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1. Introduction

The response of banks’ revenue to changes in marginal costs has long been a topic of intrinsic interest, as well as suggesting an empirical test for competitive conduct (Rosse and Panzar, 1977; Panzar and Rosse, 1987; Vesala, 1995). The linear homogeneity of marginal costs in factor prices permits a parsimonious implementation of this model with modest data requirements, an approach that has recently seen dramatically growing popularity. Google Scholar lists more than 500 papers using the Panzar–Rosse method just since 2012, including 21 in Web of Science journals. Most of these studies have utilized banking data, but the method has also been applied to non-banking samples. For a historical overview of the Panzar–Rosse literature, see Bikker et al. (2012). Some recent banking studies applying the Panzar–Rosse approach include Barbosa et al. (2015), Anginer et al. (2014), Huang et al. (2014), Moch (2013), Weill (2013), Hoxha (2013), Akin et al. (2013), Fosu (2013), Liu et al. (2012), Chen and Liao (2011), Olivero et al. (2011), Jeon et al. (2011) and Goddard and Wilson (2009).

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The contrast between the latter claim from Bikker et al. (2012) and the possibility of positive H under imperfect competition noted by Panzar and Rosse (1987) and Vesala (1995) raises a puzzle. The aim of this study is to explore and resolve that puzzle. Our main finding is that neither the sign nor the magnitude of H can reliably identify the degree of market power. This result
follows from analyzing the equilibrium response of revenue to changes in marginal costs in five alternative oligopoly settings. We show that positive $H$ can arise with economically plausible parameter ranges in all five highly non-competitive scenarios, thereby demonstrating that positive $H$ cannot rule out a high degree of market power, contrary to the claim in Bikker et al. (2012). The possibility of $H > 0$ under conditions of substantial market power turns out robust to the timing of banks’ actions, relative costs, choice of strategic variable (price or quantity), degree of product differentiation, strategy (static or dynamic), and degree of heterogeneity in banks’ conduct (collusive versus fringe). These five scenarios are not exotic, but are among the most standard oligopoly situations considered in the theoretical industrial organization literature. In short, $H > 0$ appears not to be a pathological or rare outcome for non-competitive situations, but can arise under a wide variety of highly non-competitive conditions. In combination with the results from the existing literature, this leads to the conclusion that the $H$ statistic can take either sign for any degree of competition. Consequently, the Panzar–Rosse revenue test can be used neither as a quantitative measure nor as a one-sided measure of market power. This conclusion has stark implications for an extensive body of research literature, especially empirical banking studies with important policy implications.

Our results provide a bridge between recent industrial organization literature and the empirical banking literature. The recent industrial organization literature has moved away from employing the Panzar–Rosse method in empirical studies, largely because of the concern that reduced-form models are generically incapable of achieving econometric identification of the relevant parameters (Reiss and Wolak, 2007). Our findings point to an additional form of non-identification that is specific to the Panzar–Rosse method and that invalidates the $H$ statistic as a measure of market power. Our main conclusion is that even if $H$ could be econometrically identified (an important issue typically ignored in empirical banking applications), it still would provide ambiguous conclusions about the degree of competition, either alone or in conjunction with various auxiliary tests.

More specifically, our results extend Hyde and Perloff (1995). On the basis of a simulation study, Hyde and Perloff (1995) show that the log-linear reduced-form revenue equation is generally misspecified. Where one could possibly conclude that this misspecification could be mitigated by resorting to more robust econometric techniques (such as nonparametric estimation of input price elasticities), our study shows that such a strategy will not help due to the theoretical limitations of the $H$-statistic. The main issue is that even the $H$-statistic based on a correctly specified revenue equation can be used neither as a quantitative measure nor as a one-sided measure of market power.

The remainder of this study is organized as follows. Section 2 illustrates that a positive value of $H$ can arise under various conditions of substantial market power. The implications of our theoretical findings are discussed here as well. Finally, Section 3 concludes.

2. New theoretical results

Whereas the empirical Panzar–Rosse (P–R) literature has focused almost exclusively on a simple notion that negative values of the $H$ statistic denote monopoly power while positive values denote more competitive conditions, a growing body of theoretical analysis has pointed out important exceptions to this pattern. For example, Shaffer (1983) and Bikker et al. (2012) showed that $H < 0$ can occur even under highly competitive conditions – the former in the case of blocked entry (fixed numbers of banks), and the latter in the case of constant average cost. Hyde and Perloff (1995, p. 482) similarly noted that, “[...] for many models, [the Panzar–Rosse] test cannot distinguish between collusion and competition [...]” and “We were unable to generate an example in which their test clearly distinguished between collusion and competition [...].”

This section extends the known deviations of $H$ from its commonly assumed properties by analyzing the implications of five standard market structures for the equilibrium response of bank revenue to marginal cost. The plan of our analysis is as follows. In each of five highly noncompetitive cases, we show that $H > 0$ can arise for each bank, thus disproving the notion that $H > 0$ always rules out significant market power. The noncompetitive nature of each case is established by construction (duopoly with blocked entry) and confirmed by showing a positive Lerner index (Lerner, 1934). The Lerner index compares a firm’s output price with its associated marginal costs, where Lerner (1934) referred to marginal cost pricing as the “social optimum that is reached in perfect competition”. A positive Lerner index is formally equivalent to the presence of market power, in the sense of pricing above the competitive level.2

2.1. Theorems

Throughout, when we refer to $H$ without a bank subscript, we mean the generic $H$ statistic. With a subscript, we refer to a bank-specific $H$ statistic in the context of the proof. TR stands for total revenue, MC for marginal costs, and AC for average costs.

We begin by analyzing a standard Stackelberg duopoly, and show that it provides the first direct example supporting a claim of Panzar and Rosse (1987) and Vesala (1995) that $H > 0$ is possible in a conventional static oligopoly. We show:

Lemma 2.1. In a Stackelberg duopoly with linear costs (i) The $H$ statistic is the same for the leader and the follower and can take either sign; (ii) The Lerner index is the same for the leader and the follower and has a positive sign.

Proof. Let $P = a - bx_1 - bx_2$ for banks $i = 1, 2$ producing $x_i$ at constant marginal costs $c$ and fixed costs $f_i$. Firm 1 earns profit

$\pi_1 = x_1(P - c) - f_1 = x_1(a - c - bx_1 - bx_2) - f_1$ and bank 2 earns profits

$\pi_2 = x_2(P - c) - f_2 = x_2(a - c - bx_1 - bx_2) - f_2$. The Stackelberg follower (bank 1) chooses output $x_1$ to maximize its profit, given the output selected by the other bank, with first-order condition $a - c - bx_1 - 2bx_2 = 0$, so $x_1 = (a - c - bx_2)/2b$. The leader (bank 2) chooses output $x_2$ to maximize its profit, conditional on the reaction function of bank 1. Its first-order condition is $(a - c)/2 - bx_2 = 0$ or $x_2 = (a - c)/2b$. Then $x_1 = (a - c)/4b$ and $P = (a + 3c)/4$. The second-order condition is $-b < 0$ for each bank, which is satisfied for $b > 0$ (downward-sloping demand curve). To ensure that both banks are financially viable in equilibrium, we assume that $a > c$.4

2 Pricing above marginal cost results in a lower quantity consumed than with marginal-cost pricing, due to downward-sloping aggregate demand; the combination of higher price and lower quantity reduces consumer welfare compared to marginal-cost pricing. It is in this sense that a positive Lerner index indicates imperfect competition (market power). The equilibrium value of the Lerner index depends on the model (Cournot, Bertrand, Stackelberg, etc.) and on specific parameter values within the model, thus making it relevant to confirm whether a particular model (such as used in our proofs) is consistent with a positive Lerner index. This role of the Lerner index in our theoretical analysis is not intended to convey in any sense that the Lerner index may be a superior empirical measure of market power in practice, as empirical estimates of the Lerner index may suffer from measurement error, specification error, or other problems.

3 Constant marginal costs are economically relevant in many industries (Johnston, 1960).

4 We allow for fixed costs, but we notice that first-order and second-order conditions, prices, quantities, revenues, Lerner index and $H$ statistic are the same in equilibrium either with or without fixed costs. Fixed costs only appear in the bank’s profit function and do not affect the bank’s choices of production or pricing, except possibly the overall decision to enter or exit a particular market.
If both banks maintain the same leader–follower sequence, we can solve directly for the impact of a change in c on each bank’s output, equilibrium price, and revenue: $\partial q_1/\partial c = -1 - 4b$, $\partial q_2/\partial c = -1/2b$, and $\partial P/\partial c = 3/4$. For bank 1, $\partial TR_1/\partial c = q_1P_1/\partial c + x_1P_1/\partial c$. Consequently, $\partial TR_1/\partial c = (2a - 6c)/16b$. Similarly, we find $\partial TR_2/\partial c = (2a - 6c)/8b$. Using Lemma A.1 from Appendix A, we calculate the H statistic explicitly for each bank and the market average, yielding $H_1 = (cTR_1)/\partial TR_1/\partial c = 2c(a - 3c)/[a(c + a + 3c)]$, $H_2 = (cTR_2)/\partial TR_2/\partial c = 2c(a - 3c)/[a(c + a + 3c)]$, and $\bar{H} = (H_1 + H_2)/2 = (2a - 3c)/[a(c + a + 3c)]$. Hence, $H_1 = H_2 = \bar{H} = 0$. There are two cases: $c < a < 3c$ (yielding $H > 0$) or $a > 3c$ (resulting in $H > 0$). Hence, H can take either sign for Stackelberg duopolists, depending on the relative magnitudes of $a$ and $c$. Each bank’s Lerner index (i.e., the relative markup of price over marginal cost; see Lerner, 1934) is $L = (a - c)/(a + 3c)$, with $L < 0$ for $a > c$, conbanking the existence of market power in the sense of pricing above the competitive level.5 6

As an alternate non-competitive market structure, we next consider a homogeneous Cournot duopoly with asymmetric costs and positive Lerner indices, and derive corresponding properties in that setting:

**Lemma 2.2.** In a homogeneous Cournot duopoly with asymmetric costs and linear demand (i) The H statistic can take either sign for the low-cost bank and the market average, while it is always positive for the high-cost bank; (ii) The Lerner index is positive for the low-cost bank, the high-cost bank and the market average.

**Proof.** Let $P = a - bx_1 - bx_2$, where $x_i$ is the output quantity of bank $i = 1, 2$. Let the total costs of $x_1$ equal $c_1 + x_1c_2$ and the total costs of $x_2$ equal $c_2 + x_2c_2$. We assume that $a > c$, so that the first bank has lower marginal cost. By standard first-order conditions for profit maximization, $x_1 = (a - c(2 - x))/3b$, $x_2 = (a - c(2x - 1))/3b$, and $P = (a + c(x + 1))/3$. Second-order conditions are satisfied for each bank, as $-2b < 0$ for $b > 0$ (downward-sloping demand curve). To ensure that both banks are financially viable in equilibrium, we assume that $a > c(2x - 1)$. Now $TR_1 = a^2 + ac(2x - 2) + c^2(2x^2 - x - 2)/9b$, $\partial TR_1/\partial c = (a(2x - 1) + 2c(x - a - 2))/9b$, and, according to Lemma A.1, $H_1 = (cTR_1)/\partial TR_1/\partial c = (a(2x - 1) + 2c(x - a - 2))/[a(2x^2 - x - 2)]^2 + ac + ac + c(2x - a - 2)$. Similarly, we find $H_2 = (cTR_2)/\partial TR_2/\partial c = (a(2x - 1) + 2c(x^2 - x - 2))/[a(2x^2 - x - 2)]^2 + ac + c(2x - a - 2)$. The assumption $a > c(2x - 1)$ implies that the denominator of $H_1$ is positive for $x > 1$. The numerator of $H_1$ is positive for $a > -2c(x - a - 2)/[2(2x - 1)]$. If $a > c(2x - 1)$, the latter condition is met for $x > (1 + \sqrt{3})/2 \approx 1.37$. Thus, $H_1 > 0$ for $a > c(2x - 1)$ and $x > (1 + \sqrt{3})/2$, while $H_1 < 0$ for $a > c(2x - 1)$ and $x < (1 + \sqrt{3})/2$.5 6 The Lerner index for the low-cost bank is $L_1 = [a + c(x - a)/a + c(x + a)]$, with $L_1 > 0$ for $a > c(2x - 1)$ and $x > 1$, conbanking market power under the stronger condition $a > c(2x - 1)$ and $x > 1$. Moreover, $\partial L_1/\partial x = 3c/[a + c(x + a)] > 0$, so greater inequality in marginal costs results in a higher value of the low-cost bank’s Lerner index. Regarding $H_2$, the second term in the denominator (and thereby the entire denominator) will be positive for $a > c(2x - 1)$. The numerator of $H_2$ is positive for $a > c(4x^2 + 2x - a)/2(x - a)$. The latter condition is met for $a > c(2x - 1)$ provided that $a > (1 + \sqrt{97})/12 \approx 0.90$. Hence, $H_2 > 0$ for $a > c(2x - 1)$ and $x > 1$. The high-cost bank’s Lerner index equals $L_2 = [(a - c(2x - 1))/[(a + c(x + a)]]$, with $L_2 > 0$ for $a > c(2x - 1)$ and $x > 1$. We have $\bar{H} = (H_1 + H_2)/2 > 0$ in two cases: for $H_1 > 0$ and $H_2 > 0$, and for $H_2 > 0$ and $H_1$ not too negative. We omit the expression for $\bar{H}$ because of its complexity. The market average Lerner index equals $\bar{I} = (2a - c(x + 1))/[(a + c(x + a)]$, with $\bar{I} > 0$ for $a > c(x + 1)/2(2x - 1)$ and $x > 1$. Consequently, we also have $\bar{I} > 0$ under the stronger condition $a > c(2x - 1)$ and $x > 1$.7

Next, we provide a related result for a differentiated Bertrand duopoly with positive Lerner index in the spirit of Singh and Vives (1984).

**Lemma 2.3.** In the differentiated Bertrand duopoly (i) The H statistic can take either sign for both banks and the market average; (ii) The Lerner index is positive for both banks and the market average.

**Proof.** In the model of a linear differentiated duopoly using the notation of Singh and Vives (1984), demand functions are $x_1 = a_1 - b_1p_1 + c_1p_2$ and $x_2 = a_2 - c_1p_1 - b_2p_2$ where $a_1 > 0$ and $b_i > 0$ for $i = 1, 2$. Unlike Singh and Vives (1984), we focus only on the case $c > 0$ so that the two goods are (possibly imperfect) substitutes (i.e., the banks are competing in the same industry). We notice that a bank’s own-price effect (given by $b$) should be no weaker than the cross-price effect from a rival (parameterized by $c$), so $b > c$. Firm 1 produces at constant marginal costs $m_1 > 0$, with fixed costs $f_1$. Unlike Singh and Vives (1984), we do not suppress the marginal cost, so profit for bank 1 is $\pi_1 = x_1(p_1 - m_1) - f$. In the Bertrand equilibrium, $p_1 = (2a_1b_2 + 2b_1m_1 + a_2c + b_2c_2)/(4b_1b_2 - c_2^2)$, $x_1 = b_1(a_1 + 2a_2b_2 + 2b_2c_2m_1)/(4b_1b_2 - c_2^2)$. Similar expressions can be found for $p_2$ and $x_2$. Second-order conditions are satisfied, because $-2b_i < 0$ for $b_i > 0$ (downward-sloping demand curve). In the fully symmetric case (where $a_1 = a_2$ and $b_i = 0$), we find $TR_i = TR_2 = TR = [-b(a + bm)]/(b - c + m)/(b - c + m)^2$ and $\partial TR_i/\partial m = [b(ac + 2b(c - b)m)]/(c - 2b^2)^2$. We need $a > (b + c)b$ to ensure that both banks are financially viable in equilibrium. Using Lemma A.1, we find $H_1 = H_2 = H = m(ac + 2b(c - b)m)/(a + bm)(a + c(b - b)m)$. $H > 0$ requires $a > 2b(b - c)m/c$, which is stronger than the requirement $a > (b + c)m$ because $b > c$. We have $H < 0$ for $a < 2b(b - c)m/c$. The Lerner index is the same across banks and equals $L = [a + c(b - b)m]/(a + bm)$, with $L > 0$ for $a > (b + c)m$, conbanking the existence of market power in the sense of pricing above the competitive level.1 The asymmetric case is more complicated, but results in similar conclusions.  □

Further, we also find the possibility of $H > 0$ in a dynamic equilibrium with positive Lerner index:

**Lemma 2.4.** In a dynamic open-loop duopoly equilibrium with sticky prices as analyzed by Fershtman and Kamien (1987) (i) The H statistic is the same for both banks and can take either sign; (ii) The Lerner index is the same for both banks and has a positive sign.

**Proof.** In the model and notation of Fershtman and Kamien (1987), the unique stationary open-loop duopoly equilibrium occurs at $p = [a + (a + 2c)(s + r)/(4s + 3r)]$ and $x = (a - c(s + r)/(4s + 3r))$ where $p$ is price, $s$ an exogenous parametric speed of adjustment of prices, $x$ the rate of output of bank $i$, $a$ the

5 See also Angner et al. (2014), Fu et al. (2014), Efthymiou and Yildirim (2014), Bos et al. (2013), Koetter et al. (2012), Cipolloni and Fiordelisi (2012), Chen and Liao (2011) and Turk Aris (2010).

6 Notice that $H < 0$ in the traditional case of a symmetric Cournot or conjectural variation oligopoly.

7 We notice that $H > 0$ is also possible in the more general setting of an n-bank asymmetric Cournot competition.

8 This contrasts the well-known property of homogeneous Bertrand oligopoly that ‘two is enough for competition’ if marginal costs are constant.
reservation price of aggregate demand, and \( r \) an exogenous discount rate of future earnings.\(^9\) To ensure positive production in equilibrium, it is necessary to have \( a > c \). The total costs of production equal \( f^0 + cx_k + x_k^2/2 \), so marginal costs \( MC_k \) equal \( c + x_k \).

According to Lemma A.1, \( H_i = (MC_i/TR_i)\partial TR_i/\partial MC_i \). A change of variable gives the identity \((\partial TR_i/\partial MC_i)\partial MC_i/\partial c = \partial TR_i/\partial c \). Furthermore, \( \partial TR_i/\partial c = p(\partial x_i/\partial c + x_i\partial p/\partial c) = (r + s)(a - r(1 + s))/((3r + 4s)^2) \). and \( \partial MC_i/\partial c = 1 + x_k\partial c/\partial c = (2r + 3s)/(3r + 4s) \). This results in \( H_1 = H_2 = H = (ar - 4c(s + r))(a(s + r) + c(3r + 2s))/((a - c)(3r + 2s) + 2c(s + r)) \), which implies that \( H \) has the same sign as \( ar - 4c(s + r) \), with \( H > 0 \) for \( a > 4c(1 + s/r) \) and \( H < 0 \) for \( c < a < 4c(1 + s/r) \). The Lerner index is the same across banks and equals \( L = s(a - c)/(a(2s + r) + 2c(s + r)) \), with \( L > 0 \) for \( a > c \). \( \square \)

The next result illustrates that failure of \( H \) can also arise with various numbers of banks in the market, instead of only two as in the previous propositions.

**Lemma 2.5.** In a stable cartel with a Cournot fringe and linear costs and demand (i) The \( H \) statistic is the same for the fringe and the cartel and can take either sign; (ii) The \( L \) index is the same for all banks (whether in the fringe or in the cartel) and has a positive sign.

**Proof.** Assume a linear market inverse demand function \( p = a - b(x_k + x) \) (where \( x_k \) is the fringe’s output quantity and \( x \) the cartel’s output quantity) and constant marginal costs \( c \), where \( a > c \) is required for banks to earn non-negative profit in equilibrium. Consider the endogenous stable cartel containing \( k \) banks out of a total of \( n \) banks in the market. Shaffer (1995) shows that the unique stable value equals \( k = (n + 2)/2 \) for even \( n = 2 \) and \( k = (n + 3)/2 \) for odd \( n > 3 \). Total revenue for each bank in the Cournot fringe is \( TR_f = \pi_k \), where the equilibrium values are \( p = a + (1 + 2n - 2k)c/2(1 + n - k) \) and \( x_f = (a - c)/(2b(1 + n - k)) \). Thus, \( TR_f = (a - c)(a + c(1 + 2n - 2k))/4b(1 + n - k)^2 \), with \( TR_f > 0 \) for \( a > c \). Total revenue for the cartel is \( TR_i = k\pi_k \), where the equilibrium price \( p \) is given above and equilibrium implies \( x_c = (a - c)/2b \). Thus, \( TR_i = (a - c)(a + (1 + 2n - 2k)c)/4b(1 + n - k)^2 \)\(^\text{10}\).

Now \( \partial TR_i/\partial c = [(c - 1 + 2k - 2n) + a(n - k)]/2b(1 + n - k)^2 \) and \( \partial TR_i/\partial k = [(c - 1 + 2k - 2n) + a(n - k)]/2b(1 + n - k)^2 \). According to Lemma A.1, we thus find \( H_1 = H_2 = H = c(2k - 2n - 1) + a(n - k))/[(a - c)(a + c(1 + 2n - 2k))]. We have \( H > 0 \) for \( c < a < 2c(2n - 1)/(n - k) \), which boils down to \( k > 2c(n - 1)/(n - 2) \) for even \( n > 2 \) and \( a > 2c(n - 1)/(n - 3) \) for odd \( n > 3 \). Furthermore, \( H < 0 \) for \( c < a < 2c(2n - 1)/(n - 2) \) for odd \( n > 3 \). The Lerner index equals \( L = p/c \), which in equilibrium reduces to \( L = (a - c)/[a + c(1 + n - 1)] \) for even \( n > 2 \) and to \( L = (a - c)/[a + c(n - 2)] \) for odd \( n > 3 \). We thus find \( L > 0 \) for \( a > c \). Note that the Lerner index is the same for each bank, whether in the cartel or in the fringe, because all banks face the same marginal costs and sell at the same price in equilibrium. \( \square \)

Although our theoretical results literally apply only to Stackelberg, asymmetric Cournot, differentiated Bertrand duopoly, dynamic open-loop duopoly with sticky prices, and stable cartel with a Cournot fringe, the broader point is that there are many conditions under which positive values of \( H \) may be observed despite substantial market power. That is, we do not claim that these are the only situations in which \( H > 0 \) despite substantial market power. Indeed, the relative ease of finding these cases suggests that \( H > 0 \) under a variety of other non-competitive settings as well.

Our propositions identify five sets of sufficient conditions, not necessary conditions, for this outcome. The five scenarios include highly diverse situations: when banks act sequentially or simultaneously, whether there are different marginal costs or potentially identical marginal costs, choose quantity or price, produce differentiated products or homogeneous products, have a static or dynamic strategy, exhibit collusive or fringe behavior. Thus, the possibility of \( H > 0 \) under conditions of substantial market power is robust to the timing of banks’ actions, relative costs, choice of strategic variable (price or quantity), degree of product differentiation, type of strategy (static or dynamic), and degree of heterogeneity in banks’ conduct (collusive versus fringe). These findings provide emphatic support to the claims of Panzar and Rosse (1987) and Vesala (1995) that \( H > 0 \) can arise under imperfect competition, and provide a correction to the claim of Bikker et al. (2012) that the \( P-R \) test provides a one-tail test of competition.

Our theoretical results show that the \( L \) index and the \( H \) statistic (interpreted in the conventional way) can yield contrasting results, in the sense that a positive markup of price over marginal cost (positive \( L \) index) can be observed in the same sample that generates a positive \( H \) statistic. We are aware of only one previous study that explores any formal theoretical connection between \( H \) and the \( L \) index, and the result shown there is opposite to those here: Shaffer (1983) shows that in the special case of no entry or exit, no change in demand, and isoelastic demand, we would expect to see a negative \( H \) statistic but a positive \( L \) index.

The contrast between that earlier result and the findings here illustrates the absence of a universally valid relation between \( H \) and the \( L \) index. Indeed, our theoretical results illustrate the fact that the algebraic linkage between \( H \) and the \( L \) index involves various other factors that may typically vary from sample to sample and that, moreover, often cannot be observed, such as banks’ choice of strategic variable between price and quantity or banks’ sequence of decisions (i.e., simultaneous choice or leader/follower).

Another way of understanding this issue is to note that the \( L \) index is, apart from any fixed costs, generally a monotonic function of the degree of excess profitability sustained by market power, whereas the \( H \) statistic is not an ordinal or monotonic measure of market power (Bikker et al., 2012).\(^11\) Generically, there can be no monotonic relation between any ordinal measure and any non-ordinal measure.\(^12\)

### 2.2. Economic relevance of \( H > 0 \) in the presence of market power

In Lemmas 2.1, 2.2, 2.4 and 2.5 the parameter \( a \) denotes the reservation price, which is the highest price a buyer is willing to pay. In each of the four propositions, the reservation price must exceed a scalar multiple of marginal costs to achieve \( H > 0 \). It seems intuitively plausible that such a condition holds in practice.

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\(^9\) Similar to Fershtman and Kamien (1987), we do not provide second-order conditions here. Notice that dynamic games do not exhibit second-order conditions that correspond directly to the second-order conditions of static games.

\(^10\) We refer to Shaffer (1995) for the details about the derivation of equilibrium prices and quantities, as well as the stable value of \( k \).

\(^11\) To illustrate the \( H \) statistic’s lack of ordinality, consider two markets. Lack of ordinality means that market 1 is not always more competitive than market 2 when \( H_1 > H_2 \).

\(^12\) Some readers may worry that duality theory guarantees a fixed relationship between the cost function and the revenue function, and thus between \( L \) and \( H \), implying some sort of contradiction in our theoretical findings. But duality only implies a fixed relationship between cost and revenue functions under perfect competition (McFadden, 1978, pp. 4 and 66). The research program of empirically measuring the degree of competition, by contrast, is motivated by the fact that competition is not always perfect, in which case duality does not guarantee any fixed relationship between cost and revenue functions. The possibility of imperfect competition is also widely acknowledged in theories of monopoly, oligopoly, dynamic enforcement mechanisms, etc., as well as in the global prominence of antitrust policies.
For example, many people willingly pay 20% interest rates on credit cards issued by banks that pay less than 3% costs of funds, while other consumers accept loans at even higher interest rates from consumer finance companies (up to about 36% per year). Credit cards issued by banks that pay less than 3% costs of funds, a discount rate 0.0344, are competitive (the latter encompassing various subcategories of monopoly, perfect cartel, static oligopoly, etc.). Primary references for each theoretical result are shown. The key feature evident in this enumeration is that theoretical analysis has linked each category of conduct with both positive and negative values of $\zeta$. In particular, competitive outcomes can generate $H$ of either sign, depending on technological and other factors, monopolistic competition can generate $H$ of either sign, and various types of substantially non-competitive conduct can also generate $H$ of either sign. In short, the sign of $H$ cannot yield any distinct conclusion about the degree of competition. For example, a positive empirical estimate of $H$ cannot distinguish among these various cases and, in particular, cannot rule out significant market power. This leads to our main theorem, which follows immediately from Table 1:

**Theorem 2.1.** The $H$ statistic can be either positive or negative for any degree of competition. Consequently, the Panzar–Rosse revenue test can be used neither as a quantitative measure nor as a one-sided measure of market power.

### 2.4. Discussion

Empirical applications of the $H$ statistic have typically relied on a subset of the theoretical results known about the $H$ statistic to zero-price quantity. If the two banks produce highly substitutable outputs (which seems likely, because – controlling for price and terms – a dollar borrowed from one bank is just as good as a dollar borrowed from another bank) then $b \approx c$ and the restriction on $a$ is not severe. At the other extreme, if the two banks produce outputs that consumers consider not at all substitutable ($c = 0$), then the market functions as two independent monopolists, and the standard monopoly result $H < 0$ applies.

### Table 1

$H$ statistic and competitive conditions

<table>
<thead>
<tr>
<th>Competitive conditions:</th>
<th>Positive</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Competitive</td>
<td>$H = 1$ in long-run competitive equilibrium with U-shaped AC (Rosse and Panzar, 1977; Vesala, 1995)</td>
<td>$H &lt; 0$ possible in long-run competitive equilibrium with flat AC (locally constant returns to scale) (Bikker et al., 2012)</td>
</tr>
<tr>
<td>Monopolistic competition</td>
<td>$H &gt; 0$ possible (Rosse and Panzar, 1977, 1995, Result 2.2, p. 55)</td>
<td>$H &lt; 0$ possible in short-run competitive equilibrium (Shaffer, 1982a; Shaffer, 1983)</td>
</tr>
<tr>
<td>Substantially noncompetitive</td>
<td>Lemma 2.1: $H &gt; 0$ possible in Stackelberg duopoly</td>
<td>$H &lt; 0$ for monopoly (Rosse and Panzar, 1977; Panzar and Rosse, 1987)</td>
</tr>
<tr>
<td></td>
<td>Lemma 2.2: $H &gt; 0$ possible in asymmetric Cournot duopoly</td>
<td>$H &lt; 0$ for perfect cartel (Vesala, 1995, Result 2.4, p. 58)</td>
</tr>
<tr>
<td></td>
<td>Lemma 2.3: $H &gt; 0$ possible in differentiated Bertrand duopoly.</td>
<td>$H &lt; 0$ for static oligopoly if industry demand is elastic at the equilibrium level of aggregate output (Panzar and Rosse, 1987, pp. 453–455; Vesala, 1995, pp. 57–58)</td>
</tr>
</tbody>
</table>

For example, many people willingly pay 20% interest rates on credit cards issued by banks that pay less than 3% costs of funds, while other consumers accept loans at even higher interest rates from consumer finance companies (up to about 36% per year uncompounded) or payday loan companies (even higher rates), although in all cases the lenders have much lower costs of lending, often even correcting for default rates.

The economic relevance of the condition $\zeta > (1 + \sqrt{3})/2 \approx 1.37$ in Lemma 2.2 can be assessed by analyzing the year-end 2010 US bank Call Report data. We take the ratio of total expenses to total assets as an approximation of average costs, which should be correlated with marginal cost. The quantities that yield an average costs ratio of at least 1.37 are the upper and lower 23% quantile. Therefore, if the ratios of marginal costs are similar to the ratios of this proxy of average costs, the top and bottom 23% of all US banks would satisfy the $\zeta$-criterion to guarantee $H > 0$. In the asymmetric Cournot case. This would mean that the condition is satisfied for at least 23% and 23% of all US banks – and actually substantially more because, for example, banks in the top 10% quantile have $\zeta > 1.37$ compared to many banks not in the bottom 23% quantile, while banks in the bottom 10% satisfy the $\zeta$-criterion for many banks not in the top 23% quantile.

The lower bound on the reservation price in Lemma 2.4 involves a discount rate $0 < r < 1$ and a speed of price adjustment $0 < s < \infty$. In practice $r$ will be a small positive number, say $r = 3%$. It is more difficult to determine a realistic value of $s$. We therefore proceed in a different way by observing that the speed of adjustment $s$ must satisfy $s < r/(a/4c) - 1$ to get $H > 0$. With $a > 4c$ this upper bound is positive, while the lower bound on $a$ seems realistic as argued before.

In Lemma 2.3 the parameter $a$ corresponds to the quantity that would be demanded if prices were zero, which for a normal good would exceed the observed output quantity at positive prices. Defining $z = (b - c)m$, where we have $a > z$ for financial viability of either bank, we see that $H > 0$ requires $a > (2b/c)z$. Since $b > c$, this implies that $H > 0$ requires $a > 2z$. Since $a$ corresponds to the quantity demanded at zero price, $H > 0$ requires that this zero-price demand quantity exceed twice the break-even
interpret their findings, ignoring other theoretical results that are equally valid but carry opposing implications. The general problem caused by this ‘conceptual cherry picking’ is one of necessary versus sufficient conditions. Theoretical analysis of reduced form revenue functions – such as the seminal studies by Rosse and Panzar (1977) and Panzar and Rosse (1987) – always begins with a particular assumption about conduct, and derives the associated value or sign of $H$. The pattern of conduct is thus shown to be a sufficient condition for particular values of $H$. Using empirical estimates of $H$ to identify the degree of market power requires the opposite condition to be true – that is, that a particular pattern of conduct must be necessary for the observed value of $H$ (or, equivalently, that the value of $H$ must be sufficient for a particular pattern of conduct). Due to the huge variation in possible conduct, however, it is generally not possible to link specific values of $H$ uniquely to a certain type of conduct. Appendix B provides more intuition to understand why it is generally not possible to link specific values of $H$ uniquely to a certain type of conduct.

Can the revenue test be salvaged by auxiliary analysis? The new theoretical results indicate that independent information about the nature of equilibrium – whether choosing price versus quantity, concurrent versus sequential choice, static versus dynamic strategy, different or identical marginal costs, homogeneous versus differentiated products, or collusive versus fringe behavior – must be known before we can rule out the possibility of strong market power despite $H > 0$. Further, while necessary, such information is not enough to guarantee a reliable interpretation of the sign or magnitude of $H$. Instead, exact functional forms and parameter values of bank-specific costs and demand must be known on a case-by-case basis. The associated data requirements are not merely onerous, but generally infeasible. There appears to be no practical way of rescuing $H$ as a reliable measure of market power. One could always pose this issue as a question for further research, but an obstacle is that recent research has been uncovering additional shortcomings of $H$ that expand the known dimensionality of essential auxiliary tests, rather than converging to a workable solution (Bikker et al., 2012).

3. Conclusions

Motivated by a recent proliferation of the Panzar–Rosse method’s use in banking studies, we have analyzed the response of equilibrium revenue to changes in marginal costs under five market structure scenarios not previously explored in this context. We identified five highly non-competitive market structures in which the sum of revenue elasticities with respect to input prices, $H$, can take a positive value within economically plausible parameter ranges, providing the first formal proof of a property claimed by Panzar and Rosse (1987) and Vesala (1995) and contrary to a pattern previously established for specific forms of oligopoly as well as to interpretations of empirical findings in the established literature.

Our five scenarios are not exotic, but are among the most standard oligopoly situations considered in the theoretical industrial organization literature. Moreover, they collectively demonstrate that the possibility of $H > 0$ (under substantial market power) is robust to whether banks have identical or different costs, identical or differentiated products, simultaneous or sequential actions, price or quantity as the strategic variable, and collusive or fringe behavior. Furthermore, $H > 0$ can arise in both static and dynamic equilibria. In short, $H > 0$ appears not to be a pathological or rare outcome for non-competitive situations, but can arise under a wide variety of highly non-competitive conditions.

In combination with the results from the existing literature, we thus conclude that the $H$ statistic can be either positive or negative for any degree of competition. Consequently, the Panzar–Rosse revenue test cannot be used as a measure of competition (neither as a quantitative measure, nor as a one-sided measure). This conclusion echoes and reinforces an earlier and more narrowly supported conclusion by Hyde and Perloff (1995, p. 482) regarding the Panzar–Rosse revenue test, that “we cannot imagine where it would be useful”.

It seems difficult or even impossible to salvage the reduced-form revenue test by means of auxiliary analysis due to the wide variety of conditions that such auxiliary tests must assess. Future research might usefully explore this question in more detail, though our conjecture is that no such method can be found – or, even if it can, that it would be much more convenient to use a different method that is not subject to these severe problems.

Appendix A. Additional theoretical results

The following lemma is used in proving each of our propositions. This result was obliquely mentioned in Bikker et al. (2012) but no proof was shown, and the authors have not seen a proof in the literature. Because of its importance in theoretical analysis of the PR model, we offer a proof here. TR stands for total revenue, MC for marginal costs, and $w_j$ for the $j$th input price.

Lemma A.1. $H = (MC/TR) \frac{\partial TR}{\partial MC}$.

Proof. Linear homogeneity of MC in input prices (Rosse and Panzar, 1977, p. 7) implies

$$\sum_j w_j \frac{\partial MC}{\partial w_j} = 1.$$  \hspace{1cm} (A.1)

By definition, $H = \sum_j (w_j/\text{TR}) \frac{\partial TR}{\partial w_j}$. Now

$$H \cdot \frac{TR}{MC} = \sum_j \left(\frac{w_j}{MC}\right) \frac{\partial TR}{\partial w_j} = \sum_j \left(\frac{w_j}{MC}\right) \frac{\partial TR}{\partial MC} \frac{\partial MC}{\partial w_j} = \left(\frac{\partial TR}{\partial MC}\right) \sum_j \left(\frac{w_j}{MC}\right) \frac{\partial MC}{\partial w_j} = \frac{\partial TR}{\partial MC},$$

from which the result follows. \hfill $\square$

Appendix B. The $H$ statistic and pass-through

The structure of our argument is as follows. First, we show that pass-through is a component of $H$ that can affect its sign. Then we cite results from the theoretical literature on pass-through showing the absence of a reliable linkage between market power and pass-through. Finally, we combine these two elements to indicate informally why the linkage between market power and $H$ is not generally predictable except in isolated special cases.

According to Lemma A.1,

$$H = \left(\frac{MC}{TR}\right) \frac{\partial TR}{\partial MC} = \left(\frac{MC}{PQ}\right) \frac{\partial PQ}{\partial MC} + Q \frac{\partial P}{\partial MC},$$ \hspace{1cm} (B.1)
where \( P \) is the output price, \( MC \) is the marginal cost of production, \( Q \) is the (bank-specific) quantity of output, and \( TR \) is the total revenue earned by the bank. Here, \( \frac{\partial Q}{\partial MC} < 0 \) because of downward-sloping demand, and typically \( \frac{\partial P}{\partial MC} \geq 0 \) due to some pass-through of higher costs by the bank to its customers. The sign of \( H \) overall is thus determined by the relative magnitudes of \( \frac{\partial Q}{\partial MC} \) and \( \frac{\partial P}{\partial MC} \). Relevant to the relation between market power and \( H \), the relationship between market power and \( \frac{\partial P}{\partial MC} \) has been extensively analyzed in a separate literature on pass-through or (tax) incidence, with many important results emerging quite recently (e.g. Weyl and Fabinger, 2013).

In the PR model, an implicit assumption utilized in its theoretical derivation and empirical implementation is that changes in marginal costs are the result of exogenous changes in input prices. In the theoretical literature on pass-through, the usual assumption is that a bank’s change in marginal costs results from either an exogenous change in a tax imposed on the bank, or an exogenous change in exchange rates in the case of a bank that sells its output in more than one country. In either setting, the calculations and conclusions are unaffected by the source of the change. Accordingly, we can apply results from the pass-through literature to characterize the effect of any change in marginal costs on output prices charged by a bank in equilibrium, which is one component of \( H \). Collectively, the results that have been shown in the pass-through literature illustrates several different factors that undermine any potential for a stable relationship between market power and \( H \). Any one of these factors suffices to render \( H \) an unreliable index of market power. The existence of multiple such factors implies that, even if we could somehow eliminate one or two of the problems, \( H \) would still fail to establish the degree of market power for other reasons.

One important result is that the degree of pass-through, or even the sign of pass-through, can vary under perfect competition, depending on the characteristics of the cost function (Hens, 1997). This finding may be viewed as intuitively analogous to the results shown in Bikker et al. (2012) that the relation between \( H \) and competition can also vary for different cost functions. Clearly, if perfect competition can be associated with different pass-throughs and/or different values of \( H \), then neither the observed level of pass-through nor the estimated value of \( H \) can be used to distinguish reliably between perfect competition and imperfect competition.

Another important result from the pass-through literature is that the effect on pass-through of varying degrees of market power (either measured by the number of banks in the market, or parameterized as the elasticity-adjusted Lerner index or conjectural variation) depends on the shape of the demand curve: the pass-through rate is an increasing function of market power if demand is iso-elastic, but a decreasing function of market power if demand is linear (Sjøm et al., 2012). Because the pass-through \( \frac{\partial P}{\partial MC} \) is a component of \( H \), this result provides an additional reason why \( H \) cannot be regarded as a monotonic measure of the degree of competition.

More generally, a very comprehensive recent analysis of pass-through by Weyl and Fabinger (2013) shows that the pass-through rate depends on a variety of factors that can upset any definitive ranking of pass-through according to the degree of market power alone. For instance, \( \frac{\partial P}{\partial MC} = 1 / \left[ 1 + (\epsilon_0 / \epsilon_3) \right] \) for perfect competition, where \( \epsilon_0 \) is the demand elasticity and \( \epsilon_3 \) is the supply elasticity; \( \frac{\partial P}{\partial MC} = 1 / \left[ 2 + (\epsilon_2 / \epsilon_5) + \epsilon_0 P / P \right] \) for monopoly, where the demand curvature \( P \) can take either sign; and for general symmetric imperfect competitors, \( \frac{\partial P}{\partial MC} = 1 / \left[ 1 + \theta (\partial Q / \partial P) + \theta (\epsilon_0 \theta / \epsilon_3) \right] \) where \( \theta \) is a conduct parameter that can be either a constant or a more general function of other factors.

For example, \( \theta = (P - MC) \theta_0 / P \) for monopolistic competition. Additional complications arise for heterogeneous imperfectly competitive banks. The application to our research question is that there is no monotonic ranking of pass-through (and hence of \( H \)) according to the degree of market power, except in a very few special cases that do not adequately describe real-world behavior.

References


