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Winning Isn't Everything: How Sports Competition Rules Can Make You Want to Lose

MARC PAULY

Winning isn't everything, but wanting to win is.
(Vince Lombardi, American football player and coach)

Commentators of the matches called the players' behavior "a disgrace," "infringing on the spirit of sports," and "disastrous for badminton." In two badminton matches of the 2012 London Olympics, eight players were disqualified from the female doubles competition for deliberately trying to lose their matches, by serving into the net on purpose or missing the birdie deliberately. The matches were accompanied by the booing of an angry crowd. The reason these players wanted to lose their games was to face an easier opponent in the next round of the competition: In the first-group round of the competition, players were divided into round-robin groups where every team plays every other team once. The winner and the runner-up from each group then continued on to the next round of the competition, where the winner of one group would play the runner-up of another group. The idea behind this competition format is that the winner of a group should have to face an easier opponent in the next round than the runner-up. However, sometimes players prefer to play the winner rather than the runner-up of another group, thereby inducing them to end

up second rather than first in their group. This is precisely what happened in the badminton scandal, and it created incentives for players to play badly.

Besides the moral outrage over these events, some commentators such as the Indonesian Olympic team manager also questioned the design of the competition format. Admittedly, in the case of the team manager, this suggestion was somewhat self-serving because two of the eight players who were disqualified were Indonesian. However, also in academic circles, economists specializing in an area called mechanism design started discussing how the badminton competition could be made incentive-compatible: can we design a competition format in which players never have an incentive to lose a game on purpose? A competition format where playing to one's best ability is always optimal if a player wants to win the competition?

Simple Knockout and Round-Robin Competitions

To model sports competitions mathematically, we make use of the well-known notion of a tournament T (see, e.g., Laslier (1997)) over the set of players (or teams) P : xTy

signifies that x beats y in a direct match between the two teams. In this simple model we will not allow for draws, for any two different teams x and y we have either xTy or yTx . A sports competition C then maps n given players and m tournaments to the player winning the competition. Formally, we have $C : P \times T_P^m \rightarrow P$, where T_P is the set of all tournaments over P .

The two most basic competition formats are knockout and round-robin. Figure 1 depicts a knockout competition for eight players, which we shall call K_8 . At each stage, two players play a match, and the winner, provided by the tournament, continues to the next round. The knockout competition format K_i for i players can be defined recursively:

$$K_2(x_1, x_2, T) = T(x_1, x_2)$$

$$K_{2j}(x_1, \dots, x_{2j}, T) = T(K_j(x_1, \dots, x_j, T), K_j(x_{j+1}, \dots, x_{2j}, T))$$

for $j > 1$, where $T(x, y)$ is the winner of the match between x and y according to tournament T .

Besides knockout competitions, the other basic competition format is a round-robin competition where every team of a pool plays every other team, and the winner is the team that has won most matches. To define round-robin competitions mathematically, we will make use of the *Copeland score* of a player x in a tournament T , the number of players beaten by x : $score(x, T) = |\{y \in P | xTy\}|$. Then we define the winner of the round-robin competition format by $R_n^1(x_1, \dots, x_n, T) = x_i$ iff i is the smallest index for which $score(x_i, T) \geq score(x_j, T)$ for all j . Because it is possible that there is no unique winner, we have used some arbitrary feature (in this case the player's index) to break ties. In soccer, a common way to break ties is to look at the goal difference associated with the team. As the badminton example illustrates, some competition formats also make use of the second-ranked player of a round-robin competition. For this purpose, let $R_n^2(x_1, \dots, x_n, T) = x_i$ iff there is some k such that $R_n^1(x_1, \dots, x_n, T) = x_k$ with $k \neq i$ and i is the smallest index for which $score(x_i, T) \geq score(x_j, T)$ for all j other than k .

Both round-robin and knockout competitions are frequently used in practice: National soccer leagues usually use round-robin competitions to determine the national champion. At the same time, countries such as England and Germany also have cup competitions that are organized according to a knockout format. The winners of these two competitions may then be paired in a direct match for the super cup.

A nice feature of both knockout and round-robin competitions is that they are strategy-proof in the sense discussed earlier: A team can never do better by losing than by winning. How can we make this notion of strategy-proofness or nonmanipulability mathematically precise?

Figure 1. The knockout competition K_8 .

The formal analogue of strategy-proofness is called *monotonicity*. We say that tournament T' weakly improves tournament T for player a , denoted as $T' \geq_a T$, iff for all players x and y , (1) if $x \neq a$ and $y \neq a$ then xTy iff $xT'y$, and (2) aTx implies $aT'x$. Thus, T and T' are almost the same except that a may win more matches in T' than in T . If a competition is strategy-proof, this means that a player should not do worse when winning more games, that is, in tournaments that are weak improvements for her. This leads us to capture strategy-proofness in the following way: A competition C is monotonic iff for all players $a, x_1, \dots, x_n$ and tournaments $T_1, \dots, T_m, T'_1, \dots, T'_m$, whenever $C(x_1, \dots, x_n, T_1, \dots, T_m) = a$ and $T'_i \geq_a T_i$ for all i , then $C(x_1, \dots, x_n, T'_1, \dots, T'_m) = a$.

Both round-robin and knockout competitions are monotonic. In round-robin competitions, winning more matches will increase your Copeland score, and hence if your Copeland score was maximal at the beginning, it will continue to be maximal. In knockout competitions, if a player wins the competition, she must have won all her matches in the competition, and winning (more) matches against players she does not meet in the competition will not change this. Hence, the good news is: National soccer league and cup competitions are safe from strategic manipulation in the way just defined. Playing to one's best ability is always optimal. But what about more complex competition formats, such as the one used for badminton in the 2012 Olympics?

Complex Competitions: Olympics and the Soccer World Cup

Note that the question just raised about the strategy-proofness of complex competition formats applies not only to badminton nor only to the Olympics. The competition format used in the badminton competition is one that frequently occurs also in other sports, maybe most prominently in the FIFA soccer world-cup competitions. The 2014 soccer world cup in Brazil featured 32 teams in its final competition phase. These were divided into eight groups of four teams each, named group A through H. Within each group, a round-robin competition was played. After that, in the round of sixteen, winner and runner-up of a group were paired, respectively, with the runner-up and winner of another group. For example, the winner of group A (Brazil) played the runner-up of group B (Chile), and the runner-up of group A (Mexico) played the winner of group B (the Netherlands). Formally, this competition format can be expressed by

$$K_{16}(R^1(x_1, x_2, x_3, x_4, T), R^2(x_5, x_6, x_7, x_8, T), \dots, R^1(x_5, x_6, x_7, x_8, T), R^2(x_1, x_2, x_3, x_4, T), T')$$

where group A is constituted by x_1 through x_4 and group B is constituted by x_5 through x_8 . Note that we need a second tournament variable T' here, because in the knockout phase, two teams from the same group may meet each other for a second time, but possibly with a different outcome. The reader is invited to show that such a format is not monotonic, thereby establishing that the soccer world-cup competition is not strategy-proof, just like the

badminton case. The simpler case of a competition format involving only two round-robin groups and K_4 is formally shown to fail monotonicity in Pauly (2014), and for the example of the 1954 world cup C_{1954} I will present a similar informal argument below.

As far as I know, the strategic manipulability of soccer world-cup competitions has thus far not led to a scandal similar to the badminton scandal of the 2012 Olympics. However, some soccer fans may be reminded of the non-aggression pact in the match between Austria and Germany in the 1982 world cup in Spain. In German, this match has been called the *Schande von Gijón* (the dishonor of Gijón, the city where the match took place). During this last match of the round-robin group, Germany scored a goal and subsequently neither team made any effort to change the score of 1:0, because this score guaranteed that both teams could continue on to the next round. Losing with more goals against would have prevented Austria from proceeding to the next round, whereas not winning would have prevented Germany from proceeding to the next round. In contrast to the badminton scandal, the teams were not disqualified.

But are there not competition formats that are guaranteed to prevent strategic manipulation? In fact, there are many. We already saw that pure knockout and pure round-robin competitions are strategy-proof (monotonic). Furthermore, the history of soccer world-cup competitions also contains examples of more complex competition formats that are strategy-proof (see Ekuobase and Ebietomere (2012) for a good overview of FIFA competition formats, or see Wikipedia). The very first competition format of 1930 combined round-robin and knockout. It featured 13 teams, with a first round of three groups of three teams and one group of four teams. Each group used a round-robin format. Only the four group winners were allowed to continue on to round 2, which consisted of a knockout competition (semifinals and final). We can model this competition by the following function $C_{1930}(x_1, \dots, x_{13}, T, T') =$

$$K_4(R^1(x_1, x_2, x_3, x_4, T), R^1(x_5, x_6, x_7, T), \\ R^1(x_8, x_9, x_{10}, T), R^1(x_{11}, x_{12}, x_{13}, T), T')$$

This competition format is easily seen to be monotonic: If a team wins the competition, it must be the winner of its own round-robin group, and it must also win all its knockout matches in round 2. Winning more matches will not change this fact. A further example of a strategy-proof competition format is the one used in 1950. Here, FIFA only used round-robin competitions: starting with four groups of four teams each, the round-robin winners of the four groups continued on to a second round of round-robin, the winner of which won the competition. Formally, $C_{1950}(x_1, \dots, x_{16}, T, T') =$

$$R^1(x_1, x_2, x_3, x_4, T), R^1(x_5, x_6, x_7, x_8, T), \\ R^1(x_9, x_{10}, x_{11}, x_{12}, T), R^1(x_{13}, x_{14}, x_{15}, x_{16}, T), T')$$

Again, and by a similar argument to the one just presented, this format is easily seen to be monotonic as the reader may check.

Second-Best is Good Enough

Trouble, however, started with the competition format used for the soccer world cup of 1954. Sixteen teams were divided into four round-robin groups, and the best two teams of each group were allowed to continue on to a knockout competition. $C_{1954}(x_1, \dots, x_{16}, T, T') =$

$$K_8(R^1(x_1, x_2, x_3, x_4, T), R^2(x_5, x_6, x_7, x_8, T), \\ R^1(x_5, x_6, x_7, x_8, T), R^2(x_1, x_2, x_3, x_4, T) \\ R^1(x_9, x_{10}, x_{11}, x_{12}, T), R^2(x_{13}, x_{14}, x_{15}, x_{16}, T), \\ R^1(x_{13}, x_{14}, x_{15}, x_{16}, T), R^2(x_9, x_{10}, x_{11}, x_{12}, T), T')$$

This 1954 format is essentially the same as the one used in 2014, except that there were twice as many teams in 2014 as in 1954. The year 1954 marks an interesting turning point in FIFA's competition format: For the first time, second-placed teams of round-robin rounds were allowed to continue in the competition. This feature has remained an element of FIFA competitions ever since, even though other elements of the competition format have changed in subsequent years. From our perspective, this change is of central importance, for it introduced for the first time the possibility of strategic manipulation into world-cup competitions.

The fact that C_{1954} is not monotonic can be seen as follows: Suppose team x_2 wins the competition and that x_2 is the runner-up of its round-robin group. Suppose also that x_5 wins its round-robin group and that x_6 is the runner-up. Then x_2 wins the competition because it manages to win also against x_5 . However, suppose now that x_2 wins more matches and becomes the winner of its group. Then it will subsequently have to play against x_6 (rather than x_5) against which it may lose and hence not win the competition anymore. This is precisely the formal analogue of the scenario that caused the badminton players to want to lose their matches on purpose.

The source of the manipulability thus resides in the fact that second-ranked teams of round-robin groups are allowed to continue on in the competition. Why did FIFA introduce this change in 1954? Why do many competition formats include this feature? Probably because it leads to more "forgiving" competitions, where mistakes and mediocre performances in the first matches are punished less drastically. In this way, teams have more time to get into the competition and to improve. So if we want to preserve this feature, allowing second-ranked players to continue in the competition, are there ways to avoid strategic manipulation?

The badminton scandal could have been avoided had the second phase of the competition been another round-robin round rather than a knockout round. In that case, both first- and second-ranked players would have continued to the final round-robin round. There would have been no incentive to finish second, because both first- and second-ranked players would have continued to face the same opponents in the next round. But although a double round-robin competition does solve the badminton problem, it is not immune to strategic manipulation of a subtler kind: Suppose that in a soccer competition, we have three teams

Table 1. A Soccer Manipulation Scenario

Rank	Team	# Matches	Points	Goal Difference
1	X	1	3	+4
2	Y	2	3	−3
3	Z	1	0	−1

in a round-robin group, X, Y, and Z. Suppose further that X beats Y with 4 goals to 0, and Y beats Z with 1 goal to 0. Then under the current soccer scoring rules, Table 1 presents the resulting ranking of the teams.

Now consider the final match between X and Z in a competition format where the winner and the runner-up continue on to the next round, which is again a round-robin round. Suppose that Z is a weaker team than Y, and that X would prefer to have to play Z in the next round rather than Y. In that case, X would prefer to lose to Z by a small margin (say 0:1) rather than to win. Hence, even replacing a second-round knockout competition by a round-robin competition does not eliminate the possibilities of strategic manipulation. The possibilities for strategic manipulation are yet subtler than the badminton scandal illustrates.

Is there then no way to use second-ranked round-robin players while preserving monotonicity? In a paper (Pauly 2014), I have shown that if we want to allow runners-up of round-robin groups to continue in the competition, then there are no reasonable competition formats that avoid all possible strategic manipulations. More in detail, the result says the following: Suppose we want a competition format C that starts with two round-robin groups A and B involving a number of different teams where the overall winner of the competition is determined by some mechanism M from the four teams who are ranked first and second in the two groups (see Figure 2). We want the competition format C to be strategy-proof (i.e., monotonic). Furthermore, we want M to be a reasonable mechanism, that is, it should select the overall winner from the first- and second-ranked players of the two round-robin groups in a reasonable way. What does this mean? First, no team should receive a special advantage because of who it is (e.g., because it is European rather than American). Second, the winner of one round-robin group should not have an advantage over the winner of the other group, and similarly with second-ranked players. Third, every one of the four teams should be allowed to win the overall competition at least sometimes (otherwise, there would be no reason to allow the team to continue to the next round in the first place because it can never win). Fourth and finally, the winner should only be determined based on the matches these four teams play against each other; no matches against other teams are taken into account. The impossibility result states that there is no competition format C satisfying these conditions.

Open Issues and Conclusions

The mathematical model used in this article is a very simple model to investigate strategic properties of sports competitions. Strictly speaking, it only describes win-lose games

Figure 2. A complex competition format.

and does not allow for draws. Extending the model to allow for draws is easily done, and I conjecture that the conclusions reached in the simpler model will remain the same. A more interesting extension of the model would add random devices. In this way, we can describe, for example, competition formats where we toss a coin between the winners of two round-robin groups. The winner of the coin toss then is allowed to decide whether to play the winner or the runner-up of the other group. This also creates an incentive for teams to win their matches, because it offers them the chance to choose their opponent from the other group. I have not yet seen such a competition format being used in practice.

Another extension of the simple model might be to model simultaneous matches explicitly. As a consequence of the Gijón match between Austria and Germany, the world soccer association FIFA implemented a change in the competition format to avoid strategic play: the last two group matches are now played simultaneously. There are, however, at least three problems with this solution. First, it does not solve the problem of the badminton scandal. If in the last two matches, only one of the matches is relevant for determining the winner and the runner-up of the group, the situation of the badminton scandal can still arise. Second, the problem of strategic play may repeat itself on a smaller scale. A team may learn during halftime about the score in the other match and might try to play accordingly. A third problem is that viewers are forced to choose between the games to watch, so it becomes impossible to watch all world-cup matches in real time, and consequently advertising revenues per match might decrease.

Also, we have seen that there are actually different degrees of strategic manipulation: The strategic play that took place in the badminton scandal was more dramatic

than the example illustrated in Table 1. Using round-robin rather than knockout in phase 2 of a competition avoids one kind of manipulation but not the other. The current model is too crude to make such distinctions.

What conclusions can we draw from these considerations? First, the design of competition formats in sports matters. Depending on the competition format, various kinds of strategic manipulation of the players or teams may occur, so it is important to design sports competitions with care. Second, the use of runners-up of round-robin groups is a source of “unethical” conduct in sports competitions. The badminton scandal and the soccer scandal of Gijón demonstrate the problems that can arise when runners-up are allowed to continue to the next round. It can be shown mathematically that strategic manipulation is essentially unavoidable in competition formats that make use of runners-up. Third, does this mean that we should never use competition formats that use runners-up? No, for two reasons: On the one hand, we have seen that even when using runners-up, there are ways to avoid at least some forms of strategic manipulation. Simultaneous game play and the use of round-robin groups in later competition stages are examples. On the other hand, avoiding strategic manipulation is only one goal in the design of competition formats. Other goals may include increasing or decreasing the number of games to be played, not wanting simultaneous

matches, wanting to have a final match to determine the overall winner, allowing players who lose a game the chance to continue on in the tournament, and so on. Depending on how we weigh these different goals, we will end up with different competition formats. So the moral of the story of strategic manipulation in sports is: We should think carefully about the design of our competition formats.

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