Tradeable CO2 emission permits in Europe
Koutstaal, Paul Rudolf

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
1996

Link to publication in University of Groningen/UMCG research database

Citation for published version (APA):

Copyright
Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the “Taverne” license. More information can be found on the University of Groningen website: https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment.

Take-down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): http://www.rug.nl/research/portal. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.
CHAPTER 5
COORDINATION OF ENVIRONMENTAL POLICY
IN A SECOND-BEST WORLD

5.1 INTRODUCTION

In the former chapters, attention has focused on a system of tradeable carbon permits which operates within one country. However, the enhanced greenhouse effect is an environmental problem which occurs worldwide. Tradeable permits and other economic instruments like taxes can also play a role at an international level. The main advantage of these instruments is that, for a given emission reduction, total abatement costs will be minimised because all sources of carbon dioxide will limit their emissions up to the point where their marginal abatement costs are equal. In the case of a tax, emitters will reduce their emissions up to the point where their marginal abatement costs are equal to the tax. With a system of tradeable permits such as has been described in chapter 2, marginal abatement costs will be equal to the price of the permits. To minimize worldwide costs of CO₂ abatement, marginal costs would have to be minimized not only among sources within countries but also among countries. When the marginal abatement costs are higher in one country than in another, it would be efficient to reduce emissions in the country with the low marginal costs further and to increase the emissions in the country with the high marginal costs. Introducing a tax on carbon dioxide which is equal in all countries would in theory equalize marginal costs in all countries and result in the lowest aggregate reduction costs, as has been shown by Hoel (1991).

This straightforward result and its clear implication for policy which follows from the standard analysis does not necessarily hold if a more realistic world is taken as a starting point. One of the complications which has to be faced is that the use of fossil fuels, the main source of anthropogenic CO₂ emissions, is already taxed for various reasons in most countries of the OECD at different tax rates. The question arises how a carbon tax should be combined with these existing taxes on fossil fuels. This
question is not only of academic interest, but it has also practical policy implications. Countries with high current implicit taxes on fossil fuels will argue that they have already limited their emissions and should therefore be exempted from a tax which is equal to the carbon tax introduced in countries with current low taxes. Hoeller en Coppel have investigated the differences in taxes (and subsidies) on fossil fuels in the OECD countries (1992). They show that introducing a uniform carbon tax in the OECD countries on top of the existing implicit taxes on fossil fuels leads to higher total abatement costs than equalising the existing taxes and introducing a uniform tax. Their analysis however is only partial: they only look at abatement costs of different countries and do not take into account the welfare consequences of the revenue raised by the existing taxes and the carbon tax. As has been shown by Hoel (1993), in a general equilibrium model in which there is no deadweight loss from taxation, carbon taxes should be uniform as well across countries in order to maximise collective welfare.

In this chapter, the issue of how to combine carbon taxes with existing taxes on fossil fuels is addressed in the setting of a second-best world, using a simple general equilibrium model (described in section 2). A second-best problem can best be described as an allocation problem with a constraint on the policies feasible which makes it impossible to reach the first-best optimum (Bohm 1988, p. 282). In the context of the optimal taxation problem explored here, the constraint on government policies is that lump-sum taxes are not possible and neither can proportional excise taxes be used because not all goods and endowments can be taxed. Therefore, revenue has to be raised by means of distorting taxation, like taxation on fossil fuels.

The problem is set in an international context since we are interested in solutions which are optimal in the sense that they maximise welfare (net benefits) for participating countries, collectively and individually. Account should be taken of the way countries behave with respect to each others policies. There is a wide range of strategies available to countries: from free-rider behaviour and non-cooperation to optimal cooperation with sidepayments. To simplify the analysis we restrict our model to a world existing of two countries which are involved in global pollution; i.e. the damage caused by the pollutant in a country is independent of the country in which it was emitted.
This contribution differs from the earlier literature on the subject by extending the second-best equilibrium analysis of pollution to an international context. Second-best general equilibrium models including pollution for national economies have been developed by Sandmo (1975), Auerbach (1987), Pezzey (1992) and Bovenberg and v/d Ploeg (1992).

The object of this chapter is to determine the welfare maximising tax structure when two countries cooperate in abating pollution and use a tax on a polluting good (the carbon tax) as the instrument or, alternatively, a system of national or international tradeable permits. Moreover, it is examined whether and how sidepayments are necessary to induce countries to cooperate. The two countries are assumed to be equal except with respect to the damage suffered from pollution and the government budget. Our interest is in how these two variables affect the changes in tax structure and the sidepayment when countries cooperate.

The structure of this chapter is as follows. In the next section, the model used is introduced for one country. In section 3, the model is extended to two countries and the non-cooperative and cooperative equilibria (with and without sidepayments) are determined. Moreover, the optimal tax structures in both countries are established. Unfortunately, the analysis does not provide clear answers to the question of how different government budgets and damage functions affect the tax structures and the sidepayment. Therefore in section 4 a more specific functional form is used to simulate the equilibria and to get answers to the questions asked above.

This chapter serves as a basis for the analysis in the next chapter in which the role of tradeable permits in cooperation in a second-best world is examined.

5.2 GENERAL EQUILIBRIUM MODEL OF A TAX ON AN EXTERNALITY IN A SECOND-BEST WORLD

In this section, the model used in the remainder of this paper is presented. The model used is comparable to the one used by Sandmo [1975], Auerbach [1987], Pezzey [1992] and Bovenberg and v/d Ploeg [1992]. The economy consists of one
representative consumer, with the following utility function:

$$U = u(x, y, l) \quad \text{subject to} \quad (1+t_x)x + (1+t_y)y \leq M - l \quad 5.1$$

$$u_x, u_y, u_l > 0$$

$$u_{xx}, u_{yy}, u_{ll} < 0$$

x = dirty good
y = clean good
t_y = tax on good y
t_x = tax on good x
l = leisure
M = given time endowment

The consumer can consume two products, x and y which are produced by sacrificing leisure. His budget constraint is determined by the production function $$(1+t_x)x + (1+t_y)y = M - l$$ which has constant returns to scale. Prices of the two products x and y and of labour (the wage rate) are normalised at unity without loss of generality (Bovenberg & v/d Ploeg 1992). The consumer maximises his utility under the constraint of the production function without taking account of the externalities, i.e. the pollution generated by the consumption of the dirty good x. Maximising this utility function under the constraint of the production function gives demand functions for good x, good y and leisure l which are functions of t_x and t_y. Although labour (M-l) is not taxed, the amount of leisure 'consumed' by the consumer will react to changes in the tax rates on good x and good y. When the taxes on x and y increase, real income of the consumer will fall and consequently he will work less and take more leisure. From the demand functions, the indirect utility function for the consumer can be derived, $V = V(t_x, t_y)$.

The government has to raise a given revenue requirement R by means of the taxes on good x and good y. A lump sum tax is not available, Leisure l (or labour M-l is the untaxed good. This defines a so called second-best world; second-best because distorting consumption taxes have to be used instead of non-distorting taxes like lump-
sum taxes or proportional excise taxes on all goods and leisure/labour.

Furthermore, the government must limit environmental damage which results from the consumption of the dirty good x. Direct abatement is not possible in this model, so all abatement has to come from a reduction in the consumption of x. This is the prevailing situation as regards the emission of CO₂, which can only be reduced by limiting the consumption of fossil fuels. The problem for the government is to choose the tax structure which will maximize consumer welfare, taking into account both the distortions of taxation and environmental damage. First the (standard) optimal tax rules for the second-best world will be established ignoring pollution caused by the consumption of x. Subsequently, the optimal tax rates are determined when the environmental damage caused by the consumption of x is taken into account.

The optimal tax rates (ignoring pollution) are determined by maximizing the following function:

\[ V(t_x, t_y) \text{ subject to } t_x x + t_y y \geq R \]  \hspace{1cm} 5.2

Consumer utility is maximized (by maximizing the indirect utility function V) subject to the revenue constraint of the government, which states that the amount of revenue raised by the taxes on x and y is at least R (which is exogenously determined).

The first order conditions are:

\[ L_{t_x} = V_{t_x} + \mu [ x + t_x x_{tx} + t_y y_{tx} ] = 0 \]  \hspace{1cm} 5.3

\[ L_{t_y} = V_{t_y} + \mu [ y + t_y y_{ty} + t_x x_{ty} ] = 0 \]  \hspace{1cm} 5.4

\[ L_{\mu} = t_x x + t_y y - R = 0 \]  \hspace{1cm} 5.5

Equations 5.3 and 5.4 present the standard formulas for optimal tax rates in situations
where no lump sum taxation is possible\(^1\). \(\mu\), the Lagrange multiplier, can be interpreted as the shadow costs in terms of utility of raising an additional dollar of revenue \(R\) by the government.

Next, the optimal tax rates in the presence of pollution emanating from the consumption of \(x\) are determined by maximizing the following function:

\[
V(t_x, t_y) + D(x) \text{ subject to } t_x x + t_y y \geq R
\]

in which \(D(x)\) is the damage from pollution which is a result of the consumption of good \(x\). Damage is negative benefit, so \(D(x)\) is negative. Furthermore, damage increases when \(x\) increases, therefore \(D_x < 0\). It is assumed that \(D_{xx} < 0\), so marginal damage rises when \(x\) increases.

The first order conditions are:

\[
L_{tx} = V_{tx} + D_x x_{tx} + \mu[x + t_x x_{tx} + t_y y_{tx}] = 0
\]

\[
L_{ty} = V_{ty} + \mu[y + t_y y_{ty} + t_x x_{ty}] + D_x x_{ty} = 0
\]

Equations 5.7 and 5.8 can be rewritten as:

\[
V_{tx} + \mu[x + (t_x + 1/\mu D_x) x_{tx} + t_y y_{tx}] = 0
\]

\[
V_{ty} + \mu[y + t_y y_{ty} + (t_x + 1/\mu D_x) x_{ty}] = 0
\]

Marginal environmental damage \(D_x\) can be internalised in the decisions of the representative consumer by adding an appropriate pollution tax to the existing revenue raising tax on good \(x\). Let \(t_x^R\) be the optimal tax when pollution was ignored (the

\(^1\) The optimal tax problem has originally been discussed by Ramsey. See Auerbach 1987 for a recent overview of the optimal tax problem.
Ramsey tax) and $t_x^{RP}$ the tax when pollution is taken into account (the Ramsey-Pigou tax): tax $t_x$ in equations 5.7 and 5.8 and 5.7a and 5.8a. Comparing equations 5.3 and 5.4 with 5.7a and 5.8a, it follows that $t_x^{RP}$ can be written as [Auerbach, 1987, p. 113]:

$$t_x^{RP} = t_x^R - \frac{1}{\mu} \frac{\delta D}{\delta x}$$  \hspace{1cm} 5.9

Equation 5.9 states that the optimal tax on good x in the presence of pollution is composed of two elements:

1] A tax on x which is calculated by the standard optimal tax formulas as given in equations 5.3 and 5.4 (tax $t_x^R$, the Ramsey part of the tax on x).

2] A tax which corrects for the damage resulting from the pollution caused by the consumption of x (the Pigouvian part of the tax on x, termed subsequently $t_x^P$) which is equal to the marginal damage $\delta D/\delta x$ divided by the marginal disutility of government revenue $\mu$. When $\mu$, the shadow costs in terms of utility of raising an additional dollar of government revenue rises (for example, because government raises its revenue requirement), this Pigouvian part of the tax declines. As Bovenberg and v/d Ploeg state (1992, p.9): The government can afford less tax differentiation aimed at environmental protection as the revenue raising objective of the tax system becomes relatively more important. This also implies that in this second-best world, where tax revenue is exogenously given and independent of environmental damage, relative environmental protection will be weaker the higher the revenue requirement of the government is. The consequences for the absolute level of pollution are not clear. With a rise in R, $t_x^R$ will increase while $t_x^P$ will fall. Under certain conditions (see appendix) total tax $t_x$ will increase when R is raised. Assuming that x is a normal good, consumption will fall and environmental quality will improve.

From equation 5.7a and 5.8a, it follows that the optimal tax rules for the tax on good y, the non-polluting good, are not affected by the internalisation of environmental damage in the tax system. Note however that the level of $t_y$ will change: the environmental tax will bring in revenue, which reduces the absolute level of the Ramsey taxes on both x and y.

123
In the following sections, this model will be extended and used to analyze the consequences for the optimal taxes for two countries.

5.3 NON-COOPERATION AND COOPERATION IN A SECOND-BEST WORLD

When pollution is transboundary, as is the case with carbon dioxide, environmental policy is confronted with the problem of coordinating environmental policy between independent states. In contrast with environmental problems which are confined within the boundaries of one country, the difficulty is that there is no single authority which can make a cost-benefit analysis and implement an international abatement scheme. It is realistic to assume that countries will base their own policies upon the behaviour in other countries with respect to the transboundary pollution. For example, the European Community has formulated a CO₂ policy which includes a tax on energy and carbon, but it is only to be implemented if Japan and the U.S. will reduce their emissions as well. Therefore one should determine how countries would react to reduction strategies in other countries. In section 3.1, the non-cooperative Nash equilibrium will be examined and in 3.2 the cooperative equilibrium. In 3.3, the cooperative equilibrium is extended by allowing the countries to use sidepayments.

5.3.1 NON-COOPERATIVE NASH-EQUILIBRIUM

At the outset, the model presented in the former section must be amended to account for the transboundary character of pollution. It is assumed that there are two countries, in both of them both x and y are consumed and both governments have to fulfil their (given) revenue requirement by taxing the two consumption goods x and y. It is assumed that the environmental damage which results from the consumption of x occurs everywhere, regardless of the location where x is consumed. Therefore, the damage function for both countries, D₁ and D₂, are not only a function of the consumption of x in the own country, but also of the consumption of x in the other
country:

\[ D_1 = D_1((x_1 + x_2)) \quad 5.10 \]
\[ D_2 = D_2(x_1 + x_2) \quad 5.11 \]

The social welfare function, equation 5.6, then becomes (for country 1):

\[ V_1(t_{x1}, t_{x2}) + D(x_1 + x_2) \] s.t. \[ t_{x1} x_1 + t_{y1} y_1 \geq R_1 \] 5.12

In order to determine the optimal taxes and pollution both countries end up with when they do not coordinate their policies (the non-cooperative Nash-equilibrium) it is assumed that both countries take the pollution in the other country as given. Taking \( x_2 \) as given, country 1 will maximize its social welfare function (5.12) (and vice-versa for country 2), yielding the following first order conditions:

\[ L_{tx1} = V_{tx1} + D_{1x1} x_{tx1} + t_{y1} y_{tx1} + \mu_1 [x_1 + t_{x1} x_{tx1} + t_{y1} y_{tx1}] = 0 \] 5.13
\[ L_{ty1} = V_{ty1} + D_{1x1} y_{ty1} + t_{x1} x_{ty1} + \mu_1 [y_1 + t_{y1} y_{ty1} + t_{x1} x_{ty1}] = 0 \] 5.14

Comparing these first-order conditions with those in the single country case, the only difference is the occurrence of \( x_2 \) in the derivative of the damage function. Damage is not only caused by the pollution resulting from consumption of \( x \) in the own country but also by the given level of pollution imported from the other country.

In order to determine the way country 1 will react to a change in emissions in country 2, we take the total differential of the first-order conditions (including \( t_{x1} x_1 + t_{y1} y_1 = R_1 \)) with \( t_{x2} \) as a parameter change. Solving the system gives:

\[ \frac{dt_{x1}}{dt_{x2}} = (y_1 + y_{ty1} t_{y1}) D_{1x1x2} x_{2x2} \left[ (y_1 + y_{ty1} t_{y1}) x_{1x1} - (x_1 + x_{tx1} t_{x1}) x_{1y1} \right] / |H| \] 5.15

\( H \) is the Hessian matrix.
The right hand side of equation 5.15 is negative (see appendix). Therefore, when tax\( t_2 \) rises, tax\( t_1 \) decreases. When country 2 raises its tax on good \( x \) and therefore reduces consumption of \( x \) and emissions, the marginal damage resulting from the consumption of \( x \) will diminish (in absolute terms), not only in country 2 but also in country 1. Consequently, country one can increase pollution and therefore consumption of \( x \) and it will be able to lower the Pigouvian part of its tax on \( x \). In diagram 5.1 curve \( R_1 \) represents this reaction of country 1, and \( R_2 \) for country 2 if country 1 changes its tax on \( x \). (It is assumed that an interior solution exists, for proof of the existence of the equilibrium see the appendix). The non-cooperative Nash equilibrium is point N in diagram 5.1.

![Diagram 5.1 Cooperative and non-cooperative equilibrium](image)

5.3.2 COOPERATIVE EQUILIBRIUM

In section 3.1 countries did not cooperate. Here it is assumed that countries negotiate in order to agree on abatement policies. Cooperation geared to a Pareto-optimal solution can be represented by the following function (see also Hoel 1991, p. 58):

\[
V_1(t_{x1}, t_{y1}) + D_1(x_1 + x_2) \quad s.t. \quad t_{x1} x_1 + t_{y1} y_1 \geq R_1
\]

5.16

126
\[ tx_2 x_2 + ty_2 y_2 \geq R_2 \]
\[ V_2 + D_2 \geq W_2^* \]

in which \( W_2^* \) is the welfare level of country 2 in the non-cooperative equilibrium. In other words, welfare in country 1 is optimised by setting taxes in both countries subject to the government revenue constraint and subject to welfare in the other country staying at least equal. The Lagrange function to be maximised is:

\[
L = V_1(t_{x1}, t_{y1}) + D_1(x_1 + x_2) + \mu_1[x_1 t_{x1} + y_1 t_{y1} - R_1] + \gamma[V_2(t_{x2}, t_{y2}) + D_2(x_1 + x_2) - W_2^*] + \mu_2[x_2 t_{x2} + y_2 t_{y2} - R_2]
\]

\( \gamma \) is the lagrange multiplier of the welfare constraint for country 2.

The first-order conditions are:

\[
L_{tx1} = V_1 t_{x1} + D_1 t_{x1} + \mu_1(x_1 + t_{x1} x_{tx1} + t_{y1} y_{tx1}) + \gamma D_2 t_{tx1} = 0
\]

\[
L_{ty1} = V_1 t_{y1} + D_1 t_{y1} + \mu_1(y_1 + t_{y1} y_{ty1} + t_{x1} x_{ty1}) + \gamma D_2 t_{ty1} = 0
\]

\[
L_{tx2} = \gamma(V_2 t_{x2} + D_2 t_{x2}) + \mu_2(x_2 + t_{x2} x_{tx2} + t_{y2} y_{tx2}) + D_1 t_{tx2} = 0
\]

\[
L_{ty2} = \gamma(V_2 t_{y2} + D_2 t_{y2}) + \mu_2(y_2 + t_{y2} y_{ty2} + t_{x2} x_{ty2}) + D_1 t_{ty2} = 0
\]

\[
L_{\mu1} = x_1 t_{x1} + y_1 t_{y1} - R_1 = 0
\]

\[
L_{\mu2} = x_2 t_{x2} + y_2 t_{y2} - R_2 = 0
\]

\[
L_{\gamma} = V_2 + D_2 - W_2^* = 0
\]

The maximisation procedure can be pictured in fig. 5.1 as a movement, starting in N along the constant welfare curve \( I_2 \) (where \( W_2=W_2^* \)), searching for the point where \( W_1 \) has its maximum. That is shifting \( I_1 \) upwards until it has a point of tangency with \( I_2 \).
This is point P. Given the form of the iso-welfare curves in figure 5.1 welfare in country 1 has been increased, while at the same time holding welfare in country 2 constant, by raising simultaneously both \( t_{x1} \) and \( t_{x2} \) as compared with the non-cooperative equilibrium. Therefore, pollution will be lower in the cooperative equilibrium than in the non-cooperative equilibrium.

The difference with the non-cooperative solutions (equation 5.13 and 5.14) is that the government of country 1 in choosing its tax level for \( t_{x1} \) and \( t_{y1} \) has to take into account its valuation of the marginal damage in country 2 \((\gamma D_{2x1})\) caused by consumption of \( x_1 \) (see 5.19 and 5.20). In the same way damage in country 1 caused by consumption of \( x_2 \) in country 2 is taken into account in setting \( t_{x2} \) and \( t_{y2} \). The Pareto-optimal solution (in point P in fig. 5.1) can be viewed as an agreement between the two governments to increase taxes on \( x \) reciprocally in order to reduce pollution further than in the non-cooperative case.

In the cooperative equilibrium, the Pigouvian taxes will increase, as can be seen by splitting the tax on good \( x \) in a Ramsey part and a Pigouvian part (see equation 5.9). Now, \( t_{x1} \) and \( t_{x2} \) can be written as:

\[
t_{x1} = t_{x1}R - \frac{1}{\mu_1} (D_{1x1} + \gamma D_{2x1})
\]

\[
t_{x2} = t_{x2}R' - \frac{1}{\mu_2} (D_{1x2} + \gamma D_{2x2})
\]

The Pigouvian taxes include marginal damage caused in the other country in addition to the marginal damage of consuming \( x \) in the own country. The Pigouvian taxes will probably rise and pollution will fall when countries cooperate. However, \( \mu_1 \) and \( \mu_2 \) will change as well, it is therefore not straightforward that the Pigouvian taxes are higher when countries cooperate compared with non-cooperation. The Pigouvian taxes in both countries will not be equal (except when \( \mu_1 = \mu_2 \), which is not necessarily the case).

It should be noted that the cooperative game examined in this section can produce

---

It should be noted that \( \mu_2 \) does not represent the shadow cost of taxation for country 2 in equation 3.9. Instead, it represents the effect a marginal change in the revenue requirement for country 2, \( R_2 \), has on the welfare of country 1. The constraint is not a constraint on the welfare of country 2 in this equation but a constraint on the welfare of country 1.
more than one equilibrium. Solving optimisation problem 5.17 yield the highest attainable welfare level for country 1, given the welfare constraint $W_2^*$ for country 2. Varying this constraint $W_2^*$ generates a range of welfare levels for country 1. In diagram 5.1, the contract curve, lying in between the original iso-utility curves $I_1$ and $I_2$ represents these combinations. All the combinations of welfare levels which will leave both countries better off after cooperation than with non-cooperation are solutions to the cooperative game.

5.3.3 Joint optimum and the use of sidepayments

The range of Pareto-optimal solutions with tax rates as the only instruments that are coordinated does not necessarily yield the highest attainable benefits of cooperation. In a first-best approach, the possibility to use side payments makes it possible to increase welfare in both countries by redistributing the abatement effort relative to the initial cooperative solutions and compensating the country which would be worse off in terms of welfare (see Nentjes 1994). The country which will lower its tax on $x$ and increase consumption of $x$ (and therefore its pollution) compared with the initial cooperative solution will have to compensate the other which increases its pollution tax and decreases its consumption of $x$. Without sidepayments this country would be worse off than when it did not cooperate.

However, sidepayments raise problems of their own. Sidepayments must be raised by way of the (distorting) taxes on $x$ and $y$. Therefore regard must be taken of the welfare effects of raising (and receiving) side payments by means of these taxes. The optimisation problem becomes:

$$\max V_1 + D_1$$
$$\text{s.t.: } x_1 t x_1 + y_1 t y_1 - (R_1 + S) \geq 0$$
$$\text{s.t.: } V_2 + D_2 - W_2^* \geq 0$$
$$\text{s.t.: } x_2 t x_2 + y_2 t y_2 - (R_2 - S) \geq 0$$

$W_2^*$ is the welfare level in country 2 in the non-cooperative Nash-equilibrium. $S$ is the
sidepayment made by one country to the other country. $S$ can be positive or negative: if $S$ is positive, country 1 will pay country 2. From the budget constraint for country 1, we can see that in that case the revenue requirement for country 1 increases with the sidepayment, while the revenue requirement for country 2 decreases by the same amount. An increase (decrease) of the revenue requirement has several effects. On the one hand, taxes on both goods will increase (decrease) because more (respectively less) revenue is to be raised. On the other hand the Pigouvian tax will decline (increase) because it becomes more (less) costly to levy an environmental tax. The net effect on aggregate pollution can be positive or negative, as will be seen in the next section.

In equation 5.27, the effects of sidepayments on tax levels and excess burden of taxation are taken into account explicitly. Comparing equation 5.27 with 5.17, the difference between the maximization problem in the cooperative equilibrium with and the cooperative equilibrium without sidepayments is the additional instrument of sidepayments, which makes it possible to acquire higher welfare levels through cooperation.

The first-order conditions of maximising 5.27 are mainly equal to the first-order conditions of the cooperative solution without sidepayments (5.18-5.24). Only equations 5.22 and 5.23 change:

\[
L_{\mu_1} = x_{1t}x_1 + y_{1ty_1} - (R_1+S) = 0 \quad 5.22a
\]
\[
L_{\mu_2} = x_{2tx_2} + y_{2ty_2} - (R_2-S) = 0 \quad 5.23a
\]

Furthermore, the first-order derivative of variable $S$, the sidepayment, is added:

\[
L_S = -\mu_1 + \mu_2 = 0 \quad 5.28
\]

Again, we can split the taxes on good $x$ in two parts, yielding the same formulas as in the cooperative optimum without sidepayments, see equation 5.25 and 5.26. However, the difference is that $\mu_1 = \mu_2$ (equation 5.28), therefore the Pigouvian part of the tax is now equal in both countries. Both countries will levy the same Pigouvian or ‘carbon’ tax. It should be noted that the other part of the total tax on $x$ will still differ
between the two countries. Aggregate tax levels on the polluting good $x$ will still differ between the two countries. This is the main difference with the outcome in a first-best world (without tax revenue constraints) mentioned in section 1, where it was argued that in such a first-best world tax rates are equalised between countries.

Another point worthwhile repeating is that even though $\mu_1 = \mu_2$, shadow costs of taxation in both countries will still differ. As has been noted above, $\mu_2$ does not represent the shadow cost of taxation for country 2 in equation 5.26. Instead, it represents the effect which a marginal change in the revenue requirement for country 2, $R_2$, has on the welfare of country 1. The constraint is not a constraint on the welfare of country 2 in this equation but a constraint on the welfare of country 1. When the revenue requirement in country 2 is lower, that country can gear its tax structure more to reducing consumption of $x$, which increases welfare in country 1, given its welfare constraint $W^*_2$, than when $R_2$ is higher.

Who pays whom is determined by the Lagrange multipliers $\mu_1$ and $\mu_2$. When $\mu_1$ was lower than $\mu_2$ in the initial bargaining solution without transfers, country 1 will pay country 2, which in exchange will raise its tax on good $x$. However, when $\mu_1$ is initially higher than $\mu_2$, country 1 will receive the sidepayment. All other things being equal, a decrease in $R_1$ increases the attractiveness of making a sidepayment for country 1 (because $\mu_1$ declines). The higher the revenue requirement is, the higher will be the welfare loss (dead weight loss) of raising the revenue for the sidepayment). A higher $R_2$ increases $\mu_2$, increasing the attractiveness of a positive sidepayment (country 1 pays country 2) as well. It should however be noted that changes in the revenue requirement also affect the non-cooperative Nash equilibrium and therefore $W^*_2$, the welfare constraint on country 2 in equation 5.27. Consequently, comparing cooperative equilibria in situations with different initial revenue requirements is highly problematic and does not yield clear results.

As has been mentioned above (p.12), the cooperative game discussed in section 3.2 produces more than one equilibrium. The literature on cooperatives games does provide a number of solutions to cooperative games which do yield unique outcomes\(^3\) in which

---

\(^3\) The fact that there are several possible approaches is in itself a weakness: there is no reason to prefer one of the approaches above the other.
both countries improve their welfare levels as compared with a non-cooperative solution. Examples of these are the Nash bargaining solution and the Raiffa-Kalai-Smorodinsky solution (Friedman 1986, chapter 5). The Nash bargaining solution (used also in the context of coordination of environmental policy between two countries by Hoel 1991) takes as a starting point for negotiations the welfare levels in the non-cooperative Nash equilibrium. In the Nash bargaining solution \((U_1-T_1)(U_2-T_2)\) is maximised subject to the revenue constraints where \(T_1\) and \(T_2\) are the welfare levels in the Nash equilibrium for country 1 and country 2. This approach has not been used in this general section because the second-best Nash bargaining model does not yield interpretable results.

In order to overcome these problems, in the next section the Nash-bargaining solution will be considered for a more specific functional form of the welfare function. It will be analyzed how different revenue requirements influence which country will make the sidepayment, how the tax structure in both countries is affected by cooperation with and without side-payments and what the consequences are of cooperation for the level of pollution.

5.4 SIMULATIONS

5.4.1 INTRODUCTION

As was stated in the introduction of this chapter, the aim of this research is to determine the equilibrium when two countries cooperate in reducing environmental pollution by means of coordinating taxation in a second-best world where polluting and non-polluting commodities are taxed. The general model considered in the former sections does not yield clear answers to the questions posed in the introduction of this chapter. In particular it does not answer how the tax structure will change when countries cooperate, how pollution is affected and in which way sidepayments can be used to increase welfare. In this section, the equilibrium will be simulated using a more specific model. The form chosen for the simulations is the following welfare function:
The first term of 5.29 is a Cobb-Douglas utility function. Utility is derived from two goods, x and y, and from the time not worked, leisure l. It is assumed that $\alpha = \beta = \tau = 1/3$.

The representative individual maximises this first term subject to his budget constraint:

$$U_i(x_i, y_i, l_i) + D_i(x_i+x_j) = x_i^{\alpha}y_i^{\beta}l_i^{\tau} - a(x_i+x_j)^2$$  \hspace{1cm} 5.29

$M$ is the maximum amount of time available to the consumer (his endowment). The wage level is set at unity. A Cobb-Douglas function is chosen because it has the characteristic that it yields demand functions for both goods x and y which are independent of the price of the other product (cf. Sandmo 1975). This assumption does not basically change the evaluation as we are interested in the interaction between the two countries, given the need for them to raise revenue by means of distorting taxation, and not in the cross effects of price changes per se.

The second term in 5.29 represents environmental damage. The marginal damage coefficient, $a$, is positive. First and second-order derivatives are negative.

The individual consumers maximisation yields demand functions for x and y (l is fixed):

$$x = \frac{M}{3*(1+t_x)} \hspace{0.5cm}, \hspace{0.5cm} y = \frac{M}{3*(1+t_y)}$$

Using these demand functions, the authorities maximise welfare function 5.29 subject to their revenue constraint:

$$x_t x + y_t y = R$$  \hspace{1cm} 5.31

The time endowment $M$ is set at 1000. R is a fraction of M. Both countries are assumed to have the same welfare functions and private budget constraints. The revenue requirements ($R_1$ and $R_2$) and the marginal damage coefficients ($a$ and $b$) are allowed to vary. This makes it possible to determine how the revenue requirements and marginal damage coefficients influence the sidepayment, the tax structure and the level
of pollution.

The specific cooperative solution with tax coordination as instrument examined is the Nash bargaining solution. The Nash bargaining solution is found by maximising the following function:

\[
(U_1(x_1, y_1, l_1) + D_1(x_1, x_2) - T_1) \times (U_2(x_2, y_2, l_2) + D_2(x_1, x_2) - T_2)
\]

\[
= [x_1^\alpha y_1^\beta l_1^\gamma - a(x_1 + x_2)^2 - T_1] [x_2^\alpha y_2^\beta l_2^\gamma - b(x_1 + x_2)^2 - T_2]
\]

s.t. 
\[
x_1 t_{x_1} + y_1 t_{y_1} \geq R_1
\]
\[
x_2 t_{x_2} + y_2 t_{y_2} \geq R_2
\]

\(T_1\) and \(T_2\) are the welfare levels for country 1 and country 2 in the non-cooperative Nash equilibrium. Maximising 5.32 is equal to finding the hyperbole which has as asymptotes the utility levels \(T_1\) and \(T_2\), and has a point of tangency with the frontier of the set of possible solutions to the cooperative game. This is shown in figure 5.2. The y-axis shows the utility level for country 2, the x-axis the utility level for country 1. Line AA’ represents the possible welfare levels attainable when both countries cooperate by coordinating their taxes when no sidepayments are used. The non-cooperative welfare levels (the Nash solution, point N in fig. 5.1) are the origin of figure 5.2. Therefore, the x-axis and the y-axis are the asymptotes for the hyperboles which are the iso-utility curves of function 5.32. \(P_1\) is the Nash bargaining solution for this case.

When the possibility of sidepayments is included, the maximisation problem 5.32 changes. In addition to setting the four taxes on both goods x and y in both countries,
the cooperating countries can also set the optimal sidepayment. The sidepayment enters equation 5.32 by way of the revenue constraints. \( R_1 \) becomes \( R_1 + S \), \( R_2 \) becomes \( R_2 - S \). \( S \) can be both negative or positive. When \( S \) is positive, country 1 pays country 2.

When sidepayments are possible in addition to tax coordination, the attainable utility levels will increase. This is shown in figure 4.1 by shifting line \( AA' \), the cooperative possibilities curve when no sidepayments are used, outwards to line \( BB' \). When the set of cooperative solutions increases, the Nash bargaining solution will also change, from point \( P_1 \) to point \( P_2 \) in diagram 5.2.

A striking feature of the Nash-bargaining solution with sidepayments is that the welfare level of one of the countries in the cooperative equilibrium with sidepayments can be smaller than its welfare level in the cooperative equilibrium without sidepayments. It might seem strange that the inclusion of an additional instrument (sidepayments) would result in a lower welfare level for one of the countries. However, one should realise that the Nash bargaining solution maximises a form of joint optimum subject to the constraints that both players realise a minimum welfare level, which is determined by the non-cooperative equilibrium. With the inclusion of the possibility of sidepayments, this maximum shifts. However, the initial welfare constraints will still be met: both with and without sidepayments each country will be better off.

---

4 When both countries are equal in all respects, the optimal side-payment is zero in the Nash-bargaining solution and the equilibrium does not change.
Figure 5.3 provides an illustration of this point. Curve AA' shows the possible welfare combinations which leave both countries better off as compared with the non-cooperative equilibrium (the origin) with no sidepayments. The Nash-bargaining solution is point N, the point of tangency of curve AA’ and the highest attainable hyperbole, H. Curve BB’ gives the welfare combinations including sidepayments, H’ the highest hyperbole and N' the new Nash-bargaining equilibrium. The value of the Nash-bargaining solution including sidepayments is necessarily equal to or higher than the Nash-bargaining solution without sidepayments. Country 1’s welfare level rises compared with the Nash-bargaining solution without sidepayments, country 2’s welfare declines.

![Diagram 5.3 Nash-bargaining solution and side-payments](image)

However, it is not realistic that one country would accept lower welfare levels when sidepayments are allowed compared with the equilibrium without sidepayments. A more relevant approach therefore is to take the welfare levels of the Nash-bargaining solution without sidepayments as the threat-points for determining the new Nash-bargaining solution with sidepayments. This is illustrated in figure 5.3. Point N represents the new threat points. Consequently, the Nash-bargaining solution with sidepayments is determined by the intersection between the welfare possibilities curve BB’ and the

---

5 In negotiations which would include side-payments right from the start, this problem would not arise. The step from non-cooperative equilibrium to the cooperative equilibrium will necessarily increase welfare levels in both countries. Only when side-payments are allowed after the implementation of the Nash-bargaining solution without side-payments will the problem that one country can end up worse arise. It is assumed here that negotiations will follow this two step approach.
highest attainable hyperbole which has as focus N instead of the origin. The Nash-
bargaining solution with sidepayments which has these new threat-points as asymptotes is shown by N′′. Necessarily, neither of the two countries will be worse off when sidepayments are included compared with the N-B solution without sidepayments.

This two-stage negotiation process in which countries cooperate in a first stage without sidepayments, introducing sidepayments in the second stage, is taken as the starting point for the simulations. It will be determined how different revenue requirements and different marginal damage coefficients affect the Nash-bargaining equilibrium including sidepayments. The optimal sidepayment will be calculated and the change in tax structure and consumption of good x in both countries will be established when sidepayments are used in a second round as compared with the equilibrium without sidepayments. Initially, the first-best case will be analyzed which will serve as a benchmark for the subsequent second-best analysis.

5.4.2 First-best analysis

In a first-best world, governments can levy revenue by means of non-distortionary taxation (proportional excise taxes on all goods) or lump sum taxes. These type of taxes are non-distortionary because revenue is raised by means of taxes on all goods and endowments or by a direct tax on income. Consequently, these taxes do not distort the price ratio’s between goods as would be the case in the second-best model in which taxes are levied on goods x and y and not on leisure. Such taxes distort the price ratio’s between the two goods and leisure and consequently they have an additional negative impact on welfare. In the model discussed in the former section, proportional excise taxes entail that leisure is also taxed in addition to x and y. Consequently the government budget constraint is:

\[ x_t x + y_t y + l_t l = R \] \hspace{1cm} 5.33

Alternatively, a lump sum tax can be used instead of proportional excise taxes. In that case, the budget constraint for the consumer is:
\[ x(1+t_x) + y + l = M - R - x t_x \]

in which \( t_x \) follows from maximizing \( V(t_x) + D(x(t_x)) \). The required government revenue is raised by means of the lump sum tax \( R \) which occurs in the consumer’s budget constraint. The tax on good \( x \) is a Pigouvian tax which is levied solely to reduce consumption of \( x \) because of the marginal damage resulting from the pollution caused by the consumption of \( x \). The revenue raised by the tax, \( x t_x \), is returned to the consumer by means of a lump sum transfer, see equation 5.34. The demand curve for \( x \) is different from the second-best model because the revenue raised by the tax on \( x \) is returned as a lump sum transfer. The demand curve for \( x \) is:

\[ x = \frac{(M-R)}{(2t_x+3)} \]

The demand curves for \( y \) and \( l \) (which are equal in the first-best case) now are (with \( t_y = t_l = 0 \)):

\[ y = l = \frac{1}{2}(M-R-x) \]

When a country makes a sidepayment in this first-best case, it is taken directly from the consumer’s budget through a lump sum tax. When a sidepayment is received, it is given to the consumer by means of a lump sum transfer.

The results of a number of simulations are presented in diagrams 5.4 to 5.6 (and their accompanying tables) at the end of the chapter. The x-axes show either the marginal damage coefficient or the revenue requirement for country 1. The y-axes show the changes for \( x_1, x_2 \) and \( x_1+x_2 \), which is equal to pollution in both countries, as compared with the equilibrium without sidepayments. Furthermore, the optimal sidepayments are shown.

In diagram 5.4, the revenue requirement for country 1 is varied from 150 to 450 while \( R_2 \) is set at 300. Marginal damage coefficients for both countries are set at 0.1. When both countries cooperate without sidepayments instead of non-cooperation, taxes on good \( x \) increase in both countries and pollution declines. Allowing sidepayments reduces pollution further. The sidepayment is negative as long as \( R_1 \) is smaller than \( R_2 \).
therefore country 2 pays country 1 as long as government spending in country 1 is lower than in country 2. The reason for this can be found in table 5.1 which shows the consumption levels and tax rates for goods x and y in both countries in the Nash-equilibrium and the cooperative equilibria. In the Nash-equilibrium, the taxes on x are equal in both countries. In the cooperative equilibrium without sidepayments, the Pigouvian tax in the country with the lower revenue requirement is lower than in the other country. In the first-best model examined here, taxes on good x will be equal in the cooperative equilibrium, therefore the country with the lower tax on x in the equilibrium without sidepayments (the country with the lower revenue requirement) will raise its tax on good x while the other country reduces $t_x$. The result is that taxes on x will be equal in the cooperative equilibrium with sidepayments, see table 5.1.

It should be noted that the size and sign of the sidepayment depends on the cooperative equilibrium without sidepayments. Here, the Nash-bargaining equilibrium is chosen. However, there are other equilibria which will also leave both countries better off compared with the non-cooperative Nash equilibrium. With another equilibrium as starting point the size and possibly the sign of the sidepayment will change.

In the simulation presented in diagram 5.5, revenue requirements are both set at 300 while the marginal damage coefficient of country 1, $a$, is varied from 0.9 to 1.1. The marginal damage coefficient in country 2, $b$, is set at 1. The country with the lower marginal damage coefficient levies lower taxes on good x in both the non-cooperative and the cooperative equilibrium. Consequently this country receives the sidepayment, increases its tax on the polluting good and consumes less of it while the other country decreases the tax on x and consumes more of the dirty good. Using sidepayments will reduce pollution further compared with the cooperative equilibrium without sidepayments.

---

6 This is the case because the marginal damage coefficient is set equal in both countries. Consequently, marginal damage is equal, given the levels of x consumed in both countries in the Nash equilibrium. In a first-best world, the Pigouvian tax is equal to marginal damage, therefore taxes are the same in both countries.

7 In the first-best model examined here, revenue is raised through a lump sum tax, therefore the only taxes levied on good x are the Pigouvian taxes. It has been shown in section 3.3 that the Pigouvian taxes are equal in both countries in the equilibrium with sidepayments.
In diagram 5.6, both the marginal damage coefficients and the revenue requirements differ. In country 2, government spending is set at 250. Country 1 has a marginal damage coefficient of 0.95, country 2 of 1. The revenue requirement of country 1 is allowed to vary from 220 to 420. Over the whole range of revenue requirements for country 1, country 2 makes a sidepayment to country 1. The advantage of the lower marginal damage coefficient for country 1 outweighs the higher revenue requirement of country 1, although the sidepayment from country 2 to country 1 becomes smaller the larger R₁ is (which confirms our earlier findings that an increase in R reduces the probability that the country will be on the receiving side of the transfer!). Country 2 pays the transfer, reduces tₓ (and consumption of y). Therefore it can increase its consumption of good x and therefore its pollution; country 1 receives the transfer, increases tₓ and reduces pollution.

A striking point is that as R₁ rises above 300, total pollution actually *increases*. Allowing for sidepayments can apparently mean that in the Nash-bargaining solution pollution is higher than in the equilibrium without sidepayments. This can be explained by looking more closely at what happens in both countries. Reducing pollution more in country 1 and less in country 2 is attractive because marginal damage in country 1 is lower: therefore the welfare costs of reducing pollution are lower in country 1. Consequently, country 2 pays country 1 and consumes more of good x. However, this has the additional effect that the country with the higher revenue requirement (country 1) and therefore lower income increases its income. The positive income effect on consumption of x will partially offset the reduction in x brought about by the price effect of a higher tax on good x. In total consumption of x by the consumers of the two countries will rise and therefore pollution increase. In country 1 welfare increases because it receives the sidepayment. In country 2 welfare increases because it focuses less on emission reduction. This compensates the increase in pollution.  

5.4.3 SECOND-BEST ANALYSIS

In the second-best simulations, the model described in the introduction of this section is used; consumers maximise utility function 5.29 (which include two goods, x and y, and leisure l) under the constraint of 5.30. This implies distortionary taxation:
revenue is raised only through taxing x and y while leisure remains untaxed, therefore prices are distorted and taxation causes a welfare loss in addition to the welfare loss of the income transfer. The first second-best simulation analyses the role of the revenue requirement. In diagram 5.7 (end of the chapter), R₂ is set at 300 while R₁ is allowed to vary from 100 to 500. Marginal damage coefficients are set at 0.1. As can be seen in table 5.4, in the non-cooperative equilibrium and the cooperative equilibrium without sidepayments the price ratio between good x and good y in the low revenue country, pₓ/pᵧ (the price is equal to the tax plus the unity price of 1) is larger than the price ratio in the high revenue country. Therefore the low revenue country will pay the other country (the sidepayment is positive as long as R₁ is smaller than R₂) and reduce its tax on x while the other country increases its tax on x and decreases consumption of x. This is in contrast with the first-best case analyzed above (see diagram 5.4).

In this simulation pollution rises, which is in contrast with the first-best case (see page 21). The sidepayment has the additional effect of lowering aggregate deadweight loss of taxation because the country with the lower revenue requirement will raise more revenue while the other country will raise less. This results in an overall higher consumption of x and therefore a higher level of pollution as compared with the Nash-bargaining solution without sidepayments.

In diagram 5.8, the difference between first-best and second-best is illustrated. Both countries have equal marginal damage coefficients (set at 0.1), country 1’s revenue requirement is 200 which is lower than country 2’s (R₂ = 300). The x-axis shows on the left the first-best case and on the right side the second-best case (the whole revenue requirement has to be levied through distortionary taxation). On the left hand side (first-best) country 1, which has the lower revenue requirement, is paid by country 2. As we move towards the right, towards second-best, the sidepayment increases. On the right, country 2, the country with the higher revenue requirement, receives the sidepayment from country 1. Moreover, pollution increases when no lump sum taxes

8 In this simulation, the revenue raised by the tax on x exceeds the revenue requirements in both countries. The percentage values shown on the x-axis are the percentage of the tax revenue (minus revenue requirement and sidepayment) returned to the consumers through a non-distortionary lumpsum tax. The part of the excess tax revenue which is not restored through the lumpsum tax is returned through a (distortionary) subsidy on good y (a negative tax tᵧ).
are used on the right side of the graph while it decreases on the left side, where lump sum taxes are used.

In diagram 5.9, revenue requirements are held equal at 300 while the marginal damage coefficient in country 1 is varied from 0.7 to 1.2. In country 2, the marginal damage coefficient is 1. The sidepayment is negative. The country with the higher marginal damage (country 2) pays the other country. Pollution declines. The results are the same as in the first-best case, which is not surprising. The country which has a high marginal damage will initially reduce emissions further than the other country, excepting higher abatement costs. When sidepayments are possible, it will do less while it pays the other country to do more.

5.4.4 SUMMARY

In this section, simulations have been used to determine the changes in tax structures and pollution when countries cooperate in pollution control. Two types of cooperation have been examined: with and without sidepayments. The cooperative solution analyzed here is the Nash-bargaining equilibrium. Therefore the results derived here only show the role of sidepayments and the changes in tax structures for this specific cooperative solution. The most striking conclusion is that it is not necessarily the case that including sidepayments in cooperative agreements will lead to lower pollution levels compared with cooperative agreements without sidepayments. Simulations show that both in first-best and in second-best models the use of sidepayments can lead to higher pollution levels, although both countries will (necessarily) increase their welfare. This can occur in the first-best models in situations where the revenue requirement in one country is higher while marginal damage is lower than in the other country. In the second-best models, pollution increases when marginal damage coefficients are equal and the revenue requirements differ.

Furthermore, the simulations show that by introducing sidepayments starting from an initial Nash-bargaining equilibrium in terms of cooperatively set tax rates in the first-best case, ceteris paribus, the country with the higher revenue requirement will make the sidepayment. This country will reduce its tax on good x and consume more
of x (and raise its tax on y substantially), while the other country will raise its tax on
good x. These conclusions are reversed in the second-best case. The country with the
lower revenue requirement will pay the other country. When the two countries are
equal except as regards the marginal damage done by the pollution, the country with
the lower marginal damage will receive the sidepayment. This holds in both the first-
best and the second-best case.

5.5 CONCLUSIONS

In this chapter, it has been investigated how countries can cooperate in reducing
transboundary pollution like the enhanced greenhouse effect, which is caused to a large
extent by CO₂-emissions. In order to take into account the complicating problem that
countries already tax fossil fuels, the main source of CO₂ emissions, a two country
second-best model has been used. The essence of this second-best model is that the
authorities have to use distortionary taxation to raise the revenue they need because it
is assumed that no first-best non-distortionary taxation can be used. Therefore, the
polluting good is already taxed, like the existing taxes on fossil fuels, and an initial
second-best solution is assumed to exist before the pollution problem is discovered and
a pollution tax is introduced as an instrument to reduce environmental damage. The
revenue requirement of each country is assumed to be an exogenous variable (which
is not necessarily equal in both countries). The pollution which results from the
consumption of the dirty good occurs in both countries, regardless of the country from
which it emanates, and reduces welfare. The damage caused in the two countries can
differ between the two countries.

The tax on the polluting good can be split up in a Pigouvian tax which is levied
to reduce pollution and a Ramsey part which is intended to raise revenue. The main
conclusion from the first sections (section 2 and 3) is that countries can increase their
welfare when they cooperate in reducing emissions instead of acting on their own.
With cooperation they will increase the Pigouvian part of the tax which they levy on
the dirty good. However, these Pigouvian taxes differ between the two countries when
their revenue requirements differ.

Cooperation can be extended when countries use sidepayments when they cooperate. In that case, one country pays the other country, reduces its Pigouvian tax and consumes more of the dirty good (and therefore pollutes more) while the other country raises its Pigouvian tax and consumes less of the dirty good. With sidepayments, the Pigouvian taxes in both countries will be equal, even if the revenue requirements and the damage functions differ. The Ramsey taxes however will still differ (with different revenue requirements) therefore the aggregate taxes on the polluting good differ as well between the two countries (this in contrast with cooperation in a first-best world).

Unfortunately, the general model does not tell us how the tax structure in both countries will change when they cooperate (with and without sidepayments). Therefore a more specific functional form has been used (a Cobb-Douglas utility function) to run several simulations with different damage functions and revenue requirements. The cooperative equilibrium analyzed is the Nash-bargaining solution, the results therefore are specific for this cooperative equilibrium. Simulations have been done with both non-distortionary taxation (first-best) and distortionary taxation (the second-best case). An interesting conclusion is that including sidepayments in agreements on emission abatement can actually increase pollution compared with agreements which do not include sidepayments. This can occur both in a first-best and in a second-best world.

The simulations show that in the first-best case, ceteris paribus, the country with the higher revenue requirement (and the higher tax on x) makes the sidepayment. This country reduces its tax on the polluting good and consumes more of it, while the other country raises its tax on the dirty good. These conclusions are reversed in the second-best case. The country with the lower revenue requirement pays the other country and pollution will increase.

When the two countries are equal except as regards the marginal damage done by pollution, the country with the lower marginal damage will receive the sidepayment. This holds in both the first-best and the second-best case.
Cooperation in a first-best world with sidepayments

$R_1 = 300$, $M C_1 = M C_2 = 0.1$

Diagram 5.4

Coordination in a first-best world with sidepayments

$R_1 = R_2 = 300$, $M C_2 = 1$

Diagram 5.5
Diagram 5.8

Diagram 5.9
<table>
<thead>
<tr>
<th>R2 = 300</th>
<th>MD1 = MD2 = 0.1</th>
</tr>
</thead>
</table>

### Table 5.1 Coordination with sidepayments in a first-best world.

#### Non-cooperative Nash-equilibrium

<table>
<thead>
<tr>
<th>Country 1</th>
<th>Cooperative equili. without sidepayments</th>
<th>Cooperative equilibrium with sidepayments</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>x1</td>
<td>y1</td>
</tr>
<tr>
<td>150</td>
<td>10.45</td>
<td>341.77</td>
</tr>
<tr>
<td>200</td>
<td>10.03</td>
<td>394.99</td>
</tr>
<tr>
<td>250</td>
<td>9.59</td>
<td>370.20</td>
</tr>
<tr>
<td>300</td>
<td>9.14</td>
<td>345.43</td>
</tr>
<tr>
<td>350</td>
<td>8.67</td>
<td>320.77</td>
</tr>
<tr>
<td>400</td>
<td>8.18</td>
<td>295.91</td>
</tr>
<tr>
<td>450</td>
<td>7.67</td>
<td>271.16</td>
</tr>
</tbody>
</table>

| R1 | x2 | y2 | tx2 | x2 | y2 | tx2 | Side |
| 150 | 8.61 | 345.70 | 39.16 | 5.49 | 347.25 | 62.22 | 5.64 | 342.43 | 59.68 | 9.5 |
| 200 | 8.78 | 345.61 | 38.39 | 5.67 | 347.16 | 60.18 | 5.78 | 343.91 | 58.50 | 6.4 |
| 250 | 8.95 | 345.52 | 37.60 | 5.87 | 347.07 | 58.16 | 5.92 | 345.39 | 57.31 | 3.3 |
| 300 | 9.14 | 345.43 | 36.80 | 6.07 | 346.96 | 56.14 | 6.07 | 346.96 | 56.14 | 0 |
| 350 | 9.34 | 345.33 | 35.99 | 6.29 | 346.85 | 54.13 | 6.23 | 348.48 | 54.93 | -3.2 |
| 400 | 9.54 | 345.23 | 35.17 | 6.52 | 346.74 | 52.15 | 6.40 | 350.00 | 53.69 | -6.4 |
| 450 | 9.77 | 345.12 | 34.34 | 6.77 | 346.61 | 50.17 | 6.58 | 351.46 | 52.43 | -9.5 |

### Table 5.2 Coordination in a first-best world with sidepayments.

#### Non-cooperative Nash-equilibrium

<table>
<thead>
<tr>
<th>Country 1</th>
<th>Cooperative equili. without sidepayments</th>
<th>Cooperative equilibrium with sidepayments</th>
</tr>
</thead>
<tbody>
<tr>
<td>MD1</td>
<td>x1</td>
<td>y1</td>
</tr>
<tr>
<td>0.90</td>
<td>2.60</td>
<td>498.25</td>
</tr>
<tr>
<td>0.95</td>
<td>2.46</td>
<td>498.30</td>
</tr>
<tr>
<td>1.00</td>
<td>2.33</td>
<td>498.33</td>
</tr>
<tr>
<td>1.05</td>
<td>2.22</td>
<td>498.37</td>
</tr>
<tr>
<td>1.10</td>
<td>2.11</td>
<td>498.40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Country 2</th>
<th>Cooperative equili. without sidepayments</th>
<th>Cooperative equilibrium with sidepayments</th>
</tr>
</thead>
<tbody>
<tr>
<td>MD1</td>
<td>x2</td>
<td>y2</td>
</tr>
<tr>
<td>0.90</td>
<td>2.22</td>
<td>498.39</td>
</tr>
<tr>
<td>0.95</td>
<td>2.28</td>
<td>498.36</td>
</tr>
<tr>
<td>1.00</td>
<td>2.33</td>
<td>498.33</td>
</tr>
<tr>
<td>1.05</td>
<td>2.38</td>
<td>498.31</td>
</tr>
<tr>
<td>1.10</td>
<td>2.43</td>
<td>498.29</td>
</tr>
</tbody>
</table>

**Table 5.1** Coordination with sidepayments in a first-best world.

**Table 5.2** Coordination in a first-best world with sidepayments.

**R1 = R2 = 300  MD2 = 1.0**
Table 5.3 Coordination in a first-best world with sidepayments.

<table>
<thead>
<tr>
<th>R1</th>
<th>x1</th>
<th>y1</th>
<th>tx1</th>
<th>x1</th>
<th>y1</th>
<th>tx1</th>
<th>side</th>
</tr>
</thead>
<tbody>
<tr>
<td>220</td>
<td>2.60</td>
<td>388.70</td>
<td>148.67</td>
<td>1.75</td>
<td>389.12</td>
<td>220.85</td>
<td>1.69</td>
</tr>
<tr>
<td>260</td>
<td>2.51</td>
<td>368.75</td>
<td>146.20</td>
<td>1.68</td>
<td>369.16</td>
<td>219.30</td>
<td>1.62</td>
</tr>
<tr>
<td>300</td>
<td>2.41</td>
<td>348.80</td>
<td>143.73</td>
<td>1.60</td>
<td>349.20</td>
<td>217.92</td>
<td>1.55</td>
</tr>
<tr>
<td>340</td>
<td>2.31</td>
<td>328.84</td>
<td>141.23</td>
<td>1.51</td>
<td>329.24</td>
<td>216.67</td>
<td>1.48</td>
</tr>
<tr>
<td>380</td>
<td>2.21</td>
<td>308.89</td>
<td>138.71</td>
<td>1.43</td>
<td>309.29</td>
<td>215.57</td>
<td>1.41</td>
</tr>
<tr>
<td>420</td>
<td>2.11</td>
<td>288.95</td>
<td>136.14</td>
<td>1.34</td>
<td>289.33</td>
<td>214.71</td>
<td>1.34</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>R1</th>
<th>x2</th>
<th>y2</th>
<th>tx2</th>
<th>x2</th>
<th>y2</th>
<th>tx2</th>
<th>side</th>
</tr>
</thead>
<tbody>
<tr>
<td>220</td>
<td>2.32</td>
<td>373.84</td>
<td>160.49</td>
<td>1.49</td>
<td>374.25</td>
<td>249.54</td>
<td>1.56</td>
</tr>
<tr>
<td>260</td>
<td>2.35</td>
<td>373.82</td>
<td>157.87</td>
<td>1.54</td>
<td>374.23</td>
<td>242.64</td>
<td>1.59</td>
</tr>
<tr>
<td>300</td>
<td>2.39</td>
<td>373.80</td>
<td>155.21</td>
<td>1.58</td>
<td>374.21</td>
<td>235.89</td>
<td>1.62</td>
</tr>
<tr>
<td>340</td>
<td>2.44</td>
<td>373.78</td>
<td>152.50</td>
<td>1.63</td>
<td>374.19</td>
<td>229.17</td>
<td>1.66</td>
</tr>
<tr>
<td>380</td>
<td>2.48</td>
<td>373.76</td>
<td>149.77</td>
<td>1.67</td>
<td>374.16</td>
<td>222.50</td>
<td>1.69</td>
</tr>
<tr>
<td>420</td>
<td>2.53</td>
<td>373.74</td>
<td>147.01</td>
<td>1.73</td>
<td>374.14</td>
<td>215.87</td>
<td>1.73</td>
</tr>
</tbody>
</table>

Country 1

Country 2

R2 = 250  MD1 = 0.95  MD2 = 1.0
Table 5.4  Coordination in a second-best world with sidepayments.

R2 = 300  MD1 = MD2 = 0.1
Table 5.5 Coordination with sidepayments, from first- to second-best.
R1 = 200  R2=300  MD1 = MD2 = 0.1

<table>
<thead>
<tr>
<th>Country 1</th>
<th>Non-cooperative Nash-equilibrium</th>
<th>Cooperative equilibrium without sidepayments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FB x1  tx1  y1  ty1  px/py1</td>
<td>FB x1  tx1  y1  ty1  px/py1</td>
</tr>
<tr>
<td>0.0</td>
<td>10.03 38.38 394.99 0.00 39.38</td>
<td>6.83 57.10 396.59 0.00 58.10</td>
</tr>
<tr>
<td>0.1</td>
<td>10.03 37.77 401.15 -0.03 40.00</td>
<td>6.83 56.17 402.91 -0.03 59.03</td>
</tr>
<tr>
<td>0.2</td>
<td>10.03 37.16 407.32 -0.06 40.62</td>
<td>6.82 55.26 409.24 -0.06 59.97</td>
</tr>
<tr>
<td>0.3</td>
<td>10.02 36.56 413.48 -0.09 41.25</td>
<td>6.82 54.35 415.57 -0.09 60.91</td>
</tr>
<tr>
<td>0.4</td>
<td>10.02 35.96 419.65 -0.12 41.88</td>
<td>6.82 53.45 421.89 -0.12 61.87</td>
</tr>
<tr>
<td>0.5</td>
<td>10.02 35.35 425.82 -0.14 42.51</td>
<td>6.81 52.56 428.22 -0.15 62.84</td>
</tr>
<tr>
<td>0.6</td>
<td>10.01 34.76 431.99 -0.17 43.15</td>
<td>6.81 51.66 434.55 -0.17 63.81</td>
</tr>
<tr>
<td>0.7</td>
<td>10.01 34.17 438.16 -0.20 43.79</td>
<td>6.80 50.78 440.88 -0.20 64.80</td>
</tr>
<tr>
<td>0.8</td>
<td>10.00 33.57 444.34 -0.22 44.44</td>
<td>6.80 49.81 447.22 -0.23 65.80</td>
</tr>
<tr>
<td>0.9</td>
<td>9.99 32.99 450.51 -0.25 45.10</td>
<td>6.79 49.03 453.55 -0.25 66.81</td>
</tr>
<tr>
<td>1.0</td>
<td>9.98 32.40 456.69 -0.27 45.76</td>
<td>6.78 48.17 459.89 -0.28 67.83</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Country 2</th>
<th>Non-cooperative Nash-equilibrium</th>
<th>Cooperative equilibrium without sidepayments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FB x2  tx2  y2  ty2  px/py2</td>
<td>FB x2  tx2  y2  ty2  px/py2</td>
</tr>
<tr>
<td>0.0</td>
<td>8.78  38.39 345.61 0.00 39.39</td>
<td>5.67  60.18 347.16 0.00 58.10</td>
</tr>
<tr>
<td>0.1</td>
<td>8.78  38.25 346.84 -0.01 39.53</td>
<td>5.67  59.94 348.55 -0.01 59.03</td>
</tr>
<tr>
<td>0.2</td>
<td>8.78  38.10 348.07 -0.01 39.66</td>
<td>5.68  59.70 349.93 -0.02 60.03</td>
</tr>
<tr>
<td>0.3</td>
<td>8.78  37.96 349.29 -0.02 39.80</td>
<td>5.68  59.43 351.31 -0.02 61.03</td>
</tr>
<tr>
<td>0.4</td>
<td>8.78  37.81 350.52 -0.03 39.93</td>
<td>5.68  59.17 352.69 -0.03 62.03</td>
</tr>
<tr>
<td>0.5</td>
<td>8.78  37.66 351.75 -0.03 40.06</td>
<td>5.68  58.91 354.07 -0.04 63.03</td>
</tr>
<tr>
<td>0.6</td>
<td>8.78  37.51 352.97 -0.04 40.19</td>
<td>5.68  58.64 355.45 -0.05 64.03</td>
</tr>
<tr>
<td>0.7</td>
<td>8.78  37.36 354.20 -0.05 40.31</td>
<td>5.68  58.37 356.83 -0.05 65.03</td>
</tr>
<tr>
<td>0.8</td>
<td>8.78  37.20 355.42 -0.06 40.43</td>
<td>5.68  58.09 358.21 -0.06 66.03</td>
</tr>
<tr>
<td>0.9</td>
<td>8.79  37.04 356.65 -0.06 40.56</td>
<td>5.68  57.81 359.59 -0.07 67.03</td>
</tr>
<tr>
<td>1.0</td>
<td>8.80  36.89 357.87 -0.07 40.68</td>
<td>5.70  57.52 360.97 -0.08 68.03</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Country 2</th>
<th>Cooperative equilibrium with sidepayments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FB x2  tx2  y2  ty2  px/py2 Side</td>
</tr>
<tr>
<td>0.0</td>
<td>5.78  58.51 343.91 0.00 59.51 6.4</td>
</tr>
<tr>
<td>0.1</td>
<td>5.78  58.35 345.02 -0.01 59.72 6.3</td>
</tr>
<tr>
<td>0.2</td>
<td>5.77  58.28 346.39 -0.01 60.03 6.8</td>
</tr>
<tr>
<td>0.3</td>
<td>5.76  58.25 347.94 -0.02 60.42 5.1</td>
</tr>
<tr>
<td>0.4</td>
<td>5.74  58.25 349.78 -0.03 60.89 4.1</td>
</tr>
<tr>
<td>0.5</td>
<td>5.73  58.29 351.86 -0.04 61.45 2.9</td>
</tr>
<tr>
<td>0.6</td>
<td>5.70  58.35 354.32 -0.04 62.11 1.4</td>
</tr>
<tr>
<td>0.7</td>
<td>5.68  58.46 357.27 -0.06 62.93 -0.5</td>
</tr>
<tr>
<td>0.8</td>
<td>5.65  58.51 360.41 -0.07 63.77 -2.4</td>
</tr>
<tr>
<td>0.9</td>
<td>5.62  58.57 364.03 -0.08 64.74 -4.6</td>
</tr>
<tr>
<td>1.0</td>
<td>5.59  58.58 367.97 -0.09 65.77 -6.9</td>
</tr>
</tbody>
</table>
### Table 5.6 Coordination in a second-best world with sidepayments.

R1 = R2 = 300 \hspace{1cm} MD2 = 1.0

<table>
<thead>
<tr>
<th>MD1</th>
<th>x1</th>
<th>tx1</th>
<th>y1</th>
<th>ty1</th>
<th>px/py1</th>
<th>x1</th>
<th>tx1</th>
<th>y1</th>
<th>ty1</th>
<th>px/py1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>3.31</td>
<td>99.80</td>
<td>363.36</td>
<td>-0.08</td>
<td>109.88</td>
<td>2.38</td>
<td>139.03</td>
<td>364.29</td>
<td>-0.08</td>
<td>153.04</td>
</tr>
<tr>
<td>0.8</td>
<td>2.92</td>
<td>113.31</td>
<td>363.75</td>
<td>-0.08</td>
<td>124.74</td>
<td>2.04</td>
<td>162.35</td>
<td>364.63</td>
<td>-0.09</td>
<td>178.69</td>
</tr>
<tr>
<td>0.9</td>
<td>2.60</td>
<td>127.35</td>
<td>364.07</td>
<td>-0.08</td>
<td>140.19</td>
<td>1.77</td>
<td>187.72</td>
<td>364.90</td>
<td>-0.09</td>
<td>206.59</td>
</tr>
<tr>
<td>1.0</td>
<td>2.33</td>
<td>141.94</td>
<td>364.33</td>
<td>-0.09</td>
<td>156.23</td>
<td>1.54</td>
<td>215.31</td>
<td>365.13</td>
<td>-0.09</td>
<td>236.94</td>
</tr>
<tr>
<td>1.1</td>
<td>2.11</td>
<td>157.05</td>
<td>364.56</td>
<td>-0.09</td>
<td>172.86</td>
<td>1.35</td>
<td>245.31</td>
<td>365.31</td>
<td>-0.09</td>
<td>269.94</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MD1</th>
<th>x1</th>
<th>tx1</th>
<th>y1</th>
<th>ty1</th>
<th>px/py1</th>
<th>Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>2.02</td>
<td>163.82</td>
<td>364.64</td>
<td>-0.09</td>
<td>180.30</td>
<td>-68</td>
</tr>
<tr>
<td>0.8</td>
<td>1.83</td>
<td>181.09</td>
<td>364.84</td>
<td>-0.09</td>
<td>199.30</td>
<td>-43</td>
</tr>
<tr>
<td>0.9</td>
<td>1.67</td>
<td>198.56</td>
<td>365.00</td>
<td>-0.09</td>
<td>218.52</td>
<td>-21</td>
</tr>
<tr>
<td>1.0</td>
<td>1.54</td>
<td>215.31</td>
<td>365.13</td>
<td>-0.09</td>
<td>236.94</td>
<td>0</td>
</tr>
<tr>
<td>1.1</td>
<td>1.43</td>
<td>232.38</td>
<td>365.24</td>
<td>-0.09</td>
<td>255.72</td>
<td>19</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MD1</th>
<th>x2</th>
<th>tx2</th>
<th>y2</th>
<th>ty2</th>
<th>px/py2</th>
<th>x2</th>
<th>tx2</th>
<th>y2</th>
<th>ty2</th>
<th>px/py2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>1.95</td>
<td>169.85</td>
<td>364.72</td>
<td>-0.09</td>
<td>186.94</td>
<td>1.14</td>
<td>292.50</td>
<td>365.53</td>
<td>-0.09</td>
<td>321.86</td>
</tr>
<tr>
<td>0.8</td>
<td>2.10</td>
<td>158.03</td>
<td>364.57</td>
<td>-0.09</td>
<td>173.94</td>
<td>1.29</td>
<td>258.04</td>
<td>365.38</td>
<td>-0.09</td>
<td>283.94</td>
</tr>
<tr>
<td>0.9</td>
<td>2.22</td>
<td>149.82</td>
<td>364.44</td>
<td>-0.09</td>
<td>164.02</td>
<td>1.42</td>
<td>233.54</td>
<td>365.25</td>
<td>-0.09</td>
<td>257.00</td>
</tr>
<tr>
<td>1.0</td>
<td>2.33</td>
<td>141.94</td>
<td>364.33</td>
<td>-0.09</td>
<td>156.23</td>
<td>1.54</td>
<td>215.32</td>
<td>365.13</td>
<td>-0.09</td>
<td>236.96</td>
</tr>
<tr>
<td>1.1</td>
<td>2.43</td>
<td>136.23</td>
<td>364.24</td>
<td>-0.08</td>
<td>149.95</td>
<td>1.65</td>
<td>201.34</td>
<td>365.02</td>
<td>-0.09</td>
<td>221.57</td>
</tr>
</tbody>
</table>

**Country 1**

<table>
<thead>
<tr>
<th>MD1</th>
<th>x2</th>
<th>tx2</th>
<th>y2</th>
<th>ty2</th>
<th>px/py2</th>
<th>Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>1.40</td>
<td>236.57</td>
<td>365.26</td>
<td>-0.09</td>
<td>260.33</td>
<td>68</td>
</tr>
<tr>
<td>0.8</td>
<td>1.46</td>
<td>226.84</td>
<td>365.20</td>
<td>-0.09</td>
<td>249.63</td>
<td>43</td>
</tr>
<tr>
<td>0.9</td>
<td>1.51</td>
<td>219.93</td>
<td>365.16</td>
<td>-0.09</td>
<td>242.02</td>
<td>21</td>
</tr>
<tr>
<td>1.0</td>
<td>1.54</td>
<td>215.32</td>
<td>365.13</td>
<td>-0.09</td>
<td>236.96</td>
<td>0</td>
</tr>
<tr>
<td>1.1</td>
<td>1.57</td>
<td>211.72</td>
<td>365.10</td>
<td>-0.09</td>
<td>232.99</td>
<td>-19</td>
</tr>
</tbody>
</table>
APPENDIX TO CHAPTER 5: NON-COOPERATIVE NASH-EQUILIBRIUM

Sign of the reaction curve in the non-cooperative Nash equilibrium (section 5.3)

The reaction curve of the non-cooperative Nash equilibrium is given by equation 5.7:

$$\frac{dt_1}{dt_2} = (y_1 + y_{ty1}) D_{t_{x1}t_{x2}} x_{tx2} - (x_{tx1} + x_{tx1}) x_{ty1}$$

$$(y_1 + y_{ty1})$$ and $$(x_{tx1} + x_{tx1})$$ are the changes in revenue of a marginal change in tax $t_{x1}$ or $t_{y1}$, $\partial R/\partial t$. The Lagrange multiplier $\mu$ is the change in welfare due to a slight change in the constraint $R$. A lower revenue requirement means a higher welfare level, therefore $\mu = \partial V/\partial R > 0$. $\mu$ can be written as:

$$\mu = \frac{\partial V/\partial t}{-\partial R/\partial t} \geq 0$$

$V_t \leq 0$, therefore $\partial R/\partial t \geq 0$.

If revenue would rise with a fall in one of the taxes, this tax would not be optimal: lowering the tax would raise the welfare level while at the same time the revenue constraint would be fullfilled as well. Therefore, tax revenue will rise in equilibrium when one of the taxes is raised.

$D_{x1x2}$ is equal to $D_{x1x1}$ which was assumed to be negative. $x_{tx}$ and $y_{ty}$ are negative in both countries, $x_{ty1}$ is assumed to be positive (a rise in the price of one good raises demand for the other good). As a result, $dt_1/dt_2$ is negative.

Existence of the non-cooperative Nash equilibrium (section 5.3)

The existence of the non-cooperative Nash equilibrium is proven by showing that that the equilibrium is asymptotically stable (see Fudenberg and Tyrole 1993, p.24). Sufficient condition for this is that:
The second derivatives of the welfare functions on the right hand side of equation A.4 also contain the second derivative of the damage functions. These cancel out the terms on the left hand side. Consequently, A.4 can be written as:

\[
[V_{1\text{tx1tx1}} + D_{1\text{tx1tx1}} + \mu_1(2x_{\text{tx1tx1}} t_{\text{tx1tx1}} y_{\text{tx1tx1}} y_{\text{tx1tx1}} ty_{\text{tx1tx1}})] > 0
\]

\[
[V_{2\text{tx2tx2}} + D_{2\text{tx2tx2}} + \mu_2(2x_{\text{tx2tx2}} t_{\text{tx2tx2}} y_{\text{tx2tx2}} y_{\text{tx2tx2}} ty_{\text{tx2tx2}})] > 0
\]

Both terms are negative when the utility functions are concave and the constraints are convex. Consequently, inequality A.5 holds and therefore the Nash equilibrium is asymptotically stable.