The microeconomics of strategic activism

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Chapter 5

A Signalling Theory of Consumer Boycotts*

5.1 Introduction

The last three decades have seen a dramatic increase in the popularity of consumer boycotts (Friedman, 1999, Baron, 2003).[1] From an economic viewpoint, consumer boycotts are puzzling. First, how do individual consumers manage to coordinate their actions with fellow concerned consumers? Second, how do they avoid that some consumers free-ride on the boycotting efforts borne by others? But even if consumers were able to overcome these difficulties, then still we would not expect to see boycotts: a firm facing the threat of a boycott would preempt the boycott by catering to concerned consumers' demands.

We present a theory that explains the prevalence of consumer boycotts. In our model, a firm knows that consumers are concerned about certain aspects of its production process. Yet, it does not know how much they care. It is only optimal for the firm to alter its behaviour if consumers are very concerned. This gives consumers an incentive to overstate their concern by taking an action that is costly (in utility terms) to themselves: refraining from buying the product.

The setting we consider is the following. Consumers discover that the firm is selling a product that does not adhere to the highest possible ethical standards. For ease of discussion, we consider the case that the firm's good is harmful to the

*This chapter is based on joint work with Pim Heijnen.

[1] There are various reasons for a consumer to boycott. Consumers can object to the conditions under which a product is made (Nike using child labour), to the materials used in the product (Armani using fur), or the impact of the production process on the environment (McDonald's using Brazilian beef from cattle whose grazing grounds are cleared Amazonian rainforests), to name a few.

environment. Our model can easily apply to other issues, such as labour standards. While the firm can change its practices and produce its product in a cleaner fashion, this change cannot be implemented overnight. Moreover, changing the production process is costly and the firm does not know how concerned consumers are about the firm's current practices. Consumers are only willing to pay a lot more for the new, clean variant if they are very concerned. So, the firm only wants to alter its behaviour if the level of consumer concern is sufficiently high.

We assume that consumers' level of concern is not known to the firm. The firm only observes aggregate sales. Based on this observation it has to decide whether or not to switch to the clean technology. If sales are low, consumers care enough about the environment to warrant this switch. This in turn gives consumers sometimes an incentive to decrease their purchases below the quantity they would buy if the firm were fully informed about their level of concern. We interpret this reduction as a consumer boycott: consumers only reduce their purchases below their full-information quantity to induce the firm to alter its behaviour.

We show that every equilibrium has some consumer types boycotting. Quite surprisingly, free-riding does not hinder boycotting. The reason behind this result is that in equilibrium each consumer's decision whether or not to join the boycott is pivotal. If a single consumer deviates from boycotting (and buys the full-information quantity), then the firm subsequently believes that consumers' level of concern is sufficiently low to not warrant a switch to the clean technology.

Consumers can exploit the fact that the firm is not fully informed: a fully informed firm would switch less often to the clean technology. Yet, boycotting is costly and the informational rents accruing to consumers who engage in such signalling are very small. The firm makes substantially lower profits compared to the profits in the full-information benchmark. One could thus argue that the environment is the main beneficiary of the lack of information.

The theoretical literature on boycotts, most notably Baron (2001), Innes (2006), and Baron and Diermeier (2005), focuses on how interest groups interact with firms.[2] This literature sees boycotts as a threat interest groups can employ when negotiating with firms. While this approach certainly has its merits, it leaves several questions unanswered.

Firstly, the interest groups are to some extent capable of communicating the plan of the boycott to consumers. This implies that coordination and free-riding problems are partially sidestepped. Secondly, the interest groups are effectively able to alter consumer preferences via persuasive campaigning. As a result consumers demand less

[2] Note that interest groups do not play a role in our analysis.

of the product. However, this downward shift in demand does not entail a conscious and costly decision to boycott the product. Lastly, the mere threat of a boycott is often sufficient to bring about the desired change. Consequently, in equilibrium there need not be boycotts.[3] Thus, while this strand of the literature explains how interest groups could make use of the threat of boycotts, it does not provide a satisfactory explanation of the *incidence* of consumer boycotts.

In our framework we address these issues. Firstly, in our model the absence of communication and the presence of free-riding do not preclude the emergence of boycotts. Secondly, consumer preferences remain stable and boycotting is truly costly. Lastly, only an *actual* boycott can persuade a firm to change its behaviour.

The organization of this chapter is as follows. Section 5.2 describes the basic model with a single consumer. The model is analyzed in Section 5.3. We interpret the results in Section 5.4. In Section 5.5 we extend the model to multiple consumers. Section 5.6 offers some concluding remarks.

5.2 The Basic Model

One consumer is interested in buying a specific good from a monopolist firm. As long as this consumer perceives the good as being environmentally neutral he derives the following utility from buying x units of the good:

$$u(x,p) = x - \frac{1}{2}x^2 - px, \tag{5.1}$$

where p is the price the firm charges.

Quite suddenly, several media inform the consumer that the firm's production practices are damaging to the environment. The firm does not know how the newly discovered environmental externality associated with the good impacts the consumer's preferences. The firm does know that the consumer's level of environmental concern α enters the utility function as follows:[4]

$$u_\alpha(x,p) = x - \frac{1}{2\alpha}x^2 - px. \tag{5.2}$$

Note that this expression reduces to (5.1) if $\alpha = 1$, i.e. a *consumer of type* 1 does not care about the environment and derives the same utility from the polluting good as from a clean good. This consumer type therefore does not alter his behaviour in

[3] This is a common theme in the literature on the interaction between interest groups and firms. See also Heijnen and Schoonbeek (2008)'s model of persuasive advertising by interest groups, where the threat of advertising by an interest group causes the firm to change its practices.

[4] The specification (5.2) is the quality adjusted linear-demand model as discussed in Sutton (1997) with just one firm. In Sutton (1997) α is interpreted as the square of the quality level.

response to the news. By contrast, a consumer of type 0 refrains from buying the polluting good altogether because of the pollution.[5] A consumer with an intermediate α decreases his demand for the good because of the pollution, but would still buy a positive quantity in a one-shot environment as can be inferred from the demand $x(p;\alpha)$ associated with (5.2):

$$x(p;\alpha) = \alpha(1-p). \tag{5.3}$$

The firm does not know α, but does know that α is a draw from the uniform distribution with support $[0,1]$. We denote the associated cumulative distribution function by F.

Since the news about the good's environmental impact has been released rather abruptly, the firm does not have the opportunity to alter the price of the good prior to the first time the market reopens. So, the newly informed consumer has to pay $p^* = \frac{1}{2}$ per unit.[6] This is the profit-maximizing price should the consumer's preferences be governed by equation (5.1). The firm can, of course, charge a different price in the future. Moreover, the firm might opt to switch to an alternative production technology in the future. This technology does not generate pollution. The consumer will consequently base his purchasing decision on the utility function (5.1) if this clean technology is in place. However, switching to the clean technology comes at a cost: the firm has to incur a fixed cost $\tilde{K} > 0$ to switch to the clean technology. This cost is common knowledge. Switching to the new technology takes time: in the current period the firm has only the old, polluting technology at its disposal. Naturally, whether or not the firm switches to the clean technology depends on its *belief* regarding α. The consumer has consequently an incentive to understate his type: all types (weakly) prefer a good produced with the clean technology and, since $\tilde{K}$ is a fixed cost, this variant will not be more expensive than the old variant. The consumer can understate his type by buying less than he would if the firm knew α, i.e. buying less than equation (5.3) prescribes. To investigate whether this can occur in equilibrium, we look for *pooling equilibria* in which relatively high types mimic the behaviour of some relatively low type.

The situation sketched above is a four-stage game of incomplete information. After nature has drawn α, the consumer has to decide how many units of the good to buy. In this first stage the consumer can try to signal a relatively high level

[5] We adopt the convention that type 0's utility becomes minus infinity if this type buys a positive amount.

[6] The assumption that the firm cannot adjust the price in the first period can be relaxed. This does not alter our main qualitative result (boycotts do occur), but it renders any further analysis impossible.

of environmental concern by buying a relatively small amount of the good. The consumer's purchasing rule in this stage is denoted $x_1(\alpha)$. In the second stage the firm has to decide whether or not to switch to the clean technology. In the third stage the firm sets a price for the good. In the last stage the consumer again decides how much to buy. We refer to the first two stages as period one. Period two comprises the last two stages. There is no discounting. The total payoff of the consumer can thus be written as follows:

$$U_\alpha = \left(x_1 - \frac{1}{2\alpha}x_1^2 - p_1x_1\right) + \left(x_2 - \frac{1}{2\delta + 2(1-\delta)\alpha}x_2^2 - p_2x_2\right), \qquad (5.4)$$

where p_1 $(= \frac{1}{2})$ and x_1 are the price and quantity prevailing in the first period, and p_2 and x_2 are their counterparts prevailing in the second period. If the firm does not switch to the new technology then $\delta = 0$. When $\delta = 1$ the firm switches. We assume that the firm switches to the clean technology if it is indifferent between switching and not switching.

The total profits accruing to the firm are:

$$\Pi = p_1x_1 + p_2x_2 - \delta\tilde{K}. \qquad (5.5)$$

We look for perfect Bayesian Nash equilibria (PBNEs, see for instance Fudenberg and Tirole, 1991, chapter 8) of the above game of incomplete information. Strategies must thus be best responses given the history of play and given beliefs and beliefs are updated using Bayes' Rule whenever applicable. Of course, a PBNE must be consistent with backward induction. A PBNE consists of a first period purchasing rule $x_1^*(\alpha)$, a switching rule $\delta^*(x_1)$, a second period price $p_2^*(\delta, x_1)$, and a second period purchasing rule $x_2^*(\alpha, \delta, p_2)$. Our analysis in the next section makes use of methods developed by von der Fehr (1992).

5.3 Analysis

One can distinguish two possible kinds of PBNEs in games such as the one we study. In a *fully separating PBNE* the firm can infer the value of α, i.e. the consumer's type, from the action x_1 of the consumer. This is not possible in a pooling PBNE. In a pooling PBNE various consumer types use the same action, say $\tilde{x}_1$, in the first period. As a consequence, the firm does not know the precise value of α after selling an amount $\tilde{x}_1$. A PBNE is a fully pooling PBNE if all types use the same action. In a partly pooling PBNE only a strict subset of the set of all possible types use the same action. The kind of PBNEs we are dealing with are either fully separating or partly pooling. For convenience's sake, we skip the 'fully' and the 'partly' in the remainder of the chapter. We introduce the following:

Definition 5.1 *A consumer of type α is said to* mimic *consumer type β, $\beta \neq \alpha$, if type α buys the amount which maximizes type β's period one-utility in the first stage. An equilibrium of the game* exhibits mimicking *if one or more consumer types are mimicked by a positive mass of types in this equilibrium.*[7]

Using (5.3) and the fact that $p_1 = \frac{1}{2}$ one sees that consumer type β maximizes his period one-utility by buying an amount $\frac{\beta}{2}$. Thus a type $\alpha > \beta$ mimics another type β if he buys an amount $\frac{\beta}{2}$. If this occurs, then consumer α tries to signal a level of environmental concern above its true level by buying less than $\frac{\alpha}{2}$. If type α does not try to signal a high level of environmental concern, then he simply buys an amount $\frac{\alpha}{2}$.

Our definition of a consumer boycott is closely related to mimicking:

Definition 5.2 *A* consumer boycott *occurs if a consumer buys less than the amount that maximizes his period one-utility.*

We discuss this definition in detail in Section 5.4.

In this section we characterize all PBNEs of the game. We present results which pertain to both separating PBNEs and pooling PBNEs in Subsection 5.3.1. We also characterize the separating PBNEs of the game in that subsection. Subsection 5.3.2 deals with pooling PBNEs. It turns out that if the switching cost of the firm is not prohibitively high, then the game has infinitely many pooling PBNEs but no separating PBNEs. To rule out 'unattractive' pooling equilibria we apply an equilibrium refinement developed by Grossman and Perry (1986). This is done in Subsection 5.3.3. We occasionally call a PBNE loosely an equilibrium.

5.3.1 General Properties of PBNEs

The first general result describes the equilibrium actions in the last period:

Lemma 5.1 *In any PBNE the firm charges a price $p_2^* = \frac{1}{2}$ in the second period. A consumer of type α buys an amount*

$$x_2^*(\alpha) = \delta\frac{1}{2} + (1-\delta)\frac{\alpha}{2} \tag{5.6}$$

in the second period.

[7] Since mimicking a higher type is clearly suboptimal for any consumer type, we can safely discard this possibility. To save on space we do not mention this possibility in the remainder of the chapter.

Proof. The second period profits, either $\pi_2 = \alpha(1-p_2)p_2$ or $\pi_2 = (1-p_2)p_2 - \tilde{K}$, are maximized by charging $p_2^* = \frac{1}{2}$, irrespective of (the firm's belief regarding) α. Because the game ends after the second period, the consumer simply maximizes (5.1) or (5.2) (whichever applies) when deciding how much to buy, yielding (5.6). ∎

All PBNEs share an important aspect regarding the first period purchasing rule of the consumer:

Lemma 5.2 *In any PBNE the consumer's first period purchasing rule $x_1^*(\cdot)$ is non-decreasing in type.*

Proof. Suppose the claim does not hold, i.e. $x_1^*(\alpha_1) > x_1^*(\alpha_2)$ for some consumer types α_1, α_2 such that $\alpha_1 < \alpha_2$. Because type 0 buys zero units in any equilibrium, we know that $\alpha_1 > 0$. Since $x_1^*(\alpha_2) < x_1^*(\alpha_1) \leq \frac{\alpha_1}{2} < \frac{\alpha_2}{2}$, the firm switches to the new technology if it observes $x_1^*(\alpha_2)$: if the firm did not switch after observing $x_1^*(\alpha_2)$, then type α_2 would be better off purchasing $\frac{\alpha_2}{2}$. We have to investigate two possible cases: the firm also switches after observing $x_1^*(\alpha_1)$, or the firm does not switch after observing $x_1^*(\alpha_1)$. In the first case $x_1^*(\alpha_2)$ can only be optimal if

$$u_{\alpha_2}(x_1^*(\alpha_2), \tfrac{1}{2}) + u(\tfrac{1}{2}, \tfrac{1}{2}) \geq u_{\alpha_2}(x_1^*(\alpha_1), \tfrac{1}{2}) + u(\tfrac{1}{2}, \tfrac{1}{2}).$$

But, because $u_{\alpha_2}(x, \frac{1}{2})$ is an increasing function of x as long as $x < \frac{\alpha_2}{2}$, the above inequality cannot hold. Now suppose that $x_1^*(\alpha_1)$ does not induce the firm to switch. Optimality of $x_1^*(\alpha_2)$ then requires

$$u_{\alpha_2}(x_1^*(\alpha_2), \tfrac{1}{2}) + u(\tfrac{1}{2}, \tfrac{1}{2}) \geq u_{\alpha_2}(\tfrac{\alpha_2}{2}, \tfrac{1}{2}) + u_{\alpha_2}(\tfrac{\alpha_2}{2}, \tfrac{1}{2}).$$

This inequality is equivalent to:

$$x_1^*(\alpha_2)\big(1 - \tfrac{x_1^*(\alpha_2)}{\alpha_2}\big) + \tfrac{1}{4} \geq \tfrac{\alpha_2}{2}, \tag{5.7}$$

where we have used the fact that $u(\frac{1}{2}, \frac{1}{2}) = \frac{1}{8}$. Type α_1's action $x_1^*(\alpha_1)$ is only optimal if this type has no incentive to mimic type α_2, i.e. if:

$$u_{\alpha_1}(x_1^*(\alpha_1), \tfrac{1}{2}) + u_{\alpha_1}(x_1^*(\alpha_1), \tfrac{1}{2}) \geq u_{\alpha_1}(x_1^*(\alpha_2), \tfrac{1}{2}) + u(\tfrac{1}{2}, \tfrac{1}{2}).$$

Since $x_1^*(\alpha_1)$ does not induce switching, $x_1^*(\alpha_1)$ must equal $\frac{\alpha_1}{2}$. The above inequality therefore reduces to:

$$x_1^*(\alpha_2)\big(1 - \tfrac{x_1^*(\alpha_2)}{\alpha_1}\big) + \tfrac{1}{4} \leq \tfrac{\alpha_1}{2}. \tag{5.8}$$

Let $\psi(\alpha) := x_1^*(\alpha_2)\big(1 - \frac{x_1^*(\alpha_2)}{\alpha}\big)$ for $\alpha > 0$. One has:

$$\psi(\alpha_2) - \psi(\alpha_1) = x_1^*(\alpha_2)\big(\tfrac{x_1^*(\alpha_2)}{\alpha_1} - \tfrac{x_1^*(\alpha_2)}{\alpha_2}\big) = \frac{(\alpha_2 - \alpha_1)(x_1^*(\alpha_2))^2}{\alpha_1\alpha_2} < \frac{\alpha_2 - \alpha_1}{4},$$

where the inequality follows from the facts that $x_1^*(\alpha_2) < \frac{\alpha_1}{2}$ and $x_1^*(\alpha_2) < \frac{\alpha_2}{2}$. However, combining (5.7) and (5.8) one sees that

$$\psi(\alpha_2) - \psi(\alpha_1) \geq \frac{\alpha_2}{2} - \frac{\alpha_1}{2} > \frac{\alpha_2 - \alpha_1}{4}.$$

This contradiction proves the claim. ■

In a separating PBNE the firm is able to infer the precise value of α. Whether or not it is then optimal to switch to the clean technology follows from a simple comparison of the second period-profit for given α in the two scenarios. Only if the consumer turns out to care a great deal about the environment (α sufficiently low, say $\alpha \leq \hat{\alpha}$) does the firm prefer to switch to the clean technology. However, it turns out that if $\tilde{K}$ is not too large, then there is always some higher consumer type $\alpha > \hat{\alpha}$ who has an incentive to mimic $\hat{\alpha}$. This incentive destroys possible separating PBNEs. This observation is formalized as follows:

Proposition 5.1 *The game has no separating PBNEs if $\tilde{K} < \frac{1}{4}$. If $\tilde{K} \geq \frac{1}{4}$, then there is a unique separating PBNE in which consumer type α buys a quantity $\frac{\alpha}{2}$ and the firm never switches.*

Proof. Suppose there does exist a separating PBNE. In such an equilibrium the firm can infer the true value of α from the first period equilibrium purchasing rule denoted $\hat{x}_1(\cdot)$. If the firm can infer α from $\hat{x}_1(\alpha)$, then the consumer has no incentive to distort its purchases. Consequently, $\hat{x}_1(\alpha) = \frac{\alpha}{2}$ and the firm attaches probability 1 to type α after selling a quantity $\hat{x}_1(\alpha)$. Because its gross profit with the clean technology is $\frac{1}{4}$, switching to the clean technology is optimal if and only if

$$\frac{\alpha}{4} \leq \frac{1}{4} - \tilde{K}.$$

So, in a separating PBNE the firm switches to the clean technology as long as

$$\alpha \leq \hat{\alpha} := 1 - 4\tilde{K}. \tag{5.9}$$

Note that $\hat{\alpha} \in (0,1)$ if $\tilde{K} < \frac{1}{4}$. Suppose the latter inequality holds. Consider the consumer type $\hat{\alpha} + \epsilon$ with $\epsilon > 0$ small. We show that this type is better off buying $\frac{\hat{\alpha}}{2}$ than its candidate equilibrium quantity $\frac{\hat{\alpha}+\epsilon}{2}$. If consumer type $\hat{\alpha} + \epsilon$ deviates to $\frac{\hat{\alpha}}{2}$, then this consumer can buy the clean good in period two. His total payoff when he deviates reads:

$$u_{\hat{\alpha}+\epsilon}(\tfrac{\hat{\alpha}}{2}, \tfrac{1}{2}) + u(\tfrac{1}{2}, \tfrac{1}{2}) = \tfrac{\hat{\alpha}}{4} - \tfrac{\hat{\alpha}^2}{8(\hat{\alpha}+\epsilon)} + \tfrac{1}{8}.$$

If he does not deviate, then his total payoff is $\frac{\hat{\alpha}+\epsilon}{4}$. Comparing these two payoffs reveals that a type $\hat{\alpha}+\epsilon$ wants to mimic type $\hat{\alpha}$ as long as $2(\hat{\alpha}+\epsilon)\epsilon < \hat{\alpha}(1-\hat{\alpha})+\epsilon$. Since this inequality holds for $\epsilon > 0$ sufficiently small, we conclude that there is a positive mass of types who want to mimic type $\hat{\alpha}$. This implies that the game has no separating PBNEs if $\tilde{K} < \frac{1}{4}$. If $\tilde{K} \geq \frac{1}{4}$, then either (5.9) never holds and the firm thus never switches or the firm only switches if it faces type 0 for sure. This implies that no consumer type can beneficially distort his purchases and the optimality of the purchasing rule $\hat{x}_1(\alpha) = \frac{\alpha}{2}$ follows. ■

As can be inferred from (5.9), switching to the clean technology is never optimal for the firm if the associated cost $\tilde{K}$ is more than $\frac{1}{4}$. We focus in the remainder of the chapter on the more interesting situations in which switching can be optimal. We thus assume that $\tilde{K} < \frac{1}{4}$. To avoid cluttering the notation with fractions, we use from now on the modified cost $K := 4\tilde{K}$ instead of $\tilde{K}$. Before we continue the analysis of the game, we highlight a result that follows directly from the proof of Proposition 5.1:

Lemma 5.3 *If α is common knowledge, then consumer type α buys a quantity $\frac{\alpha}{2}$ in period one. The firm switches to the clean technology if and only if $\alpha \leq 1-K$.*

We use this result when we compare the game with a complete information benchmark situation in Section 5.4.

In the proof of Proposition 5.1 we use the observation that if the firm could infer the true value of α, then it would stick to the polluting technology if its sales were above some threshold. The optimality of such a simple threshold rule holds in any PBNE:

Lemma 5.4 *Suppose $K < 1$. In any PBNE the firm switches to the clean technology if and only if $x_1 \leq \bar{x}_1$ for some threshold $\bar{x}_1 \in (0, \frac{1}{2})$.*

Proof. Recall that consumer type α buys $\frac{\alpha}{2}$ units in the second period if the firm uses the polluting technology in that period. The firm's associated second period profit reads $\frac{\alpha}{4}$. Lemma 5.2 informs us that in any PBNE the consumer's first period purchasing rule is weakly increasing in type. Consequently, the firm's equilibrium belief of the second period profit, denoted $E(\frac{\alpha}{4}|x_1)$, is strictly increasing in x_1. Thus, there is at most one x_1 in the support of the first period purchasing rule solving

$$E(\alpha|x_1) = 1-K, \tag{5.10}$$

the condition which holds if the firm is indifferent between switching and not switching. From Proposition 5.1 we know that $\bar{x}_1 = \frac{1-K}{2}$ is the unique threshold solving this condition if the firm can infer α from x_1. However, because any PBNE exhibits mimicking, exact inference of α is not always possible in equilibrium. Suppose for contradiction that (5.10) holds for no x_1 in the support of the first period purchasing rule. If $E(\alpha|x_1) < 1 - K$ for all x_1 in the support, then the firm always switches. In this situation profits with the new technology are strictly larger than profits with the old technology for all x_1 in the support of the purchasing strategy, in particular for the maximum of the support. This maximum equals $\frac{1}{2}$, the quantity consumer type 1 buys. But then a type α who mimics wants to stop mimicking and buy $\frac{\alpha}{2}$, for the latter quantity, being less than $\frac{1}{2}$, also induces the firm to switch. If, on the other hand, $E(\alpha|x_1) > 1 - K$ for all x_1 in the support, then the firm never switches and consumer types who mimic are better off not mimicking. We conclude that (5.10) holds for precisely one x_1. ■

In equilibrium the firm correctly infers that higher first period sales imply that it faces a higher consumer type, as can be gathered from Lemma 5.2. Since facing a consumer with a sufficiently low type is the sole reason to switch to the clean technology, the firm only switches if its expectation regarding α is sufficiently low, i.e. if its sales are below some threshold.

The results presented above enable us to characterize the pooling equilibria, as is done in the next subsection.

5.3.2 Pooling Equilibria

In a pooling equilibrium some consumer types must be better off mimicking some relatively low type β instead of simply maximizing their period one-utility. This only happens if the firm switches to the clean technology after selling type β's optimal quantity, but does not switch after selling a larger quantity. Before we investigate which types are better off mimicking, we establish the following:

Lemma 5.5 *In any pooling PBNE precisely one consumer type is mimicked. If type $\bar{\beta}$ is mimicked in a particular PBNE, then the firm's switching threshold in that PBNE is $\bar{x}_1 = \frac{\bar{\beta}}{2}$.*

Proof. Let $\tilde{\alpha}$ be the highest consumer type who mimics some strictly lower type, say type $\tilde{\beta} < 1$. Then $\bar{x}_1 = \frac{\tilde{\beta}}{2}$. To see this, suppose the contrary, i.e. $\bar{x}_1 \neq \frac{\tilde{\beta}}{2}$. If $\bar{x}_1 < \frac{\tilde{\beta}}{2}$, then type $\tilde{\alpha}$'s attempt to induce switching by the firm would be futile. On the other hand, if $\bar{x}_1 > \frac{\tilde{\beta}}{2}$, then type α would be better off buying slightly more.

Suppose for contradiction that at least two different consumer types are mimicked in equilibrium, say $\tilde{\beta}_1$ and $\tilde{\beta}_2$, $\tilde{\beta}_1 < \tilde{\beta}_2$. Pick a type $\tilde{\alpha}_i > \tilde{\beta}_i$ who mimics type $\tilde{\beta}_i$, $i = 1, 2$. Lemma 5.2 tells us that $\tilde{\alpha}_1 < \tilde{\alpha}_2$. Because the firm switches to the clean technology upon selling any $x \in [0, \frac{\tilde{\beta}_2}{2}]$, type $\tilde{\alpha}_1$ is better off buying $\min\{\frac{\tilde{\beta}_2}{2}, \frac{\tilde{\alpha}_1}{2}\} > \frac{\tilde{\beta}_1}{2}$. This contradiction establishes uniqueness. ■

Lemma 5.5 formalizes a simple idea. Knowing that the firm switches to the clean technology only if sales do not exceed some threshold, a consumer type α only needs to consider two options: buying the one-shot utility maximizing quantity $\frac{\alpha}{2}$, or, if $\frac{\alpha}{2}$ exceeds the threshold, buying the threshold quantity. Any other quantity is clearly suboptimal: even if it does induce the firm to switch, it does so at a higher opportunity cost than the threshold quantity. Consequently, the consumer type associated with the threshold quantity must be the only consumer type who is mimicked.

We next establish which consumer types gain from mimicking some type $\bar{\beta}$ if the firm uses the threshold rule with $\bar{x}_1 = \frac{\bar{\beta}}{2}$:

Lemma 5.6 *Suppose the firm uses the threshold rule with $\bar{x}_1 = \frac{\bar{\beta}}{2}$. Then consumer types $\alpha \in (\bar{\beta}, h(\bar{\beta})]$, where*

$$h(z) := \frac{z + \frac{1}{2} + \sqrt{(z + \frac{1}{2})^2 - 2z^2}}{2}, \quad z \in [0, 1], \tag{5.11}$$

gain from mimicking type $\bar{\beta}$. The interval $[\bar{\beta}, h(\bar{\beta})]$ is nonempty for every $\bar{\beta} \in [0, 1]$. Its length, $h(\bar{\beta}) - \bar{\beta}$, decreases strictly from $\frac{1}{2}$ to 0 as $\bar{\beta}$ goes from 0 to 1.

Proof. A consumer type $\alpha > \bar{\beta} \geq 0$ gains from mimicking type $\bar{\beta}$ if

$$u_\alpha(\tfrac{\bar{\beta}}{2}, \tfrac{1}{2}) + u(\tfrac{1}{2}, \tfrac{1}{2}) \geq 2u_\alpha(\tfrac{\alpha}{2}, \tfrac{1}{2}).$$

Rewriting this inequality yields

$$\bar{\beta} - \frac{\bar{\beta}^2}{2\alpha} + \frac{1}{2} \geq \alpha.$$

This inequality holds for (both roots are real-valued)

$$\alpha \in \left[\frac{\bar{\beta} + \frac{1}{2} - \sqrt{(\bar{\beta} + \frac{1}{2})^2 - 2\bar{\beta}^2}}{2}, \frac{\bar{\beta} + \frac{1}{2} + \sqrt{(\bar{\beta} + \frac{1}{2})^2 - 2\bar{\beta}^2}}{2}\right].$$

Because $\bar{\beta} - \frac{\bar{\beta}^2}{2\alpha} + \frac{1}{2}\big|_{\alpha=\bar{\beta}} = \frac{\bar{\beta}+1}{2} \geq \bar{\beta}$, one has

$$\frac{\bar{\beta} + \frac{1}{2} - \sqrt{(\bar{\beta} + \frac{1}{2})^2 - 2\bar{\beta}^2}}{2} \leq \bar{\beta} \leq \frac{\bar{\beta} + \frac{1}{2} + \sqrt{(\bar{\beta} + \frac{1}{2})^2 - 2\bar{\beta}^2}}{2}.$$

Because types $\alpha \leq \bar{\beta}$ need not mimic type $\bar{\beta}$ to induce switching, only types $\alpha \in (\bar{\beta}, h(\bar{\beta})]$ gain from mimicking $\bar{\beta}$. The proofs of the statements regarding $[\bar{\beta}, h(\bar{\beta})]$ can be found in the appendix. ■

The interval $(\bar{\beta}, h(\bar{\beta})]$ contains the consumer types who gain from mimicking type $\bar{\beta}$ provided the firm switches only after selling $\frac{\bar{\beta}}{2}$ or less. Figure 5.1 illustrates this consumer behaviour. The 45°-line is the consumer's equilibrium strategy if the firm were fully informed regarding α. In the interval $(\bar{\beta}, h(\bar{\beta})]$ the player mimics type $\bar{\beta}$. This is the horizontal part of the solid curve.

The intuition behind the result that the length of this interval is decreasing in $\bar{\beta}$ is simple: as the consumer type who is mimicked increases from, say, $\bar{\beta}$ to $\bar{\beta}+\epsilon$, the consumer types in the interval $(\bar{\beta}, \bar{\beta}+\epsilon]$ need not distort their period one purchases anymore to ensure that the firm switches. This effect is only partly offset by the fact that mimicking type $\bar{\beta}+\epsilon$ is less costly than mimicking type $\bar{\beta}$ for consumer types $\alpha > \bar{\beta}+\epsilon$.

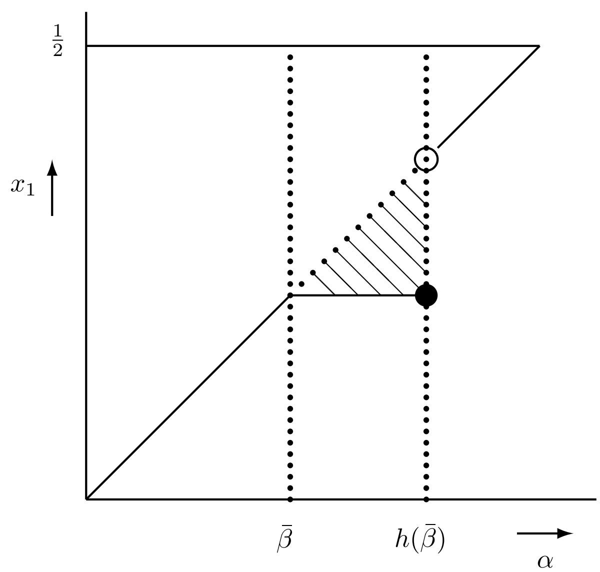

Figure 5.1: Sketch of the equilibrium behaviour of the consumer.

Of course, mimicking consumer type $\bar{\beta}$ can only occur in equilibrium if it is optimal for the firm (given its beliefs) to switch upon selling $\frac{\bar{\beta}}{2}$ or less. Since beliefs must be correct in equilibrium, the firm knows that the true type must be an element of $[\bar{\beta}, h(\bar{\beta})]$ if it sells an amount $\frac{\bar{\beta}}{2}$. Switching is thus only optimal if

$$E\big(\alpha|\alpha \in [\bar{\beta}, h(\bar{\beta})]\big) = \frac{1}{h(\bar{\beta}) - \bar{\beta}} \int_{\bar{\beta}}^{h(\bar{\beta})} \alpha \, \mathrm{d}\alpha = \frac{\bar{\beta} + h(\bar{\beta})}{2} \leq 1 - K, \tag{5.12}$$

Let $g(z) := \frac{z+h(z)}{2}$. In the appendix we show that $g'(z) > 0$, $z \in [0,1]$, $g(0) = \frac{1}{4}$, and $g(1) = 1$. Consequently, there exists at least one $\bar{\beta}$ such that $g(\bar{\beta}) \leq 1 - K$ as long as $K < \frac{3}{4}$. Equilibria exhibiting mimicking exist if this condition is met:

Proposition 5.2 *Suppose $K < \frac{3}{4}$. Then for each $\bar{\beta} \in [0, g^{-1}(1-K)]$ there exists a (pooling) PBNE exhibiting mimicking which is supported by the following strategies:*

- *Strategy of the consumer:*
  1. *First period purchasing rule:* $x_1^*(\alpha) = \begin{cases} \frac{\alpha}{2} & \text{if } \alpha < \bar{\beta} \\ \frac{\bar{\beta}}{2} & \text{if } \bar{\beta} \leq \alpha \leq h(\bar{\beta}) \\ \frac{\alpha}{2} & \text{if } \alpha > h(\bar{\beta}); \end{cases}$
  2. *Second period purchasing rule:* $x_2^*(\alpha) = \delta \frac{1}{2} + (1-\delta)\frac{\alpha}{2}$.
- *Strategy of the firm:*
  1. *Switching rule:* $\delta^*(x_1) = \begin{cases} 1 & \text{if } x_1 \leq \frac{\bar{\beta}}{2} \\ 0 & \text{if } x_1 > \frac{\bar{\beta}}{2}; \end{cases}$
  2. *Second period pricing rule:* $p_2^* = \frac{1}{2}$.

Proof. Note first that $g^{-1}(\cdot)$ exists because $g'(\cdot)$ is bounded away from zero. Fix a $\bar{\beta} \in [0, g^{-1}(1-K)]$. Consider the following map which results in a conditional cumulative distribution function for α for each $x_1 \in [0, \frac{1}{2}]$:[8]

$$\Phi_{\bar{\beta}}(\alpha|x_1) = \begin{cases} \mathbf{1}_{\{\alpha \geq 2x_1\}}(\alpha) & \text{if } 0 \leq x_1 < \frac{\bar{\beta}}{2} \\ \frac{\alpha - \bar{\beta}}{h(\bar{\beta}) - \bar{\beta}} & \text{if } x_1 = \frac{\bar{\beta}}{2} \\ \mathbf{1}_{\{\alpha \geq \tilde{\alpha}\}}(\alpha) & \text{if } \frac{\bar{\beta}}{2} < x_1 \leq \frac{h(\bar{\beta})}{2} \\ \mathbf{1}_{\{\alpha \geq 2x_1\}}(\alpha) & \text{if } \frac{h(\bar{\beta})}{2} < x_1 \leq \frac{1}{2}. \end{cases} \tag{5.13}$$

The third case of this map represents the out-of-equilibrium beliefs: the firm attaches probability 1 to consumer type $\tilde{\alpha}$, where $\tilde{\alpha}$ is such that the firm does not switch if it indeed attaches probability 1 to type $\tilde{\alpha}$.[9] The other cases deal with play on the equilibrium path: if sales are such that no mimicking has occurred, then the consumer's type is revealed and the belief is degenerate (first and last case),

[8] $\mathbf{1}_A$ is the indicator function of the set A, i.e. $\mathbf{1}_A(x) = 1$ if $x \in A$ and $\mathbf{1}_A(x) = 0$ if $x \notin A$.

[9] Of course, there is a myriad of alternative out-of-equilibrium beliefs supporting this equilibrium.

else the firm only attaches positive probability mass to types who benefit from mimicking type $\bar{\beta}$ (second case). It follows directly from Lemma 5.4, Lemma 5.6, and the discussion preceding this proposition that there exists a PBNE supported by the given strategies and the firm's beliefs $\Phi_{\bar{\beta}}$. ■

We call the equilibrium in which the firm switches if and only if it observes an $x_1 \leq \frac{\bar{\beta}}{2}$ equilibrium $\bar{\beta}$. In equilibrium $\bar{\beta}$ consumers who are gravely concerned (those with $\alpha \leq \bar{\beta}$) simply buy their one period-utility maximizing quantity, knowing that the firm, upon observing this rather modest demand, will find it optimal to switch. So, for these consumers there is no reason to boycott. Consumers who do not care much (those with $\alpha > h(\bar{\beta})$) find boycotting too costly: they gain little from engaging in this costly act. Only consumers with intermediate levels of concern (those with $\alpha \in (\bar{\beta}, h(\bar{\beta})])$ do boycott. These consumers benefit substantially from a switch to the clean technology. Such a switch can only be accomplished by buying less than the one period-utility maximizing quantity, i.e. by mimicking the lower type $\bar{\beta}$.

The above proposition does not inform us about the equilibria which could prevail if $K \in [\frac{3}{4}, 1)$. If K is in this interval and some consumer type $\bar{\beta} \geq 0$ is mimicked, then in equilibrium not every consumer type who benefits from mimicking type $\bar{\beta}$, i.e. consumer types $\alpha \in (\bar{\beta}, h(\bar{\beta})]$, can do so with probability 1. If all these types would mimic type $\bar{\beta}$ with probability 1, then the firm's updated expected value of α, that is $g(\bar{\beta}) \geq g(0) = \frac{1}{4}$, would exceed $1 - K$. Consequently, the firm would not switch to the clean technology and mimicking would be futile. On the other hand, if no mimicking would occur, then the firm would switch if $x_1 \leq \frac{1-K}{2}$. Then types $1 - K + \epsilon$ with $\epsilon > 0$ sufficiently small would gain by mimicking type $1 - K$. We conclude that if $K \in [\frac{3}{4}, 1)$ only equilibria in which some consumer types use a mixed strategy can prevail. Analysis of such equilibria is beyond the scope of this chapter. We therefore confine attention to the parameter range $K \in (0, \frac{3}{4})$.

Proposition 5.2 shows that the game has infinitely many PBNEs. The reason is that a PBNE does not impose restrictions on beliefs prevailing after out-of-equilibrium play has occurred. If one does require beliefs off the equilibrium path to meet certain criteria, one might be able to eliminate certain 'unattractive' equilibria. Grossman and Perry (1986) propose a method to restrict in a natural way the set of out-of-equilibrium beliefs. Their method leads to a refinement of a PBNE coined *perfect sequential equilibrium* (PSE). In the next subsection we discuss this equilibrium refinement and apply it to our model.

5.3.3 Perfect Sequential Equilibria

A PSE is supported by *metastrategies*. A metastrategy of a player is a rule which specifies the actions of this player, not just for any possible history, but also for any possible belief, including beliefs that do not occur when the strategies of the proposed equilibrium are used. If a player is fully informed, then his metastrategy simply prescribes his actions for any possible history. Note that a subgame perfect equilibrium of a game of full information is supported by such 'degenerate metastrategies'. Since in the present context the consumer is fully informed, we only have to specify a metastrategy $\tilde{\delta}(x_1, \Phi)$ for the firm. The metastrategy $\tilde{\delta}$ specifies whether or not the firm switches for any 'message' x_1 the consumer could send and any belief Φ the firm could entertain.

Grossman and Perry (1986) require beliefs after out-of-equilibrium play to be *credible*. Moreover, just as in PBNEs beliefs must follow from Bayes' Rule if equilibrium play occurs. These requirements limit the set of possible beliefs. In the present context credibility of beliefs requires the firm, after observing out-of-equilibrium play, to look for types who benefit from the observed deviation. If the firm can find such types it should attach positive probability to these types. Moreover, its subsequent behaviour should be optimal given such a credible belief. This means that the firm should play a best response to the observed out-of-equilibrium play combined with the credible belief.

The kind of beneficial out-of-equilibrium play sketched above clearly destroys candidate equilibria if credibility of beliefs is imposed. The set of PSEs of a game is therefore a subset of the set of PBNEs. In fact, the main result of this section is that only one of the PBNEs mentioned in Proposition 5.2 meets the requirements of sequential perfectness. Before we present this result, we discuss credible beliefs and metastrategies in more detail.

Formally, consider an out-of-equilibrium quantity purchased z. In equilibrium $\bar{\beta}$ such a z is an element of $(\frac{\bar{\beta}}{2}, \frac{h(\bar{\beta})}{2}]$. Suppose every consumer type α in $D(z) \subseteq [0,1]$ is (weakly) better off buying z than buying $x_1^*(\alpha)$ if the firm uses the metastrategy $\tilde{\delta}$, whereas every type in $[0,1] \setminus D(z)$ is (weakly) worse off if the firm uses $\tilde{\delta}$. Here, the metastrategy $\tilde{\delta}$ must be evaluated at the belief $F(\cdot|D(z))$. This belief assigns positive probability mass to the types $\alpha \in D(z)$ who benefit from buying the amount z and no probability mass to types outside $D(z)$. In fact, the cumulative distribution function $F(\cdot|D(z))$ is the posterior distribution of α conditional on $\alpha \in D(z)$. So, for example, if $D(z)$ is the nonempty interval $(\underline{d}, \overline{d}) \subseteq [0,1]$ then

$$F(\alpha|D(z)) = \frac{\alpha - \underline{d}}{\overline{d} - \underline{d}}, \quad \alpha \in D(z).$$

The belief $F(\cdot|D(z))$ abides by Grossman and Perry's (1986) definition of credibility: a belief Φ of the firm is said to be credible with respect to the action z if $\Phi(\cdot|z) = F(\cdot|D(z))$ as long as $D(z)$ has positive probability mass. If Φ is credible with respect to any out-of-equilibrium quantity z such that the prior probability of the event $D(z)$ is positive *and* $\Phi(\cdot|x_1)$ follows from Bayes' Rule for any x_1 on the equilibrium path, then Φ is credible. In general, a belief is the image of a prior distribution together with an action under a so-called *updating rule*, i.e. an updating rule is a transformation c which maps a distribution G together with an action x to a belief Ψ, $c : (G, x) \mapsto \Psi$. An updating rule is credible if it induces a credible belief. Note that the updating rule in the above example maps the pair (F, z) to $F(\cdot|D(z))$.

An updating rule c^*, a metastrategy $\tilde{\delta}^*$, and a first period purchasing rule x_1^* induce a perfect sequential equilibrium if and only if c^* induces a credible belief Φ^* and both $\tilde{\delta}^*$ and x_1^* are *sequentially perfect* for all $\alpha \in [0,1]$. The metastrategy $\tilde{\delta}^*$ is sequentially perfect if it is a best response to any action x_1 and any belief Φ. The rule x_1^* is sequentially perfect if it is a best response to $\tilde{\delta}^*$ combined with the credible updating rule c^*, i.e. if it is a best response to the map $x_1 \to \tilde{\delta}^*(x_1, c^*(F, x_1)) = \tilde{\delta}^*(x_1, \Phi^*(\cdot|x_1))$.

We now apply the above equilibrium refinement to the PBNEs of Proposition 5.2:

Proposition 5.3 *Suppose $K < \frac{3}{4}$. Then the equilibrium $g^{-1}(1-K) =: \xi = \xi(K)$ is the unique perfect sequential equilibrium.*

Proof. We first show that equilibrium $\bar{\beta}$ fails to meet the requirements of a PSE if $\bar{\beta} < \xi$. Consider two different pooling equilibria, $\bar{\beta}_1$ and $\bar{\beta}_2$, with $\bar{\beta}_1 < \bar{\beta}_2 \leq \xi$ and $(\bar{\beta}_1, h(\bar{\beta}_1)) \cap (\bar{\beta}_2, h(\bar{\beta}_2)) \neq \emptyset$. Note that $\frac{\bar{\beta}_2}{2}$ is an out-of-equilibrium quantity of equilibrium $\bar{\beta}_1$. In the appendix we show that equilibrium $\bar{\beta}_2$ (weakly) payoff dominates the equilibrium $\bar{\beta}_1$ for the consumer with strict dominance for types $\alpha \in (\bar{\beta}_2, h(\bar{\beta}_2))$. This means that if equilibrium $\bar{\beta}_1$ is a PSE, then it must be supported by an updating rule c which is such that the belief induced by c attaches positive probability to types $\alpha \in (\bar{\beta}_2, h(\bar{\beta}_2))$ after selling a quantity $\frac{\bar{\beta}_2}{2}$. Moreover, the belief attaches zero probability mass to types outside $[\bar{\beta}_2, h(\bar{\beta}_2)]$ (for these types it is never optimal to mimic type $\bar{\beta}_2$). As a consequence, if equilibrium $\bar{\beta}_1$ were a PSE, then the firm would switch in this equilibrium after selling a quantity $\frac{\bar{\beta}_2}{2}$. But then buying an amount $\frac{\bar{\beta}_1}{2}$ cannot be a best response for types $\alpha \in (\bar{\beta}_1, h(\bar{\beta}_1)) \cap (\bar{\beta}_2, h(\bar{\beta}_2))$, implying that the equilibrium $\bar{\beta}_1$ is not sequentially perfect. Because $g(\xi) = 1 - K < 1$ and $g' > 0$, we know that $\xi < 1$. Lemma 5.6 informs us that consequently $(\xi, h(\xi)) \neq \emptyset$. For every $\bar{\beta} \in [0, \xi)$ we can therefore find a $\bar{\beta}_2 > \bar{\beta}$ such that the above reasoning holds for $\bar{\beta}$ and $\bar{\beta}_2$. We conclude that equilibrium $\bar{\beta}$ is not sequentially perfect if $\bar{\beta} < \xi$.

It remains to show that equilibrium ξ is a perfect sequential equilibrium. Consider a period one deviation $z \in (\frac{\xi}{2}, \frac{h(\xi)}{2}]$. Suppose types in $(\underline{\gamma}, \overline{\gamma}) \neq \emptyset$ strictly benefit from deviating to z if the firm uses a credible updating rule and a sequentially perfect metastrategy. Deviating to z is only beneficial for types $\alpha \neq 2z$ if the firm subsequently switches. This only happens if

$$E(\alpha | \alpha \in (\underline{\gamma}, \overline{\gamma})) \leq 1 - K. \tag{5.14}$$

But we know that if the firm switches after selling a quantity z, then each type $\alpha \in (2z, h(2z)]$ wants to deviate to z, so $\underline{\gamma} = 2z$ and $\overline{\gamma} = h(2z)$. Therefore, using the fact that g is an increasing function:

$$E(\alpha | \alpha \in (\underline{\gamma}, \overline{\gamma})) = g(2z) > g(\xi) = 1 - K,$$

contradicting (5.14). This contradiction implies that Φ_ξ is credible.[10] The sequential perfectness of the (meta)strategies supporting the equilibrium ξ now follows immediately. ■

With the aid of Proposition 5.3 we argue that it makes sense to confine attention to equilibrium ξ. In the next section we interpret various features of this equilibrium and compare it to the benchmark situation in which the firm is fully informed regarding the consumer's environmental stance.

5.4 Interpretation and Discussion

Although a setting with just one consumer is obviously rather restricted, the analysis of the previous section does convey some interesting insights. We therefore discuss the properties of the unique PSE derived in the previous section (equilibrium ξ) and compare it with the equilibrium which prevails should the firm know the consumer's level of environmental concern before we move on to a setting with multiple consumers.

In equilibrium ξ some consumer types mimic a relatively environmentally concerned type to induce the firm to switch to the clean technology. Put differently,

[10] This belief is defined in the proof of Proposition 5.2.

these consumer types distort their purchases downwards to signal a relatively high level of environmental concern. According to our definition this as a consumer boycott. In addition to this strategic reason to buy less, each consumer type (except for type 1) reduces his sales compared to the amount he would buy if the good were clean. The reduction in sales stemming from the latter motive is a simple adjustment of choices to new information regarding the product. Since this adjustment has no strategic rationale, we do not include it in the consumer boycott. Using this definition of consumer boycotts, one sees that in our framework only intermediate types boycott (see Figure 1). We stress that this result is not at odds with the idea that a consumer buys less the more concerned he is: the distance between the quantity purchased if the product were environmentally neutral ($\frac{1}{2}$) and the actual quantity purchased ($x_1^*(\alpha)$) does increase monotonically in the level of concern α^{-1}.

The expected size of a consumer boycott (or, more briefly, the *boycott size*), denoted $B(K)$, equals the difference between the expected sales in the full information benchmark and the expected sales in equilibrium ξ:

$$B(K) = \int_{\xi}^{h(\xi)} \left(\frac{\alpha}{2} - \frac{\xi}{2}\right) \mathrm{d}F(\alpha) = \frac{1}{4}\big(h(\xi) - \xi\big)^2. \tag{5.15}$$

The boycott size of equilibrium $\bar{\beta}$ is the area of the striped triangle of Figure 5.1. Note that the equilibrium refinement we have employed selects the equilibrium with the smallest boycott: the length of the *boycott interval* $(\xi, h(\xi)]$ (and thus the area of the associated triangle) is smaller than the length of the boycott interval $(\bar{\beta}, h(\bar{\beta})]$ as long as $\bar{\beta} < \xi$ (this follows from Lemma 5.6.) The boycott of equilibrium ξ has the following properties:

Lemma 5.7 *Let $K < \frac{3}{4}$. The unique consumer type $\xi = \xi(K)$ who is mimicked in the PSE is located such that $\xi < 1 - K < h(\xi)$. This type decreases in K. Consequently, the length of the boycott interval $(\xi, h(\xi)]$ and therefore the size of the boycott $B(K)$ increases in K.*

Proof. The fact that $\xi < 1 - K < h(\xi)$ is obvious. The condition $g(\xi) = 1 - K$ combined with the fact that $g'(\cdot) > 0$ imply that

$$\frac{\mathrm{d}\xi}{\mathrm{d}K} = -\frac{1}{g'(\xi)} < 0.$$

The last claims follow from Lemma 5.6 and the expression (5.15) for $B(K)$. ■

Obviously, the firm's expectation of α after selling an amount $\frac{\xi}{2}$, knowing that a whole range of consumer types opt for this quantity, can only equal $1 - K$

if $1 - K$ itself is in this range. As the cost of switching K increases, the firm becomes less inclined to switch to the clean technology. Loosely speaking, a lower expectation regarding α is required to make the switch worthwhile. The consumer can only achieve this by mimicking a lower type than 'before' the increase in K. In other words, ξ decreases in K. As a consequence, more types have to distort their purchases downwards and the size of the boycott thus increases.

We use the above comparative statics results regarding ξ to assess the impact of the fact that the firm does not know the level of concern of the consumer:

Proposition 5.4 *Let $K < \frac{3}{4}$. Compared to the full information benchmark, the firm switches more often to the clean technology, expected consumer surplus is higher, and expected profits are lower.*

Proof. The first claim follows from Lemma 5.3 and the fact that $\xi < 1 - K < h(\xi)$. The other claims are proved in the appendix. ■

Proposition 5.4 reveals that consumers can exploit the fact that the firm is not fully informed: consumer surplus is higher in equilibrium ξ than in the equilibrium of the complete information benchmark. Yet, since boycotting is costly, the informational rents the consumers can appropriate are very small. The firm is substantially worse off in equilibrium ξ: it switches more often and, because some consumer types boycott, expected sales in period one are smaller. Because the firm switches more often, the environment benefits from the fact that the firm is not fully informed.

5.5 Multiple Consumers

In this section we extend the basic model we considered in the previous few sections to multiple consumers. There are $n > 1$ consumers. Each consumer $i \in \{1, 2, \ldots, n\} =: N$ has level of environmental concern α. The utility function of a consumer is again (5.2) if the old production technology is used and (5.1) if the new technology is used. The quantity purchased by consumer i in period t is denoted x_{it}, $i \in N$, $t = 1, 2$. We use X_t to indicate aggregate sales in period t.

We maintain the timing of the basic model. So, after the first market stage, but before the second market stage, the firm can switch to the clean technology. Switching costs $\tilde{K} = \frac{1}{4}K > 0$. The firm charges a price of $\frac{1}{2}$, the profit-maximizing price should the product be perceived as environmentally neutral, in the first period. Again, the firm is unaware of the amount of concern felt by consumers, but it does know that α is drawn from the standard uniform distribution. This implies that it has to use its belief regarding α instead of the true value of α when deciding whether or not to adopt the new technology. We assume that the firm only observes aggregate sales. The consumers cannot communicate with each other and a consumer thus needs to form beliefs regarding the actions of fellow concerned consumers. The probability that the firm switches to the new technology conditional on a single consumer i purchasing some quantity x in the first period, denoted $q_i(x)$, therefore plays an important role in the analysis.

We look for symmetric perfect Bayesian Nash equilibria (SPBNEs), focusing on equilibria that meet the requirements of perfect sequential equilibria. Symmetry means that all consumers employ the same strategy in equilibrium. Importantly, a symmetric equilibrium presupposes less coordination among consumers than asymmetric equilibria do.[11]

We determine these SPBNEs by using backward induction. Optimality of $p_2^*(\delta, X_1) \equiv \frac{1}{2}$ is obvious. Clearly, the only second period purchasing rule that supports an SPBNE coincides with the consumer behaviour described in Lemma 5.1, i.e. $x_2^*(\alpha) = \delta\frac{1}{2} + (1-\delta)\frac{\alpha}{2}$. One can also translate Lemma 5.2 to the current setting:

Lemma 5.8 *In any SPBNE each consumer's first period purchasing rule $x_1^*(\cdot)$ is nondecreasing in type.*

Proof. Assume for contradiction that $x_1^*(\alpha_1) > x_1^*(\alpha_2)$, for some types α_1, α_2 such that $\alpha_1 < \alpha_2$. Since a type α never buys more than $\frac{\alpha}{2}$, we know that $\alpha_1 > 0$ and that $x_1^*(\alpha_2) < x_1^*(\alpha_1) \leq \frac{\alpha_1}{2} < \frac{\alpha_2}{2}$. Note that consumer i with type α only opts to buy some quantity x smaller than $\frac{\alpha}{2}$ if $q_i(x) > q_i(\frac{\alpha}{2})$, i.e. buying x instead of $\frac{\alpha}{2}$ strictly increases the probability that the firm switches. Two cases need to be examined: $q_i(x_1^*(\alpha_1)) \geq q_i(x_2^*(\alpha_2))$ and $q_i(x_1^*(\alpha_1)) < q_i(x_2^*(\alpha_2))$. If $q_i(x_1^*(\alpha_1)) \geq q_i(x_2^*(\alpha_2))$, then buying $x_1^*(\alpha_2) < x_1^*(\alpha_1) < \frac{\alpha_2}{2}$ is clearly suboptimal for consumer type α_2. Now suppose that $q_i(x_1^*(\alpha_1)) < q_i(x_2^*(\alpha_2))$. Then optimality of $x_1^*(\cdot)$ requires for type α_1

$$\begin{aligned} &u_{\alpha_1}(x_1^*(\alpha_1), \tfrac{1}{2}) + q_i(x_1^*(\alpha_1))u(\tfrac{1}{2}, \tfrac{1}{2}) + \big(1 - q_i(x_1^*(\alpha_1))\big)u_{\alpha_1}(\tfrac{\alpha_1}{2}, \tfrac{1}{2}) \geq \\ &u_{\alpha_1}(x_1^*(\alpha_2), \tfrac{1}{2}) + q_i(x_1^*(\alpha_2))u(\tfrac{1}{2}, \tfrac{1}{2}) + \big(1 - q_i(x_1^*(\alpha_2))\big)u_{\alpha_1}(\tfrac{\alpha_1}{2}, \tfrac{1}{2}) \end{aligned}$$

and for type α_2

$$\begin{aligned} &u_{\alpha_2}(x_1^*(\alpha_2), \tfrac{1}{2}) + q_i(x_1^*(\alpha_2))u(\tfrac{1}{2}, \tfrac{1}{2}) + \big(1 - q_i(x_1^*(\alpha_2))\big)u_{\alpha_2}(\tfrac{\alpha_2}{2}, \tfrac{1}{2}) \geq \\ &u_{\alpha_2}(x_1^*(\alpha_1), \tfrac{1}{2}) + q_i(x_1^*(\alpha_1))u(\tfrac{1}{2}, \tfrac{1}{2}) + \big(1 - q_i(x_1^*(\alpha_1))\big)u_{\alpha_2}(\tfrac{\alpha_1}{2}, \tfrac{1}{2}). \end{aligned}$$

[11] If there is a unique symmetric equilibrium, then this equilibrium can be viewed as *focal*.

Rewriting these conditions yields

$$\frac{x_1^*(\alpha_1) - x_1^*(\alpha_2)}{2}\Big(1 - \frac{x_1^*(\alpha_1) + x_1^*(\alpha_2)}{\alpha_1}\Big) + \frac{1-\alpha_1}{8}\big(q_i(x_1^*(\alpha_1)) - q_i(x_1^*(\alpha_2))\big) \geq 0,$$

respectively

$$\frac{x_1^*(\alpha_2) - x_1^*(\alpha_1)}{2}\Big(1 - \frac{x_1^*(\alpha_1) + x_1^*(\alpha_2)}{\alpha_2}\Big) + \frac{1-\alpha_2}{8}\big(q_i(x_1^*(\alpha_2)) - q_i(x_1^*(\alpha_1))\big) \geq 0.$$

Adding the left-hand sides of the last two inequalities results in

$$\Big(\frac{1}{2\alpha_2} - \frac{1}{2\alpha_1}\Big)\big((x_1^*(\alpha_1))^2 - (x_1^*(\alpha_2))^2\big) + \frac{\alpha_2 - \alpha_1}{8}\big(q_i(x_1^*(\alpha_1)) - q_i(x_1^*(\alpha_2))\big),$$

an expression which is negative as long as $q_i(x_1^*(\alpha_1)) < q_i(x_2^*(\alpha_2))$, contradicting the optimality of $x_1^*(\cdot)$. ■

The firm only observes the aggregate sales X_1. Lemma 5.8 informs us that this number is (weakly) increasing in α. This observation leads to equilibrium switching rules akin to those we encounter in the single consumer scenario:

Lemma 5.9 *Suppose $K < n$. In any SPBNE the firm switches to the clean technology if and only if $X_1 \leq \bar{X}_1$ for some threshold $\bar{X}_1 \in (0, \frac{n}{2})$.*

Proof. The firm switches if and only if $E(n\alpha|X_1) \leq n - K$, i.e. $\bar{X}_1$ must solve $E(n\alpha|X_1) = n - K$. Lemma 5.8 implies that $E(\alpha|X_1)$ is strictly increasing in X_1. Consequently, the equality $nE(\alpha|X_1) = n - K$ has at most one solution $\bar{X}_1$. The arguments used in the proof of Lemma 5.4 show that if no solution $\bar{X}_1$ would exist, then at least one consumer would have an incentive to deviate to a higher quantity. ■

With the aid of Lemma 5.8 and Lemma 5.9 one can prove the multiple consumers-analogues of the main results of Section 5.3. Firstly, just like in the single consumer scenario, boycotting occurs with strictly positive probability in any SPBNE of the game. This observation is formalized as follows:

Proposition 5.5 *The game has no separating SPBNEs if $K < n$.*

Proof. If the game would have a separating SPBNE, then this equilibrium would be supported by the first period purchasing rule $\hat{x}_1(\alpha) = \frac{\alpha}{2}$ and the firm would switch if and only if $X_1 \leq \frac{n-K}{2}$, i.e. if and only if $\alpha \leq 1 - \frac{K}{n}$. Using arguments akin to those used in the proof of Proposition 5.1 one can find an $\epsilon > 0$ small such that a consumer type $1 - \frac{K}{n} + \epsilon$ has an incentive to deviate to buying $\frac{1}{2} - \frac{K}{2n} - \frac{(n-1)\epsilon}{2}$ in period one. (If a single consumer deviates to this quantity, then the firm switches.)

This deviation destroys the separating SPBNE. ■

It is not difficult to see that at most one consumer type is mimicked in an SPBNE. Proposition 5.5 informs us that each equilibrium exhibits mimicking, implying that precisely one type is mimicked in any SPBNE as long $K < n$. So, some consumer types engage in boycotting by mimicking some relatively low type in each SPBNE of the game. If in such an equilibrium consumer type $\bar{\beta}$ is mimicked, then the firm must opt to switch to the clean technology after selling $n\frac{\bar{\beta}}{2}$ or less, for else mimicking type $\bar{\beta}$ cannot be part of a consumer's equilibrium strategy if each consumer employs the same strategy. Clearly, all consumer types up to and including type $h(\bar{\beta})$ prefer to mimic type $\bar{\beta}$ should the firm switch if and only if it sells $n\frac{\bar{\beta}}{2}$ or less in period one. We summarize these findings in the following:

Proposition 5.6 *Suppose $K < \frac{3}{4}n$. Then for each $\bar{\beta} \in [0, g^{-1}(1 - \frac{K}{n})]$ there exists an SPBNE exhibiting mimicking which is supported by the following strategies:*

- *Strategy of each consumer:*

  1. *First period purchasing rule:* $x_1^*(\alpha) = \begin{cases} \frac{\alpha}{2} & \text{if } \alpha < \bar{\beta} \\ \frac{\bar{\beta}}{2} & \text{if } \bar{\beta} \le \alpha \le h(\bar{\beta}) \\ \frac{\alpha}{2} & \text{if } \alpha > h(\bar{\beta}); \end{cases}$
  2. *Second period purchasing rule:* $x_2^*(\alpha) = \delta\frac{1}{2} + (1 - \delta)\frac{\alpha}{2}$.

- *Strategy of the firm:*

  1. *Switching rule:* $\delta^*(X_1) = \begin{cases} 1 & \text{if } X_1 \le n\frac{\bar{\beta}}{2} \\ 0 & \text{if } X_1 > n\frac{\bar{\beta}}{2}; \end{cases}$
  2. *Second period pricing rule:* $p_2^* = \frac{1}{2}$.

Proof. Most claims are established in Lemma 5.6 and Proposition 5.2. It remains to show that $x_1^*(\cdot)$ is a best response to $n-1$ consumers using $x_1^*(\cdot)$ and the firm using $\delta^*(\cdot)$. Note that the SPBNE in which type $\bar{\beta}$ is mimicked is supported by beliefs of the firm which are such that it does not switch after selling an out-of-equilibrium quantity $X_1 \in (n\frac{\bar{\beta}}{2}, n\frac{h(\bar{\beta})}{2}]$. Buying $\frac{\alpha}{2}$ if $\alpha < \bar{\beta}$ is clearly a best response to the strategies of the others. If $\alpha \in [\bar{\beta}, h(\bar{\beta})]$, then buying more than $\frac{\bar{\beta}}{2}$ would induce the firm to stick to the polluting technology. By construction of $h(\cdot)$ buying more than $\frac{\bar{\beta}}{2}$ therefore decreases the deviating consumer's utility. Lastly, reducing the period one purchases below $\frac{\bar{\beta}}{2}$ to induce the firm to switch cannot be optimal for a consumer with type $\alpha > h(\bar{\beta})$, again by construction of $h(\cdot)$. ■

Surprisingly, the possibility to free-ride does not destroy equilibria in which some consumer types boycott. The reason is that the firm's decision to switch depends nontrivially on an individual consumer's decision whether or not to join the consumer boycott: each consumer's contribution to the boycott is pivotal. Proposition 5.6 is reminiscent of a result from the study of private provision of (discrete) public goods. Participating in a boycott can be viewed as contributing to a public good. Only if each consumer makes this contribution is the public good provided. Palfrey and Rosenthal (1984) show that in such settings each player does contribute to the public good as long as the cost to a player of contributing to the public good is smaller than the individual benefit of the public good. In our game both the cost and the benefit associated with the public good, i.e. associated with the switch to the clean technology, depend on the realization of α. A second departure from Palfrey and Rosenthal (1984)'s setting lies in the fact that in our game both the cost of participating in the boycott and whether boycotting is necessary is endogenous: it depends on the beliefs the firm entertains in equilibrium. Consequently, whether or not contributing to the public good in our game is necessary or optimal is determined endogenously.

We call the SPBNE in which consumer type $\bar{\beta}$ is mimicked equilibrium $\bar{\beta}$. Since the equilibrium with the highest possible $\bar{\beta}$ weakly payoff dominates the other SPBNEs for each consumer type, one can immediately conclude that:

Corollary 5.1 *Suppose $K < \frac{3}{4}n$. Then the equilibrium $\xi_n := g^{-1}(1 - \frac{K}{n})$ is the unique symmetric perfect sequential equilibrium.*

Selection of a unique equilibrium allows us to add to the comparative statics exercises of Section 5.4 the comparative statics with respect to the number of consumers. The following results can be established:[12]

Proposition 5.7 *Suppose $K < \frac{3}{4}n$. Then the consumer type ξ_n who is mimicked in the symmetric PSE increases in the number of consumers n. There exists a $\bar{K} \in (0, \frac{3}{4}n)$ such that the size of the boycott*

$$B_n(K) = \frac{n}{4}\big(h(\xi_n) - \xi_n\big)^2 \tag{5.16}$$

decreases monotonically as long as $K < \bar{K}$. If $K > \bar{K}$, then the size of the boycott increases in n for small values of n, but it decreases in n for large values of n. Lastly, the size of the boycott becomes vanishingly small as $n \to \infty$ for each K.

Proof. See the appendix. ∎

[12] The claims of Proposition 5.4 obviously continue to hold for $n > 1$ consumers.

The economic rationale behind the first result is not difficult to see. As the number of consumers increases, the cost of switching per consumer ($\frac{K}{n}$) decreases. Consequently, less consumer types need to rely on boycotting to induce the switch to the clean technology and thus ξ_n increases in n. The size of the boycott is codetermined by the number of consumers and the length of the boycott interval $(\xi_n, h(\xi_n)]$. An increase in n leads to an increase in ξ_n and therefore a decrease in the length of the boycott interval. This decrease reduces the boycott size. However, at the same time the increase in the number of consumers increases the number of boycott intervals that have to be added to arrive at $B_n(K)$. If n is relatively large, then ξ_n is also relatively large. If ξ_n is large, then the position of the indifferent consumer $h(\xi_n)$ does not change much as ξ_n changes, implying that the length of the interval $(\xi_n, h(\xi_n)]$ diminishes rapidly as more consumers are added to an already large population of consumers. Adding one boycott interval to the boycott size each time a consumer joins the population does not offset the negative impact on boycott size of the reduction in boycott interval length if n is sufficiently large. In fact, if $K < \bar{K}$, then ξ_2 is already so large that the 'length effect' dominates the 'number effect' as n increases from 2 to 3. Because the cost of switching K does not increase in the number of consumers n whereas the gain in second period profits associated with switching to the clean technology does increase in n, the firm always switches if the population of consumers is sufficiently large and the reason to boycott thus disappears as $n \to \infty$.

5.6 Concluding Remarks

Why do individual consumers boycott despite the presence of free-riding and coordination problems? Thess problems have been sidestepped by previous work on consumer boycotts, mainly by assuming that an interest group is able to negotiate with firms and orchestrate consumer boycotts. In this chapter we addressed these collective action issues (Olson, 1965), by considering a setting in which consumers are not guided by some interest group. Consumers thus have to overcome free-riding and coordination problems themselves.

In our model, the firm is uncertain about how concerned consumers are about the firm's (environmental) misconduct. If consumers are very concerned it is optimal for the firm to alter its behaviour and switch to a clean, but more costly production technology. This gives consumers an incentive to signal a high level of concern by reducing their purchases, i.e. boycott the firm. We showed that this incentive is sufficiently strong to overcome any free-riding issues.

In the case with one consumer every equilibrium has some consumer types boycotting. This result generalizes to a model with an arbitrary number of homogeneous consumers. Compared to the benchmark in which the firm does know consumers' level of concern, the firm has substantially lower profits: not only because it is boycotted, but also because it switches more often to the expensive, clean technology. Strikingly, uncertainty benefits the environment as the clean technology is now adopted more often.

This chapter provides a framework in which consumers' incentives to signal a high level of (environmental) concern regarding a firm's misconduct can lead to consumer boycotts. Admittedly, we have only looked at a rudimentary form of consumer concern. The fact that consumers are homogeneous with respect to their concerns constitutes an important limitation of the model. Investigation of more elaborate forms of consumer concerns, most notably heterogeneous concerns, is bound to increase our understanding of consumer boycotts.

5.A Appendix

Omitted details regarding Lemma 5.6

Let $\ell(z) : h(z) - z$. It suffices to show that $\ell(0) = \frac{1}{2}$, $\ell(1) = 0$, and that $\ell' \leq 0$. The first two statements are easily verified. Differentiating ℓ results in

$$\ell'(z) = -\frac{1}{2} + \frac{-z + \frac{1}{2}}{2\sqrt{(z + \frac{1}{2})^2 - 2z^2}}.$$

Solving $\ell'(z) = 0$ yields the unique root $z = 0$. The fact that $\ell'(1) = -1 < 0$ completes the proof.

Omitted details regarding the function $h(\cdot)$

Using the fact that $h(z) = \ell(z) + z$ and the derivations regarding ℓ one sees that $h(0) = \frac{1}{2}$ and $h(1) = 1$. Because $h'(z) = \ell'(z) + 1$, the analysis of ℓ also implies that $h'(0) = 1$, $h'(1) = 0$, and that $h' \geq 0$.

Omitted details regarding the function $g(\cdot)$

Straightforward calculations reveal that $g(0) = \frac{1}{4}$ and $g(1) = 1$. The derivative of g is:

$$g'(z) = \frac{3}{4} + \frac{-z + \frac{1}{2}}{4\sqrt{(z + \frac{1}{2})^2 - 2z^2}}.$$

Since the only root of $g'(z) = 0$, $\frac{1}{2} + \frac{3}{10}\sqrt{5}$, lies outside the unit interval, g is monotonic on $[0, 1]$. Because $g'(0) = 1$, $g'(z)$ must be positive for $z \in [0, 1]$.

Omitted details regarding Proposition 5.3

We have to show that equilibrium ξ weakly payoff dominates the equilibria $\bar{\beta}$ for the consumer, with strict dominance for some types, $\bar{\beta} < \xi$. Consider the equilibria $\bar{\beta}_1$ and $\bar{\beta}_2$, $0 \leq \bar{\beta}_1 < \bar{\beta}_2 \leq \xi$. Denote the equilibrium payoff of consumer α in equilibrium $\bar{\beta}$ by $V(\alpha, \bar{\beta})$. Note that the payoff of type 0 is always 0. We therefore restrict attention to types $\alpha > 0$. We have to consider two cases: $\bar{\beta}_1 < \bar{\beta}_2 \leq h(\bar{\beta}_1) < h(\bar{\beta}_2)$, and $\bar{\beta}_1 < h(\bar{\beta}_1) < \bar{\beta}_2 < h(\bar{\beta}_2)$.

Suppose first that $\bar{\beta}_1 < \bar{\beta}_2 \leq h(\bar{\beta}_1) < h(\bar{\beta}_2)$. Straightforward calculations reveal that:

- $V(\alpha, \bar{\beta}_1) = V(\alpha, \bar{\beta}_2) = \frac{1+\alpha}{8}$ if $\alpha < \bar{\beta}_1$;
- $V(\alpha, \bar{\beta}_1) = \frac{\bar{\beta}_1}{4} - \frac{\bar{\beta}_1^2}{8\alpha} + \frac{1}{8}$, $V(\alpha, \bar{\beta}_2) = \frac{1+\alpha}{8}$ if $\alpha \in [\bar{\beta}_1, \bar{\beta}_2)$;
- $V(\alpha, \bar{\beta}_1) = \frac{\bar{\beta}_1}{4} - \frac{\bar{\beta}_1^2}{8\alpha} + \frac{1}{8}$, $V(\alpha, \bar{\beta}_2) = \frac{\bar{\beta}_2}{4} - \frac{\bar{\beta}_2^2}{8\alpha} + \frac{1}{8}$ if $\alpha \in [\bar{\beta}_2, h(\bar{\beta}_1)]$;

- $V(\alpha, \bar{\beta}_1) = \frac{\alpha}{4}$, $V(\alpha, \bar{\beta}_2) = \frac{\bar{\beta}_2}{4} - \frac{\bar{\beta}_2^2}{8\alpha} + \frac{1}{8}$ if $\alpha \in (h(\bar{\beta}_1), h(\bar{\beta}_2)]$;
- $V(\alpha, \bar{\beta}_1) = V(\alpha, \bar{\beta}_2) = \frac{\alpha}{4}$ if $\alpha > h(\bar{\beta}_2)$.

The claim thus follows in the first case from the following results:

- $V(\alpha, \bar{\beta}_1) \leq V(\alpha, \bar{\beta}_2)$, $\alpha \in [\bar{\beta}_1, \bar{\beta}_2)$ with strict inequality for $\alpha \in (\bar{\beta}_1, \bar{\beta}_2)$.
  This claim is equivalent to $-\alpha^2 + 2\bar{\beta}_1\alpha - \bar{\beta}_1^2 \leq 0$, $\alpha \in [\bar{\beta}_1, \bar{\beta}_2)$. The LHS of this expression attains its maximum of 0 in $\alpha = \bar{\beta}_1$, yielding the above inequality.
- $V(\alpha, \bar{\beta}_1) < V(\alpha, \bar{\beta}_2)$, $\alpha \in [\bar{\beta}_2, h(\bar{\beta}_1)]$.
  This follows from the fact that the function $x \mapsto \frac{x}{4} - \frac{x^2}{8\alpha}$ is strictly increasing for $x < \alpha$, implying that $\frac{\bar{\beta}_1}{4} - \frac{\bar{\beta}_1^2}{8\alpha} + \frac{1}{8} < \frac{\bar{\beta}_2}{4} - \frac{\bar{\beta}_2^2}{8\alpha} + \frac{1}{8}$.
- $V(\alpha, \bar{\beta}_1) \leq V(\alpha, \bar{\beta}_2)$, $\alpha \in (h(\bar{\beta}_1), h(\bar{\beta}_2)]$.
  This claim is equivalent to $\alpha \leq \bar{\beta}_2 - \frac{\bar{\beta}^2}{2\alpha} + \frac{1}{2}$, $\alpha \in (h(\bar{\beta}_1), h(\bar{\beta}_2)]$. By construction of h, we know that this inequality holds for $\alpha \in [\bar{\beta}_2, h(\bar{\beta}_2)]$.

We now look at the case $\bar{\beta}_1 < h(\bar{\beta}_1) < \bar{\beta}_2 < h(\bar{\beta}_2)$. Again, straightforward calculations show that:

- $V(\alpha, \bar{\beta}_1) = V(\alpha, \bar{\beta}_2) = \frac{1+\alpha}{8}$ if $\alpha < \bar{\beta}_1$;
- $V(\alpha, \bar{\beta}_1) = \frac{\bar{\beta}_1}{4} - \frac{\bar{\beta}_1^2}{8\alpha} + \frac{1}{8}$, $V(\alpha, \bar{\beta}_2) = \frac{1+\alpha}{8}$ if $\alpha \in [\bar{\beta}_1, h(\bar{\beta}_1)]$;
- $V(\alpha, \bar{\beta}_1) = \frac{\alpha}{4}$, $V(\alpha, \bar{\beta}_2) = \frac{1+\alpha}{8}$ if $\alpha \in (h(\bar{\beta}_1), \bar{\beta}_2)$;
- $V(\alpha, \bar{\beta}_1) = \frac{\alpha}{4}$, $V(\alpha, \bar{\beta}_2) = \frac{\bar{\beta}_2}{4} - \frac{\bar{\beta}_2^2}{8\alpha} + \frac{1}{8}$ if $\alpha \in [\bar{\beta}_2, h(\bar{\beta}_2)]$;
- $V(\alpha, \bar{\beta}_1) = V(\alpha, \bar{\beta}_2) = \frac{\alpha}{4}$ if $\alpha > h(\bar{\beta}_2)$.

The claim follows in this case using the same arguments as those used for the first case.

Because $\bar{\beta}_1$ and $\bar{\beta}_2$, $0 \leq \bar{\beta}_1 < \bar{\beta}_2 \leq \xi$, are chosen arbitrarily, we conclude that the equilibrium ξ payoff dominates the other pooling equilibria for the consumer. ∎

Proof of Proposition 5.4

We start with consumer surplus. Note that we only have to consider types $\alpha \in (\xi, h(\xi)]$. The fact that $\frac{\xi + h(\xi)}{2} = 1 - K$ implies that $h(\xi) = 2(1 - K - \xi) + \xi$. Observe that consumer types $\alpha \in (\xi, 1 - K]$ are worse off in the boycott equilibrium, whereas types $\alpha \in (1 - K, h(\xi)]$ prefer the boycott equilibrium. Because $1 - K - \xi = 2(1 - K - \xi) + \xi - (1 - K)$, the lengths of these two intervals are the same. We call the types who prefer the boycott equilibrium the *gainers* and those who prefer the benchmark equilibrium the *losers*. Note that, since the uniform distribution has no atoms, we can safely include the boundaries in the above intervals ($[\xi, 1-K]$ for the losers, $[1-K, h(\xi)]$ for the gainers).

Let us look in more detail at the difference in utility of the losers. The interval $[\xi, 1-K]$ coincides with the parametrized set $\{1-K-\gamma : 0 \leq \gamma \leq 1-K-\xi\}$. Then the difference in utility between the two scenarios (utility in equilibrium ξ minus utility in the equilibrium of the benchmark) for loser γ reads:

$$\begin{aligned} L(\gamma) =& \Big(\frac{\xi}{4} - \frac{\xi^2}{8(1-K-\gamma)} + \frac{1}{8}\Big) - \Big(\frac{1-K-\gamma}{8} + \frac{1}{8}\Big) \\ =& \frac{\xi}{4} - \frac{\xi^2}{8(1-K-\gamma)} - \frac{1-K-\gamma}{8} \leq 0. \end{aligned}$$

It is not difficult to see that

$$\frac{\xi}{4} - \frac{\xi^2}{8(1-K)} - \frac{1-K}{8} = L(0) \leq L(\gamma) \leq L(1-K-\xi) = 0.$$

We can also reparametrize the interval $[1-K, h(\xi)]$ of gainers: $[1-K, h(\xi)] = \{1-K+\gamma : 0 \leq \gamma \leq 1-K-\xi\}$. The difference in utility for gainer γ is:

$$G(\gamma) = \Big(\frac{\xi}{4} - \frac{\xi^2}{8(1-K+\gamma)} + \frac{1}{8}\Big) - \Big(\frac{1-K+\gamma}{8} + \frac{1-K+\gamma}{8}\Big).$$

The following bounds on this difference can be obtained:

$$0 = G(1-K-\xi) \leq G(\gamma) \leq G(0) = \frac{\xi}{4} - \frac{\xi^2}{8(1-K)} + \frac{1}{8} - \frac{1-K}{4}.$$

We show that the maximal gain ($G(0)$) exceeds the maximal loss ($L(0)$). Note that:

$$\begin{aligned} & G(0) + L(0) = \frac{\xi}{2} - \frac{\xi^2}{4(1-K)} + \frac{1}{8} - \frac{1-K}{8} - \frac{1-K}{4} = \\ & \frac{\xi}{4}\Big(1 - \frac{\xi}{1-K}\Big) - \frac{1-K}{4}\Big(1 - \frac{\xi}{1-K}\Big) + \frac{K}{8} = -\frac{(1-K-\xi)^2}{4(1-K)} + \frac{K}{8}. \end{aligned}$$

We thus have to show that $\frac{K}{2} \geq \frac{(1-K-\xi)^2}{1-K}$. This inequality holds as long as

$$1-K-\sqrt{(1-K)\frac{K}{2}} \leq \xi \leq 1-K+\sqrt{(1-K)\frac{K}{2}}.$$

The second inequality obviously holds. It remains to show that $\xi \geq 1-K-\sqrt{(1-K)\frac{K}{2}}$. We first determine ξ as a function of K. By definition of $g(\cdot)$ one has:

$$\frac{3\xi + \frac{1}{2} + \sqrt{(\xi+\frac{1}{2})^2 - 2\xi^2}}{4} = 1-K. \tag{5.17}$$

Rewriting this expression and taking squares at both sides yields:

$$-\xi^2 + \xi + \tfrac{1}{4} = 9\xi^2 - 21\xi + 24K\xi + \tfrac{49}{4} - 28K + 16K^2.$$

This quadratic expression in ξ has two real roots:

$$\xi_\pm = \frac{11 - 12K \pm \sqrt{1 + 16K - 16K^2}}{10}.$$

One can check that ξ_+ does not solve (5.17), so:

$$\xi = \frac{11 - 12K - \sqrt{1 + 16K - 16K^2}}{10}. \tag{5.18}$$

We thus have to prove that

$$\frac{1 - 2K - \sqrt{1 + 16K - 16K^2} + 5\sqrt{2K(1-K)}}{10} \geq 0. \tag{5.19}$$

We look at the cases $K \leq \frac{1}{2}$ and $K \in (\frac{1}{2}, \frac{3}{4})$ separately:

- $K \leq \frac{1}{2}$. The inequality (5.19) is equivalent to $(1 - 2K) + 5\sqrt{2K(1-K)} \geq \sqrt{1 + 16K - 16K^2}$. Since both sides of this inequality are positive, we can square both sides of this inequality without having to worry about signs and arrive after some rewriting at the equivalent claim

  $$(1 - 2K)\sqrt{2K(1-K)} \geq 3K^2 - 3K.$$

  Because $3K^2 - 3K < 0$ whereas $(1 - 2K)\sqrt{2K(1-K)} \geq 0$, we conclude that (5.19) holds if $K \leq \frac{1}{2}$.

- $K \in (\frac{1}{2}, \frac{3}{4})$. We now square both sides of $5\sqrt{2K(1-K)} \geq (2K - 1) + \sqrt{1 + 16K - 16K^2}$ without having to worry about signs. This results after some simplifications in

  $$-19K^2 + 19K^2 - 1 \geq (2K - 1)\sqrt{1 + 16K - 16K^2}.$$

  One easily sees that both sides of this inequality are positive. Again squaring both sides and rearranging yields

  $$17K^3 - 34K^2 + 19K - 2 \geq 0.$$

  We thus have to show that $j(z) := 17z^3 - 34z^2 + 19z - 2$ is nonnegative as long as $z \in (\frac{1}{2}, \frac{3}{4})$. Standard calculations reveal that the derivative $j'(z) = 51z^2 - 68z + 19$ equals zero if $z = \frac{1}{3}$ or $z = 1$, so $\inf_{z \in (\frac{1}{2}, \frac{3}{4})} j(z)$ is attained in either $z = \frac{1}{2}$ or $z = \frac{3}{4}$. Because both $j(\frac{1}{2}) = \frac{9}{8}$ and $j(\frac{3}{4}) = \frac{19}{64}$ are positive, we conclude that (5.19) holds for $K \in (\frac{1}{2}, \frac{3}{4})$.

We now know that $G(0) + L(0) \geq 0$.

Because $L(1-K-\xi) = G(1-K-\xi) = 0$, $G(0) + L(0) \geq 0$, and both $L(\cdot)$ and $G(\cdot)$ are continuous, it suffices to show that $G(\gamma) + L(\gamma)$ decreases monotonically. We calculate the derivative of this function:

$$G'(\gamma) + L'(\gamma) = -\frac{\xi^2}{8(1-K-\gamma)^2} + \frac{\xi^2}{8(1-K+\gamma)^2} + \frac{1}{8} - \frac{1}{4} < 0.$$

This proves that expected consumer surplus is higher in equilibrium ξ than in the benchmark equilibrium.

We now look at profits. The expected first period profit in equilibrium ξ equals

$$\int_0^1 \frac{1}{2} x_1^*(\alpha)\,\mathrm{d}\alpha = \int_0^{\xi} \frac{\alpha}{4}\,\mathrm{d}\alpha + \int_{\xi}^{h(\xi)} \frac{\xi}{4}\,\mathrm{d}\alpha + \int_{h(\xi)}^{1} \frac{\alpha}{4}\,\mathrm{d}\alpha = \frac{1}{8} - \frac{1}{2}\Big(\frac{h(\xi)-\xi}{2}\Big)^2.$$

The expected second period profit in this equilibrium is:

$$\int_0^{h(\xi)} \frac{1-K}{4}\,\mathrm{d}\alpha + \int_{h(\xi)}^{1} \frac{\alpha}{4}\,\mathrm{d}\alpha = \frac{(1-K)h(\xi)}{4} + \frac{1-\big(h(\xi)\big)^2}{8}.$$

Adding the two profit expressions yields the expected profits in equilibrium ξ, $E\Pi^*_\xi$:

$$E\Pi^*_\xi = \frac{1 + \frac{1}{2}\xi^2 + (1-K-\xi)h(\xi) - \big(h(\xi)-\xi\big)^2}{4}.$$

We can simplify this expression by using the fact that $h(\xi) = 2(1-K) - \xi = 2(1-K-\xi) + \xi$:

$$E\Pi^*_\xi = \frac{1 + \frac{1}{2}\xi^2 + (1-K-\xi)\xi - 2(1-K-\xi)^2}{4}. \tag{5.20}$$

Consider next the benchmark equilibrium. The firm's expected profit in the first period reads

$$\int_0^1 \frac{\alpha}{4}\,\mathrm{d}\alpha = \frac{1}{8}.$$

In the second period, the firm incurs the switching cost if $\alpha \leq 1-K$, but is then subsequently able to sell a quantity of $\frac{1}{2}$. If α exceeds $1-K$, then the firm continues to sell $\frac{\alpha}{2}$ in period two. The expected profit in period two thus equals

$$\int_0^{1-K} \frac{1-K}{4}\,\mathrm{d}\alpha + \int_{1-K}^{1} \frac{\alpha}{4}\,\mathrm{d}\alpha = \frac{1+(1-K)^2}{8}.$$

Adding the last two expressions results in $E\Pi^*_B$, the expected profits in the benchmark situation:

$$E\Pi^*_B = \frac{1 + \frac{1}{2}(1-K)^2}{4}. \tag{5.21}$$

Note that the expected profits in equilibrium ξ would coincide with those in the benchmark situation if both ξ and $h(\xi)$ were replaced by $1-K$. Because $\xi < 1-K$, we can write the difference between the two expected profits as follows:

$$E\Pi_B^* - E\Pi_\xi^* = \int_\xi^{1-K} \pi(x)\,\mathrm{d}x, \tag{5.22}$$

where $\pi(\cdot)$ is the derivative of the expected profits with respect to the type who is mimicked:

$$\pi(x) := \frac{\partial E\Pi_x^*}{\partial x} = \frac{5}{4}(1-K-x). \tag{5.23}$$

Because $x < 1-K$ if $x \in [\xi, 1-K)$, we conclude that expected profits are higher in the benchmark equilibrium. ■

Proof of Proposition 5.7

We prove the claims assuming n is a continuous variable. The results for discrete n then follow from a standard argument. The condition $\xi_n = g^{-1}(1-\frac{K}{n})$ implies

$$\frac{\mathrm{d}\xi_n}{\mathrm{d}n} = \frac{K}{n^2 g'(\xi_n)} = \frac{2K}{n^2(1+h'(\xi_n))} > 0,$$

proving the first claim. Differentiating $B_n(K)$ with respect to n yields

$$\begin{aligned}\frac{\mathrm{d}B_n(K)}{\mathrm{d}n} =& \frac{1}{4}\big(h(\xi_n)-\xi_n\big)^2 + \frac{n}{2}\big(h(\xi_n)-\xi_n\big)\big(h'(\xi_n)-1\big)\frac{\mathrm{d}\xi_n}{\mathrm{d}n}\\ =& \frac{h(\xi_n)-\xi_n}{2} \times \Big[\frac{h(\xi_n)-\xi_n}{2} + \frac{2K}{n} \times \frac{h'(\xi_n)-1}{h'(\xi_n)+1}\Big]\\ =& \frac{h(\xi_n)-\xi_n}{2} \times \Big[\frac{h(\xi_n)-\xi_n}{2} + \frac{2K}{n} - \frac{2K}{n} \times \frac{2}{1+h'(\xi_n)}\Big].\end{aligned}$$

Since

$$\frac{h(1)-1}{2} + \frac{2K}{n} - \frac{2K}{n} \times \frac{2}{1+h'(1)} = -\frac{2K}{n} < 0 < \frac{1}{4} = \frac{h(0)-0}{2} + \frac{2K}{n} - \frac{2K}{n} \times \frac{2}{1+h'(0)}$$

and $\frac{\mathrm{d}}{\mathrm{d}\xi_n}\big[\frac{h(\xi_n)-\xi_n}{2} - \frac{2K}{n} \times \frac{2}{1+h'(\xi_n)}\big] = \frac{h'(\xi_n)-1}{2} + \frac{4K}{n} \times \frac{h''(\xi_n)}{1+h'(\xi_n)} < 0$, we conclude that $B_n(K)$ decreases monotonically if ξ_n is sufficiently large, i.e. if K is sufficiently small, say $K < \bar{K}$.

By construction of ξ_n, we know that $\frac{h(\xi_n)-\xi_n}{2} = 1 - \frac{K}{n} - \xi_n$. Consequently:

$$B_n(K) = \Big(\sqrt{n}\big(1-\tfrac{K}{n}-\xi_n\big)\Big)^2.$$

Replacing K by $\frac{K}{n}$ in (5.18), the expression for ξ_1, yields

$$\sqrt{n}\big(1-\tfrac{K}{n}-\xi_n\big) = \sqrt{n} \times \frac{-1+2\frac{K}{n}+\sqrt{1+16\frac{K}{n}-16(\frac{K}{n})^2}}{10}.$$

We determine the limit of this expression as $n \to \infty$:

$$\begin{aligned}
\lim_{n\to\infty} \sqrt{n}\big(1 - \tfrac{K}{n} - \xi_n\big) &= \lim_{\epsilon\downarrow 0}\left(\frac{\sqrt{K}}{10} \times \frac{-1 + 2\epsilon + \sqrt{1 + 16\epsilon - 16\epsilon^2}}{\sqrt{\epsilon}}\right) \\
&= \lim_{\epsilon\downarrow 0}\left(\frac{\sqrt{K}}{10} \times \left(4\sqrt{\epsilon} + \frac{16\sqrt{\epsilon} - 32\epsilon\sqrt{\epsilon}}{\sqrt{1 + 16\epsilon - 16\epsilon^2}}\right)\right) = 0,
\end{aligned}$$

where the second equality follows from l'Hôpital's Rule. This result implies that for each relevant K $B_n(K) \to 0$ as $n \to \infty$. We also conclude that $B_n(K)$ must be decreasing in n for n sufficiently large for all K, in particular for $K > \bar{K}$. ∎