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Hafgerðingar and giant waves

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A 13th-century text in Old Norse, *Konungs Skuggsjá* (translated as *The King's Mirror*), tells about a phenomenon that may be encountered in the Greenland Sea. It is called hafgerðingar (sea fences). The horizon is raised, and from there three giant waves come rolling in. Recently Lehn and Schroeder have explained the phenomenon as a superior mirage. I extend their analysis by introducing a periodic time dependence in the properties of the inversion layer, and show that also the illusion of incoming waves and an immediate danger may so be explained.

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1. INTRODUCTION

There is still another marvel in the Greenland Sea; I am not yet quite sure what it really is. It is called hafgerðingar, and it comes closest to this: it looks as if all the storms and waves of the sea are piled up in three places or layers, from which three (long-stretched) waves are formed. These fence in the entire sea, so that nowhere can one see an opening or escape. They are higher than great mountains and resemble steep, overhanging cliffs. Only few cases are known of men who had been trapped in them and escaped with their life.

This text is from a 12th- or 13th-century manuscript in Old Norse, *Konungs Skuggsjá*, that describes in dialogue form a conversation between a father, the king, and his son, the king-to-be. They speak of many things that a future king should know, such as good manners in court, law and justice, ethics and religion. But also about navigation, the sun, the wind, and the tides. And about the marvels in Ireland, Iceland, and Greenland.

The *Konungs Skuggsjá* was translated into Latin as *Speculum Regale* and into Danish as *Det Kongelige Speil* by Hannes Finnson, in 1768 [1]. This is the *editio princeps*. In a footnote, Finnson writes that the hafgerðingar is in reality a monster.

Other important early translations are: in English as *The King's Mirror*, by Laurence Marcellus Larson (1917) [2] and in Danish as *Kongespejlet*, by Finnur Jónsson (1926) [3].

Hannes Finnson also translated the *Book of Settlements* (*Landnámabók*) [4], wherein it is told how Herjolf Bardarson sails to Greenland in the company of Eirik the Red and runs into hafgerðingar. The story is told twice, the first time in the genealogy of his grandfather (also) Herjolf, and then a second time in that of his grandfather's friend and foster brother Ingolf Arnarson. What exactly was so threatening is not told, but it must have been scary enough for one of the men aboard to compose a poem or prayer, the *hafgerðingardrapa*, the calming

of the hafgerðingar. In the register of his book, Finnson mentions again that it is a sea monster and he specifies "a whale species, that takes its name from the fact that they can close in and occupy a part of the sea, so that the whole area seems surrounded by a wall."

The *Landnámabók* has been translated into English by Vigfússon and Powell (1905) [5] and recently by Pálsson and Edward [6]. Very recent is the Danish translation by Madsen [7].

Also in the *Bishop Sagas* [8] the hafgerðingar are mentioned, when Bishop Gudmund returns to Iceland from Norway. It is described how they got into heavy weather and escaped from a freak wave, thanks to adequate anticipation by the man at the steering oar. There is no mystery here; the hafgerðingar was just a heavy storm.

In 1871 the renowned zoologist Japetus Steenstrup presented a talk [9] for the Society of Nordic Antiquity and History, wherein he argued that the enormous waves associated with the hafgerðingar were caused by submarine earthquakes.

Twenty years later he was contradicted by his nephew, Knud Steenstrup [10], who as a geologist had made no fewer than nine journeys to Greenland. Young Steenstrup states that in open sea you hardly notice a tsunami. But he had witnessed himself how current-against-wind conditions can create such odd waves that they come splashing in on both sides of the ship.

Lindvall [11], in 1905, gives a very similar argument. He had seen himself, and had explained to him by the captain, how violent white-capped breakers can occur where the branch of the warm Gulfstream that goes south and west around Iceland meets with the cold south-going return current. The captain also said that the waves were highest around new moon and full moon, when tides are strongest.

Recently a completely different interpretation was given by Lehn and Schroeder [12], to whom I also refer for references

not mentioned in the present work. They present pictures of mirages that create the illusion of a distant wall. They also present a temperature profile of an inversion and the corresponding ray tracing that produces such an illusion. The danger of which *The King's Mirror* speaks, is then not immediate, but often enough the inversion is a foreboding of a storm.

The organization Heimskringla, which makes available Old Norse literature, has a web page on hafgerðingar that they keep updated [13]. It features the article of Japetus Steenstrup and gives annotations and comments. Summarizing the different interpretations that have been forwarded, Asger Olsen gives as his opinion that the tsunami idea does not work because these are not felt on deep water off the coast. Lehn's mirage idea is thought unlikely because it does not explain the three giant waves, and an immediate danger is absent. At this time, the Heimskringla seems to prefer the freak wave scenario.

In the present article, I will defend the mirage idea by showing that it may also produce the illusion of moving waves, when some simple dynamics is brought into the model.

After all, temperature inversions are not completely static and permanent. They come and go and what you get to see depends strongly on characteristics such as the height of the inversion, the distance over which it extends, and the way it reverts to a "normal" atmosphere. Since the inversion resembles a system of two liquids of different densities, the separating surface can support long running waves. In the next section, I will show that such waves introduce a periodicity in the mirage that creates the illusion of broad and slowly moving waves.

In Section 3 the time scale of the periodicity is estimated via two models: (a) by analogy to water waves and (b) assuming gravity or buoyancy waves, confined to a region around the height where the density gradient is maximum. Periodic changes in mirages over sea, seen by land-based observers have been reported earlier by Lehn *et al.* [14] and by Sethuraman [15], who both interpreted their observations in the latter model.

2. RAY TRACING AND SIMULATIONS

In this analysis, I will concentrate on parameters that may introduce periodicity in the image that one gets to see through an inversion. These are, in the first place, the wavelength of a transverse running wave on the inversion surface, moving along or against the observer's direction of sight. Second, of course, the observer's eye height, when there is some (mild) ambient wind below the inversion and there are water waves.

Ray-tracing procedures have been described in previous work [16,17]. Files of vertical observation angle versus distance and whether the back-traced ray hits the water or connects to the sky, are extracted and are used in a separate program to simulate the aspect of sea and sky. Similarly, a file is extracted for observation angle versus the height of the ray in an object plane away from the observer. It serves for simulating a scenery in that particular plane, which is chosen as an image of a Viking longship in the figures and visualizations presented in this work.

I find that the illusion of moving waves is strongest, when the inversion not only produces strong looming, but in addition features a duct. That is, within a narrow band of typically 10–20' and symmetric around the true horizontal, light rays

follow oscillating trajectories. This latter aspect is absent in the analysis of Lehn and Schroeder.

The temperature profile used for the simulation is shown in Fig. 1. Its analytical form is

$$T(h) = T_{\text{st}}(h) + \Delta T \left[\frac{1}{2} + \frac{1}{2a(h_2 - h_1)} \ln \left(\frac{\cosh(a(h - h_1))}{\cosh(a(h - h_2))} \right) \right]^p. \quad (1)$$

Here $T_{\text{st}}(h)$ is the US1976 standard atmosphere, shifted to reproduce the adopted temperature at sea level. The expression inside the large square brackets is a "unit jump"-function that to a good approximation rises linearly from 0 to 1 between h_1 and h_2 . The parameter a determines the rounding at the corners in $h = h_1$ and $h = h_2$. Instead of linear, the rise in between may be made to follow any power law by choosing the value for the parameter p in the exponent correspondingly. Finally, the whole expression is multiplied by the temperature difference ΔT across the inversion.

For the profile shown in Fig. 1, the parameters are: $T(h = 0) = 0^\circ\text{C}$, $P(h = 0) = 1013.25$ hPa, $\Delta T = 5^\circ\text{C}$, $h_1 = 0$ m, $h_2 = 20$ m, $a = 0.25$ m⁻¹, $p = 5$.

For an observer at around 3.5 m above sea level, this inversion produces strong looming and brings the horizon up to 7.5 arcmin above horizontal. At the same time, somewhat lower and symmetric around horizontal is a 6 arcmin wide region that corresponds to ducted light rays. Retraced backward from the eye of the observer, these rays follow oscillating paths until somewhere, maybe 60 nautical miles away (1 deg in latitude), the inversion weakens and cannot hold the rays any longer. They then connect the observer's sight to either the sky or the water. The results depend strongly on the way one lets the inversion vanish. I have chosen a method whereby the inversion weakens by multiplying the temperature gradient with an attenuation factor beyond a certain distance away from the observer:

$$\begin{aligned} A(x) &= 1 & (x \leq x_0) \\ A(x) &= 1/[1 + (x - x_0)^2/r^2] & (x > x_0) \end{aligned} \quad (2)$$

In the present analysis, I have chosen $x_0 = 60$ nautical miles (111 km) and $r = 10$ km.

Fig. 1. Temperature profile.

Fig. 2. (a) Ray tracing for the inversion described in the text. The observer's eye height is 3.5 m. In black, the rays that make no oscillations. Within the duct, rays that connect to the air are in red; those that connect to the sea are in blue. (b) View under the inversion conditions as in panel (a). (c) Ray tracing as under (a), in the additional presence of a wave on the inversion surface with amplitude 2 m and wavelength 10 km. (d) View under the inversion conditions as in panel (c).

A. Waves on the Inversion Surface and Water Waves

If the duct is not too long, the observer will perceive a raised “wall” consisting of three components: (1) the highest looming of the horizon and below it (2) ducted light by which he sees the sky, and (3) ducted light by which he sees the sea.

Ray tracing produces pictures like the one in Fig. 2, where the three “parts of the wall,” as they are explained above, are distinguished by different coloring. The farther away a ray hits the water, the darker the color. Also, the sky seen through the duct is a shade darker than the directly observed sky.

The upper panels, 2(a) and 2(b), show the ray tracing and the view when looking out over a quiet sea from an eye height of 3.5 m. For the simulation in the lower panels, (c) and (d),

a sinusoidal wave has been added to the inversion surface. By trying out different wavelengths, it is empirically found that the longer the wave, the more effective it is in shifting up and down the different “bands” in the image that one gets to see through the mirage. This may be understood intuitively: the wave pattern on the inversion layer should have wavelengths in the same order of magnitude as the oscillations of the light rays, which in our example are around 40 km. So, even if the wave pattern is a Fourier series of many wavelengths, the ray tracing will filter out the longer (and fastest traveling) components.

The view changes continually and the observer will get the illusion of very broad, moving waves in the distance. Figure 3 shows the image that the observer sees in eight “frames” of one

Fig. 3. View for different phases of the inversion surface wave, spanning one period. Typical time scales are discussed in Section 3.

wave period, as the surface wave travels overhead. *Visualization 1*, a 44 frame MP4 animation, shows the effect more clearly. Just how menacing this may look is shown in Fig. 4, where in panels (a) and (b) we see four ships of Eirik the Red's fleet, at distances of, from left to right: one, three, two, and four nautical miles, respectively. Figure 4(a) shows the scenery for a standard atmosphere; in panel (b) we see the same ships under inversion conditions that are explained above. The ships look hardly deformed, but in the background the sea seems to stand upright and appears composed of several layers, even more that the three that are mentioned in *The King's Mirror*.

Visualization 2 shows the nearest ship as the surface wave on the inversion travels overhead while at the same time the observer goes up and down at a faster pace with an amplitude of 25 cm, as he rides the sea waves in a mild 2 Bf wind.

While the ships in Fig. 4(b), the farthest at four miles, look hardly affected by the mirage, at greater distances, they are. Figure 5 shows a ship 12 miles away. In a normal atmosphere, it would be below the horizon, but in the inversion, the duct and the looming part of the inversion together extend vertically over 12 arcmin, and the image fills a good part of it. The ship can easily be seen by the naked eye, though in real life of course fuzzier than in this idealized image. It is seen upside down and seems to be wrecked as it is tossed around by the giant waves, as illustrated by *Visualization 3*.

B. Attenuation of the Inversion

Above, the inversion was kept constant over $x_0 = 60$ nautical miles (111 km) and attenuated for larger distances as described by Eq. (2), where the attenuation length r had been taken as 10 km. At a distance of 121 km, the inversion has thus been reduced to half its strength. More specifically, the isotherm at 20 m height is kept in place also for $x > x_0$, while the higher and lower isotherms are made to bend away from the central one, their differences increasing as $A^{-1}(x)$, the inverse of the attenuation factor.

In this subsection, the effect of a different parameterization is investigated. To avoid proliferation of parameters, I will study

Fig. 4. (a) Viking Drakars at distances of, from left to right, 1, 3, 2, and 4 nautical miles in a standard atmosphere. (b) The same four ships in an inversion as in the text, with a surface wave of 10 km wavelength superimposed.

a family of attenuations, of which the above used inversion is a member. For

$$x_0 = (121 - r) \text{ km}, \quad r = 10\text{--}30 \text{ km}, \quad (3)$$

the “half-strength point” is at 121 km, in common for all of them, as shown in Fig. 6.

What the inversion looks like for an observer, whose eye height is fixed at 3.5 m, is shown in Fig. 7 for different values of r . For the smaller attenuation lengths, the picture is that of a looming horizon, while the duct connects to the sky. For $r = 21$ km a thin dark band shows up in the middle of the duct and for larger attenuation lengths, this band grows thicker.

For $r = 10$ km, this band would be seen by an observer at 4.4 m above the sea, and this height decreases with increasing attenuation length. For $r = 30$ km, it shows up for eye heights above 2.9 m. At $r = 20$ km, it becomes visible from a height of 3.55 m, only just above the fixed eye height that was taken as 3.5 m. This has the interesting consequence that a mild wind of 2 Bf, with water waves of 0.25 m amplitude (wave height 0.5 m) is sufficient to make the dark band appear and disappear with the period of the waves, which is about 5 s. The effect is shown in *Visualization 4*. In this movie, there are no waves on the inversion surface. *Visualization 5* shows the effect of including inversion surface waves of 1 m amplitude and again of 10 km wavelength.

3. RUNNING WAVES ON THE INVERSION LAYER

It remains to show that the characteristics of the inversion surface waves that were adopted above are realistic. The two important questions are: (1) Is a wavelength of 10 km possible? (2) What would the time period of such a wave be?

A. Model 1: Analogy with Water Waves

The dispersion relation that relates frequency (f), wavelength (L) and water depth (D) is in its general form [18–21]

$$\frac{1}{T} = f = \sqrt{\frac{g}{2\pi L} \tanh\left(\frac{2\pi D}{L}\right)}. \quad (4)$$

With the limits for deep and shallow water

$$\frac{1}{T} = f \Rightarrow \sqrt{\frac{g}{2\pi L}} \text{ deep water}, \quad D > L/2, \quad (5)$$

$$\frac{1}{T} = f \Rightarrow \frac{1}{L} \sqrt{gD} \text{ shallow water}, \quad D < L/25. \quad (6)$$

The phase velocity, C , is always given by

$$C = L/T. \quad (7)$$

When applying these equations to an inversion, the gravitational constant must be multiplied with a factor $-(\rho_2 - \rho_1)/\rho_1$, the jump of air density across the inversion, divided by the density below it. In the above equations, this factor was tacitly omitted, because for a water-air surface its numerical value is just ≈ 1 . For the inversion that we have defined above, it is about 0.015.

D is now the height of the inversion above (mean) sea level and has been taken at 20 m.

Within this model there is no theoretical limit on wavelength.

Fig. 5. Deformed image of a ship at 12 nautical miles distance, seen in the same inversion.

Fig. 6. Attenuation factors as given by Eqs. (2) and (3).

Table 1 shows the results for different wavelengths. As these are all much larger than D , the height of the inversion, the phase velocity, C , is close to its “shallow water” limit, which here is 1.72 m/s. These periods are very long, almost 1 h and 40 min for $L = 10$ km, but may be much shorter in the presence of an ambient wind.

Let U be the wind speed below the inversion. For an observer who travels with wind speed, the picture will be as described above. But an observer who stands still will see the

overhead wave pattern pass more swiftly, with an apparent period

$$T' = \frac{LT}{L + UT}. \quad (8)$$

A mild wind speed of 5 m/s reduces this apparent period to 25 min.

Similar periods of oscillatory behavior in mirages have been observed and reported by Lehn *et al.* [14] and by Sethuraman [15].

B. Model 2: Gravity or Buoyancy Waves

The temperature profile of Fig. 1 produces a *density profile* that shows a pronounced local change in its gradient. This is shown in Fig. 8.

A small volume of air, when vertically displaced from equilibrium over a small distance h' , will experience a restoring force of

$$\rho \Delta V \frac{\partial^2 h'}{\partial t^2} = g \frac{\partial \rho}{\partial h} h' \Delta V. \quad (9)$$

Since the derivative of the density is negative, this equation allows for oscillatory motion. The associated frequency is the so-called Brunt-Väisälä (radial) frequency, defined by

$$N = \sqrt{-\frac{g}{\rho} \frac{\partial \rho}{\partial h}}. \quad (10)$$

Such waves are called gravity waves or buoyancy waves.

Figure 9 shows the profile of the square of the Brunt-Väisälä frequency. Its maximum is denoted as N_s^2 , and for the atmosphere that we have adopted, it occurs at just under 18 m in height. The width parameter d is associated with the width of the distribution at 0.42 of its top. This may seem a bit odd to those who are used to define widths as, e.g., “width at half-maximum” or at $1/e$ of the maximum, but it has a historical background. The model becomes accessible for analytic solution [22,23] when the jump in density (Fig. 8) is parameterized as

$$\rho(z) = \rho_s \left[1 - \frac{1}{2} (\Delta \rho / \rho_s) \tanh(z/d) \right]. \quad (11)$$

This conforms to the notation in the original work by Groen (1948). Later articles and books usually incorporate the factor

Fig. 7. Inversion as seen from a fixed eye height of 3.5 m for attenuation lengths $r = 10$ –30 km. No inversion surface waves.

Fig. 8. Density profile for the atmosphere under investigation.

Table 1. Analogy with Water Waves

L (m)	T (s)	C (m/s)
100	71	1.41
500	294	1.70
1000	584	1.71
5000	2915	1.72
10,000	5829	1.72

$\frac{1}{2}$ in the definition of $\Delta\rho$, which then stands for half of the total density jump.

Here $z = (h - h_0)$ is the vertical distance from the location of the maximum in the Brunt-Väisälä frequency. Then, assuming the system to be incompressible, one finds

$$N^2 = N_s^2 \cosh^{-2}(z/d), \quad (12)$$

which, indeed, for $z = \pm d$ returns the value of $0.42N_s^2$. From our present analysis, we get: $d \approx 5.0$ m.

Solutions may be found in terms of hyperbolic trigonometric functions and constitute running waves in the horizontal direction and standing waves vertically, characterized by their number of nodes $p = m - 1$, where $m = 1, 2, 3, \dots$. The solutions are bounded by (approximately) $-d < z < d$, outside of which limits the waves become evanescent.

In the absence of an ambient wind, the radial frequency is found from the modal dispersion relation

$$\hat{\omega}_m^2 = \frac{k_x^2 N_s^2 d^2}{(k_x d + m - 1)(k_x d + m)}. \quad (13)$$

Here $k_x = 2\pi/L$ is the wavenumber for the horizontal motion. In the presence of an ambient wind speed U the radial frequency as seen from the stationary system is

$$\omega = \hat{\omega} + k_x U. \quad (14)$$

I have taken Eqs. (9)–(14) from a comprehensive survey by Snively and Pasko [24]. They may also be found in the textbook by Gossard and Hooke [25]. Equations (11)–(14) or their equivalents are given in the article by Thorpe [23], and for the lowest modes in the original article by Groen [22].

For a given wavelength, the $m = 1$ mode has the lowest frequency and travels fastest. It is this mode that I will consider. Since $k_x d \ll 1$, Eq. (13) simplifies to

Fig. 9. Profile of the Brunt-Väisälä frequency squared.

$$\hat{\omega}_m^2 = k_x N_s^2 d \approx -g \frac{\Delta\rho}{\rho} k_x,$$

where in the last step $\Delta\rho$ is taken as the density drop over the full half-width of the Brunt-Väisälä frequency distribution (Fig. 9).

Hence,

$$\frac{1}{T} = f \approx \sqrt{\frac{g(-\Delta\rho/\rho)}{2\pi L}}. \quad (15)$$

This is just the *deep-water limit* of model 1, the analogy with water waves. See Eq. (5). In this example $d \approx D/4$, a quarter of the inversion height, which is evidently small enough to make the deep-water limit applicable.

Table 2 lists the results for model 2, the ducted wave scenario. Wave periods are shorter than for model 1, just over 13 min for $L = 10$ km. In the presence of an ambient wind, they may be shorter yet.

Groen [22] discusses limitations on wavelengths that might arise from the necessary simplifications that underlie the model of ducted waves, and he concludes that wavelengths of 3000 times the full width ($2d$) of the density distribution are still permissible. In our case this would be 30 km. The adopted wavelength of 10 km is therefore quite realistic.

Finally, on this type of trapped gravity waves: they are oftentimes set in motion by thunderstorms and may travel over very long distances. One may also think of katabatic winds off the high south coast of Greenland as a mechanism to initiate them.

Table 2. Ducted Wave, $m = 1$

L (m)	T (s)	C (m/s)
100	91	1.10
500	183	2.74
1000	255	3.93
5000	562	8.89
10,000	794	12.60

4. DISCUSSION, SUMMARY, AND CONCLUSIONS

A. Modelling the Inversion

The description of hafgerðingar in the *Konungs Skuggsjá* fits best with a superior mirage that causes a strong looming of the horizon, as suggested by Lehn and Schroeder [12]. The present work extends the mirage picture by including a duct and moving modulations on the inversion surface and varying eye height resulting from water waves. In doing so, the model can also account for the illusion of the long moving waves that are mentioned in the *Konungs Skuggsjá*.

For the inversion modulations, two models of running surface waves have been considered: (a) an analogy with the well-known theory of water waves, and (b) gravity or buoyancy waves. Of these two options, the first can produce periods of oscillation that are short enough to be recognized, only when an ambient wind is postulated. There is a limit to this, because superior mirages are generally seen in mild winds, generally not exceeding 5 m/s, which is already the beginning of 3 Bf on the wind scale. The gravity wave scenario produces shorter periods more easily and therefore seems to be preferred. It must, however, be remembered that also nonperiodic changes in the inversion may influence the image that the observer gets to see, and they too may be time dependent. As an example, it has been shown that the way the inversion weakens, parameterized by an attenuation length in this work, is an extremely sensitive parameter. A cold wind, blowing into the “open end” of the inversion will easily alter it enough to make the effect visible.

Throughout this work, the inversion waves have, for the sake of simplicity, been modeled as moving along or against the direction of sight. In general, they will of course be seen under an angle. Also in that case, the main features are the same. Darker and lighter bands seen through the mirage are still horizontally stratified. Yet small variations with viewing angle may exist, and their displacement with time may tell the direction in which the inversion waves move. Figure 10 shows a panoramic view, composed of 41 independent ray-trace runs in steps of 0.1 deg, covering horizontal angles of 43–47 deg in viewing direction from the propagation direction of the inversion waves.

B. Surveying the Different Interpretations of the Hafgerðingar

Historical descriptions of hafgerðingar are few. One episode is told in the *Landnámabók*, *The Book of Settlements*. It tells about

the voyage to Greenland, in 986 A.D., of Eirik the Red in company with 24 other ships. Only 14 of them arrived in Greenland; the others were either driven back or were lost. This suggests bad weather, possibly following the sighting of the hafgerðingar. Maybe also, they ran into the violent tide rips that Steenstrup [10] and Lindvall [11] witnessed and that are steep enough to be called *fences in the sea*.

In the *Bishop's Saga* it is a strong wind and a freak wave, a *wall of water*. The hafgerðingar of the *Konungs Skuggsjá* are described as a distant apparent wall [of water] all around. The word hafgerðingar might well have been used for anything steep in or around the sea that could potentially be dangerous. In this sense, it even applies to the sea monster theory of Hannes Finnson [1,4], assuming the “species of whale” is the orca, or killer whale. Attacks on ships are extremely rare, but when you feel “fenced in” by a group of them, you must pray that they do not, even for a moment, mistake the hull of your boat for a whale.

Reviewing the various possibilities that have been put forward, the mirage option comes closest to the description of the hafgerðingar, as it is told in the *Konungs Skuggsjá*. It is even suggestive for an immediate danger, especially by the distorted images that one gets to see of distant ships.

C. More Accounts of Hafgerðingar-Like Mirages

The Greenland Sea is a good place for seeing mirages. In his book *Journal of a Voyage to the Northern Whale-Fishery*, [26], William Scoresby quotes numerous examples. Some of these come close to the *Konungs Skuggsjá* description, for example on 16 July 1822, when he writes (p. 163): “...the phenomenon was so universal, that the space in which the ship navigated seemed to be one vast circular area, bounded by a mural precipice, of great elevation...” The appearance is that of basaltic ice. It seems not to have given the illusion of huge waves; at least Scoresby does not mention it. What he does mention is the changeability of the scenery: “No sooner had one appearance been examined and sketched, than it changed...” (p. 144).

In the present issue of *Applied Optics*, Young and Frappa [27] present photographic material of mirages over Lake Geneva. They also review the several hundreds of observations made by the end of the 19th century by François-Alphonse Forel over the same lake. In his monograph *Le Léman*, volume 2 [28], Forel distinguishes between the *fata morgana* and what he calls the *fata brumosa* (pp. 544–553). The first appears as a striated zone in which vertically elongated features of the opposite shore are seen. The second has the appearance of a fog, its color the same or just

Fig. 10. Panoramic view at angles of 43–47 deg with the direction in which the inversion surface waves move.

a shade darker than the water. Young and Frappa point out that the *fata brumosa* is really a form of the *fata morgana*, wherein not the opposite shore is imaged, but instead the water surface in front. The foggy appearance results when the water surface is lighter in color than the shore in the background. Forel ends his exposé by noting that the *fata morgana* and the *fata brumosa* may coexist. Indeed, just as in the hafgerðingar simulations presented in the present work, the observer sees inside a duct images of both the water and of the sky as distinguishable horizontal stripes or bands across the striated zone.

Interestingly, Forel observes that the *fata brumosa* may give the impression of a gigantic wave. He also notes that they are not static. The whole scenery seems to move “slowly and majestically” in the same direction as the wind.

California is also a good place for seeing mirages. The 2009 November issue of the journal *Weather* features on its cover an impressive picture of a hafgerðingar-type mirage, taken by Mila Zinkova [29]. She has taken many more of such images, videos and photographs, which stand model for various aspects of the hafgerðingar. These may be found under Ref. [30].

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