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Ten-dimensional Supergravity Revisited

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Abstract. We show that the existing supergravity theories in ten dimensions can be extended with extra gauge fields whose rank is equal to the spacetime dimension. These gauge fields have vanishing field strength but nevertheless play an important role in the coupling of supergravity to spacetime filling branes.

We discuss the role of these gauge fields in the construction of string theories with sixteen supercharges and mention their relation with a conjectured hyperbolic symmetry underlying string theory and M theory.

Keywords: Supergravity, Strings, Branes

PACS: 04.65.+e, 11.25.-w

1. INTRODUCTION

The Type II supergravity theories in ten dimensions form a starting point from which all lower dimensional maximal supergravities can be derived. The Type IIB [1, 2, 3] and IIA theory [4, 5, 6], with two supercharges of equal (opposite) chirality were both constructed around 1984. The Type IIA theory follows by dimensional reduction from $D = 11$ supergravity. It was extended in 1986 to include a massive parameter [7]. The IIB theory does not appear to have a higher dimensional origin. The bosonic fields of the two theories are

$$\text{IIA : } g_{\mu\nu}, \phi, B_{(2)}, C_{(1)}, C_{(3)}, \quad (1)$$

$$\text{IIB : } g_{\mu\nu}, \phi, B_{(2)}, C_{(0)}, C_{(2)}, C_{(4)}. \quad (2)$$

The subscripts (n) indicate the rank of an antisymmetric tensor gauge field, or n -form field. The IIB 4-form satisfies a self-duality relation, which prevents the construction of a covariant action.

A natural extension in both theories is the addition of duals of the n -form fields. In this way one can associate to every n -form field an $8 - n$ -form field ($n \geq 0$). The $n + 1$ -form curvatures are then related by a duality relation to the corresponding $9 - n$ -form curvatures. These forms therefore do not introduce new degrees of freedom, but instead provide a alternative way to view the role of these propagating fields. This is particularly profitable in the coupling of these fields to extended objects or branes. A p -brane, with p spatial extensions, couples in a natural way to a $p + 1$ -form field. The dual forms are therefore useful in studying the properties of p -branes with $p \geq 4$ (for $p = 3$ the brane couples to $C_{(4)}$, which is its own dual). The introduction of dual forms for the

RR potentials $C_{(n)}$ has led to a completely “democratic” formulation of IIA and IIB supergravity, where all RR forms appear simultaneously [8].

It is also possible to introduce n -form fields with rank $n \geq 9$. These do not carry propagating degrees of freedom, and are therefore not dual to the physical supergravity fields. Nevertheless, they also have interesting applications. In [9] the $C_{(9)}$ field in the massive IIA theory played an essential role in understanding the 8-brane domain wall. The dual of the curvature $G_{(10)}$ plays the role of a cosmological constant.

Ten-form fields couple to space-time filling branes. These are related to truncations of the IIB theory to $N = 1$ supersymmetric theories. A 9-brane charge is by itself inconsistent. This can be resolved by adding opposite charge on an orientifold plane, which triggers the truncation to a Type I string theory. The introduction of 10-form fields in IIB supergravity and the corresponding truncation to $N = 1$ were considered in [11]. There two 10-forms were obtained. One is an RR form $C_{(10)}$, which will reappear in our present work. The other, called $B_{(10)}$, has the wrong tension to be understood as the S -dual of $C_{(10)}$, so that these two fields could not arise from an $SU(1, 1)$ (see Section 2) doublet. This problem will be resolved in this talk.

Since 9- and 10-forms do not carry physical degrees of freedom their number is not a priori limited. Of course they must be consistent with supersymmetry, and this turns out to lead to restrictions. The purpose of our work [10] is precisely to establish how many of these forms are possible in IIB supergravity, and to classify them in the correct $SU(1, 1)$ representations. A similar investigation of IIA is presently under way [12].

In Section 2 we will review briefly the construction of [10] in Einstein frame. The coupling to branes and the brane tensions are discussed in Section 3. In Section 4 the truncation to $N = 1$ theories is treated. Our results turn out to have an intriguing relation to recent efforts to identify the symmetry group of M- and string theory. In Section 5 we comment on this correspondence.

2. SUPERSYMMETRY, $SU(1, 1)$ AND FORM-FIELDS

The starting point of our analysis is the standard IIB supergravity as first formulated by [1, 2]. The theory exhibits an explicit $SU(1, 1)$ symmetry, which acts on the two bosonic fields. The scalars parametrize an $SU(1, 1)/U(1)$ coset. The scalars and fermions in the theory each has a charge associated with the local $U(1)$ symmetry, the gauge fields have zero charge. Under $SU(1, 1)$ the fields $B_{(2)}$ and $C_{(2)}$ form a doublet $A_{(2)}^\alpha$ (satisfying $A_{(2)}^1 = (A_{(2)}^2)^*$), while $C_{(4)}$ (also written as $A_{(4)}$) corresponds to a singlet. The scalars are conveniently written as a matrix U :

$$U = \begin{pmatrix} V_-^1 & V_+^1 \\ V_-^2 & V_+^2 \end{pmatrix}. \quad (3)$$

Here $V^\pm{}^\alpha$, with charge ± 1 and with $\alpha = 1, 2$, form doublets of $SU(1, 1)$. They are constrained by the relation

$$V_-^\alpha V_+^\beta - V_+^\alpha V_-^\beta = \varepsilon^{\alpha\beta}. \quad (4)$$

The supersymmetry transformations are, to terms bilinear in fermions:

$$\begin{aligned}
\delta e_\mu^a &= i\bar{\epsilon}\gamma^a\psi_\mu + i\bar{\epsilon}_C\gamma^a\psi_{\mu C} \quad , \\
\delta\psi_\mu &= D_\mu\epsilon + \frac{i}{480}F_{\mu\nu_1\dots\nu_4}\gamma^{\nu_1\dots\nu_4}\epsilon + \frac{1}{96}G^{\nu\rho\sigma}\gamma_{\mu\nu\rho\sigma}\epsilon_C - \frac{3}{32}G_{\mu\nu\rho}\gamma^{\nu\rho}\epsilon_C \quad , \\
\delta A_{\mu\nu}^\alpha &= V_-^\alpha\bar{\epsilon}\gamma_{\mu\nu}\lambda + V_+^\alpha\bar{\epsilon}_C\gamma_{\mu\nu}\lambda_C + 4iV_-^\alpha\bar{\epsilon}_C\gamma_{[\mu}\psi_{\nu]} + 4iV_+^\alpha\bar{\epsilon}\gamma_{[\mu}\psi_{\nu]}{}_C \quad , \\
\delta A_{\mu\nu\rho\sigma} &= \bar{\epsilon}\gamma_{[\mu\nu\rho}\psi_{\sigma]} - \bar{\epsilon}_C\gamma_{[\mu\nu\rho}\psi_{\sigma]}{}_C - \frac{3i}{8}\epsilon_{\alpha\beta}A_{[\mu\nu}^\alpha\delta A_{\rho\sigma]}^\beta \quad , \\
\delta\lambda &= iP_\mu\gamma^\mu\epsilon_C - \frac{i}{24}G_{\mu\nu\rho}\gamma^{\mu\nu\rho}\epsilon \quad , \\
\delta V_+^\alpha &= V_-^\alpha\bar{\epsilon}_C\lambda \quad , \\
\delta V_-^\alpha &= V_+^\alpha\bar{\epsilon}\lambda_C \quad .
\end{aligned} \tag{5}$$

Here we have introduced

$$\begin{aligned}
P_\mu &= -\epsilon_{\alpha\beta}V_+^\alpha\partial_\mu V_+^\beta \quad , \\
Q_\mu &= -i\epsilon_{\alpha\beta}V_-^\alpha\partial_\mu V_+^\beta \quad , \\
G_{\mu\nu\rho} &= -\epsilon_{\alpha\beta}V_+^\alpha F_{\mu\nu\rho}^\beta \quad , \\
F_{\mu\nu\rho}^\alpha &= 3\partial_{[\mu}A_{\nu\rho]}^\alpha \quad , \\
F_{\mu\nu\rho\sigma\tau} &= 5\partial_{[\mu}A_{\nu\rho\sigma\tau]} + \frac{5i}{8}\epsilon_{\alpha\beta}A_{[\mu\nu}^\alpha F_{\rho\sigma\tau]}^\beta \quad .
\end{aligned} \tag{6}$$

Q_μ is the $U(1)$ gauge field which is implicitly present in the covariantizations in (5). For further details on notation we refer to [2]. An important property of these transformations is that the commutator of two supersymmetry transformations on the bosonic gauge fields closes on translations and gauge transformations. On the fermionic fields closure also requires supersymmetry transformations, local Lorentz transformations and the equations of motion, see [2] for details.

The way to obtain extensions of the supergravity multiplet above is to use this property of closure: we assume an initial form of the supersymmetry transformation of a proposed field, including free parameters, and determine these parameters by requiring closure. Since no new degrees of freedom can be introduced, closure will also require a relation between the additional and original fields. For the 6-form and 8-form fields this leads to a unique extension. For the 10-forms no relation with fields of the original IIB multiplet exists, so that the number of 10-forms is not determined a priori. We will come back to this point later in this section.

We find that the following fields can be introduced in IIB supergravity:

6-forms There is a doublet of 6-forms $A_{(6)}^a$, with $A_{(6)}^1 = (A_{(6)}^2)^*$, satisfying the duality relation

$$F_{(7)\mu_1\dots\mu_7}^\alpha = -\frac{i}{3!}\epsilon_{\mu_1\dots\mu_7\mu\nu\rho}S^{\alpha\beta}\epsilon_{\beta\gamma}F_{(3)}^{\gamma;\mu\nu\rho} \quad , \tag{7}$$

where

$$S^{\alpha\beta} = V_-^\alpha V_+^\beta + V_+^\alpha V_-^\beta \quad . \tag{8}$$

The algebra closes on these 6-forms, giving a translation and bosonic n -form gauge transformations, with $n = 1, 3, 5$.

8-forms There is a triplet of 8-forms $A_{(8)}^{\alpha\beta}$, symmetric in α, β , satisfying a reality condition

$$(A_{(8)}^{11})^* = A_{(8)}^{22}, \quad (A_{(8)}^{12})^* = A_{(8)}^{12}, \quad (9)$$

and a duality relation

$$F_{(9)}^{\alpha\beta}{}_{\mu_1 \dots \mu_9} = i \varepsilon_{\mu_1 \dots \mu_9}{}^\sigma \{ V_+^\alpha V_+^\beta P_\sigma^* - V_-^\alpha V_-^\beta P_\sigma \}. \quad (10)$$

However, the three 8-forms are related to each other through a condition on the field-strengths,

$$\varepsilon_{\alpha\gamma} \varepsilon_{\beta\delta} V_+^\alpha V_-^\beta F_{(9)}^{\gamma\delta} = 0. \quad (11)$$

This implies that in the 8-form sector there are only two degrees of freedom, the ‘duals’ of the dilaton ϕ and the axion $C_{(0)}$. Note that the three potentials are not related by a local condition. On the 8-forms the algebra closes on translations, and on n -form gauge transformations, with $n = 1, 3, 5, 7$. The existence of a triplet of 8-forms, and the relation between these forms, was also discussed in [13, 14].

10-forms There are 10-forms in two $SU(1, 1)$ representations: a doublet $A_{(10)}^\alpha$, with the usual reality condition, and a quadruplet $A_{(10)}^{\alpha\beta\gamma}$, symmetric in α, β, γ , satisfying

$$(A^{111})^* = A^{222}, \quad (A^{112})^* = A^{122}. \quad (12)$$

There are no conditions relating the 10-forms to other fields. However, the doublet and the quadruplet differ in an important respect. The doublet does not transform under n -form gauge transformations with $n < 9$, while the fields of the quadruplet transform under all lower rank gauge transformations. Of course the volume form is also a 10-form, and so is the product of the volume form with arbitrary functions of the scalar fields. However, the volume form is essentially a product of tenbeins, and therefore not a new field in the IIB supergravity theory

3. COUPLING TO BRANES

We now wish to investigate to which kind of branes the different n -form potentials couple. In particular we would like to know the tension of the corresponding branes. These tensions can be determined from the supersymmetry rules as follows. To be concrete let us consider the 8-form potentials. After gauge-fixing the generic supersymmetry rule of the 3 different potentials is as follows (in string frame):

$$\delta A_{(8)} \sim f(\tau, \bar{\tau}) \bar{\varepsilon} \gamma (a \psi_\mu + b \gamma_\mu \lambda) + \dots, \quad (13)$$

where a, b are constants, the dots stand for other terms and the scalars have been expressed in terms of a complex scalar τ . The function $f(\tau, \bar{\tau})$ can be expressed in terms of the dilaton ϕ and axion $C_{(0)}$ via the relation $\tau = C_{(0)} + i e^{-\phi}$. For our present purposes it is sufficient to consider the case of zero axion.

The same 8-form potentials may occur as Wess-Zumino terms in a supersymmetric 7-brane action as follows:

$$\mathcal{L}_{\text{brane}} \sim \underbrace{e^{-\alpha\phi}}_{\text{brane tension at } C_{(0)}=0} \sqrt{-g} + A_{(8)} + \dots \quad (14)$$

Before fixing kappa-symmetry the first, Nambu-Goto, term and the second, Wess-Zumino, term are separately supersymmetric. After gauge-fixing kappa-symmetry the (linear) supersymmetry variations of the two terms should cancel. Clearly, this is only possible if the function $f(\tau, \bar{\tau})$ is proportional to the brane tension $e^{-\alpha\phi}$. To achieve this one must consider particular combinations of the 8-form potentials. This enables us to read the brane tensions from the supersymmetry rules. This indeed works for two of the three 8-form potentials, leading to the combinations $C_{(8)}$ and $B_{(8)}$ in Table 1. They couple to the D7-brane and the S-dual $\widetilde{\text{D7}}$ -brane (with exotic brane tension g_s^{-3}), respectively. However, for the third 8-form potential, called the combination $D_{(8)}$ in the Table, we find that $a = 0$ in (13) and hence there is no corresponding supersymmetric brane action since the Nambu-Goto term always transforms to a term linear in the gravitino.

TABLE 1. The triplet of 8-form potentials and their 7-branes

potential	associated brane	tension
$C_{(8)}$	D7	g_s^{-1}
$D_{(8)}$	—	—
$B_{(8)}$	$\widetilde{\text{D7}}$	g_s^{-3}

A similar analysis can be performed for the 10-form potentials. The result is summarized in Tables 2 and 3. Note that, unlike $D_{(8)}$, the 10-form potentials $D_{(10)}$ and $E_{(10)}$ transform to the gravitino. Nevertheless we cannot associate a supersymmetric 9-brane action with these potentials since they do not transform to the correct combination of gravitino and dilatino to establish supersymmetry.

TABLE 2. The quadruplet of 10-form potentials and their 9-branes

potential	associated brane	tension
$C_{(10)}$	D9	g_s^{-1}
$D_{(10)}$	—	—
$E_{(10)}$	—	—
$B_{(10)}$	exotic	g_s^{-4}

4. TRUNCATION TO $N = 1$

We can truncate our results for $N = 2$ supergravity to find the $N = 1$ algebra [10]. Since in $D = 10$ the $N = 1$ supergravity is unique, there is only one independent truncation, all others being related by field redefinitions. In spite of this, since there are two inequivalent

TABLE 3. The doublet of 10-form potentials and their 9-branes

potential	associated brane	tension
$\mathcal{D}_{(10)}$	solitonic	g_s^{-2}
$\mathcal{E}_{(10)}$	exotic	g_s^{-3}

$N = 1$ string theories, it is instructive to truncate the $N = 2$ theory in two different ways leading to the low energy limits of $D = 10$ heterotic and type I string theory. Hence we perform the "heterotic" and the "type I" truncations [11].

The heterotic truncation can be derived from the IIB algebra by setting

$$\varepsilon = \varepsilon_C. \quad (15)$$

This projects out the following fields from the IIB spectrum:

$$C_{(0)}, C_{(2)}, C_{(4)}, C_{(6)}, C_{(8)}, B_{(8)}, C_{(10)}, E_{(10)}, \mathcal{E}_{(10)}. \quad (16)$$

Further, $B_{(10)}$ and $\mathcal{D}_{(10)}$ turn out to be dependent fields of the form $e^{x\phi} \varepsilon_{(10)}$ (where $\varepsilon_{(10)}$ is the volume form) for some x in the truncated theory. Therefore, the field contents of the heterotic truncation of the $D = 10$, IIB supergravity is given by

$$\phi, B_{(2)}, B_{(6)}, D_{(8)}, D_{(10)}. \quad (17)$$

The supersymmetry algebra which is realised on these fields can easily be obtained by setting all truncated and dependent fields to zero in the IIB algebra as presented in [10]. The type I truncation can be derived from the IIB algebra by setting

$$\varepsilon = i\varepsilon_C. \quad (18)$$

This projects out the following fields from the IIB spectrum:

$$C_{(0)}, B_{(2)}, C_{(4)}, B_{(6)}, C_{(8)}, B_{(8)}, D_{(10)}, B_{(10)}, \mathcal{D}_{(10)}. \quad (19)$$

Similarly to the heterotic case, we find that $C_{(10)}$ and $\mathcal{E}_{(10)}$ turn out to be dependent fields of the form $e^{x\phi} \varepsilon_{(10)}$ for some x in the truncated theory. Therefore the field contents of the type I truncation is given by

$$\phi, C_{(2)}, C_{(6)}, D_{(8)}, E_{(10)}. \quad (20)$$

5. VERY EXTENDED SYMMETRY GROUPS

Collecting all n -form potentials of the IIB theory we find that they transform nonlinearly under the bosonic gauge transformations in the following generic form:

$$\delta A = d\Lambda + F \wedge \Lambda, \quad F = dA + A \wedge F. \quad (21)$$

Since the gauge transformation rules only contain gauge-invariant curvatures, the bosonic gauge algebra is Abelian. Surprisingly, it turns out that the bosonic gauge transformations can all be rewritten in terms of

$$\Lambda_{(2n)} \equiv d\Lambda_{(2n-1)}. \quad (22)$$

After an appropriate (field-dependent) redefinition of the gauge fields and the parameters, all transformation rules become linear but the resulting bosonic gauge algebra is non-Abelian. Schematically, we thus obtain the following non-trivial commutators:

$$[2, 2] = 4, \quad [2, 4] = 6, \quad [2, 6] = 8, \dots \quad (23)$$

We thus see an interesting structure arising: all gauge fields can be obtained by applying a number of times the basic **2** gauge transformation. This number is the so-called level of the gauge field. A similar structure arises in the IIA case where the basic building blocks are the RR 1-form **1** and the NS 2-form **2**:

$$[1, 1] = 0, \quad [1, 2] = 3, \quad [1, 3] = 0, \quad [2, 3] = 5, \quad [1, 5] = 6, \dots \quad (24)$$

FIGURE 1. The Dynkin diagrams leading to IIA and IIB representations

The above is very reminiscent to recent work on a hyperbolic E_{11} -symmetry that might underly string and/or M-theory, see [10] for a list of references. In particular, in [15], representations of the E_{11} algebra are worked out for different embeddings of a bosonic $GL(10)$ subalgebra. This leads to the Dynkin diagrams of Figure 1. In these diagrams the horizontal line represents the $GL(10)$ subalgebra whereas the empty dots are related to our basic building blocks in the following way:

$$\text{IIA} : \quad 10 \leftrightarrow \mathbf{1}, \quad 11 \leftrightarrow \mathbf{2} \quad (25)$$

$$\text{IIB} : \quad 9, 10 \leftrightarrow \mathbf{2}. \quad (26)$$

It would be interesting to pursue this relationship further.

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