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# Chapter 12

## Finite Minds and Open Minds



Jeanne Peijnenburg and David Atkinson

**Abstract** One of the most persistent complaints about Peter Klein's infinitism involves the finite mind objection: given that we are finite, how can we ever handle an infinite series of reasons? Klein's answer has been that we need not actually produce an infinite series; it is enough that such a series be available to us. In this chapter, a different reply is presented through the reconstruction of epistemic justification as a trade-off. In acting as responsible agents, we are striking a balance between the number of reasons that we can handle and the level of precision that we want our beliefs to have. If we are unable or unwilling to manage a large number of reasons, then we have to pay the price in terms of justificatory inexactitude and thereby of accepting relatively untrustworthy beliefs. As well as being intuitively attractive, this idea of a trade-off is warranted by the mathematics of epistemic justification, understood as involving probabilistic relations.

**Keywords** Finite mind · Infinitism · Inference chains · Justification · Structure of justification · Probabilistic justification · Degrees of justification · Klein · Epistemology

When Peter Klein first gave a talk on what later became known as infinitism, he was rudely interrupted. "You are kidding, aren't you?", chimed in one of the listeners, who, to judge from subsequent approving chuckles, was clearly not alone in his discombobulation. Fortunately, Peter belongs to the kind that is challenged rather than deterred by such a reaction. Questioning received wisdom not only appeals to his open and unorthodox mind, but it also seems to tickle his well-developed feel for the absurd. At any rate, he unflinchingly continued his thoughts on the subject and developed them into an epistemological program that keeps generating fruitful and inspiring discussions.

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A frequently raised complaint about Klein's infinitist program is the age-old finite mind objection. Given that humans are finite, how can they forever go on citing reasons for reasons for what they believe? In his early papers, Klein argued that the finite mind objection is based on the "Completion Requirement," according to which a belief can only be called justified if the agent has actually completed the process of reasoning. This requirement, says Klein, is contrary to the spirit of infinitism, and moreover it is too demanding:

Of course, the infinitist cannot agree to [the Completion Requirement] because to do so would be tantamount to rejecting infinitism. More importantly, the infinitist should not agree because the Completion Argument demands more than what is required to have a justified belief. (Klein 1998, 920; Cf. Klein 1999, 314)

Klein sees epistemic justification as being essentially incomplete; it is provisional at heart and always open to further improvement. In later work, he has fleshed this out by means of two distinctions: that between propositional and doxastic justification, and that between objective and subjective availability. Propositional justification depends on the objective availability of reasons in an endless chain, where objective availability means that one proposition *is* a reason for another, even if we are not aware of it. Doxastic justification, on the other hand, hinges on an availability that is subjective: the agent must be able to actually "call on" a reason in the endless chain.<sup>1</sup> Although in its entirety the chain can never be subjectively available to a finite mind, the agent can take a few steps on the infinite trajectory. How many steps are taken, or need to be taken, is a pragmatic matter and depends on the context:

We don't have to traverse infinitely many steps on the endless path of reasons. There just must be such a path and we have to traverse as many as are contextually required. (Klein 2007, 13)

We sympathize with this view, but tend to approach the subject in a somewhat different way. Where Klein denies that infinite epistemic chains can be completed, we assert that there is a sense in which they can. Moreover, we have a different opinion about what it means that justification is context dependent. As Klein sees it, we follow a path of reasons and stop at a point where a reason is sufficiently obvious or very likely to be true. Or in the words of Nicholas Rescher,

In any given context of deliberation, the regress of reasons ultimately runs out into 'perfectly clear' considerations which are (contextually) so plain that there just is no point in going further . . . Enough is enough. (Rescher 2010, 47)

On our view, by contrast, the fact that a reason is sufficiently plain or clear or highly likely or even self-evident is irrelevant for any decision about stopping or continuing. What *is* relevant for such a decision is the size of the contribution that the reason makes to the probability of the belief that we aim to justify by means of an

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<sup>1</sup>For the distinction between objective and subjective availability, see Klein (1999, 299–300); Klein (2003, 722); Klein (2005, 136). For the difference between propositional and doxastic justification, see Klein (2007, 6–11).

epistemic chain. If this contribution is small enough to be neglected, then we simply ignore the reason in question, no matter how high or low its probability is.

These divergences from Klein stem from our framing epistemic justification as something that involves probabilistic relations, rather than, for example, entailments. Consider the following infinite epistemic chain:

$$A_0 \leftarrow A_1 \leftarrow A_2 \leftarrow \dots$$

We call  $A_0$  the target proposition or target belief (disregarding for the moment the distinction between propositional and doxastic justification).  $A_n \leftarrow A_{n+1}$  means that  $A_{n+1}$  is a reason for  $A_n$ , or that the (belief in) proposition  $A_{n+1}$  epistemically justifies the (belief in) proposition  $A_n$ . In earlier work we have argued that infinite epistemic chains are vicious if the relations between the propositions or beliefs are those of entailment. However, if justification is probabilistic, in the sense that  $A_{n+1}$  makes  $A_n$  more probable, then the chains are generally benign. In particular, the following two statements can be proven:

1. The target  $A_0$  may have a unique and non-zero probability, notwithstanding the fact that it is justified by an infinite chain.
2. The effect of distant propositions on the unique and non-zero probability of the target diminishes as their distance from the target increases, and an infinitely distant proposition has no effect at all.

Claim 1 goes against the idea that the probability of the target in an infinite probabilistic chain must be either zero (sometimes known as the probability diminution argument) or indeterminate. Notable representatives of the former position are David Hume (1739/2000/2006) and C. I. Lewis (1929), while the position that it is indeterminate has been defended by, for example, Rescher (2010). Claim 1 of course also gainsays anyone who believes that a grounding proposition is needed for determining the probability of the target. In this sense it nullifies another prominent argument against infinitism, dubbed by Peter Klein ‘the no starting point objection’ (2000, 204).<sup>2</sup> Claim 2 is particularly interesting for finite chains. Effectively, it states that the further away the grounding proposition is from the target, the smaller is its contribution to the latter’s probability. Applied to infinite chains, Claim 2 means that in the limit the impact of any grounding proposition will vanish completely.<sup>3</sup>

Together, the claims indicate how we differ from Peter Klein. The first claim means that a probabilistic chain can be completed, in that it yields a well-defined and positive probability value for the target proposition. The second one articulates the fact that the influence of a particular reason on the probability value of the target

<sup>2</sup>Laurence Bonjour, for example, raises this objection when he remarks that in an infinite chain “justification could never get started and hence no belief would be genuinely justified” (Bonjour 1976, 282). Carl Ginet makes similar remarks, but uses the term “structural objection” (Ginet 2005).

<sup>3</sup>In this chapter, we are talking about subjective probability, since we are dealing with beliefs, but in fact our formalism applies also to objective probability. It can, for example, be used in the analysis of causal chains, on condition that causality is interpreted probabilistically.

lessens as the distance between the target and this reason increases. At a certain point, the influence of the reason on the target will be small enough to be neglected. Where exactly that point is located depends on pragmatic considerations; but, as we will further explain below, we can make these considerations as precise as we wish.

Elsewhere we have formally demonstrated Claims 1 and 2 on the basis of the probability calculus.<sup>4</sup> Interestingly enough, it is precisely this formal demonstration that appears to have triggered the most resistance. Not that the demonstration itself is flawed—everybody appears to agree that it is not. Rather, the complaint is that the actual enterprise of justifying beliefs is not properly modelled by our formal approach. Even if one assumes, as most epistemologists do, that the phrase ‘ $A_{n+1}$  justifies  $A_n$ ’ implies that  $A_{n+1}$  makes  $A_n$  more probable, the consequences of the probability calculus may not be applied lock, stock, and barrel. Thus Jeremy Gwiazda has complained that what we call the completion of an infinite justificatory regress is in fact merely the computation of the limit of a convergent series (Gwiazda 2011). In a similar vein, Adam Podlaskowski and Joshua Smith have argued that although “valuable lessons” can be drawn from our results, such as 1 and 2, it is “entirely unclear” that these results meet a basic requirement, namely “providing an account of infinite chains of propositions *qua* reasons made available to agents” (Podlaskowski and Smith 2014, 212). Podlaskowski and Smith call this ‘the availability problem’:

[A] demonstration that finite agents can actually calculate the probability of a proposition’s truth—even if it belongs to an infinite chain of reasons—does not thereby show that each reason is equally *available* to a finite agent.<sup>5</sup> (Podlaskowski and Smith 2014, 216)

In a word, the criticism is that the finite mind objection still applies.

The point is well taken, moreover it is one that is familiar. Jonathan Cohen famously argued that what he calls “Pascalian probability” (read: probability according to the calculus) is not particularly suited to reasoning in court, everyday life, or even science (Cohen 1977). Cohen’s argument about probability could equally well be applied to logic, and in fact has been applied that way. Historically, logical systems have often been criticized for being too formal and too far removed from actual human reasoning. This goes for mediaeval scholastic systems exploiting Aristotelian syllogistics as much as for modern forms of mathematical logic. In his valedictory lecture, Johan van Benthem rightfully reminded us that a heedful logician typically goes back and forth between normative and descriptive considerations:

Logical theory that ignores actual behavior seems dangerously empty, lacking focus. On the other hand, I do not want to lose the potential of normative thinking either, that can help us *improve* performance, or design better ways of dealing with the world and with one another (van Benthem 2014, 18–19).

<sup>4</sup>Peijnenburg (2007); Atkinson and Peijnenburg (2010); Peijnenburg and Atkinson (2014a, b). Atkinson and Peijnenburg (2017, Chapter 5, especially §5.3).

<sup>5</sup>Michael Rescorla in this connection even uses the scare term “hyper-intellectualism” (Rescorla 2014).

Probability is in this respect like logic, and so is epistemic justification: on the one hand, we ponder abstractly about how we ought to reason when justifying our beliefs, and on the other hand, we have to keep an eye on how we actually do go about finding reasons for what we believe. The fact itself that epistemic justification has a normative and a descriptive side is presumably uncontroversial. Controversies are rather about the question as to how much weight should be apportioned to each side. An approach like ours, which authorizes the completion of infinite epistemic chains, may seem to unduly stress the normative part.

The above, however, insufficiently takes into account the consequences our view has for everyday reasoning and for finite chains. Of course it is true that people cannot forever continue giving reasons for their beliefs—most of us already lose track after three or four steps. The point is that our approach appertains to ordinary finite chains as well, since it enables us to determine how long an epistemic chain needs to be, even without any knowledge of a foundational proposition. Here is how it works. Imagine the shortest chain there is, a belief  $A_0$  is justified by  $A_1$ :

$$A_0 \leftarrow A_1$$

In this finite chain,  $A_0$  is the target and  $A_1$  is the ground. The arrow is interpreted as before, namely as implying, as a necessary but not sufficient condition, that  $A_1$  makes  $A_0$  more probable. It is important to realize that the unconditional probability of  $A_0$ , namely  $P(A_0)$ , is not solely a function of the conditional probability of  $A_0$  given  $A_1$ , that is, of  $P(A_0|A_1)$ . In determining the unconditional probability of  $A_0$  we must also take into account what this probability would be if  $A_1$  were *false*:  $P(A_0|\neg A_1)$ . Now  $P(A_0)$  must lie between  $P(A_0|A_1)$  and  $P(A_0|\neg A_1)$ . If neither of these conditional probabilities is zero, the unconditional probability of  $A_0$  cannot be zero either.

Suppose that the value of  $P(A_0|A_1)$  is  $x$  and the value of  $P(A_0|\neg A_1)$  is  $y$ , and let  $x$  and  $y$  differ greatly; for example  $x$  is very close to one and  $y$  is very close to zero. Assuming we don't know the unconditional probability of  $A_1$ , we face a great deal of uncertainty as regards the value of  $P(A_0)$ . The only thing we can be sure of is that this value lies somewhere in the wide interval between  $x$  and  $y$ . However, our results are of help here, for they imply that the interval shrinks as the chain is lengthened. To illustrate this, let us first add one link to the chain, by giving a reason for what originally was the ground  $A_1$ :

$$A_0 \leftarrow A_1 \leftarrow A_2$$

Here  $A_1$  is justified by  $A_2$ , which now does duty as the new grounding proposition. The unconditional probability of  $A_1$  must lie between the conditional probability of  $A_1$  given  $A_2$  and that of  $A_1$  given  $\neg A_2$ . If neither of these conditional probabilities is zero, then the unconditional probability of  $A_1$  cannot be zero. Importantly, this has the effect of further restricting the interval in which the unconditional probability of  $A_0$  must lie: it will now be strictly smaller than it was in the absence of  $A_2$ . A similar story would apply if we were to expand the series further, by adding  $A_3$  as a new

ground. The probability of  $A_3$  would restrict the domain in which the probability of  $A_2$  may lie, which in turn diminishes the interval in which the probability of  $A_1$  may lie, which thereby further narrows down the interval for  $P(A_0)$ . The more propositions there are, the smaller is the interval within which the probability of  $A_0$  must lie. In the limit of an infinite probabilistic regress, this interval has shrunk to a point. The probability of  $A_0$  has then been determined uniquely in terms of all the conditional probabilities along the chain.

All of this means that one can determine in advance how many reasons an agent needs in order to approach the true probability of the target within a given error margin. If this number of reasons happens to be too large to fit into the agent's finite mind, then she will have to relax the level and be content with a degree of justification that is less accurate. But if the number of reasons is rather small, so that they are mentally encompassed with ease, then the satisfaction level can always be tightened up and brought closer to the target's true probability. Epistemic justification thus boils down to striking a balance. In acting as responsible epistemic agents, we are instigating a trade-off between the number of reasons that we can handle and the level of accuracy that we want to reach. If we are unable or unwilling to manage a large number of reasons, we have to pay in terms of a lack of precision and hence of trustworthiness of (our belief in) proposition  $A_0$ . Taking the short route thus comes at a price, but in situations where precision is not important, we can take it easy and should do so on pain of exerting ourselves unnecessarily.

The point is a general one, and it can be made in qualitative or quantitative terms. In the latter case, we may choose between using precise numbers or intervals possibly with vague boundaries. Here is an example with precise numbers. Suppose we have taken a few steps in the chain and then stopped. We first set the probability of the ground equal to one and then to zero. Assuming we know the values of the conditional probabilities, we can now determine the minimum and the maximum value of the probability of  $A_0$ , let them be 0.65 and 0.67 respectively. Then our best estimate of  $P(A_0)$  will lie in the middle, at 0.66, for in that manner we have minimized the possible error that this estimate can have, to wit 0.01. The unknown true limiting probability of  $A_0$ , whatever its precise value is, cannot deviate from 0.66 by more than 0.01, since we know for a fact that it lies somewhere between 0.65 and 0.67. If we proclaim ourselves satisfied with a number that deviates by no more than 0.01 from the true value, then we need go no further in inquiring as to any support that the target might have beyond the minimal required to reach this error of 0.01. This is because any extension of the chain, obtained by adding a proposition that supports the erstwhile ground, would only decrease the error. So in this case we know exactly how many reasons we need in order to approach the true value of the target to a level that satisfies us. Since we are content with a value that deviates no more than 0.01 from the true value, we require no more than the reasons we already have. And if our mind is capacious enough to store these reasons, then we have accomplished our task: we have justified  $A_0$  to a satisfactory level, staying neatly within the limitations of our finite mind.

Why can we be so sure that any extension of the chain will always decrease the error margin surrounding the probability of  $A_0$ ? How do we know that the margin will never widen, or that no fluctuations will occur further up in the chain? The answer is that these features follow directly from the fact, articulated in our Claim 2, that distant reasons are less important than those that are nearby.<sup>6</sup>

The structure of the probabilistic epistemic chain is such that it enables us to say how many reasons we need to call on in order to approach the probability of the target to a satisfactory level. To do that, we do not need to know the length of the chain; we need not even know whether it is finite or infinite. Nor do we have to know the probability of the ground. The only thing we need are the (precise or imprecise) values of a certain number of conditional probabilities (sometimes more, sometimes fewer, depending on the speed of convergence) that suffice to take us to within a desired level of accuracy with respect to the true, but unknown probability of the target. Once we are there, we can safely ignore the rest of the chain.

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<sup>6</sup>For details see Atkinson and Peijnenburg (2017).



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