collective good freely enjoyed by all agents with similar policy preferences. The expected value of this collective good depends on whether or not a lobby advocating the policy at the other end of the political spectrum is formed. If both left-wing and right-wing citizens manage to form one or more lobbies advocating their favourite policy, then in the lobbying stage competition between lobbies with diametrically opposed interests ensues. Competition increases donations and reduces the likelihood that lobbying is successful. In fact,

[6] In these common agency games the policy maker is the agent and the lobbies act as the agent's principals. Each lobby offers the policy maker a policy-contingent transfer. The policy maker subsequently implements a policy and receives the associated transfers. There is no strategic role for the policy maker: the lobbies de facto play a game 'through' their common agent.

[7] `http://en.wikipedia.org/wiki/National_Rifle_Association`, accessed February 13, 2009.

from an *ex ante* perspective, a citizen who prefers one of the two extreme policies faces a prisoner's dilemma: lobbying is better than refraining from civic action, yet competing with a lobby advocating the policy at the other end of the political spectrum is the worst possible outcome.

The observations of the previous paragraph imply that the value of the collective good is endogenous. Moreover, because funds are supplied by individual agents, the value of the collective good is reduced by *privately* incurred costs. These two factors - the endogenous nature of the collective good, the privately incurred costs - affect the incentives of individual citizens to initiate a lobby nontrivially. Whether or not large groups are less successful than small groups in furthering their interests, i.e. whether or not the Olson paradox (Olson, 1965) holds, depends therefore on the finer details of the economic environment. Esteban and Ray (2001) also study the group size paradox in a setting with a collective good with a partly private nature. It turns out that their result - the probability that a group wins a rent-seeking contest incrases in the group's size as long as the private component of the exogenous prize is sufficiently small - also holds in our setting with an *endogenous* prize.

Of course, citizens who favour the socially optimal policy could also engage in lobbying to counter the lobbying activities of citizens with 'extreme preferences'. In our basic model such 'moderate citizens' are assumed to refrain from lobbying. As a robustness check, we allow these citizens to also initiate lobbies in a simplified version of the model.[8] In this variant the policy maker is highly susceptible to lobbying efforts. This implies that it is very likely that the policy maker implements one of the extreme policies should moderate citizens refrain from lobbying. Yet, even in this worst case scenario for moderate citizens does lobbying by such citizens seldomly occur. In fact, if the number of citizens is sufficiently large, then citizens who prefer the socially optimal policy never initiate lobbies. The reason behind the spare use of lobbying by moderate citizens is that the potential gains stemming from lobbying are much smaller than those for citizens with extreme preferences: for instance, a left-wing citizen experiences much more disutility from implementation of the right-wing policy than a moderate citizen does.

We compare the outcomes of our lobbying game with the outcomes of a voting game similar to the citizen-candidates models of Osborne and Slivinski (1996) and Besley and Coate (1997). In our voting game each citizen, those with extreme preferences as well as those who prefer the socially optimal policy, can choose to become a candidate in an election. Running as a candidate is costly. The winner of the election

[8] Allowing moderate citizens to initiate lobbies in the full-fledged model renders this model untractable.

can implement his most preferred policy. Just like in the lobbying game, the inability of a citizen to communicate with fellow citizens leads to an equilibrium in mixed strategies. Since the socially optimal policy is (by definition) favoured by the median voter, a candidate advocating this policy attracts more votes than a candidate advocating another policy. Consequently, moderate citizens have the strongest incentives to become candidates. Members of the smallest group with extreme preferences (either the left-wing citizens or the right-wing citizens) never become candidates. The other group of citizens with extreme preferences can sometimes vote for a candidate who advocates their preferred policy. Yet, such a candidate only wins the election in the unlikely event that no moderate citizen runs for policy maker. The electoral route is therefore far less compelling than the lobbying route for citizens with extreme preferences and relatively few allies, especially if the level of political polarization is large.

The rest of this chapter is organized as follows. In the next section we introduce the basic model. Section 6.3 contains the analysis of the model, including derivation of equilibria, comparative statics results, and the aforementioned robustness check. We extend the model by endogenizing the contest design in Section 6.4. We also present welfare results in that section. A comparison of the lobbying route with the electoral route can be found in Section 6.5. Section 6.6 offers some concluding remarks. Proofs are relegated to the appendix.

6.2 The Model

The economy contains three types of agents: left-wing agents (L-*agents*), moderate agents (M-*agents*), and right-wing agents (R-*agents*). The M-agents have the same policy preferences as P, the incumbent policy maker. The policy maker's most preferred policy is called policy M. The L-agents prefer a more leftist policy (policy L) being implemented than the one preferred by P, whereas the R-agents prefer policy R. This policy is to the right of policy M. An i-agent suffers a disutility from policy $x \in \{L, M, R\}$ being implemented equal to the political distance between i and x, $i = L, M, R$. For the sake of simplicity we assume that the political distance between L and M is equal to the political distance between R and M. This distance is $\Delta > 0$. Because the policy issue is unidimensional, the political distance between L and R is simply 2Δ. So, if for instance L is implemented, then an R-agent experiences a disutility of 2Δ, whereas an M-agent experiences a disutility of Δ. The policies L and R, being located at the opposing endpoints of the political spectrum, are polar policies. The number Δ can therefore be interpreted as the level of political polarization. There are $n_i > 1$ i-agents, $i = L, M, R$.

Without any intervention by agents in the political process the policy maker will implement policy M. However, any L-agent or R-agent can start a special interest group which can lobby the policy maker. Initiating such a *lobby* comes at a monetary cost $f > 0$, incurred fully by the agent who decided to initiate a lobby. This start-up cost includes the costs of forming an organization, expenditures on public relations activities, and personnel costs. We suppose that getting organized is a prerequisite for successful lobbying as it "reduce[s] transaction costs in lobbying activity, coordinate[s] campaign-giving decisions, and communicate[s] political 'offers' to the politicians" (Mitra, 1999, p. 1120). Thus, without a full-fledged organization like-minded agents cannot coordinate their lobbying efforts and, more importantly, any attempt by an individual agent to influence the political process will be in vain. Importantly, an agent who has initiated a lobby cannot recoup her start-up cost f should another agent also have initiated a lobby advocating the same policy.

A lobby tries to raise money from agents. This money is subsequently used to influence P's policy choice. We assume that lobbies can effortlessly convert donations into lobbying efforts. These efforts are nonrefundable. If a lobby is successful, then the policy maker implements the lobby's most preferred policy (either policy L or policy R) instead of policy M. For the moment we assume that agents who prefer policy M do not engage in lobbying activities. We return to possible lobbying by M-agents in Subsection 6.3.4. There we show that these agents have much weaker incentives to form a lobby than L-agents and R-agents have. It is therefore reasonable to restrict attention to lobbying by agents with extreme preferences.

We model lobbying as a rent-seeking contest in the following way. The probability that P implements policy i is

$$\pi_i(D_L, D_R) = \frac{D_i}{D_L + D_R + \beta}, \quad i = L, R, \tag{6.1}$$

where D_i is the aggregate amount of donations to lobbies advocating policy i and $\beta \geq 0$. The probability that P implements policy M equals

$$\pi_M(D_L, D_R) = \frac{\beta}{D_L + D_R + \beta}. \tag{6.2}$$

The number β signifies the level of 'toughness' of the policy maker: as β increases, the probability that P sticks to policy M increases. The specification (6.1)-(6.2) is taken from Dasgupta and Nti (1998). It has two important features.

Firstly, there are no economies or diseconomies of scale. This implies that the distribution of efforts among lobbies advocating the same policy does not matter.

So, if for instance two lobbies advocate policy R, then the probability that R is implemented depends on the sum of the total donations to the two groups. The linearity of the above specification implies that lobbies do not face coordination problems (or any other type of frictions) at the lobbying stage or at the donations stage.[9] This contrasts with the first stage of the game, which is plagued by such collective action problems.

Secondly, if $\beta > 0$, then with positive probability neither contestant wins the 'prize' of the contest. Dasgupta and Nti (1998) use the specification (6.1)-(6.2) to allow for the possibility that the organizer of a contest values the contested prize himself and opts to retain it with positive probability. One of their motivating examples deals with a policy maker who cares about both welfare and any rents he might be able to obtain from holding office. In this example the prize is social welfare and β being positive means that the government cares about social welfare. So, besides the level of toughness one can interpret β as the level of benevolence of the policy maker P. We discuss the motives of our policy maker in due course. The case $\beta = 0$ coincides with the standard rent-seeking contest in which the organizer of the contest parts with the prize with certainty.

We mainly interpret an agent as an individual citizen. Yet, an agent could also be a (small) 'club' of citizens acting cooperatively. An example would be the inhabitants of a neighbourhood. Since we are interested in the formation of lobbies by agents who are unable to coordinate on a specific action, we take an agent to be the largest club of citizens such that coordination *within* a club is possible, but coordination *between* clubs is not possible. So, the members of a club do not suffer from the public good nature of initiating a lobby and can figure out a way to share the burden of the start-up cost f. Again, overcoming the coordination problems between agents by getting organized is crucial. Without an organization coordination is impossible.

An agent can donate any amount to any lobby. Of course, an i-agent will only donate, if she donates at all, to special interest groups lobbying for i. Since the distribution of donations among lobbies advocating policy i does not matter, we only have to look at the total donations of an i-agent to the collection of special interest groups lobbying for i, $i = L, R$. These collections play an important role in the analysis and we therefore introduce the following:

Definition 6.1 *The collection of interest groups lobbying for i is called* coalition i. *We say that* coalition i is formed *if at least one lobby advocating policy i emerges, i.e. if coalition i is non-empty, $i = L, R$.*

[9] This allows us to put these two stages together in one big stage without loss of generality.

Note that coalition i can contain any number of lobbies between 0 and n_i. Coalition i is *not* formed only if this number is 0. Note that an i-agent is unable to support her favourite policy in the second stage if coalition i is not formed.

Each agent attaches a constant marginal utility of $\lambda > 0$ to income. Note that a high λ indicates a low wealth level, i.e. the economy can be considered as poor. A rich economy is associated with a low λ. We therefore call $1/\lambda$ the level of wealth.[10] Taking into account the policy choice x, the possibly incurred start-up cost, and any individual donations $d \geq 0$, an L-agent's utility is:

$$u_L(x, d, \delta) = \begin{cases} -\lambda d - \delta\lambda f & \text{if } x = L \\ -\Delta - \lambda d - \delta\lambda f & \text{if } x = M \\ -2\Delta - \lambda d - \delta\lambda f & \text{if } x = R, \end{cases} \tag{6.3}$$

where $\delta = 0$ if the agent under consideration has not initiated a lobby and $\delta = 1$ if she has initiated a lobby. Likewise, if x is implemented, then an R-agent donating d has utility:

$$u_R(x, d, \delta) = \begin{cases} -\lambda d - \delta\lambda f & \text{if } x = R \\ -\Delta - \lambda d - \delta\lambda f & \text{if } x = M \\ -2\Delta - \lambda d - \delta\lambda f & \text{if } x = L. \end{cases} \tag{6.4}$$

All agents are risk-neutral.

The following game unfolds. In the first stage all agents independently and simultaneously decide whether or not to initiate a lobby. In the second stage each agent can donate to lobbies in coalition L or coalition R provided the coalition of her liking is formed. As f is sunk at this stage, whether or not an agent has initiated a lobby does not affect the amount she donates. In the last stage the policy that is going to be implemented is determined according to (6.1)-(6.2) and payoffs are realized. At each stage, past actions are observable.

We look for subgame perfect equilibria. A strategy of an agent consists of the probability with which the agent initiates a lobby in the first stage and the amount she donates in the second stage. Of course, these donations depend on which coalitions are formed in the first stage. The focus will be on symmetric equilibria, i.e. each L-agent uses the same strategy and each R-agent uses the same strategy. Imposing symmetry enables us to select a unique equilibrium. More importantly, it means that we focus on the only equilibrium of the game that does not presuppose coordination between agents: it is reasonable to assume that agents can infer that

[10] It is common practice in the rent-seeking literature to measure expenditures in units commensurate with the prize. We do not normalize λ to 1 to be able to emphasize the link between interest group formation and the level of wealth in the economy.

other agents with the same preferences and the same abilities will act in a similar fashion as themselves.[11]

6.3 Analysis

We introduce the following:

Condition 6.1 *The political distance is sufficiently large to warrant civic action. More specifically,* $\Delta > \beta\lambda$.

If this condition fails to hold, then either the policy maker is too tough or the political issue at stake is not important enough (relative to income) to make donations worthwile. As a consequence, no lobbies are initiated if $\Delta \leq \beta\lambda$. We concentrate on the more interesting situations in which lobbying can occur.

6.3.1 Equilibrium Donations

We determine each agent's expected utility gross of any start-up costs in equilibria of subgames starting in stage two. Four cases need to be considered: neither coalition is formed, only coalition L is formed, only coalition R is formed, and both coalitions are formed. Let A be the set of formed coalitions, so $A \in \{\emptyset, \{L\}, \{R\}, \{L, R\}\}$. We denote an i-agent's expected utility gross of start-up costs in the equilibrium of 'continuation game A' by $v_i(A)$, $i = L, R$. We call this number an i-agent's payoff in A. An i-agent's equilibrium donation in continuation game A is denoted $d_i(A)$. These numbers are derived below.

Neither coalition is formed. The policy maker implements M after having received no donations whatsoever. The two types of agents have the same payoff in this continuation game: $v_L(\emptyset) = v_R(\emptyset) = -\Delta$.

Only L is formed. In this case the policy maker implements either L (with probability $\frac{D_L}{D_L+\beta}$) or M (with the complementary probability). Let D_L^{-j} be the aggregate donations to coalition L excluding the donations of L-agent j. We derive j's best response to D_L^{-j}. If this agent donates d^j, then her expected (gross) utility equals

$$-\Delta \frac{\beta}{(D_L^{-j} + d^j) + \beta} - \lambda d^j.$$

Agent j's first-order condition reads

$$\Delta \frac{\beta}{\left(D_L^{-j} + d^j + \beta\right)^2} = \lambda.$$

[11] The symmetric subgame perfect equilibrium is thus *focal*.

Observe that agent j wants to donate up to the point where the marginal gain of the *total* donations equal λ. We discuss the implications of this feature of the model below. Invoking symmetry yields an L-agent's equilibrium donation:

$$d_L(\{L\}) = \frac{\sqrt{\beta\lambda\Delta} - \beta\lambda}{\lambda n_L}. \tag{6.5}$$

Aggregate equilibrium donations to L are $D_L(\{L\}) = \big(\sqrt{\beta\lambda\Delta} - \beta\lambda\big)/\lambda$. It follows that L is implemented with probability $1 - \sqrt{\frac{\beta\lambda}{\Delta}}$, whereas M is implemented with probability $\sqrt{\frac{\beta\lambda}{\Delta}}$. The payoffs in continuation game $\{L\}$ are

$$v_L(\{L\}) = -\sqrt{\beta\lambda\Delta}\Big(1 + \frac{1}{n_L}\Big) + \frac{\beta\lambda}{n_L},$$
$$v_R(\{L\}) = -2\Delta + \sqrt{\beta\lambda\Delta}.$$

Only R is formed. The analysis of this case mirrors the analysis of the previous case. We can immediately conclude that

$$d_R(\{R\}) = \frac{\sqrt{\beta\lambda\Delta} - \beta\lambda}{\lambda n_R}, \tag{6.6}$$

and that

$$v_L(\{R\}) = -2\Delta + \sqrt{\beta\lambda\Delta},$$
$$v_R(\{R\}) = -\sqrt{\beta\lambda\Delta}\Big(1 + \frac{1}{n_R}\Big) + \frac{\beta\lambda}{n_R}.$$

Both L and R are formed. Let D_R, the aggregate donations to coalition R, be given and suppose that the donations to L excluding those of L-agent j sum to D_L^{-j}. If this agent donates d^j her expected gross utility reads

$$-\Delta\frac{2D_R + \beta}{(D_L^{-j} + d^j) + D_R + \beta} - \lambda d^j.$$

Similarly, R-agent k donating an amount d^k enjoys an expected gross utility of

$$-\Delta\frac{2D_L + \beta}{D_L + (D_R^{-k} + d^k) + \beta} - \lambda d^k,$$

where D_R^{-k} is the aggregate donations of other R-agents. The associated first-order conditions reveal that in equilibrium the following holds:

$$\lambda = \Delta\frac{2D_R + \beta}{\big((D_L^{-j} + d^j) + D_R + \beta\big)^2} = \Delta\frac{2D_L + \beta}{\big(D_L + (D_R^{-k} + d^k) + \beta\big)^2}.$$

In equilibrium the two denominators are the same and hence aggregate donations are $D_L(\{L,R\}) = D_R(\{L,R\}) = \frac{1}{2}(\Delta/\lambda - \beta)$. Invoking symmetry yields

$$\begin{aligned} d_L(\{L,R\}) &= \frac{\Delta - \beta\lambda}{2\lambda n_L}, \\ d_R(\{L,R\}) &= \frac{\Delta - \beta\lambda}{2\lambda n_R}. \end{aligned} \tag{6.7}$$

The associated probability that P implements i is $\frac{1}{2}(1-\beta\lambda/\Delta)$, $i = L, R$. The policy maker sticks to M with probability $\beta\lambda/\Delta$. The payoffs in this continuation game are:

$$\begin{aligned} v_L(\{L,R\}) &= -\Delta\Big(1 + \frac{1}{2n_L}\Big) + \frac{\beta\lambda}{2n_L}, \\ v_R(\{L,R\}) &= -\Delta\Big(1 + \frac{1}{2n_R}\Big) + \frac{\beta\lambda}{2n_R}. \end{aligned} \tag{6.8}$$

From the above findings we distill the following:

Lemma 6.1 *Assume Condition 6.1 holds. If coalition i ($i = L, R$) is formed, then in the symmetric equilibrium an i-agent donates more if the other coalition is also formed ($d_i(\{L,R\}) > d_i(\{i\})$), but the probability that i is implemented is smaller if the other coalition is also formed (*$\Pr(P \text{ implements } i | \{L,R\}) < \Pr(P \text{ implements } i | \{i\})$*). An i-agent ranks her payoffs in the various continuation games as follows:*

$$\begin{aligned} v_L(\{L\}) > v_L(\emptyset) > v_L(\{L,R\}) > v_L(\{R\}), \\ v_R(\{R\}) > v_R(\emptyset) > v_R(\{L,R\}) > v_R(\{L\}). \end{aligned} \tag{6.9}$$

As has already been noted, an individual agent has an incentive to donate up to the point where the marginal political gain of the *total* donations equals λ. This implies that any distribution of donations among i-agents such that these donations sum to $D_i(A)$ constitutes an equilibrium strategy profile of i-agents in continuation game A.[12] More importantly, it also implies that there is no collective action problem at this stage: the choices of individual i-agents lead to the same aggregate donations as the choice of a planner maximizing the collective payoff of all i-agents.

Lemma 6.1 states that a coalition of lobbies receives more donations if the other coalition is also formed, i.e. competition between lobbies amplifies donations. At the same time, the probability that the policy maker gives in to the equilibrium

[12] This observation is also made in Baik (1993). Baik furthermore shows that only the "hungriest" agents, i.e. those with the lowest λ or, equivalently, the highest Δ, will expend effort in a rent-seeking contest with a public-good prize. One easily sees that, ignoring lump-sum transfers between i-agents, the other equilibria of continuation game A lead to the same outcomes.

efforts expended by coalition i is smaller if the other coalition is also present. Yet, the probability that P implements one of the two extremal policies is larger if both coalitions are formed. These results lead to the ranking (6.9).

6.3.2 Equilibrium Lobby Formation

In a symmetric equilibrium in mixed strategies an L-agent starts a lobby with some probability p_L and an R-agent with some probability p_R. Obviously, existence of such equilibria depends on the size of the start-up cost f. We now investigate possible mixed strategy equilibria assuming f is such that a symmetric equilibrium in mixed strategies does exist. A sufficient condition ensuring that the game has a unique symmetric equilibrium in mixed strategies is presented below.

Let ϕ_i be the probability that coalition i is *not* formed if each individual i-agent initiates a lobby with probability p_i in the first stage of the game, i.e. $\phi_i := (1-p_i)^{n_i}$, $i = L, R$. If an i-agent opts for a mixed strategy, then she must be indifferent between not initiating a lobby and initiating a lobby. The following equality therefore holds for $i = L$ in equilibrium:

$$(1-p_L)^{n_L-1}\phi_R v_L(\emptyset)+\big(1-(1-p_L)^{n_L-1}\big)\phi_R v_L(\{L\})+(1-p_L)^{n_L-1}(1-\phi_R)v_L(\{R\})+$$
$$\big(1-(1-p_L)^{n_L-1}\big)(1-\phi_R)v_L(\{L,R\}) = \phi_R v_L(\{L\})+(1-\phi_R)v_L(\{L,R\})-\lambda f. \quad (6.10)$$

The left-hand side is an L-agent's expected utility if she decides to refrain from initiating a lobby and all other citizens use the mixed strategies specified by (p_L, p_R). It consists of her payoffs in the four possible continuation games. These payoffs are weighted according to the probability with which they occur should every agent except the L-agent under consideration abide by the above mixed strategies. The right-hand side is her expected utility if she does initiate a lobby. Of course, besides the start-up cost, the right-hand side only contains the payoffs in continuation games in which coalition L is formed. Note that, since the distribution of donations among lobbies advocating the same policy does not matter, neither side of the equilibrium condition depends on the number of lobbies advocating a given policy. Interchanging L and R in the above equilibrium condition leads to the equilibrium condition for an R-agent.

Some tedious algebra, which is relegated to the appendix, reduces the two equilibrium conditions to:

$$\phi_L^{1-\frac{1}{n_L}}\left(1-\nu-\frac{1-\nu^2}{2n_L}+\frac{(1-\nu)^2}{2n_L}\phi_R\right) = F, \quad (6.11)$$
$$\phi_R^{1-\frac{1}{n_R}}\left(1-\nu-\frac{1-\nu^2}{2n_R}+\frac{(1-\nu)^2}{2n_R}\phi_L\right) = F, \quad (6.12)$$

where $\nu := \sqrt{\frac{\beta\lambda}{\Delta}} \in (0,1)$ and $F := \frac{\lambda f}{\Delta}$. Recall from Lemma 6.1 that $\frac{\beta\lambda}{\Delta}$ is the equilibrium probability that P implements M if both coalitions are formed, whereas $\sqrt{\frac{\beta\lambda}{\Delta}}$ is the equilibrium probability that P implements M if one coalition is formed. The number ν thus measures the likelihood that lobbying fails. The ratio F is a simple cost-benefit ratio: it is the loss in utility associated with the cost of initiating a lobby divided by the gain in utility if the most preferred policy is implemented instead of policy M. Armed with these interpretations, one sees that the left-hand sides of (6.11)-(6.12) equal the expected gain from initiating a lobby for an L-agent and an R-agent respectively, whereas the right-hand sides are equal to the cost of initiating a lobby. The expected gain from initiating a lobby for, say, an R-agent is the probability-weighted sum of two terms: the increase in payoff $v_R(\{L, R\}) - v_R(\{L\})$ and the increase in payoff $v_R(\{R\}) - v_R(\emptyset)$.

We now present the following:

Proposition 6.1 *Assume Condition 6.1 holds. If*

$$F < 1 - \nu - \frac{1-\nu^2}{2n_i}, \quad i = L, R, \tag{6.13}$$

then the game has a unique symmetric subgame perfect equilibrium. In this equilibrium an i-agent initiates a lobby with probability $p_i^* \in (0,1)$ in the first stage, where $p_i^* = 1 - (\phi_i^*)^{\frac{1}{n}}$, $i = L, R$. The pair (ϕ_L^*, ϕ_R^*) is the unique solution to (6.11)-(6.12). The equilibrium donations in the continuation games $\{L\}$, $\{R\}$, and $\{L, R\}$ are given in (6.5), (6.6), and (6.7).

The above result states that all agents initiate a lobby with positive probability as long as the cost of such an action (measured in cost per benefit units) is not too large compared to a measure of the expected gain from lobbying. This measure, i.e. the right-hand side of (6.13), is proportional to an i-agent's gain from lobbying if the opposing group of agents form their coalition with certainty and all other i-agents refrain from lobbying. To arrive at this measure for, say, $i = L$ one has to substitute $\phi_L^* = 1$ and $\phi_R^* = 0$ in the left-hand side of (6.11). The condition is reminiscent of the "privileged group" condition of Olson (1965) which states (in the present setting) that the private expected benefit from the prize to one agent exceeds the cost of initiating a lobby. Proposition 6.1 reveals that even in this privileged group situation collective action need not ensue.

If F or ν is too large, then agents never initiate lobbies. The first type of agents who refrain from such actions are those with the smallest number of political allies. For instance, if $n_R < n_L$, then R-agents are more likely to remain inactive in stage one: in that case condition (6.13) is more stringent for $i = R$. The reason is that the anticipated per agent donation in stage two is very high if the number of political allies is small as can be gathered from (6.5), (6.6), and (6.7). Note that the game has a unique symmetric subgame perfect equilibrium for any number of agents $n_L > 1$ and $n_R > 1$ if $F < 1 - \nu - \frac{1-\nu^2}{4}$.

Because initiating a lobby advocating a certain policy beyond the first one advocating that policy merely entails expending start-up costs (and is thus purely wasteful), agents do not opt to initiate a lobby with certainty even if F and ν are both very small. This implies that lobbying for i need not occur in equilibrium (in fact, such lobbying occurs with probability $1 - \phi_i^* < 1$). Thus, because agents are unable to coordinate actions in the first stage, they run the risk of not having the opportunity to lobby with the policy maker in the second stage. This risk does not vanish as the conditions for lobbying improve (F and/or ν decrease). On the other hand, miscoordination can also lead to excessive lobby formation.

6.3.3 Comparative Statics

Proposition 6.1 ascertains that precisely one symmetric equilibrium can prevail. This result allows us to perform comparative statics. Our first comparative statics result deals with changes in p_i^* and ϕ_i^* as the parameters Δ, f, λ, and β vary:

Proposition 6.2 *Assume Condition 6.1 and the inequalities (6.13) hold. Then the equilibrium probability p_i^* that an i-agent initiates a lobby, $i = L, R$, increases in the level of political polarization Δ, decreases in the start-up cost f, increases in the level of wealth $1/\lambda$, and decreases in the level of toughness of the policy maker β.*

The probability that at least one lobby advocating policy i is formed (i.e. $1 - \phi_i^*$), $i = L, R$, reacts in the same manner on changes in these parameters.

Proposition 6.2 is intuitive. As the saliency of the political issue at stake relative to income increases, that is either Δ or $1/\lambda$ increases, undertaking civic action becomes more worthwile and hence an agent becomes more inclined to initiate a lobby. On the other hand, if it becomes less likely that the policy maker can be swayed by lobbying efforts, i.e. P's toughness β increases, agents are more likely to refrain from forming lobbies. Naturally, an increase in the start-up cost f makes agents more reluctant to initiate a lobby.

Contrary to the comparative statics with respect to Δ, f, $1/\lambda$, and β, changes in the number of agents on either side of the political spectrum do not lead to straightforward changes in the equilibrium strategies. As the next result shows, the behaviour of the equilibrium strategies as the number of agents varies depends non-trivially on the values of F and ν.

Proposition 6.3 *Assume Condition 6.1 and the inequalities (6.13) hold. Then there exist maps* $\Phi : \mathbb{N} \times [0,1] \to [0,1]$, $\Psi : \mathbb{N} \times [0,1] \to [0,1]$, *where* $\Psi(n,\tau) \leq \Phi(n,\tau)$ *for all* $n \geq 2$, $\tau \in [0,1]$, *such that the following holds:*

- ϕ_L^* *increases in* n_L *and* ϕ_R^* *decreases in* n_L *if* $F < \Psi(n_L, \nu)$,
- ϕ_L^* *decreases in* n_L *and* ϕ_R^* *increases in* n_L *if* $F > \Phi(n_L, \nu)$,
- ϕ_R^* *increases in* n_R *and* ϕ_L^* *decreases in* n_R *if* $F < \Psi(n_R, \nu)$,
- ϕ_R^* *decreases in* n_R *and* ϕ_L^* *increases in* n_R *if* $F > \Phi(n_R, \nu)$.

Both Φ *and* Ψ *increase in their first argument and decrease in their second argument. Moreover, for* $\nu < 1$ *sufficiently large, there exists a non-empty interval* $I(\nu)$ *such that* ϕ_i^*, $i = L, R$, *decreases in* n_i *for* n_i *small but increases in* n_i *for* n_i *large as long as* $F \in I(\nu)$.

Figure 6.1: If $\nu = 0.8$, then ϕ_i^* is nonmonotonic in n_i if $F \in I(0.8)$.

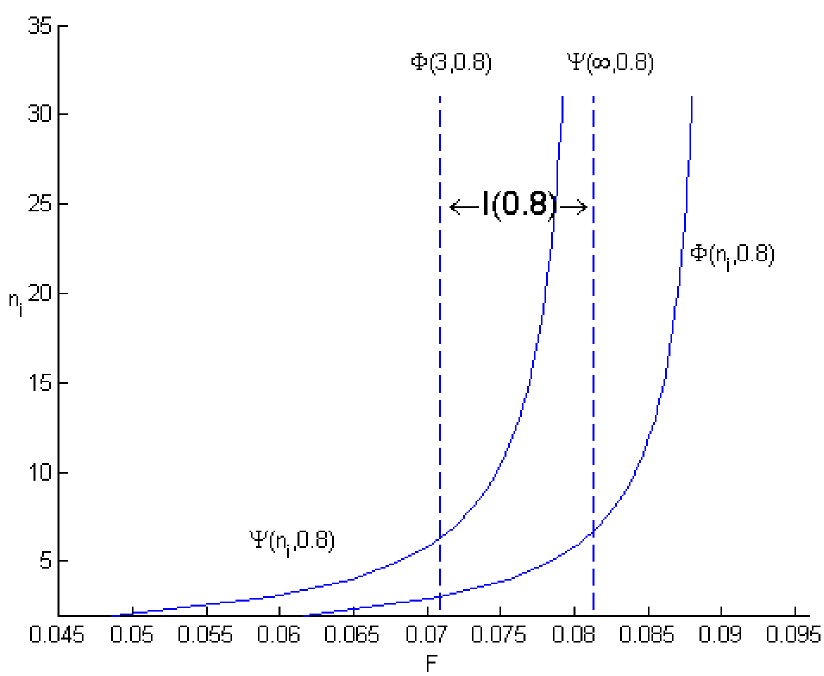

Figure 1 illustrates some of the claims of Proposition 6.3. It depicts (F, n_i)-space partitioned into various regions. To the left of the curve labelled $\Psi(n_i, 0.8)$, F is smaller than $\Psi(n_i, 0.8)$ and hence ϕ_i^* is increasing in n_i in this region. In the region to the right of the curve labelled $\Phi(n_i, 0.8)$, F exceeds $\Phi(n_i, 0.8)$ and ϕ_i^* is therefore decreasing in n_i. If $F \in I(0.8)$, say $F = 0.075$, then ϕ_i^* is nonmonotonic in n_i. To see this observe that the vertical line $F = 0.075$ crosses the graph of $\Phi(\cdot, 0.8)$ as well as the graph of $\Psi(\cdot, 0.8)$. This has two implications. Firstly, because $0.075 > \Phi(3, 0.8)$, ϕ_i^* decreases as n_i increases from 2 to 3, i.e. ϕ_i^* is decreasing in n_i if n_i is sufficiently small. Secondly, at the same time $0.075 < \Psi(\infty, 0.8)$, implying that ϕ_i^* must be increasing in n_i for n_i large.

The result that ϕ_L^* and ϕ_R^* move in opposite direction as n_L or n_R changes is not difficult to understand. If, say, ϕ_R^* increases due to an increase in n_R, then the expected gain in utility of an L-agent from initiating a lobby increases. The reason is that a special interest group lobbying for policy L has a smaller probability of facing competition (by a lobby advocating policy R) in the last stage. The prospect of competing with a lobby advocating policy R (continuation game $\{L, R\}$) is particularly unattractive for an L-agent contemplating initiating a lobby as it combines large donations with a low probability of getting the favorite policy implemented. In fact, it is not difficult to see that $v_L(\{L, R\}) - v_L(\{R\}) < v_L(\{L\}) - v_L(\emptyset)$. The probability that a special interest group lobbying for policy L does not face competition, which is thus the rosier prospect for our L-agent, increases as ϕ_R^* increases. These observations imply that ϕ_L^* and ϕ_R^* move in opposite direction.

The intuition behind the result that the response of ϕ_i^* to changes in the number of agents depends crucially on F and ν is more involved. To explain this intuition we first list the three effects that affect an agent's incentives in the first stage. Firstly, an agent's incentive to initiate a lobby depends on the expected gain of such an action should no other agent with the same political stance initiate a lobby. We call this the *gain effect*. As can be gathered from (6.11)-(6.12), the expected gain equals $\left(1 - \nu - \frac{1-\nu^2}{2n_L} + \frac{(1-\nu)^2}{2n_L}\phi_R\right)$ in case $i = L$. (The expectation with respect to the contest probabilities is taken.) Because total donations are borne by more agents as n_L grows, the expected gain increases in n_L. Secondly, the usefulness of initiating a lobby depends on whether or not other agents also initiate lobbies: if one particular i-agent initiates a lobby, then other i-agents prefer to free-ride on this particular agent's action rather than initiate their own lobby. This *free-riding effect* is the term $\phi_i^{1-\frac{1}{n_i}}$ in (6.11)-(6.12). Lastly, an agent becomes less inclined to initiate a lobby if F increases. This is the *cost effect*.

Let us now consider the impact F has on changes in ϕ_L^* as n_L changes. We know from Proposition 6.2 that ϕ_L^* and ϕ_R^* are relatively small if F is small. This has two implications. Firstly, the change in the gain effect for an L-agent as n_L is replaced by $n_L + 1$ is small if ϕ_R^* is small. Secondly, a small ϕ_L^* means that p_L^* is relatively large. As a consequence, the probability that a lobby advocating policy L is formed increases quite a bit as the number of L-agents increases by 1 should p_L^* remain constant. But, if initiating a lobby becomes only marginally more attractive (the first implication), agents have an incentive to exploit the large change in the free-riding effect (the second implication) and thus reduce the probability with which they initiate a lobby. Hence a higher ϕ_L^* prevails. By contrast, if F is relatively large, then the change in the free-riding effect is small and does not outweigh the (large) gain effect. As a consequence, ϕ_L^* decreases in n_L.

To understand how ν affects changes in ϕ_L^* as n_L increases, we use insights provided by Esteban and Ray (2001). Esteban and Ray (2001) investigate the collective action problem when the prize a group can obtain by participating in a rent-seeking contest can have both a *private component* and a *public component*. The private component of the prize has to be divided between the members of the winning group. The benefit of the private component to an individual member of a group thus diminishes as the group size increases. By contrast, the benefit of the public component to a group member does not depend on the group size. They show that the equilibrium probability that a specific group wins the rent-seeking contest increases in this group's size if the public component is sufficiently large relative to the total size of the prize.

In the present setting, the agents face a collective action problem in the first stage of the game similar in spirit to the one Esteban and Ray (2001) consider. If an agent initiates a lobby with a higher probability, i.e. exerts more individual effort (in expected terms), then the probability that the prize is obtained in the second stage increases.[13] The probabilities p_L and p_R can thus be seen as individual efforts exerted in the 'contest' taking place in the first stage. Moreover, the prize of this 'contest' (the gain effect) has a private component and a public component. The private component of the gain effect is $-\frac{1-\nu^2}{2} + \frac{(1-\nu)^2}{2}\phi_R^*$, the public component is $1 - \nu$. The relative size of the public component equals

$$\frac{1}{1 - \frac{1}{2}(1 - \phi_R^*) - \frac{\nu}{2}(1 + \phi_R^*)}.$$

This number increases in ν. Consequently, a large ν means a relative important public component and therefore, invoking Esteban and Ray's intuition, the probability that the L-agents win, i.e. $1 - \phi_L^*$, increases in n_L. This means that ϕ_L^* decreases in n_L if ν is large. This conclusion is in line with the second bullet point of Proposition 6.3.

[13]Esteban and Ray (2001) allow the cost of effort to be nonlinear. In our setting this (expected) cost is linear and equals $p_i \lambda f$, $i = L, R$.

The first bullet point informs us that the probability that the L-agents win decreases in n_L if the public component is relatively small.

Esteban and Ray's results are intuitive in our setting: if the private component is relatively important, then the gain diminishes rapidly when the number of L-agents increases. The incentive to initiate a lobby then also diminishes rapidly and a higher ϕ_L^* prevails. Conversely, if the public component is relatively important, the decrease in the incentive to initiate a lobby is more than offset by the increase in the number of agents who initiate a lobby with positive probability and hence a lower ϕ_L^* prevails.

6.3.4 Agents with Moderate Preferences

In this section we perform a robustness check. It could be that agents who have moderate preferences, i.e. the M-agents, have an incentive to counter the lobbying activities of agents with extreme preferences by also becoming involved in lobbying. To investigate this possibility we look at a situation in which L-agents, R-agents, as well as M-agents can initiate special interest groups which can lobby with the policy maker. We assume that the policy maker retains the prize with zero probability, i.e. $\beta = 0$. The policy maker implements policy M if no lobbying occurs. We thus focus on the standard contest success function as used in Tullock (1980). Observe that this situation is a worst case scenario for M-agents: if they refrain from civic action, then the policy maker P will not implement their favorite policy as soon as at least one of the extreme collections is formed. If β were positive, then P would implement policy M with positive probability for any configuration of formed collections. The case $\beta = 0$ thus brings about the largest incentive for an M-agent to initiate a lobby.

The n_M M-agents experience a disutility of Δ if either policy L or policy R is implemented and experience no disutility if policy M is implemented. Moreover, these agents have the same wealth level $(1/\lambda)$ as L-agents and R-agents have. Just like L-agents and R-agents, an M-agent incurs a start-up cost of f if she initiates a lobby. If a lobby advocating policy M emerges, we say that *coalition* M is formed. Again, the distribution of resources among lobbies advocating the same policy does not matter. We maintain the timing of the basic model and look for symmetric subgame perfect equilibria (in mixed strategies).

A detailed analysis of the various continuation games is relegated to the appendix. The payoffs of an agent of type $i \in \{L, M, R\}$ in each continuation game $A \in \{\emptyset, \{L\}, \{M\}, \{R\}, \{L, M\}, \{L, R\}, \{M, R\}, \{L, M, R\}\}$ (denoted $v_i(A)$) are gathered in Table 1. This table reveals that M-agents, just like L-agents and R-

Table 6.1: Payoffs in the various continuation games of the game with M-agents.

	$v_L(\cdot)$	$v_M(\cdot)$	$v_R(\cdot)$
$\emptyset$	$-\Delta$	0	$-\Delta$
$\{L\}$	0	$-\Delta$	-2Δ
$\{M\}$	$-\Delta$	0	$-\Delta$
$\{R\}$	-2Δ	$-\Delta$	0
$\{L,M\}$	$-\frac{\Delta}{2}-\frac{\Delta}{4\lambda n_L}$	$-\frac{\Delta}{2}-\frac{\Delta}{4\lambda n_M}$	$-\frac{3}{2}\Delta$
$\{L,R\}$	$-\Delta-\frac{\Delta}{2\lambda n_L}$	$-\Delta$	$-\Delta-\frac{\Delta}{2\lambda n_R}$
$\{M,R\}$	$-\frac{3}{2}\Delta$	$-\frac{\Delta}{2}-\frac{\Delta}{4\lambda n_M}$	$-\frac{\Delta}{2}-\frac{\Delta}{4\lambda n_R}$
$\{L,M,R\}$	$-\Delta-\frac{\Delta}{2\lambda n_L}$	$-\Delta$	$-\Delta-\frac{\Delta}{2\lambda n_R}$

agents, face a prisoner's dilemma. However, M-agents suffer less from this dilemma than L-agents and R-agents do. The reason is that the political distance between M-agents and the other agents is always Δ, implying that M-agents experience a policy disutility of at most Δ. The political distance between L-agents and R-agents is 2Δ, implying that the policy disutility of agents with extreme preferences can be twice as large. This difference in possible disutility brings about a lower marginal gain from donating to a lobby favouring M for M-agents compared to the marginal gain from donating to a lobby of their liking for L-agents and R-agents should all three coalitions be formed. In fact, M-agents refrain from donating to coalition M altogether if both rival coalitions are formed (continuation game $\{L, M, R\}$). Moreover, M-agents are less keen than L-agents and R-agents to engage in civic action in the first stage of the game. Using techniques similar to those used in Subsection 6.3.2 allows us to formalize this observation:

Proposition 6.4 *If $F \geq \frac{1}{2}\left(1-\frac{1}{2n_M}\right)$, then M-agents never initiate lobbies. Consequently, the unique symmetric subgame perfect equilibrium of Proposition 6.1 prevails provided Condition 6.1 and the inequalities (6.13) hold. Moreover, if the number of i-agents (n_i) is sufficiently large, $i = L, M, R$, then M-agents never initiate lobbies, irrespective of the value of F.*

So, even in the worst case scenario $\beta = 0$ the conditions for coalition M to be formed with positive probability are much more stringent than the conditions for coalition L or coalition R to be formed with positive probability. In fact, even if F is relatively small does a sufficient condition à la the inequalities (6.13) not obtain. Furthermore, if the economy is very populated (n_L, n_M, n_R large), then M-agents never initiate lobbies no matter how small the start-up cost. We think that Proposition 6.4 provides ample reasons to restrict attention to situations in which only agents with extreme preferences can get organized.

6.4 Rent-seeking Motives and Welfare

In this section we endogenize the policy maker's level of toughness β. Recall that it was argued that β is linked to P's benevolence. We establish this link by approximating agents' behaviour by their limiting behaviour as the number of agents at both sides of the political spectrum goes to infinity. Restricting attention to such large populations is reasonable: the vast majority of countries in the world have large numbers of inhabitants who strongly object their government's policy plans. We once again assume that M-agents remain inactive. The following lemma characterizes the limiting behaviour of the equilibrium probabilities ϕ_L^* and ϕ_R^*:

Lemma 6.2 *Assume Condition 6.1 and the inequalities (6.13) hold. Then* $\phi_L^* \to \frac{F}{1-\nu}$ *as* $n_L \to \infty$ *(for fixed* n_R*) and* $\phi_R^* \to \frac{F}{1-\nu}$ *as* $n_R \to \infty$ *(for fixed* n_L*).*

This lemma follows directly from the equilibrium conditions (6.11)-(6.12). From (6.13) we know that this number is strictly between zero and one. So, even with infinitely many allies does an agent run the risk of not being able to lobby the policy maker in the second stage. Intuitively, if the number of agents is very large, then the individual donations become negligible. An individual agent's expected loss associated with her not initiating a lobby therefore converges to $\phi(1-\nu)\Delta$. Since in equilibrium the probability that coalition i is not formed equates an individual agent's cost of initiating a lobby (λf) with the aforementioned loss, ϕ^* must converge to $\frac{F}{1-\nu}$.

We now look at an extension of the game analyzed in the previous section. In this extended game the policy maker sets β in a stage preceding the stage in which lobbies can be formed. In other words, the policy maker designs the contest. We assume that the policy maker is able to commit himself to the chosen design in the last stage of the game. For reasons of analytical tractability we confine attention to settings with a large number of agents at both ends of the political spectrum (the setting with a *large population*), allowing us to use the limiting behaviour presented in Lemma 6.2.

In the extended game the policy maker P cares about social welfare and about the efforts expended by lobbies in the rent-seeking contest. Let $\alpha \geq 0$ be the weight the policy maker attaches to lobbies' efforts relative to total social welfare.[14] If $\alpha = 0$, then the policy maker is purely benevolent and does not care about the lobbies'

[14] Grossman and Helpman (1996) and Prat (2002) propose models which link the size of (campaign) funds of a political party/politician to the probability that that political party/politician

efforts at all. A positive α indicates that P can, to some extent, be influenced by these efforts. If these efforts constitute illegal monetary transfers from the lobbies to P, then one can interpret α as P's level of corruptibility. For the sake of brevity we restrict attention to this interpretation. Needless to say, the model applies equally well to situations in which the efforts of lobbies embody perfectly legal perks.

Whether policy M is the socially optimal policy depends on the number of agents at each point of the political spectrum. Denote the fraction of the population who prefer policy i to policy M by μ_i, so $\mu_i := \frac{n_i}{n_L+n_M+n_R}$, $i = L, R$. Then, after normalizing the total size of the population to 1, total social welfare amounts to $S_M := -(\mu_L+\mu_R)\Delta$ if M is implemented, $S_L := -(1-\mu_L-\mu_R)\Delta-2\mu_R\Delta = -(1-\mu_L+\mu_R)\Delta$ if L is implemented, and $S_R := -(1+\mu_L-\mu_R)\Delta$ if R is implemented. Comparing these expressions reveals that policy M is the social optimum if and only if the following holds:

Condition 6.2 *Extreme policies are never supported by a majority, i.e.* $\mu_L < \frac{1}{2}$ *and* $\mu_R < \frac{1}{2}$.

We return to the policy maker's motives. The payoff of P in continuation game $A \in \{\emptyset, \{L\}, \{R\}, \{L, R\}\}$, $W(A)$, reads:

$$W(A) = \alpha\big(D_L(A) + D_R(A)\big) + \sum_{j\in\{L,M,R\}} \Pr(P \;\text{ implements }\; j|A) \times S_j. \tag{6.14}$$

If the number of agents at each end of the political spectrum is large, then the probability that continuation game A occurs can be approximated by the limiting probabilities given in Lemma 6.2. The associated expected utility of the policy maker, $\mathcal{W}$, is:

$$\mathcal{W} = \big(\tfrac{F}{1-\nu}\big)^2 w(\emptyset)+\tfrac{F}{1-\nu}\big(1-\tfrac{F}{1-\nu}\big)w(\{L\})+\tfrac{F}{1-\nu}\big(1-\tfrac{F}{1-\nu}\big)w(\{R\})+\big(1-\tfrac{F}{1-\nu}\big)^2 w(\{L,R\}). \tag{6.15}$$

Some tedious algebra, which can be found in the appendix, simplifies the above expression to:

$$\mathcal{W} = \Delta(1-\nu-F)\big((1+\nu-F)z-2F\big) - \Delta\mu(\nu+F)^2, \tag{6.16}$$

where $z := \frac{\alpha-\lambda}{\lambda}$ and $\mu := \mu_L + \mu_R$. The parameter z indicates how much the policy maker values money more or less than agents do. In particular, because $\alpha = (1+z)\lambda$, a positive z means that P values money more than agents do, whereas a negative z

wins the next election. These models provide a rationale for a positive α, even if the policy maker cannot wield the funds for private use.

means P is less interested in money than agents are. The parameter μ is the fraction of the population who opposes policy M or, more briefly, the fraction of opposers.

The expected utility of the policy maker depends on β only via ν. Differentiating $\mathcal{W}$ with respect to ν thus gives P's first order condition from which one deduces the following:

Proposition 6.5 *Suppose Condition 6.2 holds. If $z > -\mu$ and $F < \frac{\mu+z}{1+z}$, then in the unique equilibrium of the extended game with a large population the policy maker P sets*

$$\nu^* = \frac{1-\mu}{\mu+z}F, \tag{6.17}$$

yielding

$$\beta^* = \frac{\lambda}{\Delta} \times \big(\frac{\lambda(1-\mu)}{\alpha-\lambda(1-\mu)}\big)^2 f^2. \tag{6.18}$$

This equilibrium level of toughness β^* decreases in the level of political polarization Δ, increases in the start-up cost f, decreases in the level of wealth $1/\lambda$, decreases in the fraction of opposers μ, and decreases in P's level of corruptibility α.

If either $F \geq \frac{\mu+z}{1+z}$ or $z \leq -\mu$, then P sets β so large that both L-agents and R-agents refrain from initiating lobbies.

When designing the contest the policy maker faces a trade-off between higher expected aggregate donations and lower social welfare: a higher β, and hence a higher ν, leads to lower expected aggregate donations,[15] but at the same time to a higher probability that P implements the welfare maximizing policy. Observe that the policy maker, by choosing a level of toughness β, not only designs the contest, but also chooses the probability with which the contest takes place in the second stage. In other words, Proposition 6.5 provides an *endogenous* probability that lobbying takes place. The optimally designed contest responds to changes in the parameters in an intricate manner as is explained below.

An increase in the level of political polarization Δ increases the expected political gain from donating in the second stage. As a result, aggregate donations (in the continuation games with formed collections) go up as Δ increases. To fully benefit from these larger donations, P increases the likelihood that the contest ensues by reducing β. Note that there is also a downside to decreasing β in response to an increase in Δ: the loss in social welfare from implementing either extreme policy instead of policy M (the socially optimal policy) is aggravated.[16] This effect is,

[15] Expected aggregate donations equal $\frac{\Delta}{\lambda}\big((1-F)^2 - \nu^2\big)$, so the donations component of $\mathcal{W}$ is $(1+z)\Delta\big((1-F)^2 - \nu^2\big)$.

[16] Condition 6.2 ensures that $\frac{\partial S_i}{\partial \Delta} < \frac{\partial S_M}{\partial \Delta}$, $i = L, R$.

however, more than offset by the first effect. Similar effects play a role if the level of wealth $1/\lambda$ increases.

The total donations a coalition receives from agents do not depend on the start-up cost f. However, because the probability that a coalition lobbies the policy maker decreases in f, the *ex ante* expected aggregate donations decrease in f. The (monetary) gains for the policy maker associated with a specific level of toughness thus decrease in f, whereas the (welfare) cost of the same β does not decrease in f. A higher β^* thus prevails.

We use the observation that the policy maker influences the probability that the contest takes place by choosing β to explain the fourth comparative statics result. As μ increases the social welfare associated with policy M (S_M) decreases. At the same time the expected social welfare associated with implementing either extreme policy with probability $\frac{1}{2}$ $(\frac{1}{2}S_L + \frac{1}{2}S_R)$ does not change as μ increases. Consequently, the welfare cost of having the contest decreases in μ. This gives the policy maker an incentive to increase the probability that the contest takes place. As a result β^* decreases in μ.

The last claims of Proposition 6.5 are intuitive. An increase in the level of corruptibility α boosts the policy maker's incentives to accrue donations and hence a lower β prevails. Clearly, if getting organized is very costly ($F \geq \frac{\mu+z}{1+z}$) or if the policy maker does not care much about money ($z \leq -\mu$), P sets β such that agents do not find it worthwile to initiate lobbies (Condition 6.1 is violated). In other words, the policy maker does not organize the contest *de facto*.

Obviously, policy makers are in general not in a position to communicate β^* and hence their level of corruptibility to the public.[17] Yet, one could argue that agents can infer something about this number from the average level of corruption they witness in the economy they inhabit. If the policy maker's level of corruptibility α is close to this average level, then the agents can obtain an accurate guess of β^*. Moreover, agents can also have learnt β^* via past lobbying activities involving the current policy maker.

Proposition 6.5 allows us to assess the impact of lobbying on welfare. Because aggregate donations do not depend on the number of agents and hence donations are negligible when the number of agents is large, we can ignore these expenditures and use total expected social welfare, denoted $\mathcal{S}$, as our welfare measure. One obtains:

Proposition 6.6 *Suppose Condition 6.2 holds and that $z > -\mu$ and $F < \frac{\mu+z}{1+z}$. Then in the equilibrium of the extended game with a large population total expected*

[17] The quotes mentioned in the introduction suggest that the two major candidates in the 2008 U.S. presidential election both have a high β^*.

social welfare equals

$$\mathcal{S}^* = -\Delta + \Delta(1-\mu)\Big(\frac{1+z}{\mu+z}\Big)^2 F^2. \tag{6.19}$$

Expected social welfare $\mathcal{S}^*$ decreases in the level of political polarization Δ, increases in the start-up cost f, decreases in the level of wealth $1/\lambda$, and decreases in P's level of corruptibility α.[18]

Since the above comparative statics results are immediate consequences of those presented in Proposition 6.5 combined with the fact that social welfare increases in the level of toughness, we only briefly discuss the intuition behind the results pertaining to the most interesting parameter, namely α.

Proposition 6.6 establishes an intuitive result: corruptible policy makers are bad for social welfare. As α increases, i.e. the policy maker becomes more corruptible, the policy maker's incentive to attract the lobbies' funds grows stronger. To lure agents into lobbying activities the policy maker chooses a low β, thereby indicating that it is quite likely that he will be swayed by future lobbying activities. This in turn does make civic action more probable, leading to a lower probability that the socially optimal policy (policy M) is implemented. Thus, the lower the policy maker's benevolence, the lower (expected) welfare will be.

6.5 Elections

In this section we investigate the possibility that agents with an extreme policy preference do not use lobbying to get their most preferred policy implemented, but instead try to influence the political process more directly. More specifically, we look at an election game in the spirit of the *citizen-candidates* model of Osborne and Slivinski (1996) and Besley and Coate (1997). In this game each agent (i.e. each citizen) can opt to become a candidate in an election. The winner of the election becomes the next policy maker. The policy maker can implement the policy of his liking. Citizens have no office-holding motive.[19]

In the first stage of the game all citizens decide simultaneously whether or not to run for this office. After the set of candidates has become common knowledge, each candidate shakes hands and kisses babies. During this campaigning each candidate's true most preferred policy is revealed. Since we assume that candidates

[18] Since a change in μ would entail a change in the preferences of some agents, the comparative statics result with respect to μ is meaningless and therefore omitted. We are aware of the confusion the result with respect to $1/\lambda$ might cause. It should be clear that $\mathcal{S}$ only measures welfare *partially*.

[19] This contrasts with the model discussed in the previous sections in which the office holder was able to extract rents from the lobbies.

cannot commit to a specific policy,[20] citizens take a candidate's most preferred policy as the policy which is going to be implemented should this candidate win the election. Candidates incur a campaign cost of $c > 0$. The marginal utility of income is still λ. In the second stage all citizens (including candidates) cast their vote. Voting is *strategic*: a citizen maximizes her expected utility when casting her vote.[21] If a citizen is indifferent between several candidates, then she votes for each of those candidates with equal probability. The candidate who receives the largest number of votes wins the election, i.e. the winner is determined according to the *plurality rule*. In the last stage the winner implements his most preferred policy and payoffs are realized. If nobody decides to run for office each policy $i \in \{L, M, R\}$ is implemented with probability $\frac{1}{3}$.

To make the comparison of the results of this model with the results of our lobbying model fair, we assume that two or more candidates with the same most preferred policy, say policy M, are able to select one of them to be *the* candidate advocating policy M. In other words, citizens can vote for at most one candidate advocating policy i, $i = L, M, R$. However, because candidates need campaigning to signal their political stance, all citizens who decided to run for office in the first stage do incur the campaign cost c.

We look for symmetric subgame perfect equilibria of the game. Obviously, no symmetric subgame perfect equilibria in pure strategies exist. Let q_i be the probability that an i-agent chooses to become a candidate and let $\psi_i := (1 - q_i)^{n_i}$ be the probability that no candidate favours policy i, $i = L, M, R$. We assume that Condition 6.2 holds. It is not difficult to see that this condition implies that the median voter prefers policy M and hence that a candidate who favours this policy (an *M-candidate*) wins any election in which such a candidate participates.

Suppose that $n_R < n_L$, implying that an *L-candidate* obtains more votes than an *R-candidate*. If all agents opt to become a candidate with positive probability, then the equilibrium conditions regarding ψ_L, ψ_M, and ψ_R of the voting game (derivations can be found in the appendix) are:

$$\psi_L^{1-\frac{1}{n_L}} \psi_M (2 - \psi_R) = C, \quad \psi_M^{1-\frac{1}{n_M}} (1 - \tfrac{1}{3}\psi_L \psi_R) = C, \quad \psi_R^{1-\frac{1}{n_R}} \psi_M \psi_L = C, \quad (6.20)$$

where $C := \frac{\lambda c}{\Delta}$ (the equilibrium conditions if $n_L < n_R$ mirror the above conditions). The number C is the cost-benefit ratio of the election game. Unfortunately, a sufficient condition akin to the inequalities (6.13) for an equilibrium in purely mixed

[20] This is common practice in the literature on citizen-candidates.

[21] The conclusions are not altered if voting is *sincere* (agents vote for the candidate whose favorite policy is closest to their own favorite policy) as long as $n_M > \max\{n_L, n_R\}$.

strategies to arise cannot be obtained from the conditions (6.20). We can, however, deduce the equilibrium of the election game for an economy with a large population, i.e. $n_i \to \infty$, $i = L, M, R$:

Proposition 6.7 *Suppose Condition 6.2 holds and $C < 1$. Then the election game with a large population has a unique (symmetric) equilibrium with the following probabilities that no i-candidate participates in the election, $i = L, M, R$:*

- *If $\mu_R < \mu_L$ and $C \leq \frac{3}{4}$, then $(\psi_L^*, \psi_M^*, \psi_R^*) = (\frac{3}{4}, \frac{4}{3}C, 1)$,*
- *If $\mu_R < \mu_L$ and $C > \frac{3}{4}$, then $(\psi_L^*, \psi_M^*, \psi_R^*) = (C, 1, 1)$,*
- *If $\mu_R > \mu_L$ and $C \leq \frac{3}{4}$, then $(\psi_L^*, \psi_M^*, \psi_R^*) = (1, \frac{4}{3}C, \frac{3}{4})$,*
- *If $\mu_R > \mu_L$ and $C > \frac{3}{4}$, then $(\psi_L^*, \psi_M^*, \psi_R^*) = (1, 1, C)$.*

Thus, the extreme policy with the smallest number of supporters is never advocated by a citizen-candidate.

Proposition 6.7 reveals that agents who have the smallest number of political allies never become candidates. The reason is that these agents would surely lose the election from any candidate with other preferences. They thus only gain from being a candidate if the other two policies are not represented by candidates. The expected gain from becoming a candidate is consequently small, in fact too small to make becoming a candidate worthwile. These agents therefore only influence the policy outcome by engaging in lobbying activities. Moreover, if C is small, then it is *ex ante* improbable that a candidate who advocates an extreme policy wins the election. In particular, if $C \leq \frac{3}{4}$ and $\mu_R < \mu_L$, then R-agents never become a candidate, the *ex ante* probability that an L-candidate wins equals $\frac{1}{3}C$, and the *ex ante* probability that an M-candidate wins equals $1 - \frac{4}{3}C$.[22] These numbers imply that an M-candidate wins the election (and hence policy M is implemented) with probability close to one if C is very small. Consequently, even if policy L has a large number of supporters (and more than policy R), then the lobbying route has a much larger associated probability that L will be implemented than the electoral route if C is small. Observe that C decreases in the level of political polarization Δ. This means that the electoral route is in particular a bad option for agents with extreme preferences if the level of political polarization is large. This contrasts the results obtained for the extended lobbying game: one can show that the *ex ante* equilibrium probability that policy L is implemented increases in Δ.[23]

[22] These probabilities do not add to one, because with positive probability nobody becomes a candidate.

[23] This probability equals $(1 - \phi_L^*)(\frac{1}{2} + \frac{1}{2}\phi_L^*) = \frac{1}{2}\big(1 - (\phi_L^*)^2\big)$. Consequently, the derivative of

6.6 Concluding Remarks

We have developed a theory that explains the formation of special interest groups that lobby policy makers. In our model, citizens who oppose the policy maker's plan to implement the socially optimal policy can initiate a special interest group. In contrast with the existing literature a citizen who contemplates initiating a lobby has to start from scratch, without the means to coordinate actions with fellow citizens. Coordination failures together with citizens' incentives to free-ride on the civic actions undertaken by others lead to an equilibrium characterized by a random number of lobbies. The probability that a certain policy alternative is advocated by a lobby depends on the level of political polarization, the cost of organizing a special interest group, the number of citizens at each position in the political spectrum, and the policy maker's affinity for the lobbies' efforts. Our model affirms the common wisdom that citizens who strongly object to the policy maker's proposals are most inclined to initiate lobbies. More surprisingly, the probability that a certain policy is advocated by a lobby need not be monotonic in the number of citizens who prefer that policy. The fact that a citizen who initiates a lobby supplies a collective good (to fellow citizens with the same political stance) with an endogenous value and a partly private nature drives the comparative statics results regarding the probability that a lobby advocating a certain policy is formed.

We have argued that lobbying offers a citizen with extreme policy preferences and few political allies a relatively good shot at having her preferred policy implemented. Comparing the outcomes of our lobbying model with the outcomes of a citizen-candidates model of electoral competition featuring the same agents reveals that citizens with extreme preferences only have electoral success in the unlikely event that no citizen with the median voter's preferences becomes a candidate in the election. The electoral route is especially unattractive for citizens with extreme preferences if the level of political polarization is large.

It goes without saying that our analysis has its limitations. Only three policy alternatives are feasible in the model. It has therefore only limited applicability in case the policy maker has a continuum of policies at his disposal, for instance if he has to set a tax or subsidy. It would be interesting to extend the model to a

this probability with respect to Δ is $2\phi_L^* \frac{\partial(1-\phi_L^*)}{\partial\Delta}$. Since $1 - \phi_L^* = \frac{(\mu+z)-(1+z)F}{(\mu+z)-(1-\mu)F}$, one has

$$\frac{\partial(1-\phi_L^*)}{\partial\Delta} = \frac{-(\mu+z)^2}{\big((\mu+z)-(1-\mu)F\big)^2} \times \frac{\partial F}{\partial\Delta}.$$

Because $\frac{\partial F}{\partial\Delta} < 0$, we conclude that the probability that policy L is implemented indeed increases in Δ.

continuum of policies, perhaps by using the common agency framework that has already earned its place in the literature on lobbying.

6.A Appendix

Throughout the appendix we use $\neg i$ to signify the polar policy of i. So, if $i = R$, then $\neg i = L$ and vice versa. We abbreviate special interest group/lobby to SIG.

Omitted details regarding Lemma 6.1

By Condition 6.1 there exists a $\nu \in (0,1)$ such that $\beta\lambda = \nu^2\Delta$. We use ν to rewrite $d_i(\{L,R\}) - d_i(\{i\})$:

$$d_i(\{L,R\}) - d_i(\{i\}) = \frac{\Delta - \nu^2\Delta}{2\lambda n_i} - \frac{\Delta\sqrt{\nu^2} - \nu^2\Delta}{\lambda n_i} = \frac{\Delta}{2\lambda n_i}(\nu^2 - 2\nu + 1).$$

The expression between brackets is obviously positive. We next compare the probabilities that P implements i in the two relevant continuation games:

$$\Pr(P \text{ implements } i|\{L,R\}) - \Pr(P \text{ implements } i|\{i\}) =$$

$$\Big(\frac{1}{2} - \frac{\nu^2}{2}\Big) - \Big(1 - \nu\Big) = -\frac{1}{2}\big(\nu^2 - 2\nu + 1\big) < 0.$$

The number ν also helps ranking the payoffs $v_i(A)$:

$$v_i(\{i\}) - v_i(\emptyset) = \Delta\bigg(- \nu\Big(1 + \frac{1}{n_i}\Big) + \frac{\nu^2}{n_i} + 1\bigg) = \Delta\Big(1 - \nu\Big)\Big(1 - \frac{\nu}{n_i}\Big) > 0,$$

$$v_i(\emptyset) - v_i(\{L,R\}) = \Delta\Big(\frac{1}{2n_i} - \frac{\nu^2}{2n_i}\Big) = \Delta(1-\nu)\frac{1+\nu}{2n_i} > 0,$$

$$v_i(\{L,R\}) - v_i(\{\neg i\}) = \Delta\Big(1 - \frac{1}{2n_i} + \frac{\nu^2}{2n_i} - \nu\Big) = \Delta\Big(1 - \nu\Big)\Big(1 - \frac{1+\nu}{2n_i}\Big) > 0.$$

Derivation of equations (6.11)-(6.12)

Using the fact that $(1-p_i)^{n_i - 1} = \phi_i^{1-\frac{1}{n_i}}$ and rearranging terms shows that (6.10) is equivalent to

$$\phi_i^{1-\frac{1}{n_i}}\phi_{\neg i}v_i(\{i\}) - \phi_i^{1-\frac{1}{n_i}}\phi_{\neg i}v_i(\emptyset) + \phi_i^{1-\frac{1}{n_i}}(1-\phi_{\neg i})v_i(\{L,R\})$$
$$- \phi_i^{1-\frac{1}{n_i}}(1-\phi_{\neg i})v_i(\{\neg i\}) = \lambda f.$$

Plugging in the values of $v_i(\{i\}) - v_i(\emptyset)$ and of $v_i(\{L,R\}) - v_i(\{\neg i\})$ derived above yields

$$\phi_i^{1-\frac{1}{n_i}}\Delta(1-\nu)\bigg(\phi_{\neg i}\Big(1 - \frac{\nu}{n_i}\Big) + (1-\phi_{\neg i})\Big(1 - \frac{1+\nu}{2n_i}\Big)\bigg) = \lambda f,$$

from which (6.11)-(6.12) follow immediately.

Proof of Proposition 6.1

Let $n_L, n_R > 1$ be given. Abbreviate (ϕ_L, ϕ_R) to ϕ and likewise (n_L, n_R) to n. We have to show that, given the restriction (6.13) on F, the system

$$X_i(\phi, n) := \phi_i^{1-\frac{1}{n_i}}\left(1 - \frac{1+\nu}{2n_i} + \frac{1-\nu}{2n_i}\phi_{\neg i}\right) - \frac{F}{1-\nu} = 0, \quad i = L, R \tag{6.21}$$

has a unique solution $\phi^* = \phi^*(n)$. Observe that the LHS of each equation increases monotonically from 0 to $1 - \frac{1+\nu}{2n_i} + \frac{1-\nu}{2n_i}\phi_{\neg i}$ as ϕ_i increases from 0 to 1. Consequently, for each $\phi_{\neg i} \in [0,1]$ there is a unique $\tilde{\phi}_i = \tilde{\phi}_i(\phi_{\neg i}) \in [0,1]$ solving $X_i(\phi_i, \phi_{\neg i}) = 0$ if $\frac{F}{1-\nu} \leq 1 - \frac{1+\nu}{2n_i}$. Thus, both $X_L(\phi_L, \phi_R) = 0$ and $X_R(\phi_L, \phi_R) = 0$ separately have a root ($\tilde{\phi}_L(\phi_R)$ respectively $\tilde{\phi}_R(\phi_L)$) in $[0,1]$ if (6.13) holds. So, if (6.13) holds, then the map $\tau = (\tau_L, \tau_R) : [0,1]^2 \to [0,1]^2$, $(\phi_L, \phi_R) \mapsto (\tilde{\phi}_L, \tilde{\phi}_R)$ is well-defined. By the contraction mapping principle, (6.21) has a unique solution ϕ^* if there exists an $\eta \in (0,1)$ such that

$$\|\tau(\phi^2) - \tau(\phi^1)\|_2^2 \leq \eta \|\phi^2 - \phi^1\|_2^2,$$

for all $\phi^1, \phi^2 \in [0,1]^2$. Note that

$$\tau_i(\phi) = \left(\frac{F}{1-\nu} \times \frac{1}{1 - \frac{1+\nu}{2n_i} + \frac{1-\nu}{2n_i}\phi_{\neg i}}\right)^{\frac{n_i}{n_i - 1}}.$$

Let $a_i := 1 - \frac{1+\nu}{2n_i}$ and $b_i := \frac{1-\nu}{2n_i}$. Observe that, by (6.13), $\frac{F}{1-\nu} < a_i$. Assume without loss of generality that $\phi_{\neg i}^1 < \phi_{\neg i}^2$. Then:

$$|\tau_i(\phi^2) - \tau_i(\phi^1)| < \left(\frac{a_i}{a_i + b_i \phi_{\neg i}^1}\right)^{\frac{n_i}{n_i - 1}} - \left(\frac{a_i}{a_i + b_i \phi_{\neg i}^2}\right)^{\frac{n_i}{n_i - 1}}.$$

It follows from the mean value theorem that $|\tau_i(\phi^2) - \tau_i(\phi^1)| < \epsilon |\phi_{\neg i}^2 - \phi_{\neg i}^1|$ for some $\epsilon \in (0,1)$ if the derivative of the function

$$g_i : y \mapsto y + \left(\frac{a_i}{a_i + b_i y}\right)^{\frac{n_i}{n_i - 1}}$$

is strictly positive on $[0,1]$. To see this, note that for $\phi^1, \phi^2 \in [0,1]^2$, $\phi_{\neg i}^1 < \phi_{\neg i}^2$, one has

$$g_i(\phi_{\neg i}^2) - g_i(\phi_{\neg i}^1) = g_i'(\xi)(\phi_{\neg i}^2 - \phi_{\neg i}^1) \geq \min_{y \in [0,1]} g_i'(y)(\phi_{\neg i}^2 - \phi_{\neg i}^1) \Rightarrow$$

$$\tau_i(\phi^1) - \tau_i(\phi^2) \leq (1 - \min_{y \in [0,1]} g_i'(y))(\phi_{\neg i}^2 - \phi_{\neg i}^1),$$

for some $\xi \in (\phi_{\neg i}^1, \phi_{\neg i}^2)$. Because

$$g_i'(y) = 1 - \frac{n_i}{n_i - 1} a_i^{\frac{n_i}{n_i - 1}} b_i \left(\frac{1}{a_i + b_i y}\right)^{\frac{n_i}{n_i - 1} + 1}$$

increases in y, it suffices to show that $g_i'(0) > 0$. Note that:

$$g_i'(0) = 1 - \frac{n_i}{n_i - 1} a_i^{\frac{n_i}{n_i-1}} b_i \left(\frac{1}{a_i}\right)^{\frac{n_i}{n_i-1}+1} = 1 - \frac{b_i}{a_i}\frac{n_i}{n_i - 1} = 1 - \frac{n_i(1-\nu)}{(n_i-1)(2n_i-1-\nu)}.$$

The claim that $g_i'(0) > 0$ follows from the fact that $\frac{n_i(1-\nu)}{(n_i-1)(2n_i-1-\nu)}\Big|_{n_i=2} = \frac{2(1-\nu)}{3-\nu} < 1$ combined with the observation that

$$\frac{(n+1)(1-\nu)}{n(2n+1-\nu)} \Bigg/ \frac{n(1-\nu)}{(n-1)(2n-1-\nu)} = \frac{(n^2-1)(2n-1-\nu)}{n^2(2n+1-\nu)} < 1, \quad n \geq 2.$$

We conclude that

$$\|\tau(\phi^2) - \tau(\phi^1)\|_2^2 = |\tau_L(\phi^2) - \tau_L(\phi^1)|^2 + |\tau_R(\phi^2) - \tau_R(\phi^1)|^2 <$$

$$\eta|\phi_R^2 - \phi_R^1|^2 + \eta|\phi_L^2 - \phi_L^1|^2 = \eta\|\phi^2 - \phi^1\|_2^2,$$

where $\eta = \max\{1 - g_L'(0), 1 - g_R'(0)\} \in (0,1)$ and hence that (6.21) has a unique solution $\phi^* = (\phi_L^*, \phi_R^*)$. ■

Proof of Proposition 6.2

To assess the sign of the four partial derivatives mentioned in the proposition, we need the sign of $\frac{\partial \phi_i^*}{\partial \nu}$ and the sign of $\frac{\partial \phi_i^*}{\partial F}$.

We start with $\frac{\partial \phi_i^*}{\partial \nu}$, focusing on $i = L$. The case $i = R$ can be tackled in a similar fashion. Differentiating the system (6.21) with respect to ν yields

$$\frac{\partial X}{\partial \phi}\frac{\partial \phi^*}{\partial \nu} + \frac{\partial X}{\partial \nu} = 0,$$

where

$$\frac{\partial X}{\partial \phi} := \begin{bmatrix} \frac{\partial X}{\partial \phi_L} & \frac{\partial X}{\partial \phi_R} \end{bmatrix} := \begin{bmatrix} \frac{\partial X_L}{\partial \phi_L} & \frac{\partial X_L}{\partial \phi_R} \\ \frac{\partial X_R}{\partial \phi_L} & \frac{\partial X_R}{\partial \phi_R} \end{bmatrix}, \quad \frac{\partial \phi^*}{\partial \nu} := \begin{pmatrix} \frac{\partial \phi_L^*}{\partial \nu} \\ \frac{\partial \phi_R^*}{\partial \nu} \end{pmatrix}, \quad \text{and} \quad \frac{\partial X}{\partial \nu} := \begin{pmatrix} \frac{\partial X_L}{\partial \nu} \\ \frac{\partial X_R}{\partial \nu} \end{pmatrix}. \tag{6.22}$$

Cramer's Rule informs us that

$$\frac{\partial \phi_L^*}{\partial \nu} = \det\begin{bmatrix} -\frac{\partial X}{\partial \nu} & \frac{\partial X}{\partial \phi_R} \end{bmatrix} \Big/ \det \frac{\partial X}{\partial \phi}, \tag{6.23}$$

where all partial derivatives must be evaluated in $\phi = \phi^*$. We calculate the value of all relevant partial derivatives in the equilibrium in turn:

$$\frac{\partial X_i}{\partial \phi_i}\Big|_{\phi=\phi^*} = \left(1 - \frac{1}{n_i}\right)(\phi_i^*)^{-\frac{1}{n_i}}\left(1 - \frac{1+\nu}{2n_i} + \frac{1-\nu}{2n_i}\phi_{\neg i}^*\right) = \left(1 - \frac{1}{n_i}\right) \times \frac{F}{1-\nu} \times \frac{1}{\phi_i^*}, \tag{6.24}$$

$$\frac{\partial X_i}{\partial \phi_{\neg i}}\Big|_{\phi=\phi^*} = (\phi_i^*)^{1-\frac{1}{n_i}} \times \frac{1-\nu}{2n_i}, \quad i = L, R, \tag{6.25}$$

$$\frac{\partial X_i}{\partial \nu}\Big|_{\phi=\phi^*} = -\frac{(\phi_L^*)^{1-\frac{1}{n_L}}}{2n_L} \times (1+\phi_R^*) - \frac{F}{(1-\nu)^2}. \tag{6.26}$$

We first investigate the denominator of (6.23) using (6.24)-(6.25):

$$\det \frac{\partial X}{\partial \phi}\Big|_{\phi=\phi^*} = \Big(1-\frac{1}{n_L}\Big)\Big(1-\frac{1}{n_R}\Big)\Big(\frac{F}{1-\nu}\Big)^2 \times \frac{1}{\phi_L^*\phi_R^*} - (\phi_L^*)^{1-\frac{1}{n_L}}\frac{1-\nu}{2n_L} \times (\phi_R^*)^{1-\frac{1}{n_R}}\frac{1-\nu}{2n_R}.$$

Define $\Gamma_i := \Big(1-\frac{1+\nu}{2n_i}+\frac{1-\nu}{2n_i}\phi_{\neg i}^*\Big)^{-1}$, $i = L, R$. Note that:

$$n_L n_R \det \frac{\partial X}{\partial \phi}\Big|_{\phi=\phi^*} = (n_L-1)(n_R-1) \times \Big(\frac{F}{1-\nu}\Big)^2 \frac{1}{\phi_L^*\phi_R^*} - \Gamma_L\Gamma_R \times \Big(\frac{F}{1-\nu}\Big)^2 \times \Big(\frac{1-\nu}{2}\Big)^2.$$

Because $\Gamma_i < \Big(1-\frac{1+\nu}{2n_i}\Big)^{-1}$, one has:

$$\frac{1-\nu}{2} \times \Gamma_i < (1-\nu) \times \frac{n_i}{2n_i-1-\nu} < n_i - 1.$$

We conclude that

$$\det \frac{\partial X}{\partial \phi}\Big|_{\phi=\phi^*} > 0. \tag{6.27}$$

We now determine the sign of the numerator of (6.23). Substituting (6.24)-(6.26) into this determinant yields:

$$\det\Big[-\tfrac{\partial X}{\partial \nu} \quad \tfrac{\partial X}{\partial \phi_R}\Big]\Big|_{\phi=\phi^*} = \Big(\frac{(\phi_L^*)^{1-\frac{1}{n_L}}}{2n_L} \times (1+\phi_R^*) + \frac{F}{(1-\nu)^2}\Big)\Big(1-\frac{1}{n_R}\Big) \times \frac{F}{1-\nu} \times \frac{1}{\phi_R^*}$$
$$-\Big(\frac{(\phi_R^*)^{1-\frac{1}{n_R}}}{2n_R} \times (1+\phi_L^*) + \frac{F}{(1-\nu)^2}\Big) \times (\phi_L^*)^{1-\frac{1}{n_L}} \times \frac{1-\nu}{2n_L} = \Big(\frac{F}{1-\nu}\Big)^2 \times$$
$$\Big[\Big(\frac{\Gamma_L}{2n_L} \times (1+\phi_R^*) + \frac{1}{1-\nu}\Big)\Big(1-\frac{1}{n_R}\Big)\frac{1}{\phi_R^*} - \Gamma_L \times \frac{1-\nu}{2n_L} \times \Big(\frac{\Gamma_R}{2n_R} \times (1+\phi_L^*) + \frac{1}{1-\nu}\Big)\Big],$$

where we have used the fact that $(\phi_i^*)^{1-\frac{1}{n_i}} = \Gamma_i \times \frac{F}{1-\nu}$. By the definition of Γ_i it follows that

$$\frac{\Gamma_i}{2n_i} = \frac{1}{2(n_i-1)+(1-\nu)(1+\phi_{\neg i}^*)}.$$

We use this to rewrite the part of $\det\Big[-\frac{\partial X}{\partial \nu} \quad \frac{\partial X}{\partial \phi_R}\Big]\Big|_{\phi=\phi^*}$ between square brackets:

$$\Big[\cdots\Big] = \frac{1}{1-\nu} \times \Big(\frac{n_R-1}{n_R\phi_R^*} - \frac{1-\nu}{2(n_L-1)+(1-\nu)(1+\phi_R^*)}\Big) +$$
$$\frac{1}{2(n_L-1)+(1-\nu)(1+\phi_R^*)} \times \Big(\frac{(1+\phi_R^*)(n_R-1)}{n_R\phi_R^*} - \frac{(1-\nu)(1+\phi_L^*)}{2(n_R-1)+(1-\nu)(1+\phi_L^*)}\Big)$$

Since

$$\frac{(n_R-1)(1+\phi_R^*)}{n_R\phi_R^*} > \frac{n_R-1}{n_R\phi_R^*} > \frac{1}{2}, \quad \frac{1-\nu}{2(n_L-1)+(1-\nu)(1+\phi_R^*)} < \frac{1}{2},$$

and

$$\frac{(1-\nu)(1+\phi_L^*)}{2(n_R-1)+(1-\nu)(1+\phi_L^*)} < \max_{y\in[0,2]} \frac{y}{2+y} = \frac{1}{2},$$

we can conclude that the term between square brackets is positive, implying that the numerator of (6.23) is positive and hence that

$$\frac{\partial\phi_i^*}{\partial\nu} > 0.$$

We turn our attention to $\frac{\partial\phi_i^*}{\partial F}$, focusing on $\frac{\partial\phi_L^*}{\partial F}$. By Cramer's Rule we have

$$\frac{\partial\phi_L^*}{\partial F} = \det\begin{bmatrix} -\frac{\partial X_L}{\partial F} & \frac{\partial X_L}{\partial\phi_R} \\ -\frac{\partial X_R}{\partial F} & \frac{\partial X_R}{\partial\phi_R}\end{bmatrix}\Big|_{\phi=\phi^*} \Big/ \det\frac{\partial X}{\partial\phi}\Big|_{\phi=\phi^*}. \tag{6.28}$$

It remains to determine the sign of the numerator. Because $\frac{\partial X_i}{\partial F} = -\frac{1}{1-\nu}$, this expression equals

$$\det\begin{bmatrix} -\frac{\partial X_L}{\partial F} & \frac{\partial X_L}{\partial\phi_R} \\ -\frac{\partial X_R}{\partial F} \frac{\partial X_R}{\partial\phi_R} & \end{bmatrix}\Big|_{\phi=\phi^*} = \frac{1}{1-\nu}\times\left(1-\frac{1}{n_R}\right)\times\frac{F}{1-\nu}\times\frac{1}{\phi_R^*}$$
$$-\frac{1}{1-\nu}(\phi_L^*)^{1-\frac{1}{n_L}}\times\frac{1-\nu}{2n_L} = \frac{F}{(1-\nu)^2}\times\left(\frac{n_R-1}{n_R\phi_R^*} - \Gamma_L\times\frac{1-\nu}{2n_L}\right),$$

where we have used the fact that $(\phi_L^*)^{1-\frac{1}{n_L}} = \Gamma_L\times\frac{F}{1-\nu}$. The fact that $\Gamma_L < \left(1-\frac{1+\nu}{2n_L}\right)^{-1}$ implies that

$$\frac{n_R-1}{n_R\phi_R^*} - \Gamma_L\times\frac{1-\nu}{2n_L} > \frac{n_R-1}{n_R\phi_R^*} - \frac{1-\nu}{2(n_L-1)+1-\nu} > 0.$$

So, the numerator of (6.28) is positive and therefore:

$$\frac{\partial\phi_i^*}{\partial F} > 0.$$

The following results are straightforward:

$$\frac{\partial p_i^*}{\partial\phi_i} = -\frac{1}{n_i}(\phi_i^*)^{-1+\frac{1}{n_i}} < 0, \quad \frac{\partial F}{\partial\Delta} = -\frac{\lambda f}{\Delta^2} < 0, \quad \frac{\partial\nu}{\partial\Delta} = -\frac{1}{2\Delta}\sqrt{\frac{\beta\lambda}{\Delta}} < 0,$$

$$\frac{\partial F}{\partial f} = \frac{\lambda}{\Delta} > 0, \quad \frac{\partial\nu}{\partial f} = 0, \quad \frac{\partial F}{\partial(1/\lambda)} = -\frac{\lambda^2 f}{\Delta} < 0, \quad \frac{\partial\nu}{\partial(1/\lambda)} = -\frac{\lambda}{2}\sqrt{\frac{\beta\lambda}{\Delta}} < 0,$$

$$\frac{\partial F}{\partial\beta} = 0, \quad \frac{\partial\nu}{\partial\beta} = \frac{1}{2}\sqrt{\frac{\lambda}{\beta\Delta}} > 0.$$

The first four claims now follow immediately:

$$\frac{\partial p_i^*}{\partial \Delta} = \frac{\partial p_i^*}{\partial \phi_i}\frac{\partial \phi_i^*}{\partial F}\frac{\partial F}{\partial \Delta} + \frac{\partial p_i^*}{\partial \phi_i}\frac{\partial \phi_i^*}{\partial \nu}\frac{\partial \nu}{\partial \Delta} > 0,$$

$$\frac{\partial p_i^*}{\partial f} = \frac{\partial p_i^*}{\partial \phi_i}\frac{\partial \phi_i^*}{\partial F}\frac{\partial F}{\partial f} + \frac{\partial p_i^*}{\partial \phi_i}\frac{\partial \phi_i^*}{\partial \nu}\frac{\partial \nu}{\partial f} < 0,$$

$$\frac{\partial p_i^*}{\partial (1/\lambda)} = \frac{\partial p_i^*}{\partial \phi_i}\frac{\partial \phi_i^*}{\partial F}\frac{\partial F}{\partial (1/\lambda)} + \frac{\partial p_i^*}{\partial \phi_i}\frac{\partial \phi_i^*}{\partial \nu}\frac{\partial \nu}{\partial (1/\lambda)} > 0,$$

$$\frac{\partial p_i^*}{\partial \beta} = \frac{\partial p_i^*}{\partial \phi_i}\frac{\partial \phi_i^*}{\partial F}\frac{\partial F}{\partial \beta} + \frac{\partial p_i^*}{\partial \phi_i}\frac{\partial \phi_i^*}{\partial \nu}\frac{\partial \nu}{\partial \beta} < 0.$$

The last claim of Proposition 6.2 is an obvious corollary to the former claims. ■

Proof of Proposition 6.3

Assume for the moment that n_L and n_R are continuous variables. A standard argument shows that all results derived below also hold for n_L and n_R discrete. Differentiating the system (6.21) with respect to n_L yields

$$\frac{\partial X}{\partial \phi}\frac{\partial \phi^*}{\partial n_L} = -\frac{\partial X}{\partial n_L},$$

where

$$\frac{\partial \phi^*}{\partial n_L} := \begin{pmatrix} \frac{\partial \phi_L^*}{\partial n_L} \\ \frac{\partial \phi_R^*}{\partial n_L} \end{pmatrix}, \quad \frac{\partial X}{\partial n_L} := \begin{pmatrix} \frac{\partial X_L}{\partial n_L} \\ \frac{\partial X_R}{\partial n_L} \end{pmatrix},$$

and $\frac{\partial X}{\partial \phi}$ is defined above (see (6.22)). Applying Cramer's Rule leads to

$$\frac{\partial \phi_L^*}{\partial n_L} = \det \begin{bmatrix} -\frac{\partial X}{\partial n_L} & \frac{\partial X}{\partial \phi_R} \end{bmatrix} \Big/ \det \frac{\partial X}{\partial \phi}, \tag{6.29}$$

where all partial derivatives must be evaluated in $\phi = \phi^*$. We calculate the value of the relevant partial derivatives which have not been calculated in the previous proof:

$$\frac{\partial X_L}{\partial n_L}\Big|_{\phi=\phi^*} = \frac{(\phi_L^*)^{1-\frac{1}{n_L}}}{n_L^2}\left(\log \phi_L^*\left(1 - \frac{1+\nu}{2n_L} + \frac{1-\nu}{2n_L}\phi_R^*\right) + \frac{1+\nu}{2} - \frac{1-\nu}{2}\phi_R^*\right) =$$
$$\frac{\log \phi_L^*}{n_L^2} \times \frac{F}{1-\nu} + \frac{1}{n_L}\left(-\frac{F}{1-\nu} + (\phi_L^*)^{1-\frac{1}{n_L}}\right) = \frac{1}{n_L}\left((\phi_L^*)^{1-\frac{1}{n_L}} - \frac{F}{1-\nu}\left(1 - \frac{\log \phi_L^*}{n_L}\right)\right), \tag{6.30}$$

where the second equality follows from the fact that $X_L(\phi^*, n) = 0$. Obviously, $\frac{\partial X_R}{\partial n_L} = 0$. Using the expressions (6.24), (6.25), and (6.30) one sees that

$$\det \begin{bmatrix} -\frac{\partial X}{\partial n_L} & \frac{\partial X}{\partial \phi_R} \end{bmatrix}\Big|_{\phi=\phi^*} =$$
$$\frac{1}{n_L}\left(\frac{F}{1-\nu} \times \left(1 - \frac{\log \phi_L^*}{n_L}\right) - (\phi_L^*)^{1-\frac{1}{n_L}}\right) \times \left(1 - \frac{1}{n_R}\right) \times \frac{F}{1-\nu} \times \frac{1}{\phi_R^*}. \tag{6.31}$$

Consequently, the sign of the numerator of (6.29) is the same as the sign of

$$\frac{F}{1-\nu}\times\Big(1-\frac{\log\phi_L^*}{n_L}\Big)-(\phi_L^*)^{1-\frac{1}{n_L}}=\frac{F}{1-\nu}\times\Big(1-\frac{\log\phi_L^*}{n_L}-\Gamma_L\Big)=$$
$$\frac{F}{1-\nu}\times\Big(1-\frac{\log\big(\frac{F}{1-\nu}\Gamma_L\big)}{n_L-1}-\Gamma_L\Big),$$

where the second and third expression follow from the fact that $X_L(\phi^*,n)=0$. We know from the previous proof that det $\frac{\partial X}{\partial\phi}>0$. Therefore the sign of $\frac{\partial\phi_L^*}{\partial n_L}$ is the same as the sign of $H_L(n_L):=1-\frac{\log\big(\frac{F}{1-\nu}\Gamma_L\big)}{n_L-1}-\Gamma_L$. Recall that $\Gamma_L=\Big(1-\frac{1+\nu}{2n_L}+\frac{1-\nu}{2n_L}\phi_R^*\Big)^{-1}$ depends on n_L. Observe that:

$$\Big(1-\frac{\nu}{n_L}\Big)^{-1}<\Gamma_L<\Big(1-\frac{1+\nu}{2n_L}\Big)^{-1}.$$

Since H_L decreases in Γ_L, these inequalities imply that

$$1-\Big(1-\frac{1+\nu}{2n_L}\Big)^{-1}-\frac{\log\Big(\frac{F}{1-\nu}\big(1-\frac{1+\nu}{2n_L}\big)^{-1}\Big)}{n_L-1}<H_L(n_L)$$

and that

$$H_L(n_L)<1-\Big(1-\frac{\nu}{n_L}\Big)^{-1}-\frac{\log\Big(\frac{F}{1-\nu}\big(1-\frac{\nu}{n_L}\big)^{-1}\Big)}{n_L-1}.$$

Note that if $1-\Big(1-\frac{\nu}{n_L}\Big)^{-1}-\frac{\log\Big(\frac{F}{1-\nu}\big(1-\frac{\nu}{n_L}\big)^{-1}\Big)}{n_L-1}<0$, then *a forteriori* $H_L(n_L)<0$. One straightforwardly verifies that the first inequality holds if and only if

$$F>(1-\nu)\times\Big(1-\frac{\nu}{n_L}\Big)\times\exp\Big[(n_L-1)\Big(1-\big(1-\frac{\nu}{n_L}\big)^{-1}\Big)\Big]=:(1-\nu)\tilde{\Phi}(n_L,\nu).$$

The function $\tilde{\Phi}$ increases in its first argument as the following derivation shows:

$$\begin{aligned}\frac{\partial\tilde{\Phi}}{\partial n_L}=&\frac{\nu}{n_L^2}\exp\Big[(n_L-1)\Big(1-\big(1-\frac{\nu}{n_L}\big)^{-1}\Big)\Big]\\&+\big(1-\frac{\nu}{n_L}\big)\Big(1-\big(1-\frac{\nu}{n_L}\big)^{-1}+(n_L-1)\big(1-\frac{\nu}{n_L}\big)^{-2}\times\frac{\nu}{n_L^2}\Big)\\&\times\exp\Big[(n_L-1)\Big(1-\big(1-\frac{\nu}{n_L}\big)^{-1}\Big)\Big]\\=&\frac{\nu}{n_L}\times\big(1-\frac{1}{n_L}\big)\times\Big(\big(1-\frac{\nu}{n_L}\big)^{-1}-1\Big)\exp\Big[(n_L-1)\Big(1-\big(1-\frac{\nu}{n_L}\big)^{-1}\Big)\Big]>0.\end{aligned}$$

It follows that ϕ_L^* decreases in n_L as long as $n_L\leq\bar{n}$ if $F>(1-\nu)\tilde{\Phi}(\bar{n},\nu)$ for some $\bar{n}>2$. At the same time F should abide by (6.13). We now show that both conditions can hold simultaneously. Since $\tilde{\Phi}$ increases in n_L, it suffices to show that

$\tilde{\Phi}(n_L,\nu) < 1 - \frac{1+\nu}{4}$, i.e. $\tilde{\Phi}(n_L,\nu)$ smaller than (6.13) evaluated in $n_L = 2$, as long as $n_L \leq \bar{n}$ for some $\bar{n} \geq 3$. Observe that:

$$\tilde{\Phi}(3,\nu) < 1 - \frac{1+\nu}{4} \Leftrightarrow \exp\left[\frac{-2\nu}{3-\nu}\right] < \frac{3}{4}.$$

This inequality obviously holds for $\nu < 1$ sufficiently large, proving that both conditions hold simultaneously for ν sufficiently large.

The above argument implies that ϕ_L^* is everywhere decreasing in n_L (i.e. $\bar{n} = \infty$) if $F > (1-\nu)\lim_{n_L\to\infty}\Phi(n_L,\nu) = (1-\nu)\exp[-\nu]$. This condition is compatible with (6.13) if $\exp[-\nu] < \frac{3-\nu}{4}$, an inequality which is clearly satisfied for $\nu < 1$ sufficiently close to 1.

Secondly, $H_L(n_L) > 0$ if

$$1 - \left(1 - \frac{1+\nu}{2n_L}\right)^{-1} - \frac{\log\left(\frac{F}{1-\nu}\left(1 - \frac{1+\nu}{2n_L}\right)^{-1}\right)}{n_L - 1} > 0.$$

Solving this inequality for F yields:

$$F < (1-\nu) \times \left(1 - \frac{1+\nu}{2n_L}\right) \times \exp\left[(n-1)\left(1 - \left(1 - \frac{1+\nu}{2n_L}\right)^{-1}\right)\right] =: (1-\nu)\tilde{\Psi}(n_L,\nu).$$

Differentiating $\tilde{\Psi}$ with respect to n_L results in:

$$\frac{\partial\tilde{\Psi}}{\partial n_L} = \left(1 - \frac{1}{n_L}\right) \times \frac{1+\nu}{2n_L} \times \left(\left(1 - \frac{1+\nu}{2n_L}\right)^{-1} - 1\right) \exp\left[(n_L - 1)\left(1 - \left(1 - \frac{1+\nu}{2n_L}\right)^{-1}\right)\right] > 0.$$

Thus, ϕ_L^* increases in n_L as long as $n_L \geq \bar{n}$ if $F < (1-\nu)\tilde{\Psi}(\bar{n},\nu)$ for some $\bar{n} > 2$. If $F < (1-\nu)\tilde{\Psi}(2,\nu)$, i.e. if $F < (1-\nu)\frac{3-\nu}{4}\exp\left[-\frac{1+\nu}{3-\nu}\right]$, then ϕ_L^* is even monotonically increasing in n_L.

Combining the above results, one sees that ϕ_L^* is monotonically increasing in n_L if $F < (1-\nu)\tilde{\Psi}(2,\nu)$, whereas ϕ_L^* is monotonically decreasing in n_L if $F > (1-\nu)\lim_{n\to\infty}\tilde{\Phi}(n,\nu)$. (The fact that $\frac{1+\nu}{3-\nu} > \nu$ for every $\nu \in (0,1)$ implies that $\tilde{\Psi}(2,\nu) < \lim_{n\to\infty}\tilde{\Phi}(n,\nu)$. So, the two parameter regions are disjoint.) For $F \in ((1-\nu)\tilde{\Psi}(2,\nu), (1-\nu)\lim_{n\to\infty}\tilde{\Phi}(n,\nu))$ ϕ_L^* need not be monotonic. Indeed, one can choose $\nu < 1$ sufficiently large such that $\tilde{\Phi}(3,\nu) < \lim_{n\to\infty}\tilde{\Psi}(n,\nu)$ for $\nu < 1$. Then for $F \in (\tilde{\Phi}(3,\nu), \lim_{n\to\infty}\tilde{\Psi}(n,\nu)) =: I(\nu)$ ϕ_L^* decreases in n_L for n_L small and increases in n_L for n_L large. See also Figure 1.

Note that the sign of

$$\frac{\partial\phi_R^*}{\partial n_L} = \det\begin{bmatrix}\frac{\partial X}{\partial\phi_L} & -\frac{\partial X}{\partial n_L}\end{bmatrix}\Big|_{\phi=\phi^*} \Big/ \det\frac{\partial X}{\partial\phi}\Big|_{\phi=\phi^*}$$

is the same as the sign of

$$\det\begin{bmatrix}\frac{\partial X}{\partial\phi_L} & -\frac{\partial X}{\partial n_L}\end{bmatrix}\Big|_{\phi=\phi^*} = (\phi_R^*)^{1-\frac{1}{n_R}} \times \frac{1-\nu}{2n_R} \times \frac{1}{n_L}\left((\phi_L^*)^{1-\frac{1}{n_L}} - \frac{F}{1-\nu} \times \left(1 - \frac{\log\phi_L^*}{n_L}\right)\right).$$

The sign of this expression is minus the sign of (6.31). This implies that ϕ_R^* increases in n_L iff ϕ_L^* decreases in n_L.

One readily verifies that $\tilde{\Psi}(n,\cdot) \leq \tilde{\Phi}(n,\cdot)$, $n \geq 2$. It is now immediate that the claims of Proposition 6.3 dealing with changes in n_L hold for $\Psi : (n,\nu) \mapsto (1-\nu)\tilde{\Psi}(n,\nu)$ and $\Phi : (n,\nu) \mapsto (1-\nu)\tilde{\Phi}(n,\nu)$. (One readily verifies that Ψ and Φ decrease in their second argument.) The proofs of the claims regarding changes in n_R mirror the above proofs. ■

Omitted details regarding Table 1

The payoffs of the continuation games with zero or one group are immediate. The payoffs of the continuation game $\{L,R\}$ follow from setting $\beta = 0$ in (6.8). Consider continuation game $\{L,M\}$. The expected utility (gross of any start-up costs) of an M-agent donating d_M is $-\frac{D_L}{D_L+D_M}\Delta - \lambda d_M$, where D_i denotes the aggregate donations to coalition i, $i = L,M,R$. The expected utility of an L-agent donating d_L equals $-\frac{D_M}{D_L+D_M}\Delta - \lambda d_L$. The FOCs inform us that $D_M(\{L,M\}) = D_L(\{L,M\}) = \frac{\Delta}{4\lambda}$. Imposing symmetry and using the fact that coalition i obtains the prize with probability $\frac{1}{2}$ in equilibrium yields the payoffs. The payoffs of continuation game $\{M,R\}$ can be derived in a similar manner.

The last continuation game we have to investigate is $\{L,M,R\}$. In this continuation game the expected utility of an L-agent donating d_L is $-\frac{D_M+2D_R}{D_L+D_M+D_R}\Delta - \lambda d_L$, that of an M-agent donating d_M is $-\frac{D_L+D_R}{D_L+D_M+D_R}\Delta - \lambda d_M$, and that of an R-agent donating d_R is $-\frac{2D_L+D_M}{D_L+D_M+D_R}\Delta - \lambda d_R$. The associated FOCs read:

$$\frac{D_M+2D_R}{(D_L+D_M+D_R)^2}\Delta = \lambda, \quad \frac{D_L+D_R}{(D_L+D_M+D_R)^2}\Delta = \lambda, \quad \frac{2D_L+D_M}{(D_L+D_M+D_R)^2}\Delta = \lambda,$$

from which one infers that $D_M + 2D_R = D_L + D_R = 2D_L + D_M$. This leads to $D_M(\{L,M,R\}) = 0$ and $D_L(\{L,M,R\}) = D_R(\{L,M,R\}) = \frac{\Delta}{2\lambda}$. Observe that in equilibrium coalition L and coalition R both win the contest with probability $\frac{1}{2}$, whereas policy M is abandoned with certainty. Imposing symmetry gives the last row of the table.

Proof of Proposition 6.4

We first have to derive the equilibrium condition regarding the probability p_i that an i-agent initiates a SIG, $i = L,M,R$. We start with the condition for L-agents. (As before, $\phi_i := (1-p_i)^{n_i}$ denotes the probability that coalition i is not formed.) An L-agent is indifferent between not initiating a SIG and initiating a SIG if and only if:

$$\begin{aligned}
&\phi_L^{1-\frac{1}{n_L}}\phi_M\phi_R v_L(\emptyset) + (1-\phi_L^{1-\frac{1}{n_L}})\phi_M\phi_R v_L(\{L\}) + \phi_L^{1-\frac{1}{n_L}}(1-\phi_M)\phi_R v_L(\{M\}) \\
&+ \phi_L^{1-\frac{1}{n_L}}\phi_M(1-\phi_R)v_L(\{R\}) + (1-\phi_L^{1-\frac{1}{n_L}})(1-\phi_M)\phi_R v_L(\{L,M\}) \\
&+ (1-\phi_L^{1-\frac{1}{n_L}})\phi_M(1-\phi_R)v_L(\{L,R\}) + \phi_L^{1-\frac{1}{n_L}}(1-\phi_M)(1-\phi_R)v_L(\{M,R\}) \\
&+ (1-\phi_L^{1-\frac{1}{n_L}})(1-\phi_M)(1-\phi_R)v_L(\{L,M,R\}) = \\
&\phi_M\phi_R v_L(\{L\}) + (1-\phi_M)\phi_R v_L(\{L,M\}) + \phi_M(1-\phi_R)v_L(\{L,R\}) \\
&+ (1-\phi_M)(1-\phi_R)v_L(\{L,M,R\}) - \lambda f.
\end{aligned}$$

Tedious algebra reduces this equality to

$$\phi_L^{1-\frac{1}{n_L}}\Big(\frac{1}{2}(1+\phi_M) - \frac{(1-\phi_M)\phi_R + 2(1-\phi_R)}{4n_L}\Big) = F.$$

Exchanging the subscripts L and R results in the equilibrium condition regarding p_R. In sum, the equilibrium condition for i-agents, $i = L, R$, reads:

$$\phi_i^{1-\frac{1}{n_i}}\Big(\frac{1}{2}(1+\phi_M) - \frac{(1-\phi_M)\phi_{\neg i} + 2(1-\phi_{\neg i})}{4n_i}\Big) = F. \tag{6.32}$$

An M-agent is indifferent between not initiating a SIG and initiating a SIG if and only if:

$$\begin{aligned}
&\phi_L\phi_M^{1-\frac{1}{n_M}}\phi_R v_M(\emptyset) + (1-\phi_L)\phi_M^{1-\frac{1}{n_M}}\phi_R v_M(\{L\}) + \phi_L(1-\phi_M^{1-\frac{1}{n_M}})\phi_R v_M(\{M\}) \\
&+ \phi_L\phi_M^{1-\frac{1}{n_M}}(1-\phi_R)v_M(\{R\}) + (1-\phi_L)(1-\phi_M^{1-\frac{1}{n_M}})\phi_R v_M(\{L,M\}) \\
&+ (1-\phi_L)\phi_M^{1-\frac{1}{n_M}}(1-\phi_R)v_M(\{L,R\}) + \phi_L(1-\phi_M^{1-\frac{1}{n_M}})(1-\phi_R)v_M(\{M,R\}) \\
&+ (1-\phi_L)(1-\phi_M^{1-\frac{1}{n_M}})(1-\phi_R)v_M(\{L,M,R\}) = \\
&\phi_L\phi_R v_M(\{M\}) + (1-\phi_L)\phi_R v_M(\{L,M\}) + \phi_L(1-\phi_R)v_M(\{M,R\}) \\
&+ (1-\phi_L)(1-\phi_R)v_M(\{L,M,R\}) - \lambda f.
\end{aligned}$$

This condition is equivalent to:

$$\phi_M^{1-\frac{1}{n_M}}\chi\Big(\frac{1}{2} - \frac{1}{4n_M}\Big) = F, \tag{6.33}$$

where $\chi := \phi_L(1-\phi_R) + (1-\phi_L)\phi_R$ is the probability that precisely one of the extreme coalitions is formed. From (6.33) one infers that M-agents never initiate SIGs ($\phi_M^* = 1$) if $F \geq \frac{1}{2}\big(1 - \frac{1}{2n_M}\big)$. We know from Proposition 6.1 that if M-agents refrain from civic action, then i-agents employ the mixed strategy solving (6.11)-(6.12) provided $F < 1 - \frac{1}{2n_i}$ (this is inequality (6.13) evaluated at $\beta = 0$), $i = L, R$. This proves the first claim.

Note that the conditions (6.32)-(6.33) reduce to

$$\phi_i(1+\phi_M) = 2F, \quad i = L, R, \quad \phi_M\chi = 2F$$

as $n_i \to \infty$, $i = L, M, R$, implying that $\phi_L = \phi_R =: \bar{\phi}$ in the limit. One easily verifies that the system

$$\bar{\phi}(1+\phi_M) = 2F, \quad \phi_M \times 2\bar{\phi}(1-\bar{\phi}) = 2F$$

does not have a solution $(\bar{\phi}, \phi_M) \in [0,1]^2$. The second claim now follows from the continuity of the left-hand sides of (6.32)-(6.33) in n_L, n_M, and n_R. ■

Derivation of equation (6.16)

Using the probabilities that a specific policy is implemented in the various continuation games (see Lemma 6.1), one sees that expected social welfare in continuation game $\{i\}$, $S(\{i\})$, equals

$$S(\{i\}) = (1-\nu)S_i + \nu S_M = -(1-\nu)(1-\mu_i+\mu_{\neg i})\Delta - \nu\mu\Delta, \quad i = L, R,$$

whereas if both coalitions are formed social welfare is

$$S(\{L,R\}) = \frac{1}{2}(1-\nu^2)S_L + \frac{1}{2}(1-\nu^2)S_R + \nu^2 S_M = \nu^2(1-\mu)\Delta - \Delta.$$

Lastly, if neither coalition is formed, then social welfare is $S(\emptyset) = -\mu\Delta = W(\emptyset)$. Note that $S(\{L\}) + S(\{R\}) = -2(1-\nu)\Delta - 2\nu\mu\Delta$. This expression and the fact that $D_i(\{i\}) = \frac{\Delta}{\lambda}\nu(1-\nu)$, $i = L, R$, yields

$$W(\{L\}) + W(\{R\}) = 2\Big(\frac{\alpha}{\lambda}\nu(1-\nu) - (1-\nu) - \nu\mu\Big)\Delta.$$

Because $D_L(\{L,R\}) + D_R(\{L,R\}) = \frac{\Delta}{\lambda}(1-\nu^2)$, one has:

$$W(\{L,R\}) = \Big(\frac{\alpha}{\lambda}(1-\nu^2) - 1 + \nu^2(1-\mu)\Big)\Delta.$$

It follows that in the expression for $\mathcal{W}$ the term $\mu\Delta$ is multiplied by

$$-\Big(1-\frac{F}{1-\nu}\Big) \times \frac{F}{1-\nu} \times (2\nu) - \Big(\frac{F}{1-\nu}\Big)^2 - \Big(1-\frac{F}{1-\nu}\Big)^2 \times \nu^2 = -(\nu+F)^2. \quad (6.34)$$

The rest of $\mathcal{W}$ reads

$$\Big(1-\frac{F}{1-\nu}\Big)\Delta(1-\nu)\Big(2\frac{F}{1-\nu} \times \Big(\frac{\alpha}{\lambda}\nu - 1\Big) + (1+\nu)\Big(1-\frac{F}{1-\nu}\Big) \times \Big(\frac{\alpha}{\lambda} - 1\Big)\Big) =$$

$$\Delta(1-\nu-F)\big((1+\nu)z - (2+z)F\big). \quad (6.35)$$

Adding (6.34) times $\mu\Delta$ to (6.35) yields the desired result.

Proof of Proposition 6.5

Differentiating (6.16) with respect to ν and dividing the result by Δ yields the FOC:

$$-(1+\nu)z + (2+z)F + z(1-\nu-F) - 2\mu(\nu+F) = 0,$$

from which (6.17) follows. This number is only optimal if the SOC holds, i.e. if $z > -\mu$. The subgame starting after P sets $\nu = \nu^*$ has the unique symmetric equilibrium presented in Proposition 6.1 only if Condition 6.1 and the inequalities (6.13) hold. Condition 6.1 for $\nu = \nu^*$ reads $\nu^* < 1$. Taking the limit as $n_i \to \infty$, $i = L, R$, of the inequalities (6.13) and evaluating the result in $\nu = \nu^*$ gives $\nu^* < 1 - F$, a restriction which is more difficult to satisfy than the restriction $\nu^* < 1$. Obviously, $\nu^* < 1 - F \Leftrightarrow F < \frac{\mu+z}{1+z}$. Using the definition of ν, one sees that $\beta^* = \frac{\lambda}{\Delta} \times \left(\frac{1-\mu}{\mu+z}\right)^2 f^2$. The comparative statics results with respect to Δ, f, and α are obvious. Finally:

$$\begin{aligned}\frac{\partial \beta^*}{\partial \lambda} =& \frac{\beta^*}{\lambda} + 2\frac{\lambda}{\Delta} \times \frac{1-\mu}{\mu+z} \times \frac{\alpha(1-\mu)}{\lambda^2(\mu+z)^2} f^2 = \frac{\beta^*}{\lambda} \times \left(1 + \frac{2\alpha}{\lambda(\mu+z)}\right) > 0,\\ \frac{\partial \beta^*}{\partial \mu} =& -2\frac{\lambda}{\Delta} \times \frac{1-\mu}{\mu+z} \times \frac{1+z}{(\mu+z)^2} f^2 < 0.\end{aligned}$$

■

Proof of Proposition 6.6

We first calculate expected aggregate donations ($\mathcal{D}$):

$$\begin{aligned}\mathcal{D} =& \left(\frac{F}{1-\nu}\right)^2 \times 0 + 2\left(1 - \frac{F}{1-\nu}\right) \times \frac{F}{1-\nu} \times \frac{\Delta}{\lambda}\nu(1-\nu)\\ &+ \left(1 - \frac{F}{1-\nu}\right) \times \left(1 - \frac{F}{1-\nu}\right) \times \frac{\Delta}{\lambda}(1-\nu^2) = \frac{\Delta}{\lambda}\big((1-F)^2 - \nu^2\big).\end{aligned}$$

Since $\mathcal{S} = \mathcal{W} - \alpha\mathcal{D}$, one has:

$$\begin{aligned}\mathcal{S} =& \Big[\Delta(1-\nu-F)\big((1+\nu)z - (2+z)F\big) - \Delta\mu(\nu+F)^2\Big]\\ &- \Big[(1+z)\Delta\big((1-F)^2 - \nu^2\big)\Big]\\ =& \Delta z\big((1-F)^2 - \nu^2\big) - 2\Delta(1-\nu-F)F - \Delta\mu(\nu+F)^2 - \Delta(1+z)\big((1-F)^2 - \nu^2\big)\\ =& \Delta F^2 - \Delta + \Delta\nu^2 + 2\Delta\nu F - \Delta\mu(\nu+F)^2 = -\Delta + \Delta(1-\mu)(\nu+F)^2,\end{aligned}$$

yielding, after inserting $\nu = \nu^*$, the desired expression. The comparative static results with respect to Δ, f, and $1/\lambda$ are trivial. Lastly:

$$\frac{\partial \mathcal{S}^*}{\partial z} = -2\frac{1+z}{\mu+z} \times \left(\frac{1-\mu}{\mu+z}\right)^2 F^2 < 0.$$

■

Derivation of equations (6.20)

Suppose that $n_L > n_R$. We have to consider eight possible configurations of candidates: one with zero candidates, three with one candidate, three with two candidates, and one with three candidates. Observe that in the latter the 0-candidate wins the election because R-agents vote strategically for this candidate provided $n_L < n_M + n_R$, i.e. provided $\mu_L < \frac{1}{2}$. The winner of an election in which a candidate runs unopposed is obvious. An L-candidate only wins a two-candidate election when his opponent is an R-candidate, other two-candidate elections are won by an M-candidate. These observations lead to the following expected utility of an L-agent who opts not to run for office should the strategies of all other agents be dictated by (p_L, p_M, p_R):

$$
\begin{aligned}
&-\psi_L^{1-\frac{1}{n_L}}\psi_M\psi_R\Delta - 2\psi_L^{1-\frac{1}{n_L}}\psi_M(1-\psi_R)\Delta - \psi_L^{1-\frac{1}{n_L}}(1-\psi_M)\psi_R\Delta - \\
&(1-\psi_L^{1-\frac{1}{n_L}})(1-\psi_M)\psi_R\Delta - \psi_L^{1-\frac{1}{n_L}}(1-\psi_M)(1-\psi_R)\Delta \\
&-(1-\psi_L^{1-\frac{1}{n_L}})(1-\psi_M)(1-\psi_R)\Delta,
\end{aligned}
$$

The expected utility of this agent if she does become a candidate reads:

$$
-(1-\psi_M)\psi_R\Delta - (1-\psi_M)(1-\psi_R)\Delta - \lambda c.
$$

Equating the above two expected utilities and rewriting results in the equilibrium condition of L-agents given in (6.20). An M-agent who is not a candidate has the following expected utility:

$$
\begin{aligned}
&-\frac{2}{3}\psi_L\psi_M^{1-\frac{1}{n_M}}\psi_R\Delta - (1-\psi_L)\psi_M^{1-\frac{1}{n_M}}\psi_R\Delta - \psi_L\psi_M^{1-\frac{1}{n_M}}(1-\psi_R)\Delta \\
&-(1-\psi_L)\psi_M^{1-\frac{1}{n_M}}(1-\psi_R)\Delta.
\end{aligned}
$$

If an M-agent becomes a candidate, then she is certain to win the election. The expected utility of an M-agent who becomes a candidate is thus simply $-\lambda c$. The equilibrium condition for M-agents mentioned in (6.20) immediately follows. Finally, an R-agent who is not a candidate has an expected utility of

$$
\begin{aligned}
&-\psi_L\psi_M\psi_R^{1-\frac{1}{n_R}}\Delta - 2(1-\psi_L)\psi_M\psi_R^{1-\frac{1}{n_R}}\Delta - \psi_L(1-\psi_M)\psi_R^{1-\frac{1}{n_R}}\Delta \\
&-(1-\psi_L)(1-\psi_M)\psi_R^{1-\frac{1}{n_R}}\Delta - 2(1-\psi_L)\psi_M(1-\psi_R^{1-\frac{1}{n_R}})\Delta \\
&-\psi_L(1-\psi_M)(1-\psi_R^{1-\frac{1}{n_R}})\Delta - (1-\psi_L)(1-\psi_M)(1-\psi_R^{1-\frac{1}{n_R}})\Delta.
\end{aligned}
$$

If this agents is a candidate, then her expected utility equals

$$
-2(1-\psi_L)\psi_M\Delta - \psi_L(1-\psi_M)\Delta - (1-\psi_L)(1-\psi_M)\Delta - \lambda c.
$$

Equating the last two expressions yields the last equilibrium condition given in (6.20).

Proof of Proposition 6.7

Consider the case $n_L > n_R$. With a large population the equilibrium values of ψ_L, ψ_M, and ψ_R solve

$$\psi_L \psi_M (2 - \psi_R) = C, \quad \psi_M (1 - \tfrac{1}{3} \psi_L \psi_R) = C, \quad \psi_R \psi_M \psi_L = C.$$

The first and the last equality can only hold simultaneously if $\psi_R = 1$. One easily verifies that the first two equalities (i.e. the equalities which must hold if L-agents and M-agents do not use a pure strategy) are solved uniquely by $(\psi_L, \psi_M) = (\frac{3}{4}, \frac{4}{3}C)$ if $\psi_R = 1$. These numbers are sound probabilities as long as $C \leq \frac{3}{4}$, proving the first bullet point.

The three conditions cannot hold simultaneously if $C > \frac{3}{4}$, implying that at most two types of agents become candidates with positive probability. Three possible cases thus need to be considered: If we exclude R-agents from being candidates ($\psi_R = 1$), then $(\psi_L, \psi_M) = (C, 1)$ is the only feasible solution (recall that the condition for i-agents need not hold with equality if $\psi_i = 1$), for the solution $(\psi_L, \psi_M) = (1, \frac{3}{2}C)$ is not an element of the unit square if $C > \frac{3}{4}$. If one imposes $\psi_M = 1$, then one arrives at the same feasible solution. The other possibility, $(\psi_L, \psi_R) = (1, C)$, does not constitute equilibrium strategies, because $\psi_L = 1$ is not a best response to $\psi_R = C$. In the last case ($\psi_L = 1$), the remaining conditions are solved by $(\psi_M, \psi_R) = (\frac{4}{3}C, \frac{3}{4})$, again not an element of the unit square. We conclude that $(\psi_L^*, \psi_M^*, \psi_R^*) = (C, 1, 1)$. The case $n_R > n_L$ mirrors the above analysis. ∎