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MRP Planned Orders in a Multiple-Supplier Environment with Differing Lead Times

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This study examines a deterministic material requirements planning (MRP) problem where lead times at subsequent ordering moments differ. Adequate replenishment methods that can cope with lead time differences are lacking because of the order crossover phenomenon, that is, replenishment orders are not received in the sequence they are ordered. This study specifies how to handle order crossovers and recalculate planned order releases after an update of gross requirements. The optimal (s, S) policy is based on dynamic programming. The state space is kept to a minimum due to three fundamental insights. The performance of the optimal solution approach is compared with two heuristics based on relaxations and a benchmark approach in which order crossovers are ignored. A numerical analysis reveals that average cost savings up to 25% are possible if the optimal policy is used instead of the benchmark approach. The contribution of this study is threefold: (1) it generalizes theory on MRP ordering, allowing for lead time differences and order crossovers; (2) it develops new fundamental insights and an optimal solution procedure, leading to substantial cost saving; and (3) it provides good-performing heuristics for a general and realistic replenishment problem that can replace the current replenishment methods within MRP.

Key words: different lead times; discrete lot-sizing; MRP; multiple suppliers; order crossovers

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1. Introduction and Literature Review

Although material requirements planning (MRP: Silver et al. 1998, Vollmann et al. 2005) is one of the best known techniques in the field of inventory planning and has been made available in all enterprise resource planning systems, it lacks support for frequently encountered situations where essential parameters such as item lead times are known to change during the planning horizon. Appropriate material planning is a key element in becoming more competitive (Plenert 1999), and 75% of manufacturing companies use MRP as their main method of material planning (Jonsson and Mattsson 2006). Material planning requires adequate software support. The problem that we consider is to support the order release decisions in case that the lead times change during the planning horizon and these changes (their timing as well as magnitude) are known in advance.

Lead time changes that are known to occur during the planning horizon may originate from suppliers, buyers, or logistic service providers. For example, lead times offered by a supplier may differ over the year according to a known pattern (e.g., holidays are known in advance, but during the holiday season, lead times are twice as long due to low capacity). Buyers themselves might cause lead time differences

due to using multiple suppliers, purchasing substitute products (e.g., Cohen et al. 1995), or changing the requested lead time. They know the timing and magnitude of the lead time differences in advance. Finally, logistics service providers may offer extended lead times during the wet season due to bad conditions of the transportation roads, but the timing of the wet season and associated lead time are known in advance.

The problem that inspired this research has been encountered in a tobacco company that has to decide on the amount of tobacco leaves to order in different countries of origin. In some of the countries, the raw materials have to be ordered directly from the farmers, sometimes even before the growing season starts. In other countries, co-operations of farmers sell harvested tobacco on the market, where it is available for sale for several months. The lead times they offer may differ. This results in huge differences in lead times at the various ordering moments. The tobacco company faces known changes in lead time and price over the planning horizon, has limited sourcing possibilities, and aims to trade off costs for setups, purchasing, shipping, holding, and backordering. All these factors are taken into account in this paper. Other factors, such as substitutability of different types of leaves and political stability of the countries of origin, are also

important in the practice of tobacco procurement but will be left out of our analysis in order to focus on the fundamental problem of coping with known lead time changes in order release.

MRP uses dynamic lot-sizing methods, such as part-period balancing, lot-for-lot ordering, or more advanced dynamic programming algorithms (Tempelmeier 2002, Wagner 2011), in order to plan the release of orders. However, known lead-time changes have not received much attention in MRP dynamic lot-sizing problems thus far (Minner 2003). Some recent papers consider lead time differences that depend on the available capacity (e.g., Ioannou and Dimitriou 2012, Jodlbauer and Reitner 2012, Louly et al. 2008, Pahl et al. 2007), but most dynamic lot-sizing methods assume constant or zero lead times (Wolsey 1995), and to the best of our knowledge no one has studied the problem of known lead time changes in the context of dynamic lot sizing in MRP. This seems strange, but the main reason might be the inherent complexity of the resulting mathematical model, as lead time changes may result in order crossovers. Orders are said to cross if they arrive in a different sequence than ordered (Hadley and Whitin 1963). This phenomenon can only occur if there are two or more orders outstanding. Due to recent developments in supply chain management, order crossovers are more prevalent than in the past (Muharremoglu and Yang 2010, Riezebos 2006). For example, electronic ordering with different lead-time options is now commonly available, not only for large consumer portals such as Amazon but also in business-to-business procurement. Moreover, substitute products from different suppliers might have different lead times, possibly resulting in order crossovers. These crossovers have become an issue in several firms (Bozarth et al. 2009, Kannan and Tan 2005, Srinivasan et al. 2011, Stock et al. 2010). However, inventory theory has primarily given attention to problems where order crossovers do not occur or their occurrence could be neglected (Hayya et al. 2008).

Most research on order crossovers has focused on stochastic order crossovers (Bradley and Robinson 2005, Muharremoglu and Yang 2010, Ramasesh et al. 1991, Robinson and Bradley 2008). However, crossovers may also result if deterministic lead times take different values over the planning horizon. For this problem and stochastic demand, Riezebos and Gaalman (2009) propose a stochastic dynamic programming formulation. Axsäter (2011) proves that in a continuous review system with stochastic demand and knowledge on a future lead-time change, order postponement may result in a cost benefit when the inventory policy anticipates correctly on future ordering possibilities. Both methods are not directly applicable in MRP, which considers demand (gross

requirements) to be known and updates become available periodically.

In the context of recent research stream related to multiple supply modes and emergency orders, Minner (2003) provides a review of multiple supplier options. Recently, Wang et al. (2010) investigate whether a buying firm should employ multiple sourcing and/or improve supplier reliability. The authors find that a mixed strategy of dual sourcing and reliability improvement can provide significant value if suppliers are very unreliable and/or capacity is low relative to demand. Zhu (2012) characterizes the joint replenishment and pricing strategies with expediting orders as an additional supply source in response to demand uncertainties. Kouvelis and Li (2013) explore the impact of yield and demand uncertainty on the cycle order size, the emergency order size, and the way to split the available good units between the fast and slow shipping modes. However, none of the above papers address the issue of order crossovers.

This study investigates how an MRP system can take into account the situation where lead times are known but vary, which causes order crossovers to occur, and use this information to reduce costs when updates of gross requirements become available over the course of time. The problem is to determine an ordering policy that minimizes the total purchasing, setup, holding, and backordering costs over a finite horizon, where all cost parameters are general time-dependent. This problem is a generalization of the problem considered in the classical papers of Zangwill (Zangwill 1966, 1968), as we consider lead times to be time-dependent as well. We extend the solution approach of Federgruen and Tzur (1993), who present an efficient $O(N \log N)$ solution method for the dynamic lot-sizing problem with backlogging and N ordering decisions in the planning horizon.

The contribution of this study is threefold. Firstly, we generalize theory on MRP ordering by including the occurrence of dynamic lead-time variation and order crossovers. Next, we develop new fundamental insights and an efficient optimal solution procedure, leading to substantial cost savings and better material planning. Finally, we develop two heuristics, compare them with a benchmark, and show how they can be applied using an extension of the MRP scheme.

The structure of this study is as follows. Section 2 introduces the model. Section 3 develops structural results for the solution approach and describes the general solution procedure. Section 4 describes two relaxations that result in near-optimal solution approaches and a benchmark policy. Use of the solution approaches is illustrated in an extended MRP scheme. Section 5 provides a numerical analysis and compares the performance of the solution

approaches. Section 6 presents our conclusions and directions for future research.

2. Model Description

We consider a typical MRP discrete-time inventory replenishment problem with deterministic demand, a finite M -period horizon, N known ordering moments (where $N \leq M$), deterministic lead times expressed as an integral number of periods, and a cost structure consisting of non-stationary (i.e., time-dependent) setup costs, purchasing costs, holding costs, and backordering costs. However, our model differs from the traditional MRP replenishment problem in one important aspect: each ordering moment may incur a different lead time, and these lead times are known in advance. The interval length between successive ordering moments may differ as well. The problem is to determine the order quantities (assumed to be non-negative) by minimizing the total costs over the planning horizon and anticipating the circumstances at the remaining ordering moments.

We test our model in the context of a rolling horizon, which is typical for MRP replenishment planning, and include this perspective already in how we define the input parameters. At each order moment, updated gross requirements may have become available for all future periods, while the actual usage for earlier periods has become known. Both the actual usage and the most recent gross requirements are represented in a demand information matrix D (an $N \times M$ matrix arranged based on the order moment of each order). Hence, the i th row in the matrix D describes the most up-to-date demand information available at order moment o_i where $o_i \leq o_{i+1}$. If at the same time several ordering options are available, for example, in case of multiple suppliers or substitute products, $o_i = o_{i+1}$ and the successive rows of D will be equal. The optimal policy consists of the N optimal decisions $\{Q_i | i = 1, \dots, N\}$, that is, the orders that ought to be issued at the successive ordering moments when aiming to minimize the total setup, purchasing, holding, and backordering costs over the remaining horizon, given the demand information available at these moments.

The sequence of events in the model is as follows. At each order moment, we observe the current inventory level, the complete ordering history, and the relevant demand information row in D . Then, we decide whether a setup will be issued and how much to order at this ordering moment, given all lead times and prices that will hold for the current and remaining ordering moments. Returns (negative order sizes) are not allowed. Shortages are backlogged. Finally, the holding cost and the backorder cost are counted, based on the end-of-period inventory levels.

We denote h_m and b_m as, respectively, the holding cost per unit and backorder cost per unit of period m , c_i as the purchasing price, K_i as the setup cost, and L_i as the lead time that holds at the i th ordering moment o_i .

In the context of different lead times at various ordering moments, it is important to project the effect of MRP planned orders at future ordering moments and calculate the amount that might be ordered, based upon the demand information currently available. For this purpose, MRP uses so-called “planned order releases” and we model these quantities with the variable $Q_{i,t}$: the amount that we expect to issue at a future order moment o_t based upon the demand information at order moment o_i , where $t \in (i, N]$. For all other $t \in [1, i]$ we define $Q_{i,t} \equiv Q_t$ as the decision on the size of these orders that has been made in previous order moments (i.e., they are now treated as scheduled receipts) or is currently under consideration.

Define $x_{i,m}$ as the inventory level at the end of period m , where the calculation is based on the demand information row vector $D_i = (D_{i,1}, \dots, D_{i,M})$. Note that for $m > o_i$ this is the projected inventory level. Our initial inventory level at the start of period 1 equals x_0 , hence $x_{i,0} = x_0 \forall i$.

When deciding on Q_i we minimize the total cost TC_i , consisting of purchasing costs, setup costs, and inventory costs from the moment this order arrives ($o_i + L_i$) till the end of the horizon:

$$\min_{Q_i \geq 0} TC_i = c_i * Q_i + K_i * I\{Q_i > 0\} + \sum_{m=o_i+L_i}^M [h_m(x_{i,m})^+ + b_m(x_{i,m})^-], \quad (1)$$

where

$$x_{i,m} = x_{i,m-1} - D_{i,m} + \sum_{t=1}^N (Q_{i,t} * I\{o_t + L_t = m\}). \quad (2)$$

The indicator function $I\{\text{condition}\}$ equals one if the condition is true; otherwise, it is zero. To summarize our notation, we have introduced the following variables and parameters:

Parameters

- N : Number of order moments in the planning horizon
- M : Number of periods in the planning horizon
- o_i : Period of the i th order moment
- L_i : Lead time at the i th order moment
- $D_{i,m}$: Demand information for period m available at the i th order moment

Costs

- b_m : Backordering cost per item short at the end of period m

- h_m : Holding cost per item on stock at the end of period m
 c_i : Purchasing cost per item at the i th order moment
 K_i : Ordering costs at the i th order moment

Variables

- Q_i : Order size at the i th order moment
 $Q_{i,t}$: Planned order size for the t -th order moment, planned at the start of o_i
 $x_{i,m}$: Inventory level at the end of period m , as calculated at the start o_i

3. Analysis

In this section, we develop structural results for the optimal policy and present the general solution procedure. The problem is reformulated as a dynamic programming problem in which the decision moment is the arrival moment. At each arrival moment, the objective is to minimize the inventory cost from the arrival moment to the end of the horizon based on the current inventory level and demand information. In Theorem 1, we will prove that the reformulated problem is equivalent to the original problem defined by Equations (1) and (2).

3.1. Fundamental Insights

The inclusion of order crossovers makes it necessary to increase the state space, as the inventory position (stock on hand plus scheduled receipts) does not provide sufficient information anymore. Orders that have not yet been issued but will arrive before the current order should be considered as well. The new model formulation from the perspective of arrival moments makes it possible to reduce the size of the state space significantly by introducing three essential components, that is, demand information transformation, stage reduction, and terminal condition revision. To simplify notation, we disregard the time-index of the cost parameters from now on, but our results still hold for non-stationary cost parameters. We need to introduce additional notation for the development of the solution approach. For readers' convenience, the list of notation introduced in this section is provided below.

- O : column vector with N components for the order moment
 L : column vector with N components for the corresponding lead time
 A : column vector with N components of arrival moments in sequence of arrival
 OI : $N \times N$ binary matrix of the information about order moments and arrival moments
 D^a : $N \times N$ matrix of demand information arranged based on the arrival moment of each order
 J : row vector with N components of the sequence of arrivals

- Θ : row vector with N components corresponding to the sequence of arrival moments
 Φ : row vector with N components corresponding to the number of orders that have not yet been issued and will arrive in a certain interval
 Δ : row vector with N components corresponding to the last arrival among those orders issued by a certain order moment

Demand information transformation. We recommend transforming the order moment information to the information related to the sequence of arrival moments. For the binary matrix OI , $OI_{ij} = 1$ if an order is placed at the i th order moment and delivered at the j th arrival moment, and $OI_{ij} = 0$ otherwise. We have

$$A = OI^{-1}(O + L). \quad (3)$$

Note that OI is a permutation of the identity matrix. If there is no order crossover at all, OI equals the identity matrix. As OI is orthogonal, $OI^{-1} = OI^T$.

Figure 1 shows an example problem with order crossovers in a six-period horizon, where

$$O = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}; L = \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}; OI = OI^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}; A = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}.$$

For our solution approach, we are interested in the order moments corresponding to the sequence of arrivals $J = [1, \dots, N]$. The indicator θ_j identifies the order moment o_{θ_j} of the order that arrives at the j th arrival moment a_j . The row vector of order moments Θ corresponding to the sequence of arrival moments J can easily be determined as either

$$\Theta = [\theta_1, \dots, \theta_N] = J \cdot OI, \quad (4)$$

or

$$\theta_j = \arg \max_{i=1}^N OI_{ij}, \quad \forall j = 1, \dots, N. \quad (5)$$

Using OI , we transform the demand information based on the order moments into that based on the arrival moments. Denote D^a as an $N \times M$ matrix of demand information arranged based on the arrival moment of each order, where $D_{j,m}^a$ is demand information for period m observed at the

Figure 1 Order Crossovers with $N = 3$ Decisions and $M = 6$ Periods

order moment of the order that arrives at a_j . We have:

$$D^a = OI^{-1}D. \quad (6)$$

Stage reduction. The second essential component is the observation that in order to decide the order quantity that arrives at the j th arrival moment, we should consider all orders that arrive during the interval $(o_{\theta_j}, a_j]$, even if they have not yet been issued, since those orders affect the inventory available at the arrival moment a_j . Denote the number of the orders that have not yet been issued and will arrive in the interval $(o_{\theta_j}, a_j]$ as ϕ_j . Note that $1 \leq \phi_j \leq j$. We have

$$\phi_j = \sum_{i=\theta_j}^N \sum_{t=1}^j OI_{i,t}, \quad \forall j = 1, \dots, N. \quad (7)$$

For Figure 1, this results in: $\Theta = [\theta_1, \theta_2, \theta_3] = [1, 2, 3]$; $\Phi = [\phi_1, \phi_2, \phi_3] = [1, 2, 3]$. Hence, when deciding the order that arrives at a_3 , $\theta_3 = 1$ reveals that it is ordered at order moment o_1 , and $\phi_3 = 3$ reveals that when deciding on the size of Q_1 , we should also consider the order quantities of two future planned orders that will arrive before a_3 : $Q_{1,2}$ and $Q_{1,3}$ although they have not yet been issued.

Moreover, we do not have to reconsider the decision for orders arriving after a_j when deciding the order quantity that arrives at a_j , since those decisions have been determined based on the information available at the corresponding decision-making moments. Our decision problem for the order quantity arriving at a_j and ordered at o_{θ_j} can then be reduced to solving a ϕ_j -stage problem instead of solving an $(N - \theta_j + 1)$ -stage problem.

Terminal condition. The third essential component is the observation that the gross requirements updates will affect the terminal condition of the decision problem in case of order crossovers. If we set the size of an order that crosses an already issued order, and gross requirements have been updated since then, the already issued order size may appear too small or too large. Then for the new order, we might decide to order more or less to correct for this mismatch. This results in additional costs in the interval $[a_j, a_{j+1})$, but reduces costs in the interval $[a_{j+1}, a_{\delta_j+1})$, where a_{δ_j} indicates the last arrival moment of any order that has been issued by o_{θ_j} . Since j is changed from 1 to N , we define $a_{N+1} \equiv M + 1$ to make the interval $[a_j, a_{j+1})$ feasible for all possible j .

LEMMA 1. *The last arrival among those orders issued by o_{θ_j} is identified by*

$$\delta_j = \max_{i=1}^{\theta_j} \max_{t=j}^N (OI_{i,t} * t) \geq j \quad \forall j = 1, \dots, N. \quad (8)$$

PROOF. Since a_{δ_j} indicates the last arrival moment of any order that has been issued by o_{θ_j} , δ_j indicates the position in the arrival sequence J where that order arrives. Now Equation (8) can be rewritten as $\delta_j = \max \left[j, \max_{i=1}^{\theta_j+1} \max_{t=j+1}^N (OI_{i,t} * t) \right]$, because by definition we have $OI_{\theta_j,j} \equiv 1$, that is, the order that is issued at o_{θ_j} arrives at position j , so we have $\delta_j \geq j$. There are $\theta_j - 1$ orders that have already been issued before the order that arrives at position j . If any of these orders (say i) arrives later than position j (say in position $t > j$), $OI_{i,t}$ equals 1 and δ_j becomes equal to t . Formula (8) checks for all orders that have been issued before whether they arrive later than the j th arriving order and takes the maximum of their arrival positions. This maximum is always larger than j . If no such order exists, δ_j remains equal to j . Hence, Equation (8) determines the last arrival among those orders issued by o_{θ_j} . $\square$

For the example in Figure 1, $\Delta = [\delta_1, \delta_2, \delta_3] = [3, 3, 3]$.

Note that the correction has an impact on all future costs till arrival moment a_{δ_j+1} . As the size of the order that arrives at a_{δ_j+1} has not yet been decided, we do not have to take costs in any further periods into account. We only need to consider the costs in the interval $[a_{j+1}, a_{\delta_j+1})$ for the terminal condition. Therefore, this lemma helps us to determine the maximum time window that needs to be considered for the current decision.

3.2. Solution Approach

We now construct a dynamic programming problem with ϕ_j stages to find the optimal value of Q_{θ_j} arriving at a_j . Firstly, we introduce an auxiliary variable v that indicates the arrival moments that need to be included in the dynamic programming problem:

$$v = n + j - \phi_j. \quad (9)$$

Let $V_n(x_{a_v-1} | D_j^a)$ be the minimal total cost from stage n to the end of ϕ_j stages when the ending inventory level of period $a_v - 1$ is x_{a_v-1} . From now on, we omit the first index of x that remains equal within the dynamic programming problem, that is, x_{a_v-1} is short for x_{θ_j, a_v-1} .

Denote Q_n^a as the size of the order that arrives at the n th stage based on the demand information D_j^a . MRP planned orders are orders with $n < \phi_j$. They ought to be non-negative as well. Of course $Q_{\phi_j}^a = Q_{\theta_j}$.

For $n = 1, \dots, \phi_j$, the dynamic programming formulation is given by

$$V_n(x_{a_v-1}|D_j^a) = \min_{Q_n^a \geq 0} \left\{ \sum_{m=a_v}^{a_{v+1}-1} \left[h \left(x_{a_v-1} + Q_n^a - \sum_{t=a_v}^m D_{j,t}^a \right)^+ + b \left(x_{a_v-1} + Q_n^a - \sum_{t=a_v}^m D_{j,t}^a \right)^- \right] + cQ_n^a + KI\{Q_n^a > 0\} + V_{n+1} \left(x_{a_v-1} + Q_n^a - \sum_{m=a_v}^{a_{v+1}-1} D_{j,m}^a | D_j^a \right) \right\}. \quad (10)$$

For the terminal condition, we use

$$V_{\phi_j+1}(x_{a_{j+1}-1}) = \sum_{i=1}^{\delta_j} \sum_{m=a_{j+1}}^{a_{j+1}+1-1} \left[h \left(x_m + Q_{\theta_{j+i}}^* - \sum_{t=a_{j+i}}^m D_{j,t}^a \right)^+ + b \left(x_m + Q_{\theta_{j+i}}^* - \sum_{t=a_{j+i}}^m D_{j,t}^a \right)^- \right] \quad (11)$$

which equals zero if $\delta_j = 0$. (Recall that we omitted the time-index of the cost parameters for ease of interpretation and that starred variables have already been optimized (e.g., $Q_{\theta_{j+i}}^*$).

Denote y_n as the inventory level just after the arrival of the n th arrival order. It is clear that $y_n = x_{a_v-1} + Q_n^a$. Then, we can rewrite Equation (10) as

$$V_n(x_{a_v-1}|D_j^a) = \min_{y_n \geq x_{a_v-1}} [G_n(y_n) + KI\{y_n > x_{a_v-1}\}], \text{ where} \\ G_n(y_n) = \sum_{m=a_v}^{a_{v+1}-1} \left[h(y_n - \sum_{t=a_v}^m D_{j,t}^a)^+ + b(y_n - \sum_{t=a_v}^m D_{j,t}^a)^- \right] + V_{n+1} \left(y_n - \sum_{m=a_v}^{a_{v+1}-1} D_{j,m}^a | D_j^a \right). \quad (12)$$

LEMMA 2 $G_n(y_n)$ is K -convex in y_n and $V_n(x_{a_v-1})$ is K -convex in x_{a_v-1} for $n = 1, \dots, \phi_j$.

PROOF. see Appendix A.

Since $G_n(y_n)$ is K -convex in y_n , the (s, S) policy is optimal and the order-up-to level S_n in stage n is given by

$$S_n = \arg\{y_n : \min G_n(y_n)\}, \\ s_n = \arg\{y_n \leq S_n : G_n(y_n) = G_n(S_n) + K\}. \quad (13)$$

After solving (12), $Q_{\phi_j}^a$ is the optimal order quantity for Q_{θ_j} .

General solution procedure. We can find the optimal quantity for each order based on the arrival moment a_j by solving the corresponding ϕ_j -stage problem. Based on the dynamic programming formulation (12), we develop the following general solution procedure:

Step 1. Set $j = N$; Determine Θ by Equation (4), Φ by Equation (7), and Δ by Equation (8).

Step 2. Solve a ϕ_j -stage problem to find Q_{θ_j} .

Step 3. Set $j = j - 1$. If $j > 0$, go to Step 2; if not, stop.

Remark. When $D_{i,m} = D_{i-1,m} \quad \forall i = 2, \dots, N$; $m = o_i, \dots, M$, (i.e., there are no updates of gross requirements for periods $o_i, \dots, M$) we can modify Step 3 of the general procedure to speed up the computation. The new step is stated as

Step 3. Set $j = j - \phi_j$. If $j > 0$, go to Step 2; if not, stop.

Now, we need to prove that the general solution procedure does indeed provide an optimal solution to the original model with order crossovers as stated in section 2.

THEOREM 1. *The general solution procedure yields the optimal solution to the original problem defined by Equations (1) and (2).*

Proof: see Appendix A.

Note that the optimal policy is an (s, S) policy that takes into account whether or not the gross requirements have been updated. The procedure can easily be extended to the case where demand is stochastic, which would result in an equivalent stochastic dynamic programming formulation.

4. Relaxations of the Problem

Most MRP systems use settings for the length of the planning horizon M and number of orders N for which our optimal solution method will find a solution in a few seconds. In case of large lead-time differences and large values for N and M , a complex order crossover structure may result for which it takes more time to find the optimal solution using our general solution procedure. Hence, this section presents some relaxations of the original problem that result in simpler and computationally more efficient heuristic solution approaches. The first approach assumes that we allow the order size for the MRP planned orders (Q_n^a with $n < \phi_j$) to be negative. The second approach assumes that the effect of order crossovers for future arrival moments can be neglected. We also need a benchmark to evaluate against the other solution

approaches. For that purpose, we use a third heuristic that totally ignores order crossovers. Finally, we show how the decision on planned order receipts and releases can easily be supported by an extension of the MRP scheme.

4.1. Heuristic I: Negative MRP Planned Order Sizes Allowed

In the general procedure, we have to check for the non-negativity of the MRP planned order sizes to find the optimal solution at each iteration (see Equation (10)), which complicates the procedure. This section relaxes the non-negativity constraint of the planned order sizes for the crossover orders.

PROPOSITION 1. *Without the non-negativity constraint, the worst-case performance of the general procedure in terms of computational time is $O(N^2 \log N)$.*

PROOF. According to the general procedure, we have to execute the procedure N times till $j = 0$. For each time, we need to solve a ϕ_j -stage dynamic programming problem at Step 2 to find the optimal solution for $Q_{\phi_j}^*$. Note that $\phi_j \leq j \leq N$. Then, for each iteration, if the cost parameters are time-independent, the worst-case performance in terms of the computational time is the same as that of the Federgruen and Tzur (1993) algorithm, that is, $O(N \log N)$. Thus, for N iterations, the complexity is $O(N^2 \log N)$. $\square$

We note that the relaxation still requires non-negative ordering decisions for the actual orders $Q_{\phi_j}^a$, as only the planned orders with $n < \phi_j$ are allowed to be negative. Therefore, this relaxation only affects the calculation of the initial inventory level just before the moment the current order arrives. If we allow for negative planned orders in our MRP projections, our calculation of the inventory level is a lower bound, since it allows the material planner to return the excess stock. Consequently, we may order more for the current order because we underestimate the inventory level at the start of the corresponding period by using the relaxation. This will lead to higher holding costs from the arrival moment onwards compared to the optimal policy for the original problem. However, we do expect that very frequently both policies will result in exactly the same solution, hence the same holding costs. This is because a negative planned order size at a crossover order moment will rarely occur. A negative order size may occur if the updated gross requirements for the previous order moment are lower than the gross requirements for the corresponding periods and this reduction is larger than the total usage (demand) between the arrival moments. Hence, we expect that the optimal

policy for the relaxation problem is a good approximation for the optimal policy of the original problem.

4.2. Heuristic II: Simplifying the Terminal Condition

The second relaxation of the problem is to simplify the terminal condition (11) in the dynamic programming problem (12). This terminal condition assures that the current order not only focuses on the demand it has to cover but in light of new demand information it also corrects for inaccurate decisions that have been made before. Heuristic II ignores the possibility of these corrections and uses Equation (14) as a terminal condition:

$$V_{\phi_j+1}(x_{a_{j+1}-1}) = 0. \quad (14)$$

We expect that using this heuristic will result in a higher cost gap in cases of either increasing uncertainty or more complex order crossover structures. On the other hand, calculations will be simplified and computation time will be shorter.

4.3. Benchmark Heuristic III: Ignoring Order Crossovers

If there is no order crossover, the optimal solution to the dynamic programming problem (12) can be directly computed by the algorithm developed by Federgruen and Tzur (1993). This is the well known (s, S) policy that orders based upon the inventory position. However, if there are order crossovers, this procedure will not result in the optimal solution. Heuristic III applies the Federgruen and Tzur algorithm that assumes that orders will arrive in the sequence they are ordered. Hence, Heuristic III is a benchmark approach, as it shows the potential savings of using our optimal policy that takes the crossover structure into account.

4.4. Example of Extended MRP Scheme

All three heuristics require the use of an MRP scheme in which known differences in lead times are represented. MRP parameters, such as safety stock and lead times, are normally constant over the horizon and shown in a parameter set. This set can easily be extended to allow for time-dependent parameters. In Table 1, we show an example where the lead times are time-dependent and expressed in weeks, purchasing and setup costs are specified for these orders as well, while the safety stock is constant over the horizon.

Note that in some weeks order release is not possible, which is indicated by an x as the lead time for that week. The fields used to decide on the planned releases have been grayed out for these weeks. We do the same for the weeks in which it is not possible to receive a new order that still has to be issued. This

Table 1 Example of an Extended MRP Scheme to Support Non-Stationary Lead Times

MRP parameter set	Constant	Time-dependent (weeks)									
		1	2	3	4	5	6	7	8	9	10
Lead time for order release		5	1	1	3	x	x	2	x	1	x
Setup costs		5	5	5	5	x	x	5	x	5	x
Purchasing costs		0.5	0.5	0.5	0.5	x	x	0.5	x	0.5	x
Safety stock	0										
MRP scheme	Initial										
Gross requirements		10	8	12	15	9	20	10	0	5	10
Scheduled receipts			11								
Projected available	20	10	13	1	−14	−23	−43	−53	−53	−58	−68
Net requirements		0	0	0	14	0	20	0	0	5	0
Planned receipt				0	23		30	0		15	0
Planned release optimal		30	0	23	0			15		0	
Planned release heuristic I		30	0	23	0			15		0	
Planned release heuristic II		30	0	23	0			15		0	
Planned release heuristic III		0	0	53	0			15		0	
MRP scheme after update	Initial										
Gross requirements		8	6	12	20	15	25	8	5	0	12
Scheduled receipts			11				30				
Projected available		12	15	3	−17	−32	−27	−35	−40	−40	−52
Net requirements			0	0	20	15	0	0	0	0	12
Planned receipt				0	40			0			12
Planned release optimal			0	40	0			0		12	
Planned release heuristic I			0	40	0			0		12	
Planned release heuristic II			0	42	0			0		12	
Planned release heuristic III			0	35	0			17		0	

helps the material planner to identify which options are left when deciding on actual and planned releases.

Table 1 also shows the results of using our solution approaches for determining the planned receipts, both for the initial situation and the situation after one week when an update of the gross requirements has been provided. Note that we use cost parameters $h_i = 0.1$; $b_i = 1$; $c_i = 0.5$; $K_i = 5$.

The data from the MRP scheme can easily be translated into the variables used in our model:

$$\begin{aligned}
 O &= [1, 2, 3, 4, 7, 9]^T, L = [5, 1, 1, 3, 2, 1]^T \\
 A &= [3, 4, 6, 7, 9, 10]^T \\
 D &= \begin{bmatrix} 10 & 8 & 12 & 15 & 9 & 20 & 10 & 0 & 5 & 10 \\ 8 & 6 & 12 & 20 & 15 & 25 & 8 & 5 & 0 & 12 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \\
 x_0 &= 20 + 11 = 31
 \end{aligned}
 \quad OI = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The weeks in which a planned release is possible are described using O . The lead times that will hold for these planned releases are given with L (weeks). Next, A identifies the weeks in which a new receipt is possible. Note that $OI \times A = O + L$. The presented demand information matrix D is incomplete (no information is shown for the last four order moments), but it suffices to illustrate the principle. The second row shows the

updated information available when issuing the second order. Actual usage in period one appeared to be 8 units instead of the 10 that we expected. Moreover, the gross requirements have been updated. The initial inventory x_0 includes the scheduled receipts.

In the example, there is no difference between the optimal solution and Heuristics I and II in the actual order issued in the first week. However, the benchmark heuristic does not order anything in the first week as it neglects the possibility of order crossover. The other policies use the crossover order that can be issued in week 1 to cover demand in week 6, which saves holding costs.

The second part of Table 1 shows the effect after one week. We assume that the optimal solution (ordering 30 units in week 1) has been implemented and show the recalculated MRP scheme which uses the updated gross requirements information. The solution approaches result in differences in the suggested planned order releases, where Heuristic I results in the same suggestion as the optimal approach.

5. Numerical Analysis

This section analyzes the effect of including order crossovers in the inventory model and examines the

differences between the optimal solution described in section 3 and the three heuristics discussed in section 4. They are compared in terms of computation time needed to find a solution and the total cost of the solution. We aim to identify conditions for application, so the experimental design includes four important factors: the solution approach, the crossover structure, the demand correlation, and the cost structure.

5.1. Experimental Design

The length of the horizon M and number of decisions N are chosen such that several order crossovers may occur. Throughout our experiments, we use $M = 40$ and $N = 10$. The first arrival moment of an order is in every experiment set to period 12 and the initial inventory just before that period is set to zero, which makes it possible to compare the impact of different crossover structures and other parameters among experiments.

Our experimental design is presented in Table 2. We use a full-factorial design to compare the solutions found by the four solution approaches for all combinations of the other experimental factors. The first experimental factor, structure of order crossovers, is denoted with the vector P . We test two cases of P : $p_j \leq 2$ (at most one order crossover each time); $p_j \leq N$ (at most $N - 1$ order crossovers each time). We use the *randperm* function in Matlab to re-sequence the arrival moments. Note that the variability in the resulting lead times increases if the crossover structure becomes more complex, but the mean lead time remains constant.

The next experimental factor is the coefficient of correlation of demands in two successive periods given by ρ . In our experiments, we use a first-order autoregressive process (AR1) to represent the gross requirements, that is, $D_m = \mu_m + \rho(D_{m-1} - \mu_{m-1})$ for $m = 1, 2, \dots, M$ and $|\rho| < 1$, where D_m is the actual demand in period m , μ_m is the mean of D_m , and ε_m is an i.i.d. random variable following a normal distribution with mean 0 and standard deviation σ . In period m , after D_m is realized, we generate an update for the

demand information in period $m+k$ as $\hat{d}_{m,m+k} = \mu_{m+k} + \rho^k(D_m - \mu_m)$, for $k = 1, 2, \dots, M - m$. We first consider stationary demand, that is, $\mu_m = 100$ for all m and $\rho = 0$. Next, we experiment with autoregression using $\rho = -0.5$ and $\rho = 0.5$. Settings for initial demand are similar to that of Lu et al. (2006). That is, $\hat{d}_{0,m} = \mu_m + \sigma_0 \rho^m$, where $\sigma_0 = 0.25\mu_1$.

Finally, we experiment with the cost structure. Holding costs are stationary at $h_i = 0.1$. We use two levels for the ratio between holding and backordering costs: $h/b = 0.1$ and 1.0 . If the ratio is small (i.e., backordering cost is much higher than holding cost), policies that frequently face shortages due to demand information inaccuracy or relaxations in the calculations are expected to perform worse. The cost ratio 1.0 expresses the situation where backordering cost is equal to holding cost. Setup and purchasing costs remain constant over the experiments, with setup cost $K_i = 5$, and purchase cost $c_i = 0.5$.

As performance measures, we use the total cost gap with the optimal solution and the computation time. The cost gap is $\Delta_{TC} = \frac{TC - TC^*}{TC^*} \times 100\%$ where TC and TC^* are the total cost over the whole horizon under the heuristic and the optimal policy, respectively. The computation time is measured in seconds CPU time, as reported by Matlab R2013a. We use a PC with Intel Core i3 3.3 GHz for the calculations. The numerical analysis of each experiment is based on 500 runs with randomly generated data (demand and order information).

5.2. Results

The most important conclusion of Tables 3 and 4 is that it is beneficial to take order crossovers into account in a solution approach. Comparing the optimal solution with the approach that completely ignores the existence of order crossovers (i.e., Heuristic III), we find that the average cost gap is 3% to 7% for simple crossover structures (Table 3) and 13% to 25% for complex crossover structures (Table 4). Therefore, substantial cost savings can be obtained when applying an approach that takes the presence of order crossovers into account.

The optimal approach takes more calculation time than the approach that completely ignores order crossovers. Table 5 shows that in case of a simple crossover structure, the calculation time doubles, but for complex crossover structures it is much higher. If we would like to have quick results, we can apply Heuristic I instead of the optimal approach, as the calculation speed is slightly larger than Heuristic III, but the cost benefit is substantial. Heuristic I is able to find solutions with an average cost gap that ranges from 2% to 8% of the optimal solution. Heuristic II finds solutions with an average cost gap between 2% and

Table 2 Experimental Design

Experimental factors			
Crossover structure (P)	Correlation (ρ)	Cost structure (h/b)	Solution approach (<i>Heur</i> or <i>Opt</i>)
$P \leq 2$	$\rho = -0.5$	$h/b = 0.1$	I: allow negative size of MRP planned orders
$P \leq N$	$\rho = 0$ $\rho = 0.5$	$h/b = 1.0$	II: terminal condition = 0 III: ignore order crossovers Optimal solution

Table 3 Effect of Cost Structure and Demand Correlation on Cost Gap of Heuristics Where $P \leq 2$

$P \leq 2$ cost gap	Heuristic I (%)		Heuristic II (%)		Heuristic III (%)	
	$h/b = 0.1$	$h/b = 1.0$	$h/b = 0.1$	$h/b = 1.0$	$h/b = 0.1$	$h/b = 1.0$
ρ						
−0.5	2.06	1.94	2.05	2.14	2.85	2.96
0	3.43	2.78	3.78	3.94	4.55	4.96
0.5	4.18	3.69	5.36	5.35	7.13	7.29

Table 4 Effect of Cost Structure and Demand Correlation on Cost Gap of Heuristics Where $P \leq 10$

$P \leq 10$ cost gap	Heuristic I (%)		Heuristic II (%)		Heuristic III (%)	
	$h/b = 0.1$	$h/b = 1.0$	$h/b = 0.1$	$h/b = 1.0$	$h/b = 0.1$	$h/b = 1.0$
ρ						
−0.5	4.58	3.28	3.41	3.39	12.81	12.98
0	6.18	5.05	5.11	5.35	16.34	16.59
0.5	8.05	7.17	7.22	7.38	25.08	25.12

Table 5 Average CPU Time of Solution Approaches Under Various Crossover Structures

Approach	$P \leq 2$	$P \leq 10$
	CPU time (sec.)	CPU time (sec.)
Heuristic I	55.65	655.88
Heuristic II	60.18	871.78
Heuristic III	30.67	42.55
Optimal	83.56	987.27

7% of the optimal solution. Hence, our two relaxation approaches are able to find near-optimal solutions and the savings compared to ignoring order crossovers range from 1% to 18%.

We use ANOVA to investigate the significance of the main effects and interaction effects. Figure 2 shows the main effects and Figure 3 the interaction effects. All main effects are significant at a level of 0.05. A paired-*t*-test shows that Heuristic I and II are significantly different, but it is clear that they both are much better than Heuristic III.

The results in Figure 3 show that the crossover structure has a strong effect on the performance of the heuristics. If the complexity is low ($P \leq 2$), the cost gap with the optimal solution is on average less than 3%, but for a high complexity the average cost gap significantly increases. The increase is mainly due to the bad performance of Heuristic III that completely ignores the presence of order crossovers. If we take a look at the performance of Heuristics I and II for different levels of crossover complexity, we see that there is a significant but small interaction effect. Heuristic I outperforms Heuristic II in case of low complexity, while the opposite is true for a highly complex crossover structure. However, as the calculation time of Heuristic II strongly increases with crossover complexity, it is doubtful whether the minor cost saving is worth the effort. Therefore, it

seems that Heuristic I is the preferable option if usage of the optimal solution is not possible.

The effect of demand correlation on the performance of the heuristics is clearly present. If the correlation is negative, the cost gap of the heuristics with the optimal solution decreases. If a positive correlation coincides with a more complex order crossover structure, costs become worse. Hence, the payoff for investments in gross requirements updates is higher.

The cost structure has only a minor impact on performance, but the interaction effect with the solution approach is significant. If the cost factor is low (i.e., holding cost is much lower than backordering cost), the cost gap with the optimal solution is slightly larger.

In general, ignoring the presence of order crossovers results in high cost gaps up to 25%. By applying the relaxation of subsection 4.1 in Heuristic I, we find near-optimal solutions in the same order of time as we would have faced when ignoring order crossovers. Using either Heuristic I or Heuristic II, a cost saving up to 18% of total inventory cost is possible.

6. Concluding Remarks

We study a very general periodic-review inventory model where lead times as well as costs are deterministic but non-stationary. The non-stationary lead times may result in order crossovers, which cannot be addressed by the currently known methods. By formulating a dynamic programming model with the decision moment at the order arrival moment, we develop a general solution procedure to obtain the optimal order quantities for the problem with non-stationary lead times. The complexity of our method is $O(N^2 \log N)$ in case order crossovers occur. Further, we develop heuristics that simplify the calculations and increase the speed of calculating. The

Figure 2 Main Effects (All Significant at 0.05 Level)

Figure 3 Interaction Effects

solution approaches are demonstrated in an extended MRP scheme to provide support to material planners facing different lead times over time, possibly resulting in order crossovers. By extensive

numerical experiments, we examine the performance of the heuristics and the optimal solution in terms of the total cost and the computation time. The results show that substantial cost gaps of up to

25% will occur if order crossovers are present but not accommodated in a solution approach. Managers should pay serious attention to this impact of lead-time changes and order crossovers on cost performance. Heuristic I is developed to achieve a relatively good cost performance in less time. The cost gap with the optimal solution is relatively small (between 3% and 8%).

The results of this study can provide an MRP solution for the multiple supplier situation, which is now more important than ever since firms retract from sole sourcing due to supply chain risk concerns. Under this situation, lead time changes and order crossovers are more frequently encountered. Our solution approach proposes to support this decision making in MRP systems and to use an (s, S) policy where order crossovers are naturally included in the analysis. Moreover, it is possible to implement our solution approach in commercial material planning systems, as many of these systems already are able to use discrete lot-sizing techniques for situations with non-stationary cost parameters.

The results of this study are also relevant for inventory theory. Most authors circumvent the presence of inventory order crossovers, mainly due to the difficulty in finding closed form solutions for the optimal order quantity. In this study, we show that the optimal solution in a periodic system with order crossovers still can be characterized as an (s, S) policy.

Our current work can be extended in the following directions. Firstly, several factors that are related to the supply side, such as limited sourcing, perishability, and yield losses can be included. Secondly, factors that are related to the transport, storage, and retrieval of inventory can be included, such as deterioration, transport-mode selection, and blending policies. Finally, demand factors can be included, such as price-sensitive demand, or modeling gross requirements as an explicit combination of order arrivals and demand forecasts. Another direction for future research is to explore the cost/time trade-off in supply chain problems in more detail, as our solution approach allows for multiple ordering options with both different lead times, setup costs, and purchase prices.

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Appendix. Proof

PROOF OF LEMMA 1. We intend to prove it by induction. For $n = \phi_j$, by Equation (12) we have:

$$G_{\phi_j}(y_{\phi_j}) = \sum_{m=a_j}^{a_{j+1}-1} \left[h(y_{\phi_j} - \sum_{t=a_j}^m D_{j,t}^a)^+ + b(y_{\phi_j} - \sum_{t=a_j}^m D_{j,t}^a)^- \right] + V_{\phi_j+1} \left(y_{\phi_j} - \sum_{m=a_j}^{a_{j+1}-1} D_{j,m}^a | D_j^a \right).$$

Since V_{ϕ_j+1} is a piece-wise linear function of V_{ϕ_j} and the slope of V_{ϕ_j+1} is increasing in y_{ϕ_j} , it is clear that V_{ϕ_j+1} is convex in y_{ϕ_j} . By following the same argument, we can prove that $G_{\phi_j}(y_{\phi_j})$ is also convex in y_{ϕ_j} . By Scarf (1960), $V_{\phi_j}(x_{a_j-1} | D_j^a)$ is K -convex in x_{a_j-1} .

Next, since $V_{n+1}(x_{a_{v+1}-1} | D_j^a)$ is K -convex in $x_{a_{v+1}-1}$, $V_{n+1}(y_n - \sum_{m=a_v}^{a_{v+1}-1} D_{j,m}^a | D_j^a)$ is K -convex in y_n by Equation (12). By following the same argument, we can show that $G_n(y_n)$ is K -convex in y_n and $V_n(x_{a_v-1} | D_j^a)$ is K -convex in x_{a_v-1} . $\square$

PROOF OF THEOREM 1. We intend to show by induction that the general solution approach is valid.

For $j = N$, we determine the optimal quantity for the last arrival order. By θ_N , we know the demand information available at the order moment of the last arrival order. By ϕ_N , we know the number of arriving orders from the time epoch that the last order is placed to the time epoch that the last order has arrived. Although we need to solve an N -stage problem in general, the choice of Q_{θ_N} does not affect those $(N - \phi_N)$ orders that arrive before the θ_N^{th} order moment. Thus, we only need to solve a ϕ_N -stage dynamic programming problem to find the optimal Q_{θ_N} .

Suppose that the procedure is valid for $j = n + 1$. We show that the procedure still holds for $j = n$. The order moment of the n th arrival order is θ_n , and ϕ_n indicates the number of arriving orders from the time epoch that the n th arrival order is placed to the time epoch that the n th arrival order has arrived. We need to solve an $(N - (n - \phi_n))$ -stage problem in order to find the optimal solution. However, for those orders that arrive after the n th arrival moment, the optimal solutions have already been obtained when we solve the problem for $j = n + 1$, based on the demand information available when determining these orders. We might have to correct for demand updates that have become available since then, but this will not affect the order sizes of these orders, only the size of the current order.

Thus, since the number of orders arriving later than the n th arrival moment is $N - n$, by the backwards induction, we reduce the $(N - (n - \phi_n))$ -stage problem to a ϕ_n -stage problem. Solving this problem leads to the optimal solution. Because we prove the procedure is also valid for $j = n$, the induction proof is completed. The general procedure holds for $n = 1, \dots, N$. $\square$

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