Strategic interactions in environmental economics

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Chapter 4

Modeling strategic responses to car and fuel taxation*

4.1 Introduction

Cars and fuels used for their propulsion are among the most heavily taxed consumer goods in many countries. In the Netherlands, for example, 42 percent of the consumer price of a new car and 71 percent of the consumer price of gas are taxes. As a result 13 percent of tax revenues accrues from car use and ownership. Similar figures are observed throughout the European Union. The large stakes at hand and the central role of car ownership and use in daily life make that car and fuel taxation is a subject of continuous public debate and lobbying.

In the economics literature four types of contributions can be identified that are relevant for analyzing car and fuel taxation. The first type of contributions are studies that estimate the price elasticity of kilometrage, a parameter which is crucial in projecting consumers' responses to tax changes. This literature shows a wide range of estimates and suggests that the elasticity is generally small but non-negligible; see *e.g.* Goodwin (1992) and Greene, Kahn and Gibson (1999). The second type of contributions is applied research that focuses on the environmental impact of car use and various types of fuel; see for example Crawford and Smith(1995) and Michealis (1995).[1] Typically, this

*This Chapter is a version of Heijnen and Kooreman (2006).

[1] Most EU countries have implemented a differential tax treatment (DTT) favoring the use of diesel. A notable exception is the United Kingdom that – apart from a slightly *higher* excise on diesel fuel, treats diesel and gasoline cars equally. The general conclusion from the literature is that there is no unambiguous net difference between diesel and gasoline in terms of the various emissions, taking into account differences in engine and fuel efficiency.

literature focuses on 'first-order' effects, and ignores any possible behavioral effects in response to changes in the tax regime.

A third type of contributions addresses the question of optimal vehicle and fuel taxation. In the case of car taxation, it is prohibitively costly (as yet) to monitor how much of each pollutant each car produces. Several articles (most notably Fullerton and West, 2002) have investigated second-best taxes in which different types of cars and different types of fuels can be taxed differently. Fullerton and West conclude, that even though the number of instruments is much smaller than the number of pollutants, welfare in the second-best case can come close to first-best welfare. de Borger (2001) examines a similar question, but with more emphasis on the decision to buy a car. The government is restricted to a lump-sum tax on car-ownership and excise on fuel. Due to the fact that pollution is directly related to the number of kilometers driven, the optimal taxes are essentially Pigovian. Calthrop and Proost (2003) provide a good summary of the optimal taxation of cars. In these kind of studies, neither the car market nor the fuel market are allowed to be imperfect.

A fourth type of contributions – more recent and small in number – considers potential responses to taxation from an industrial organization perspective. The most notable paper in this respect is Verboven (2002), who provides strong empirical evidence that car manufacturers take advantage of the favorable differential tax treatment (DTT) for diesel fuel by increasing the mark-up for diesel cars. More specifically, he finds that 75 to 90 percent of the price differential between gasoline and diesel cars can be explained by mark-up differences. Clearly, such responses might well mitigate the intended effect of the policy.

While Verboven's analysis is an important step towards a more complete analysis of the taxation effects, additional steps are necessary. First, Verboven assumes that the price elasticity of demand for kilometrage is exactly zero. While the empirical literature has generated small price elasticities for kilometrage demand on average, the assumption of completely inelastic demand is unattractive in any analysis on taxation. Even when elasticities are small, behavioral effects can be substantial when taxes are high (note that in many countries taxes almost double consumer prices).[2] Secondly, it is likely

DTT might serve as an implicit subsidy on freight transportation in an attempt to boost a country's competitiveness. Such a policy, however, is known to be inefficient from a supranational point of view.

[2] In theory, moreover, a zero price elasticity of kilometrage would allow fuel producers

that DTT also affects the pricing behavior of fuel companies, an aspect not analyzed in Verboven.

The purpose of this Chapter is to set further steps to a coherent economic analysis of car and fuel taxation by developing a model that describes the interactions between the actors involved: consumers, car producers, fuel producers and the government. The model also provides a framework for a normative analysis of optimal car and fuel taxation.

A possible long-term effect of high fuel prices is the introduction of more fuel efficient cars. However, the impact of fuel prices on car design in general and fuel efficiency in particular is doubtful: Goldberg (1998) examines the effects of minimum quality standards, in particular a requirement that the corporate average fuel efficiency (CAFE) exceeds 26 miles per gallon. Her conclusion is that an immense increase (780%) in gasoline prices is needed to have the same effect on the efficiency of cars as CAFE. I therefore take fuel efficiency as given.

Section 2 describes the consumer side of the model. I focus on the consumer's choice between two versions of a car that differ in engine type. The two types require different fuels and have (possibly) different fuel efficiency. This leads to a mixed discrete-continuous model of consumer behavior, with a condition for the optimal fuel type, and the optimal kilometrage conditional on fuel type. After allowing for preference heterogeneity, the consumer model yields population fractions of fuel types and a kilometrage distribution, given consumer car prices and consumer fuel prices (i.e. inclusive of taxes). Section 3 considers the behavior of car manufacturers and fuel producers. In both markets I consider the monopoly as well as the full competitiveness case. For each of the four possible configurations, I derive the optimal price levels, conditional on the behavior of all other actors in the market and the government. Section 4 then characterizes (Nash) equilibria conditional on tax levels. Section 5 discusses the issue of optimal car and fuel taxation, when the government focuses on environmental as well as on budgetary targets. In particular, I investigate the effects of a tax policy in which car taxes fully depend on car use. Section 6 concludes.

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to increase fuel prices (and profits) beyond limits (unless the fuel market would be fully competitive).

4.2 The consumer

Consider the following indirect utility function (see Appendix 4.A for the derivation):

$$u_{jk} = y - \rho p^*_{jk} - \tau_{jk} + a_{jk} + \frac{1}{\lambda}\mathrm{e}^{-\lambda\pi_{jk}}\theta^j_0 + v_j, \tag{4.1}$$

where j denotes the car model, $k = G, D$ engine type, gasoline or diesel, u_{jk} utility derived from owning and using car model (j,k), y is income, ρ an annualization coefficient[3], p^*_{jk} price of car (j,k), including sales tax, τ_{jk} annual lump-sum tax on car (j,k), a_{jk} mean intrinsic utility of consuming (j,k), λ a price sensitivity parameter, θ^j_0 a heterogeneity parameter, and π_{jk} fuel cost of driving one kilometer, the marginal cost of driving. It is the product of the inverse fuel efficiency of a car (in liters/km, denoted by w_{jk}) and the consumer fuel price per liter, denoted by r^*_k. Thus $\pi_{jk} = w_{jk} \cdot r^*_k$ where the consumer fuel price is calculated as $r^*_k = (r_k + t_k)(1 + t_{vat})$; r_k the before tax fuel price set by the fuel producer, t_k excise and t_{vat} value added taxes.

The error term v_j has zero expectation, and measures the unobserved utility of consuming a car of model j. Given my focus on the choice between the two types of engines, I make the simplifying assumption that cars only differ in engine type and in the error v_j. Since the latter cancels out in the utility difference $u_{jD} - u_{jG}$, I suppress the subscript j in the sequel.

From Roy's identity it follows that the demand for kilometrage equals:

$$\theta_k = \theta_0 \mathrm{e}^{-\lambda\pi_k}, \tag{4.2}$$

where θ_k denotes the amount the owner of a car with engine k drives given π_k. Note that $\pi_D < \pi_G$ implies $\theta_D > \theta_G$. The price elasticity ϵ is defined as:

$$\epsilon = \frac{\mathrm{d}\theta_k}{\mathrm{d}\pi_k}\frac{\pi_k}{\theta_k} = -\lambda\theta_k\frac{\pi_k}{\theta_k} = -\lambda\pi_k. \tag{4.3}$$

The higher the variable cost (π_k) of using engine type k, the more sensitive the consumer is to price changes.

Consumer heterogeneity is introduced through θ_0, which can be interpreted as the amount of kilometers a car owner would drive if $\pi_k = 0$. Let $\theta_0 \in \Theta \subset \mathbb{R}^+$. F is the cumulative distribution function of Θ.

[3]See Hausman (1979). The annualization coefficient is defined as follows. Suppose a product that costs one euro lasts T years and is also replaced every T years. A consumer has an implicit discount rate of r. Let ρ be such that this consumer is indifferent between paying one euro every T years or ρ euro every year. It can be shown that $\rho = \frac{r}{1+r} \times \frac{1}{1-(1+r)^{-T}}$.

There exists a unique θ_0, denoted as θ_*, such that $u_D = u_G$. Using (4.1) we obtain:

$$\theta_* = \frac{\Delta a - \rho\Delta p^* - \Delta\tau}{\frac{1}{\lambda}[\mathrm{e}^{-\lambda\pi_G} - \mathrm{e}^{-\lambda\pi_D}]}, \tag{4.4}$$

where $\Delta x = x_D - x_G$ for a variable x. Equation (4.4) is an extension of Verboven (2002, equation 2) If $\lambda \to 0$, then the denominator converges to $\Delta\pi$, as can be verified using the rule of L'Hôpital.

Consumers for which $\theta_0 < \theta_*$ choose a gasoline car. The market share of gasoline cars is defined as:

$$s_G = \mathrm{P}(\theta_0 < \theta_*) = F_\Theta(\theta_*). \tag{4.5}$$

Note that $s_G = 1 - s_D$ is the market share of diesel cars.

Let observed kilometrage demand be denoted by θ. Then:

$$\theta = \begin{cases} \theta_G & \text{if } \theta_0 < \theta_* \\ \theta_D & \text{else} \end{cases} \tag{4.6}$$

Note that in Verboven's model ($\lambda \to 0$) the demand for kilometrage is $\theta = \theta_0$, i.e. independent of fuel type.

The expected value of θ_0 given that $\theta_0 \geq \theta_*$ is:

$$\mathrm{E}[\theta_0|\theta_0 \geq \theta_*] = \frac{\int_{z\geq\theta_*} z f_\Theta(z)\mathrm{d}z}{1 - F_\Theta(\theta_*)}. \tag{4.7}$$

The average diesel car owner will drive:

$$\mathrm{E}\theta_D = \mathrm{E}[\theta_0|\theta_0 \geq \theta_*]\mathrm{e}^{-\lambda\pi_D}, \tag{4.8}$$

and will demand $w_D\mathrm{E}\theta_D$ liters of fuel. Similarly, demand for gasoline is given by:

$$w_G\mathrm{E}\theta_G = w_G\mathrm{E}[\theta_0|\theta_0 \leq \theta_*]\mathrm{e}^{-\lambda\pi_G}, \tag{4.9}$$

where

$$\mathrm{E}[\theta_0|\theta_0 \leq \theta_*] = \frac{\int_{z\leq\theta_*} z f_\Theta(z)\mathrm{d}z}{F_\Theta(\theta_*)}. \tag{4.10}$$

Note that utility goes to infinity if λ tends to zero. In this sense it does not converge to the model in Verboven (2002). However, as shown in Appendix 4.B, the difference in utility for two different cases (e.g. different prices) does converge to a finite number. I will normalize utility to zero for the case in which there is perfect competition in both markets. Utility in other cases will be the utility relative to this benchmark-case of perfect competition. Since a change in λ entails a change in the consumer's preferences, comparing utility levels for different λ's is not meaningful.

4.3 The producers

4.3.1 The car manufacturers

Given my interest in the effects of price and taxation differentials, I follow Verboven (2002) in using a stylized model of car pricing behavior that focuses on the price differentials between diesel and gasoline cars.

The firm equips the cars with either a gasoline engine or a diesel engine. The profit of the firm is given by:

$$\Pi_{car} = (p_G - c_G)s_G + (p_D - c_D)(1 - s_G), \tag{4.11}$$

where p_k is the price of car model $k = G, D$ before taxes and c_k the constant marginal cost of this model. The consumer price of car model k is given by $p_k^* = p_k(1 + t_k) + \beta_k$, where t_k denotes value added taxes on cars and β_k a lump-sum tax or subsidy on the sale of a car (not to be confused with the annual car tax).

One case I will consider is a fully competitive market with $p_G = c_G$ and $p_D = c_D$. As an alternative consider the car manufacturer to be a monopolist. The monopolist sets the price of the diesel version such that (4.11) is maximized, assuming that consumers do not substitute to other cars. Differentiating (4.11) with respect to p_D we obtain:

$$(p_G - c_G)\frac{\partial s_G}{\partial p_D} + (1 - s_G) - (p_D - c_D)\frac{\partial s_G}{\partial p_D} = 0. \tag{4.12}$$

Substituting

$$\frac{\partial s_G}{\partial p_D} = \frac{\partial s_G}{\partial \theta_*} \times \frac{\partial \theta_*}{\partial p_D^*} \times \frac{\partial p_D^*}{\partial p_D} = f_\Theta(\theta_*)\frac{-\rho(1 + t_D)}{\frac{1}{\lambda}[\mathrm{e}^{-\lambda\pi_G} - \mathrm{e}^{-\lambda\pi_D}]}$$

in (4.12) yields:

$$\Delta p = \Delta c - \frac{1 - F_\Theta(\theta_*)}{f_\Theta(\theta_*)} \times \frac{\mathrm{e}^{-\lambda\pi_G} - \mathrm{e}^{-\lambda\pi_D}}{\rho\lambda(1 + t_D)}. \tag{4.13}$$

Equation (4.13) is an extension of Verboven (2002, equation 8) that allows for a possibly non-zero price elasticity. Note that (4.13) is an implicit expression in Δp as θ_* is a function of Δp.

4.3.2 The fuel producers

As in the case of the car market, I will consider the competitive and the monopoly case in the fuel market. The profits of the firm are given by:

$$\Pi_{fuel} = (r_G - b_G)s_G w_G \mathrm{E}\theta_G + (r_D - b_D)s_D w_D \mathrm{E}\theta_D \tag{4.14}$$

In the competitive case, $r_G = b_G$ and $r_D = b_D$. In the monopoly case, the fuel producer maximizes (4.14) with respect to r_G and r_D.

In the determination of optimal fuel pricing different time dimensions might be distinguished. For example one could assume that in the short run changes in fuel prices affect kilometrage, but not the diesel/gasoline market shares. In the very long run one might also endogenize fuel efficiency (an extension not considered in this Chapter). In this Chapter, the focus will be on the long term; consumers can respond to changes in prices by switching between engine types.

4.4 Equilibria

I consider all four possible combinations of car market and fuel market forms:

Case 1: both markets are competitive

Case 2: the fuel market is competitive, the car market is monopolistic

Case 3: the fuel market is monopolistic, the car market is competitive

Case 4: both markets are monopolistic

4.4.1 Numerical aspects

Cases 2 and 4 require to solve Δp from (4.13). In general, the solution can only be obtained numerically. As a specification for $F_\Theta(\cdot)$ I choose the log-normal distribution, a distribution function that combines a parsimonious parameterization with a plausible shape. Appendix 4.C gives the required corresponding expressions for kilometrage conditional on fuel type. In all calculations I obtained (at most) one numerical solution to (4.13) corresponding with a profit maximum.

Cases 3 and 4 require to maximize (4.14) with respect to r_G and r_D. The first order conditions are highly nonlinear (recall that s_G, $\mathrm{E}\theta_G$, and $\mathrm{E}\theta_D$ all depend on r_D and/or r_G), with potentially multiple solutions. I therefore choose to maximize (4.14) using grid search.

4.4.2 Parameter values

The values of the exogenous variables in the 'base scenario' are given in Table 4.1. The distribution of θ_0 (the number of kilometers that would be driven if the marginal costs of driving were zero or if kilometrage demand would be fully price inelastic) is lognormal with parameters $\mu = 10$ and $\sigma = 0.6$, which implies a mean of 26370 kilometers and a standard deviation of 17359. In addition, I choose three different values of the price sensitivity parameters: $\lambda = 5$, $\lambda = 1$ and $\lambda = 0$. This implies a kilometrage elasticity of -0.4, -0.08 and 0, respectively, if the fuel price is 1 and the inverse fuel inefficiency is 0.08. This corresponds with the range of elasticities found in the empirical literature (cf. Goodwin, 1992). I take $\rho = 0.15$ (implying an annual discount rate of 6.64% when the lifetime is 10 years). These choices, in combination with the current Dutch tax rates, generate a kilometrage distribution that roughly mimics the observed kilometrage distribution in the Netherlands in terms of shape and moments (Personenautopanel 1999). [4]

4.4.3 Results

Table 4.1 reports the implied values of the endogenous variables for the tax base scenario (based on the current Dutch tax rates). Comparing the results for different values of λ for each case, we see that by adding the restriction that $\lambda = 0$ (i.e. the model as used in Verboven (2002)) we would have overestimated both the market share of diesel cars and the price difference between diesel and gasoline cars. The assumption that the elasticity of demand for kilometrage is not zero implies that owning a diesel car becomes more valuable and more consumers want to buy a diesel car. More price-sensitive consumers (on average) drive less and the optimal premium on diesel cars (i.e. Δp in Cases 2 and 4) is therefore lower.

The first three columns apply to Case 1 (both markets are competitive), and differ only in the value of the price sensitivity parameter λ. The larger price sensitivity in the first column leads to a lower diesel share $(1 - s_G)$, a smaller conditional kilometrage for both diesel and gasoline, and a lower total kilometrage. The price elasticities in the first column are five times as large as those in the second column (a fact that immediately follows from (4.3)). Given the competitiveness of both markets, prices are fully determined by production costs and taxes and do not depend on the price sensitivity parameter.

[4] The Kolmogorov-Smirnov statistic (KS-statistic) measures the maximal distance between the sample cumulative distribution function and the proposed true cumulative distribution function (Lindgren, 1993, pp. 479–481). The KS-statistic lies between zero (perfect fit) and one (bad fit). The data is taken from the Personenautopanel, a survey of car-owning households in the Netherlands. For each household yearly kilometrage is known. There are 2381 observations. Under the assumption of log-normality, the KS-statistic is 0.06. So, the fit is fairly good.

Table 4.1: Equilibrium values (tax base scenario)

		Case 1 fuel market comp. car market comp.			*Case 2* fuel market comp. car market mono.			*Case 3* fuel market mono. car market comp.		*Case 4* fuel market mono. car market mono.	
		$\lambda = 5$	$\lambda = 1$	$\lambda = 0$	$\lambda = 5$	$\lambda = 1$	$\lambda = 0$	$\lambda = 5$	$\lambda = 1$	$\lambda = 5$	$\lambda = 1$
Fuel prices before taxes	r_G	0.34	0.34	0.34	0.34	0.34	0.34	5.88	87.88	2.79	12.23
	r_D	0.30	0.30	0.30	0.30	0.30	0.30	2.61	14.31	2.31	13.19
Fuel prices after taxes	r^*_G	1.16	1.16	1.16	1.16	1.16	1.16	7.75	105.33	4.07	15.31
	r^*_D	0.74	0.74	0.74	0.74	074	0.74	3.48	17.41	3.13	16.08
Elasticities of km w.r.t. prices	e_G	-0.460	-0.093	0	-0.464	-0.093	0	-3.101	-8.427	-1.628	-1.225
	e_D	-0.222	-0.0440	0	-0.222	-0.044	0	-1.045	-1.044	-0.940	-0.965
Profit fuel company	Π_{fuel}	0	0	0	0	0	0	1208	7797	1097	7621
Threshold	θ_*	25920	19742	18434	41797	35356	34025	14531	2534	38656	26082
Exp. km gasoline	$\mathrm{E}\theta_G$	10140	12194	12721	13152	17602	18930	476	0	3962	4758
Exp. km diesel	$\mathrm{E}\theta_D$	33772	34505	34854	47299	49846	50688	11064	9281	21741	16138
Exp. km	$\mathrm{E}\theta$	19429	24965	26370	18030	24539	26370	8480	9279	7060	9186
Market share gasoline	s_G	0.6069	0.4276	0.3833	0.8572	0.7849	0.7657	0.2441	0.0002	0.8257	0.6109
Price difference cars before taxes	Δp	500	500	500	2606	3408	3219	500	500	2857	5844
Price difference cars after taxes	Δp^*	3087	3087	3087	6726	7785	8111	3087	3087	7160	12312
Tax revenue	R	3401	3657	3718	3393	3755	3856	3401	4868	3036	4674
Total fuel use	E	1289	1602	1680	1307	1749	1872	511	557	489	609
CO_2		4.79	5.99	6.28	4.78	6.41	6.87	1.93	2.11	1.80	2.26
Particulates		1.94	2.75	2.97	1.25	1.85	2.02	1.10	1.22	0.60	0.92
Exp. utility	U	0	0	0	-134	-259	-754	725	-16066	-2386	1773

Exogenous variables: $\rho = 0.15$, $\mu = 10$, $\sigma = 0.6$, $\Delta c = 500$, $w_G = 0.08$, $w_D = 0.06$, $b_G = 0.34$, $b_D = 0.30$, $\tau_D = 844$, $\tau_G = 416$, $t_G = 0.635$, $t_D = 0.323$, $t_{vat} = 0.19$, $\Delta\beta = 2223$

Columns 4, 5 and 6 in Table 4.1 report on Case 2, a car market monopoly combined with a competitive fuel market. The before tax mark-up difference between diesel and gasoline cars equals 81 percent of total price difference ((2606-500)/2606) if $\lambda = 1$, and 85 percent if $\lambda = 5$. Due to the relatively higher consumer price of diesel cars, the diesel share is lower under a car market monopoly. This is also the case for total kilometrage (although both conditional kilometrages increase).

The assumption of a fuel market monopoly yields to very high fuel prices, in particular when the car market is *not* a monopoly, and when price sensitivity is low (note that in this model fuel prices tend to infinity when $\lambda \rightarrow 0$). Note that a fuel market monopoly at least halves the total numbers of kilometers driven. If competition policy in both markets would induce a shift from Case 4 (both markets monopoly) to Case 1 (both markets competitive), for example, consumer utility increases, but tax revenues could decline while emissions (using total fuel use as a crude proxy) would increase. From a budgetary and environmental viewpoint some market power in the fuel and car market might therefore be a blessing in disguise.

The question which of the four cases best describes the actual situation is a difficult empirical one that I cannot address in this Chapter. On the basis of the overall plausibility of implied values of the endogenous variables, I conjecture that Case 2 (competitive fuel market, car market monopoly) with λ somewhat lower than 5 provides the best description of the stylized empirical facts: i.e. the market share of gasoline cars (approx. 85%), fuel prices (approx. €1.20 for gasoline and approx. €0.80 for diesel) and average kilometers driven (approx. $20,000$ kilometer).

Tables 4.2 and 4.3 display – in terms of elasticities – how the endogenous variables in the models respond to a change in one of the tax rates. I focus on the excise tax rate on diesel, t_D (Table 4.2) and on the annual lump-sum tax on diesel, τ_D (Table 4.3). In Case 1 (both markets are competitive) there are no strategic responses by producers, only an effect on consumer behavior. The increased tax rate on diesel decreases the total number of kilometers (elasticity -0.16) and increases total tax revenues (elasticity 0.02). The results for Case 2 (competitive fuel market, monopoly car market) show how strongly the strategic response on the part of the car manufacturer can affect the impact of the tax change. When the diesel tax rate increases, the car manufacturer responds by lowering the mark-up difference between diesel and gasoline car (i.e. by making diesel cars less expensive), as a result of which the decrease in total kilometrage and the increase in total tax revenues are much smaller than in Case 1. When the fuel market is a monopoly as well (Cases 3 and 4), the effects of strategic responses are even more pronounced. The fuel producer now responds by lowering the before tax fuel prices, in particular of gasoline. Note that this occurs both in case of an increase in the diesel fuel tax rate (Table 4.2) and in case of an increase lump-sum diesel tax (Table 4.3). This induces a substitution from diesel to gasoline cars, and a net *decrease* in total tax revenues in Case 4.

In Case 4 the fuel producer "competes" with the car manufacturer in extracting consumer surplus. As a result fuel prices and profits of the fuel producers in Case 4 are lower than in Case 3 (see Table 4.1).

In order to gain some further insight in the properties of the model and the effects of the various taxes, I have also simulated the effects of large changes in the various tax rates to reveal possibly non-linear patterns (the elasticities in Tables 4.2 and 4.3 are calculated on the basis of a one percent tax increase). Here I confine myself to Case 2 with $\lambda = 5$. While many of the effects are monotone, there also are some noteworthy nonlinear patterns, in particular the relationship between total fuel use and the excise tax rate on diesel fuel (t_D). Initially, total fuel use decreases when the diesel tax increases. For higher level of the diesel tax rate, however, total fuel use starts to *increase*. The explanation is that the high diesel tax rate induces an increasing proportion of consumers to opt for a gas version, which has a lower fuel efficiency.

4.5 The government and taxation

In setting tax rates, the government might be interested in several (partly conflicting) targets related to car ownership and use. The first one is annualized tax revenue. For each fuel type $i = D, G$, the tax revenue is the sum of three components: annual fixed ownership taxes τ_i, annualized purchase taxes $(\rho(p_i^* - p_i))$ and fuel taxes $(w_i(r_i^* - r_i)\mathrm{E}\theta_i)$. Thus total tax revenue R is:

$$R = \sum_{i=D,G} s_i \left\{\tau_i + \rho(p_i^* - p_i) + w_i(r_i^* - r_i)\mathrm{E}\theta_i\right\} \tag{4.15}$$

The second aspect is total pollution defined as:

$$E = \sum_{i=D,G} s_i w_i v_i \mathrm{E}\theta_i, \tag{4.16}$$

where v_i $(i = D, G)$ are fuel specific pollution intensities. Both fuels emit a wide range of pollutants CO_2, CO, NO_x, HCs, SO_2 and particulates.

Table 4.2: Tax elasticities w.r.t. t_D ($\lambda = 5$)

		Case 1 fuel market comp. car market comp.	*Case 2* fuel market comp. car market mono.	*Case 3* fuel market mono. car market comp.	*Case 4* fuel market mono. car market mono.
Fuel prices before taxes	r_G	0.00	0.00	-0.323	-0.065
	r_D	0.00	0.00	-0.038	-0.004
Fuel prices after taxes	r_G^*	0.00	0.00	-0.292	-0.053
	r_D^*	0.518	0.518	0.076	0.119
Elasticities of km w.r.t. prices	e_G	0.000	0.000	-0.292	-0.053
	e_D	0.518	0.518	0.076	0.119
Profit fuel company	Π_{fuel}	0	0	-0.134	-0.067
Threshold	θ_*	0.540	0.347	0.225	0.193
Exp. km gasoline	$E\theta_G$	0.345	0.152	1.090	0.177
Exp. km diesel	$E\theta_D$	0.224	0.152	0.004	0.033
Exp. km	$E\theta$	-0.158	-0.095	-0.128	-0.108
Market share gasoline	s_G	0.568	0.152	0.483	0.100
Price difference cars before taxes	Δp	0.000	-0.393	0.000	-0.240
Price difference cars after taxes	Δp^*	0.000	-0.272	0.000	-0.166
Tax revenue	R	0.021	0.003	0.013	-0.011
Total fuel use	E	-0.055	-0.026	-0.120	-0.056
Exp. utility	U	0.171	0.061	-0.142	0.011

Exogenous variables: $\rho = 0.15$, $\mu = 10$, $\sigma = 0.6$, $\Delta c = 500$, $w_G = 0.08$, $w_D = 0.06$, $b_G = 0.34$, $b_D = 0.30$, $\tau_D = 844$, $\tau_G = 416$, $t_G = 0.635$, $t_D = 0.323$, $t_{vat} = 0.19$, $\Delta\beta = 2223$

Table 4.3: Tax elasticities w.r.t. τ_D ($\lambda = 5$)

		Case 1 fuel market comp. car market comp.	*Case 2* fuel market comp. car market mono.	*Case 3* fuel market mono. car market comp.	*Case 4* fuel market mono. car market mono.
Fuel prices before taxes	r_G	0.00	0.00	-1.548	-0.111
	r_D	0.00	0.00	-0.307	-0.169
Fuel prices after taxes	r_G^*	0.00	0.00	-1.397	-0.091
	r_D^*	0.00	0.00	-0.273	-0.148
Elasticities of km w.r.t. prices	e_G	0.000	0.000	-1.397	-0.091
	e_D	0.000	0.000	-0.273	-0.148
Profit fuel company	Π_{fuel}	0	0	-0.170	-0.077
Threshold	θ_*	0.947	0.574	1.274	0.506
Exp. km gasoline	$E\theta_G$	0.605	0.250	5.486	0.387
Exp. km diesel	$E\theta_D$	0.596	0.442	0.763	0.519
Exp. km	$E\theta$	-0.139	-0.086	-0.008	-0.086
Market share gasoline	s_G	0.993	0.250	2.731	0.260
Price difference cars before taxes	Δp	0.000	0.069	0.000	0.152
Price difference cars after taxes	Δp^*	0.000	-0.020	0.000	0.105
Tax revenue	R	0.006	-0.005	-0.054	-0.018
Total fuel use	E	0.028	0.016	-0.268	0.012
Exp. utility	U	0.184	0.066	0.143	-0.025

Exogenous variables: $\rho = 0.15$, $\mu = 10$, $\sigma = 0.6$, $\Delta c = 500$, $w_G = 0.08$, $w_D = 0.06$, $b_G = 0.34$, $b_D = 0.30$, $\tau_D = 844$, $\tau_G = 416$, $t_G = 0.635$, $t_D = 0.323$, $t_{vat} = 0.19$, $\Delta\beta = 2223$

According to Khare and Sharma (2003, Table 15), per kilometer driven a gasoline car emits relatively large quantities of CO_2 while diesel cars emit relatively large amounts of particulates. These two pollutants also represent global (CO_2) and local (particulates) effects. If the emissions of both pollutants are roughly proportional to fuel use, then *ceteris paribus* a decrease in fuel use will mean less CO_2 and less particulates. If a tax reform would cause a drastic shift towards diesel, then it is possible that the emissions of particulates will nonetheless increase.

Therefore, I will use three measures for emissions: total fuel use, CO_2-emissions, and particulate-emissions. I assume that for each kilometer driven, a gasoline car emits 287.8 grams of CO_2 and 0.032 grams of particulates, while a diesel car emits 227.1 grams of CO_2 and 0.131 grams of particulates (cf. Khare and Sharma, 2003, Table 15). For instance, in the case of CO_2-emissions: $w_G v_G = 287.8$ and $w_D v_D = 227.1$

Total kilometrage, a central variable in congestion analysis, is obtained as a special case of (4.16) by equating the pollution intensities to the fuel efficiencies, i.e. $v_i = 1/w_i$:

$$K = \sum_{i=D,G} s_i \mathrm{E}\theta_i, \tag{4.17}$$

Another special case is total fuel use ($v_i = 1$).[5]

The final aspect I consider is consumer welfare, which I define as:

$$U = \sum_{i=D,G} s_i \left\{ y - \rho p_i^* - \tau_i + a_i + \frac{1}{\lambda} \mathrm{e}^{-\lambda \pi_i} \mathrm{E}(\theta_0 | i) \right\} \tag{4.18}$$

This is average indirect utility. Although consumers probably also care about emissions, the influence of an individual consumer on total emissions is negligible and will enter their utility function as a constant. This provides a strong rationale for the government to apply a corrective tax (cf. Cremer and Thisse, 1999).

In general, optimal tax rates obviously depend on how these various aspects are weighed in the government's objective function.

A frequently debated policy proposal in The Netherlands and elsewhere is

[5]Note that evaluating R and K requires to know p_D^* and p_G^* (rather than Δp^* only). In the calculations below, I set $p_G^* = 20000$ (average sales price) and then calculate p_D^* by adding Δp^* as predicted by the model.

to make all car related taxes fully dependent on car use.[6] I simulate the effects of this move from lump-sum vehicle tax to tax on car use ('variabilisation') for Cases 1 and 2. I set $\tau_D = \tau_G = \Delta\beta = 0$, retain the VAT tax (i.e. t_{vat}), and then choose $t \equiv t_G = t_D$ such that the taxation generates the same total tax revenues as in the tax base scenario. Since the favorable purchase tax treatment for gas cars disappears in this tax schedule, I eliminate the favorable diesel fuel tax treatment in the new policy scenario by imposing $t_G = t_D$; if the favorable diesel fuel tax treatment would be retained, the share of gasoline cars would be essentially zero. Table 4.4 reports the effects of such a policy. In Case 1 with a low price sensitivity ($\lambda = 1$), consumer fuel prices under variabilisation would more than double, while the share of gasoline car owners would essentially vanish. The total number of kilometers decreases, but as a consequence of substitution towards diesel only by about 10 percent. Total fuel use decreases by 15 percent, as a consequence of both the lower number of kilometers and the higher diesel fuel efficiency. Due to the substitution towards diesel, the emission of particulates rises by 8%.

In Case 1 with a high price sensitivity ($\lambda = 5$) it appears not to be possible to generate the same tax revenues under variabilisation. The higher consumer fuel price generates a strong decrease in the number of kilometers, the effect of which on total revenues is stronger than the effect of the higher per liter revenue. The maximum revenue that can be obtained in this case is two thirds of the revenue in the tax base scenario; the consumer fuel prices are then three to five times as high as the consumer fuel prices in the tax base scenario. Case 2 shows similar patterns for the two different values of the price sensitivity parameter.

If the price sensitivity of kilometrage is sufficiently low, the variabilisation policy has the possibility of inducing a tax revenue neutral shift towards lower a kilometrage and a more fuel efficient engine type. On the other hand, it also implies a drop in expected consumer utility. Note, however, that the model does not account for the possibly positive effect of lower aggregate emissions on consumer utility.

Note that by using the model in Verboven (2002) ($\lambda = 0$) I would have underestimated the size of the excise on fuel. Also in this case it is always possible to generate the same tax revenues under variabilisation.

[6] The European Parliament requested the abolishment of purchase taxes on cars in order to 'increase competitiveness of car markets between EU countries' (*NRC Handelsblad*, November 7, 2003).

Table 4.4: Effects of fully kilometrage-based taxation

		Case 1 fuel market comp. car market comp.						*Case 2* fuel market comp. car market mono.					
		$\lambda = 5$		$\lambda = 1$		$\lambda = 0$		$\lambda = 5$		$\lambda = 1$		$\lambda = 0$	
Fuel prices before taxes	r_G	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34
	r_D	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
Fuel prices after taxes	r_G^*	1.16	3.69	1.16	2.61	1.16	2.33	1.16	3.27	1.16	2.50	1.16	2.24
	r_D^*	0.74	3.64	0.74	2.57	0.74	2.28	0.74	3.38	0.74	2.45	0.74	2.19
Elasticities of km	e_G	-0.460	-1.48	-0.093	-0.209	0	0	-0.464	-1.37	-0.093	0.2000	0	0
w.r.t. prices	e_D	-0.222	-1.09	-0.044	-0.154	0	0	-0.222	-1.01	-0.044	-0.147	0	0
Profit fuel company	Π_{fuel}	0	0	0	0	0	0	0	0	0	0	0	0
Threshold	θ_*	25920	4180	19742	1941	18434	1806	41797	20949	35356	19410	34025	19333
Exp. km gasoline	$E\theta_G$	10140	809	12194	1388	12721	1596	13152	3545	17602	10820	18930	13177
Exp. km diesel	$E\theta_D$	33772	8866	34505	22609	26371	34854	47299	13506	49846	30868	50688	35689
Exp. km	$E\theta$	19429	8844	24965	22608	26370	26370	18030	8858	24539	22518	26370	26370
Market share gasoline	s_G	0.6069	0.0028	0.4276	0.000	0.3833	0.0000	0.8572	0.4667	0.7849	0.4168	0.7657	0.4139
Price difference cars													
before taxes	Δp	500	500	500	500	500	500	2606	2558	3219	4836	3408	5154
after taxes	Δp^*	3087	595	3087	595	3087	595	6726	3042	7785	5750	8111	6136
Tax revenue	R	3401	2357*	3657	3657	3718	3718	3393	2349*	3755	3755	3856	3856
Total fuel use	E	1289	531	1602	1357	1680	1582	1307	565	1749	1441	1872	1691
CO_2		4.97	2.01	5.99	5.13	6.28	5.99	4.78	2.11	6.41	5.39	6.87	5.08
Particulates		1.94	1.16	2.75	2.96	2.97	3.45	1.25	1.00	1.85	2.50	2.02	2.20
Exp. utility	U	0	-100	0	-181	0	-2436	-134	-252	-259	-686	-754	-3166
Fuel excise tax	t	-	2.76	-	1.86	-	1.62	-	2.60	-	1.76	-	1.54

Note: The left column refers to the current situation and the right column is the equilibrium outcome under variabilisation.

* maximum revenue possible under variabilisation

4.6 Concluding remarks

The analysis of car and fuel taxation is complicated by the many interactions between the actors involved: consumers, car producers, fuel producers and the government. This Chapter has provided a coherent framework in which these interactions are made explicit. The analysis emphasizes that optimal tax rates do not only depend on the precise form of the government's objective function and the empirical values of the model parameters; the market forms in both the fuel and the car market are essential as well.

As an application of the model, I analyzed the effects of a move of lump-sum vehicle taxation toward taxing vehicle use. If the price sensitivity of kilometrage is sufficiently low, such a variabilisation policy can induce a tax revenue neutral shift towards lower kilometrage and a larger share of the more fuel efficient engine type. Both these effects reduce fuel use and CO_2-emissions, but due to the substition towards diesel-powered cars particulate emissions will increase. If the price sensitivity is relatively high full variabilisation may not be possible without a loss of tax revenues.

Another potential application of the model is to analyze the effects of an increase in fuel production costs, for example as a result of increased crude oil prices. Goel and Nelson (1999) provide empirical evidence that in such cases governments tend to lower fuel taxes in order to mitigate the effect of the price increase for consumers. I consider the extension of the present model to allow for endogenous government behavior – possibly in a vote-maximization framework – as a challenging further step toward a coherent analysis of car and fuel taxation.

4.A Derivation of the indirect utility function

Consumers receive extra utility when they drive more, other things equal. This can be incorporated in two equivalent ways: assume a demand function for kilometrage and derive (indirect) utility from the demand function or assume a utility function dependent on the amount of kilometers consumed and then derive the demand function and the indirect utility.

4.A.1 Starting point: a utility function

Let (j, k) be given, then the utility of the consumer depends on the expenditure on other goods (z) as a function of θ_k^j and the utility received from consuming θ_k^j. If $y^* = y - \rho p_{jk}^* - \tau_{jk}$, then $z = y^* - \pi_{jk}\theta_k^j$. The utility received from consuming θ_k^j is $f(\theta_k^j)$, where $f(\cdot) > 0$, $f(\cdot)' > 0$ and $f(\cdot)'' < 0$. Utility is given by:

$$\bar{u}_{jk}(\theta_k^j) = y^* - \pi_{jk}\theta_k^j + a_{jk} + f(\theta_k^j). \tag{4.19}$$

Maximize with respect to θ_k^j:

$$\frac{\mathrm{d}\bar{u}_{jk}(\theta_k^j)}{\mathrm{d}\theta_k^j} = -\pi_{jk} + f'(\theta_k^j) = 0 \Longrightarrow f'(\theta_k^j) = \pi_{jk}. \tag{4.20}$$

From (4.20) follows the demand function of θ_k^j. Substituting the demand function in (4.19) gives the indirect utility.

4.A.2 Starting point: a demand function

Suppose the demand function of θ_k^j is given by:

$$\theta_k^j = \theta_0^j e^{-\lambda\pi_{jk}}, \tag{4.21}$$

where θ_0^j and λ are parameters. Note that if the variable costs are zero, the consumer will drive θ_0^j kilometers. So, θ_0^j is the maximum amount of θ_k^j the consumer will 'purchase'. Substituting (4.21) in (4.19) gives the indirect utility, but only if the function $f(\cdot)$ is chosen in such a way that the demand function that follows from (4.20) is the same as the demand function given in (4.21).

4.A.3 Equivalence

From (4.21) follows:

$$\pi_{jk} = \frac{\log\theta_k^j - \log\theta_0^j}{-\lambda} = -\frac{1}{\lambda}[\log\theta_k^j - \log\theta_0^j]. \tag{4.22}$$

Substituting this into (4.20) gives:

$$f'(\theta_k^j) = -\frac{1}{\lambda}[\log \theta_k^j - \log \theta_0^j]. \tag{4.23}$$

Integrating over θ_k^j:

$$f(\theta_k^j) = -\frac{1}{\lambda}\int [\log \theta_k^j - \log \theta_0^j]\mathrm{d}\theta_k^j = -\frac{1}{\lambda}[\theta_k^j \log \theta_k^j - \theta_k^j - \theta_k^j \log \theta_0^j]. \tag{4.24}$$

After some rearranging the following is obtained:

$$f(\theta_k^j) = -\frac{\theta_k^j}{\lambda}[\log\left(\frac{\theta_k^j}{\theta_0^j}\right) - 1]. \tag{4.25}$$

From $\theta_k^j/\theta_0^j \leq 1$ it follows that $f > 0$. Note that $f'(\theta_k^j) = (-1/\lambda)[\log \theta_k^j - \log \theta_0^j] > 0$ and $f''(\theta_k^j) = -1/(\lambda\theta_k^j) < 0$. Substitute (4.21) and (4.25) in (4.19) to obtain the indirect utility function given in (4.1).

I would like to end by making some remarks about the function $f(\cdot)$:

1. $f(0) = -\infty$
2. $f(\theta_0^j \mathrm{e}) = 0$
3. $f'(\theta_0^j) = 0$ and $f(\theta_0^j) = \theta_0^j/\lambda > 0$.

This implies, that $\forall\theta \in [0, \theta_0^j]$ $f'(\theta) \geq 0$ and $\forall\theta \in (\theta_0^j, \theta_0^j\mathrm{e}]$ $f'(\theta) < 0$. Since $f'(\theta_k^j) = \pi_{jk}$ and π_{jk} is finite and non-negative, $\theta_k^j \in (0, \theta_0^j]$.

4.B Normalizing utility

The starting point in (4.1) is:

$$u_{jk} = y - \rho p_{jk}^* - \tau_{jk} + a_{jk} + \frac{1}{\lambda}\mathrm{e}^{-\lambda\pi_{jk}}\theta_0^j + v_j. \tag{4.26}$$

For convenience I drop the j subscript and the error term v_j:

$$u_k = y - \rho p_k^* - \tau_k + a_k + \frac{1}{\lambda}\mathrm{e}^{-\lambda\pi_k}\theta_0. \tag{4.27}$$

Suppose we have two scenarios. In the first scenario, the after tax sales price of a car is $p_k^{*,I}$ and the price of fuel is π_k^I. In the second scenario, the after tax sales price of a car is $p_k^{*,II}$ and the price of fuel is π_k^{II}. It is always possible to normalize the utility in one of the scenarios to zero. Suppose utility in scenario II is equal to zero. It follows that utility in the first scenario now is:

$$y - \rho p_k^{*,I} - \tau_k + a_k + \frac{1}{\lambda}\mathrm{e}^{-\lambda \pi_k^I}\theta_0 - y + \rho p_k^{*,II} + \tau_k - a_k - \frac{1}{\lambda}\mathrm{e}^{-\lambda \pi_k^{II}}\theta_0 = \tag{4.28}$$

$$\rho(p_k^{*,II} - p_k^{*,I}) + \frac{\theta_0}{\lambda}[\mathrm{e}^{-\lambda \pi_k^I} - \mathrm{e}^{-\lambda \pi_k^{II}}] \equiv \Delta_u \tag{4.29}$$

As long as $\lambda > 0$, the calculation of Δ_u is straightforward. Suppose $\lambda = 0$. Then the difference in utility is:

$$\lim_{\lambda \to 0} \Delta_u = \rho(p_k^{*,II} - p_k^{*,I}) + \theta_0 \lim_{\lambda \to 0} \frac{\mathrm{e}^{-\lambda \pi_k^I} - \mathrm{e}^{-\lambda \pi_k^{II}}}{\lambda} \tag{4.30}$$

$$= \rho(p_k^{*,II} - p_k^{*,I}) + \theta_0 \lim_{\lambda \to 0}[-\pi_k^I \mathrm{e}^{-\lambda \pi_k^I} + \pi_k^{II}\mathrm{e}^{-\lambda \pi_k^{II}}] \tag{4.31}$$

$$= \rho(p_k^{*,II} - p_k^{*,I}) + \theta_0[\pi_k^{II} - \pi_k^I] < \infty. \tag{4.32}$$

Note that the difference in utility is just the difference in annualized car price plus the difference in fuel cost evaluated in θ_0. This is a consequence of the quasi-linear utility and the fact that if $\lambda = 0$, then a person will always drive θ_0.

4.C The lognormal distribution

If $\theta_0 \sim LN(\mu, \sigma^2)$, then $\log \theta_0 \sim N(\mu, \sigma^2)$. Let $\Phi(.|\mu, \sigma^2)$ denote the c.d.f. of a normal distribution with mean μ and variance σ^2. Then:

$$F(\theta_*) = \Pr(\theta_0 < \theta_*) = \Phi(\log \theta_* | \mu, \sigma^2) \tag{4.33}$$

and

$$f(\theta_*) = \frac{1}{\theta_* \sigma \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\log \theta_* - \mu}{\sigma}\right)^2\right]. \tag{4.34}$$

The required expressions for the expectation of θ_0 conditional on fuel type are

$$\mathrm{E}(\theta_0 | \theta_0 < \theta_*) = \exp(\mu + \frac{1}{2}\sigma^2) \times \frac{\Phi(\log \theta_* | \mu + \sigma^2, \sigma^2)}{\Phi(\log \theta_* | \mu, \sigma^2)} \tag{4.35}$$

and

$$\mathrm{E}(\theta_0 | \theta_0 > \theta_*) = \exp(\mu + \frac{1}{2}\sigma^2) \times \frac{1 - \Phi(\log \theta_* | \mu + \sigma^2, \sigma^2)}{1 - \Phi(\log \theta_* | \mu, \sigma^2)}. \tag{4.36}$$

See, for example, Aitchison and Brown (1957).