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REFLECTIONS ON THE INOPERABILITY INPUT–OUTPUT MODEL

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We argue that the inoperability input–output model is a straightforward – albeit potentially very relevant – application of the standard input–output model. In addition, we propose two less standard input–output approaches as alternatives to take into consideration when analyzing the effects of disasters or disruptions.

Keywords: Input–output analysis; Inoperability; Disaster impacts

1. INTRODUCTION

In a recent special issue of *Economic Systems Research* (see Okuyama and Santos, 2014) dealing with disaster impacts and input–output analysis, four of the six articles use or at least mention the “inoperability input–output model” (IIM). These build on a very large number of earlier expositions and applications of this methodology (e.g. Haimes and Jiang, 2001; Santos and Haimes, 2004; Anderson et al., 2007; Santos et al., 2008).¹ While this model and its applications may be very interesting and relevant to assessment of disaster impacts, we will argue that it is really a straightforward application of the standard input–output model with a small tweak. In the second part of our reflections, we will propose the generalized hypothetical extraction method and the mixed exogenous/endogenous input–output model as alternative options for analyzing disaster impacts.

2. THE WORKING OF THE INOPERABILITY INPUT–OUTPUT MODEL

The applications of IIM boil down to answering the typical input–output question: What happens system-wide when there is a change – in disaster situations, a reduction – in final demand for something? For example, in Santos and Haimes, 2004, it was a 10% reduction in demand for air transportation as a result of terrorism.²

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¹ The huge number of articles employing the IIM and its variants precludes citation of the complete set. For example, there are more than three dozen in *Risk Analysis* and there have been at least nine in this *journal*, including the 2014 special issue.

² Additional examples are in Anderson et al. (2007, p. 193) “... we have simplified the analysis of workforce unavailability and its direct impact on the productivity of each sector by converting the given output (supply)

The answer, of course, yields $\Delta \mathbf{x} = \mathbf{L}(\Delta \mathbf{f})$, where $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$. (The authors consistently use $\mathbf{c}$ to represent final demands; we will use $\mathbf{f}$.) Given a $\Delta \mathbf{f}$ vector comprised mostly of 0s (for sectors where there is no final demand change) and at least one negative number – for the sector (or sectors) that experience the demand reduction – the resulting (value) reductions for all sectors are given by $\Delta \mathbf{x} = \mathbf{L}(\Delta \mathbf{f})$.

The authors choose to address this demand reduction scenario by expressing output changes as *proportions* of initial (or ‘base’) outputs. This can be done in various ways, all leading to the same answer. In a nutshell (the authors do not derive their results in this way):

- (1) Normalize the standard equation as $\hat{\mathbf{x}}^{-1}(\Delta \mathbf{x}) = \hat{\mathbf{x}}^{-1}\mathbf{L}(\Delta \mathbf{f})$.
- (2) Define $\tilde{\mathbf{x}}$ as the (decreased) output in the ‘degraded’ situation, and $\mathbf{q} \equiv \hat{\mathbf{x}}^{-1}(\mathbf{x} - \tilde{\mathbf{x}}) = \hat{\mathbf{x}}^{-1}(\Delta \mathbf{x})$. Notice that $0 \leq q_j \leq 1$, where $q_j = 0$ represents no reduction in sector j ’s output and $q_j = 1$ represents complete elimination of sector j output. This is the model’s ‘inoperability’ measure. Note that the inoperability vector $\mathbf{q}$ is the same as the outcome of normalizing the standard equation.
- (3) If the final demand is normalized in the same way, we have $\mathbf{f}^* = \hat{\mathbf{x}}^{-1}(\mathbf{f} - \tilde{\mathbf{f}}) = \hat{\mathbf{x}}^{-1}(\Delta \mathbf{f})$, where $\tilde{\mathbf{f}}$ represents final demand in the ‘degraded’ situation.³ Introducing $\mathbf{I} = \hat{\mathbf{x}}\hat{\mathbf{x}}^{-1}$ into the normalized standard equation $\mathbf{q} = \hat{\mathbf{x}}^{-1}\mathbf{L}(\Delta \mathbf{f}) = \hat{\mathbf{x}}^{-1}\mathbf{L}(\hat{\mathbf{x}}\hat{\mathbf{x}}^{-1})(\Delta \mathbf{f})$ and regrouping terms yields $\mathbf{q} = [\hat{\mathbf{x}}^{-1}\mathbf{L}\hat{\mathbf{x}}][\hat{\mathbf{x}}^{-1}(\Delta \mathbf{f})] = \mathbf{G}\mathbf{f}^*$, where $\hat{\mathbf{x}}^{-1}\mathbf{L}\hat{\mathbf{x}} = \mathbf{G} = (\mathbf{I} - \mathbf{B})^{-1}$ is the standard Ghosh model inverse.

The authors express the result in (3) as $\mathbf{q} = (\mathbf{I} - \mathbf{A}^*)^{-1}\mathbf{f}^*$, where they label $\mathbf{A}^*$ the ‘demand-side interdependency matrix’. Santos and Haimes (2004, p. 1443) mention in passing that its elements are “similar to the form of the elements in the supply-side model coefficient matrix (i.e. the allocation coefficients proposed by Ghosh)”. Yet here and in (many) later publications $\mathbf{A}^*$ is shown explicitly as $\hat{\mathbf{x}}^{-1}\mathbf{A}\hat{\mathbf{x}}$ (for example, in Anderson et al., 2007, and Santos et al., 2008). Indeed, $\mathbf{A}^*$ is not just ‘similar’ but *exactly* the Ghosh model’s $\mathbf{B} = \hat{\mathbf{x}}^{-1}\mathbf{A}\hat{\mathbf{x}}$. This relationship between $\mathbf{A}$ and $\mathbf{B}$ is well known and has been for some time. The same goes for the fact that $\mathbf{G} = \hat{\mathbf{x}}^{-1}\mathbf{L}\hat{\mathbf{x}}$, and hence that their model can be stated as $\mathbf{q} = \mathbf{G}\mathbf{f}^*$. For example, $\mathbf{B} = \hat{\mathbf{x}}^{-1}\mathbf{A}\hat{\mathbf{x}}$ appears (without comment) in Appendix I of Augustinovic (1970); it, along with $\mathbf{G} = \hat{\mathbf{x}}^{-1}\mathbf{L}\hat{\mathbf{x}}$, was discussed in the first edition (1985) of Miller and Blair,⁴ which the authors cite (in Santos and Haimes, 2004, and in many later publications). This makes all the more surprising the statement in Yu et al. (2014, p. 85) that “One of the key developments in IIM theory is the procedure for deriving $\mathbf{A}^*$ from standard input-output tables.”

Exactly the same results can be derived in various ways and as approach (1) above indicates there is no need to introduce any ‘new’ matrices. The output from the standard analysis (i.e. $\Delta \mathbf{x} = \mathbf{L}(\Delta \mathbf{f})$), can be ‘normalized’ in one simple step, namely $\hat{\mathbf{x}}^{-1}(\Delta \mathbf{x})$. (As

constraints into equivalent demand reductions” or Santos et al. (2008, p. 43) where the authors speak of assessing “The effects of possible consumption shift scenarios across various sectors of the economy.”

³ ‘Normalizing’ final demands in input-output models to account for differing sector/industry sizes is nothing new. Early examples are in Gray et al. (1979) or Mattas and Shrestha (1991).

⁴ In those days $\hat{\mathbf{A}}$ was used for what is now generally represented as $\mathbf{B}$. ($\hat{\mathbf{A}}\downarrow$ was used by Augustinovic for $\mathbf{A}$.) The result also appears in Chen and Rose (1986), Miller (1989) and Dietzenbacher (1989).

noted, this is the authors' inoperability vector $\mathbf{q}$, in which q_j measures inoperability for sector j – its 'normalized production loss.'). This approach seems straightforward, with no need for normalization of the final demand change. The inoperability model puts the normalization at the beginning; use of the standard Leontief input–output model puts the normalization at the end.

Apparently the authors believe that they have generated results that were previously unavailable in an input–output analysis. For example: "Hence, through matrix manipulations we have introduced the concept of inoperability into the traditional I-O model which augments typical economic loss analysis" (Anderson et al., 2007, p. 186).⁵ But that seems an unfair comparison. The standard model generates output value changes and can easily produce output value proportion changes ('inoperability'); the IIM model generates 'inoperability' (output value proportions) and from those can generate output changes in value terms. (An extremely simple numerical example is given in Appendix 1.) There is a simple one-to-one correspondence between the two solutions. So both metrics are deliverable in either model, and results from applications of the standard model have often been expressed in exactly these output value proportions (percentage) terms.⁶

Additionally, recognition that elements of $\mathbf{G}$ can be viewed as percentage changes in outputs due to percentage changes in demands⁷ is not new. It was discussed extensively in Dietzenbacher (1997, p. 640):

When the final demand in sector j is increased by 1 percent of the original output of this sector, the quantity of product i (and hence the total output value in sector i) is increased by g_{ij} percent. (Notation changed from b^{ij} used by Dietzenbacher for elements of the Ghosh inverse.)

This work is also cited in Santos and Haimes (2004) and in many later IIM publications. (Note that Dietzenbacher here considers cases of increases but the point remains equally valid for decreases.) This point is emphasized again in Dietzenbacher (2005 – admittedly, this postdates Santos and Haimes, 2004, but is before Anderson et al., 2007 or Santos et al., 2008, and many later publications): "The (i,j) th element of $(0.01) \hat{\mathbf{x}}^{-1} \mathbf{L} \hat{\mathbf{x}}$ gives ... the direct and indirect output elasticity of industry i with respect to the output of industry j ." And, after observing that $\mathbf{G} = \hat{\mathbf{x}}^{-1} \mathbf{L} \hat{\mathbf{x}}$, concludes "... the elements of $\mathbf{G}$ give the *direct and indirect* output-to-output elasticities" (p. 426). (He also points out that elements in $\mathbf{B}$ give the *direct* output-to-output elasticities.)⁸ Clearly, the relationship described in the quotation is equally valid for quantifying output *decreases* due to final demand *decreases*. Applicability holds in both directions.

⁵ "Inoperability is treated as a separate metric since it complements the economic loss metric. Furthermore, two distinct rankings are generated from the inoperability and economic loss objectives ..." Santos et al. (2014, p. 63).

⁶ Early examples can be found in Rose (1977) and Schaffer (1980).

⁷ Where the change in demand in sector j is expressed as a percentage of the original output of this sector.

⁸ De Mesnard (2001 – a reference also cited numerous times in these articles) characterizes the elements in $\mathbf{B}$ in a similar way.

3. LINKS TO OTHER INPUT–OUTPUT APPROACHES

As the second part of our reflections we would like to point out two links to other approaches in the input–output literature. To our knowledge, these links have not been made before and seem to be of interest for an input–output analysis of disasters. The first link is to the (generalized) hypothetical extraction method and the second link is to input–output models with mixed exogenous/endogenous variables.

When thinking of the term inoperability model in connection with disasters, we would expect the model to be used also for cases where one or more industries have become (partially) inoperable due to serious disruptions. Although some work has been done in which input coefficients are perturbed (e.g. Barker and Rocco, 2011), a systematic analysis of what happens to the input coefficients when certain production facilities shut down seems to be lacking. This type of analysis is exactly what Dietzenbacher and Lahr (2013) describe in their generalized hypothetical extraction method (expanding the techniques originally put forward by Paelinck et al., 1965 and Miller, 1966). They discuss the case where (for whatever reason) a sub-industry ceases to exist, how this affects the input coefficients, industry outputs and values added.⁹

As an alternative for the dynamic IIM (DIIM), one may wish to consider using a mixed model (Miller and Blair, 2009, pp. 621–633). This type of model may be appropriate in situations where the gross outputs of some industries and the final demands in the other industries are specified exogenously. In order to elaborate on this, we first need to discuss briefly the dynamic version of the IIM. The model is (at least mathematically) a special case of the dynamic Leontief model (Miller and Blair, 2009, pp. 639–654). That is, $\mathbf{x}_t = \mathbf{A}\mathbf{x}_t + \mathbf{f}_t - \hat{\mathbf{k}}^{-1}(\mathbf{x}_{t+1} - \mathbf{x}_t)$, which can be rewritten as

$$\mathbf{x}_{t+1} = \mathbf{x}_t + \hat{\mathbf{k}}(\mathbf{A}\mathbf{x}_t + \mathbf{f}_t - \mathbf{x}_t). \quad (1)$$

If final demands are fixed ($\mathbf{f}_t = \mathbf{f}$), the equilibrium solution is given by $\mathbf{x}_{t+1} = \mathbf{x}_t = \mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f}$. Now suppose in $t = 0$ the output is disrupted and is (exogenously) given by $\mathbf{x}_0$, which equals the supply of goods. The demand for goods is given by $\mathbf{A}\mathbf{x}_0 + \mathbf{f}$, assuming that final demands have not changed. The gap between demand and supply amounts to $\mathbf{A}\mathbf{x}_t + \mathbf{f}_t - \mathbf{x}_t$. The coefficients $k_i > 0$ indicate the part of the gap for industry i that will be closed in the next period by increasing the industry's output. They are termed resilience or recovery coefficients. Rewriting (1) gives

$$\mathbf{x}_{t+1} = [\mathbf{I} - \hat{\mathbf{k}}(\mathbf{I} - \mathbf{A})]\mathbf{x}_t + \hat{\mathbf{k}}\mathbf{f}_t = \mathbf{M}\mathbf{x}_t + \hat{\mathbf{k}}\mathbf{f}_t. \quad (2)$$

Assuming $\mathbf{f}_t = \mathbf{f}$ and given $\mathbf{x}_0$, the solution is given by $\mathbf{x}_{t+1} = \mathbf{M}'\mathbf{x}_0 + (\mathbf{I} + \mathbf{M} + \dots + \mathbf{M}^{t-1})\hat{\mathbf{k}}\mathbf{f}$, which can be shown to converge to $(\mathbf{I} - \mathbf{M})^{-1}\hat{\mathbf{k}}\mathbf{f} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f} = \mathbf{x}$. A similar path can be given in the case where final demands change. A clear and more detailed exposition of the DIIM is given by Lian and Haimes (2006), but note that they use continuous time and thus arrive at a set of differential equations (whereas we have used discrete time and arrived at difference equations).

⁹ When revising this paper, we became aware of one other paper that proposes hypothetical extraction (and also structural decomposition analysis) as an alternative to IIM. Muldrow and Robinson (2014) compare the three techniques empirically in a case study of the 2008 Cedar Rapids Flood.

It should be stressed that the DIIM differs from the standard dynamic Leontief model, which is given by $\mathbf{x}_t = \mathbf{A}\mathbf{x}_t + \mathbf{f}_t + \mathbf{B}(\mathbf{x}_{t+1} - \mathbf{x}_t)$. Lian and Haines (2006) arrive at the DIIM by using $\mathbf{B} = -\hat{\mathbf{k}}^{-1}$. The DIIM describes the recovery path following an exogenous decrease in the outputs and this path converges to the original situation. Alternatively, the DIIM describes an adaptation path following a decrease in the final demand levels and this path converges to a new stable situation. Convergence holds for $0 < k_i \leq 1$. The standard application of the dynamic Leontief model deals with a long-term expansion of the output due to investments. The model does not aim at returning back (via a recovery path) to the original stable situation or at moving (via an adaptation path) to a new stable situation. Instead, it aims at convergence toward a steady-state *growth* path. The elements of $\mathbf{B}$ are capital coefficients and in the case $\mathbf{B} = -\hat{\mathbf{k}}^{-1}$ we have $k_i < 0$. Typically $\mathbf{x}_{t+1}$ as given by Equation 2 does not converge. So, whereas the models are given by a single equation they serve different purposes which is reflected by different signs of the parameters k_i .

For the mixed exogenous/endogenous model, the industries are divided into two groups: group 1 with exogenous outputs and endogenous final demands, and group 2 with exogenous final demands and endogenous outputs. We will use an underscore to indicate the exogenous variables. The input–output model in its partitioned form is given by

$$\begin{pmatrix} \underline{\mathbf{x}}^1 \\ \underline{\mathbf{x}}^2 \end{pmatrix} = \begin{bmatrix} \mathbf{A}^{11} & \mathbf{A}^{12} \\ \mathbf{A}^{21} & \mathbf{A}^{22} \end{bmatrix} \begin{pmatrix} \underline{\mathbf{x}}^1 \\ \underline{\mathbf{x}}^2 \end{pmatrix} + \begin{pmatrix} \mathbf{f}^1 \\ \mathbf{f}^2 \end{pmatrix}.$$

Re-arranging terms yields

$$\begin{pmatrix} \mathbf{f}^1 \\ \underline{\mathbf{x}}^2 \end{pmatrix} = \begin{bmatrix} 0 & -\mathbf{A}^{12} \\ 0 & \mathbf{A}^{22} \end{bmatrix} \begin{pmatrix} \mathbf{f}^1 \\ \underline{\mathbf{x}}^2 \end{pmatrix} + \begin{bmatrix} \mathbf{I} - \mathbf{A}^{11} & 0 \\ \mathbf{A}^{21} & \mathbf{I} \end{bmatrix} \begin{pmatrix} \underline{\mathbf{x}}^1 \\ \underline{\mathbf{f}}^2 \end{pmatrix}$$

and the solution is given by

$$\begin{pmatrix} \mathbf{f}^1 \\ \underline{\mathbf{x}}^2 \end{pmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{A}^{12} \\ 0 & \mathbf{I} - \mathbf{A}^{22} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{I} - \mathbf{A}^{11} & 0 \\ \mathbf{A}^{21} & \mathbf{I} \end{bmatrix} \begin{pmatrix} \underline{\mathbf{x}}^1 \\ \underline{\mathbf{f}}^2 \end{pmatrix}.$$

In line with the DIIM as described above, we could keep the exogenous final demands as they were before the disruption took place. That is, $\mathbf{f}_t^2 = \underline{\mathbf{f}}^2$. For the exogenous outputs we could define a recovery path. That is, $\underline{\mathbf{x}}_{t+1}^1 = \underline{\mathbf{x}}_t^1 + \hat{\mathbf{k}}(\underline{\mathbf{x}}^1 - \underline{\mathbf{x}}_t^1)$, where $\underline{\mathbf{x}}^1$ indicates the output levels before the shock (or the desired output levels). The coefficients k_i indicate the share of the output gap that is closed in the next period.

Appendix 2 contains a simple three-industry example. Although both approaches converge to the same outcome and the results do not differ too much from each other, some specific features become apparent. For example, whereas the mixed model shows a monotonic recovery path, the DIIM exhibits effects of ‘overshooting’. Because output is decreased in industry 1, the other industries (2 and 3 in the example) need to produce less intermediate inputs and therefore have a supply surplus. As a consequence their outputs are decreased in the next period but recover over time. Another difference between the two models is in their working. In the DIIM, all final demands are endogenous and the differences with the desired final demands yield the gaps that determine the exogenous output levels in the next period. In the mixed model, only some outputs are determined exogenously (as are their recovery paths), and the other outputs are determined endogenously and satisfy market clearing (i.e. supply of goods equals demand for goods). Whether

one model is more realistic than the other (or under what conditions one is more realistic than the other) is open for discussion. In any case, the two approaches show fundamental differences in their dynamics.

4. CONCLUSIONS

The IIM seems to us to be a lot of old wine in new (or possibly just relabeled) bottles. That is, the IIM is a straightforward application of the standard input–output model. This does not mean that the wine itself tastes worse. The application of input–output techniques to analyze disaster impacts was and remains to be an application of great relevance.

We would like to emphasize that much interesting work has been done. In these cases, however, it is not the input–output part that makes the application interesting (because that is very standard) but rather the modeling of aspects that are exogenously fed into the input–output part. For example, Santos et al. (2008) model final demand disruptions using probability distributions, Barker and Rocco (2011) apply interval arithmetic to study perturbations in input coefficients and final demands, and Barker and Santos (2010) link disruptions to inventory policies. In a dynamic setting, due attention has been given to the recovery path of the outputs after a disruption. For example, Orsi and Santos (2010) make the coefficients that determine the recovery path time-varying, Jonkeren and Giannopoulos (2014) focus on the shape of the recovery path, and Santos et al. (2014) use event tree analysis to analyze perturbations of the recovery path.

In the second part of this note we have provided two links to other input–output approaches, namely the generalized hypothetical extraction method and the mixed exogenous/endogenous input–output model. To our knowledge these links have not been made before and we feel that they might be interesting for the analysis of disaster impacts. Generalized extractions allow for studying what happens if certain (sub-) industries become inoperable and the mixed model provides a different perspective on the recovery process.

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Appendix 1. Numerical example for IIM and the standard IO model

$$\mathbf{Z} = \begin{bmatrix} 225 & 600 & 110 \\ 250 & 125 & 425 \\ 325 & 700 & 150 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} 2000 \\ 2200 \\ 3000 \end{bmatrix} \quad \mathbf{f} = \begin{bmatrix} 1065 \\ 1400 \\ 1825 \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} 1.1994 & .3186 & .1032 \\ .1998 & 1.1800 & .1837 \\ .2721 & .4605 & 1.1318 \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} 1.1994 & .4198 & .1548 \\ .1817 & 1.1800 & .2505 \\ .1814 & .3377 & 1.1318 \end{bmatrix}$$

Scenario: a 20% reduction in demand for sector 2.

Standard IO model:

Changes in final demand values generate changes in output values, from which changes in output proportions can be derived. Let $\Delta \mathbf{f}$ indicate the disruption or decrease in final demands, then we have:

$$\Delta \mathbf{f} = \begin{bmatrix} 0 \\ 280 \\ 0 \end{bmatrix}, \quad \Delta \mathbf{x} = \mathbf{L} \Delta \mathbf{f} = \begin{bmatrix} 106.86 \\ 330.40 \\ 128.94 \end{bmatrix} \Rightarrow \hat{\mathbf{x}}^{-1} \Delta \mathbf{x} = \begin{bmatrix} 0.0534 \\ 0.1502 \\ 0.0430 \end{bmatrix}$$

IIM:

Decreases in final demand proportions (i.e. the ratios of final demand decreases to corresponding output, in this example $280/2200 = 0.1273$) generate changes in output proportions, from which changes in output values can be derived:

$$\Delta \mathbf{f}^* = \begin{bmatrix} 0 \\ 0.1273 \\ 0 \end{bmatrix}, \quad \mathbf{q} = \mathbf{G} \Delta \mathbf{f}^* = \begin{bmatrix} 0.0534 \\ 0.1502 \\ 0.0430 \end{bmatrix} \Rightarrow \hat{\mathbf{x}} \mathbf{q} = \Delta \mathbf{x} = \begin{bmatrix} 106.80 \\ 330.44 \\ 129.00 \end{bmatrix}$$

(Differences between $\Delta \mathbf{x} = \mathbf{L}(\Delta \mathbf{f})$ and $\hat{\mathbf{x}} \mathbf{q} = \Delta \mathbf{x}$ are due to rounding when using only four-decimal accuracy.)

Appendix 2. Numerical example for DIIM and the mixed exogenous/endogenous IO model

We will use the example from the previous appendix. For the DIIM, we assume that the output of industry 1 is reduced in period 0 to 1200 due to an external shock. The final demands remain the same, but cannot generally be met by the outputs. This yields the gaps (defined as intermediate plus final demand for goods minus supply of goods). The recovery coefficient for industry 1 is assumed to be $k_1 = 0.5$. For industries 2 and 3 we assume that they can fully adapt immediately, that is, $k_2 = k_3 = 1$. The results are given in the upper part of Table A1. Observe that due to the shock in period 0, there is a gap (shortage of goods) of 710, which implies that half of that amount (355) determines the increase in the output of industry 1 in period 1. Observe also that for goods 2 and 3 we have surpluses (i.e. negative gaps), because the reduction in output of industry 1 implies that it requires less intermediate inputs. Assuming $k_2 = k_3 = 1$ implies that positive (negative) gaps equal next period's output decreases (respectively, increases).

For the mixed exogenous/endogenous model, we assume that the output of industry 1 is exogenously specified. It is 1200 in period 0 and the desired level is 2000. Half of the gap of 800 is closed in the next period, so that the output of industry 1 in period 1 is exogenously specified as 1600, and so forth. The final demands for goods 2 and 3 are specified exogenously at their original levels. The results are given in the lower part of Table A1. The gap reports the difference between the desired (or original) final demand

TABLE A1. Results for DIIM and the mixed exogenous/endogenous model (ME/EM).

Period	Output level			Gap		
	1	2	3	1	2	3
<i>DIIM</i>						
0	1200	2200	3000	710	− 100	− 130
1	1555	2100	2870	363	20	19
2	1736	2120	2889	208	27	37
3	1841	2147	2926	124	20	27
4	1903	2167	2954	76	13	18
5	1941	2179	2971	46	8	11
10	1995	2198	2998	4	1	1
<i>ME/EM</i>						
0	1200	2067	2819	667	0	0
1	1600	2133	2909	333	0	0
2	1800	2167	2955	167	0	0
3	1900	2183	2977	83	0	0
4	1950	2192	2989	42	0	0
5	1975	2196	2994	21	0	0
10	1999	2200	3000	1	0	0

levels and the actual final demand levels. Observe that the gaps for goods 2 and 3 are obviously 0.