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## Advanced analysis of branch and bound algorithms

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
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# Advanced Analysis of Branch and Bound Algorithms

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AND BOUND ALGORITHMS

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Groningen, February 2007

Marcel Turkensteen

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