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Integration of capacity, pricing, and lead-time decisions in a decentralized supply chain

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ABSTRACT

We consider a decentralized supply chain consisting of a supplier and a retailer facing price- and lead-time-sensitive demand. The decision process is modelled by a Stackelberg game where the supplier, as a leader, determines the capacity and the wholesale price, and the retailer, as a follower, determines the sale price and lead time. The equilibrium strategy of these two players is obtained. By comparing with the performance of the corresponding decentralized chain without capacity decision as a benchmark, we characterize the impact of capacity decision on the players' profit, i.e., the supplier's profit may be always significantly increased while the retailer's profit is only increased when the capacity is underestimated in the benchmark model. Further, we demonstrate that the integration of capacity decision can also significantly reduce the profit loss caused by double marginalization. Finally, we find that the revenue-sharing and two-part tariff contracts cannot coordinate the decentralized channel. Instead, we propose a franchise contract with a contingent rebate that can achieve channel coordination and a win-win outcome.

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1. Introduction

In various industries, firms frequently face capacity, pricing and lead-time decisions. The integration of these three critical decisions between manufacturing and marketing/sales is widely cited as a weapon to gain a competitive advantage in the market (e.g., Shapiro, 1977; Braham, 1987; Crittenden, 1992; O'Leary-Kelly and Flores, 2002). However, most 21st-century supply chains are decentralized. For example, in the healthcare industry, Philips Healthcare Netherlands, a world-known medical-equipment manufacturer, has several manufacturing plants in Europe and North America. To fulfill the demand in Asia, the firm has a major dealer in Hong Kong, LABandTECH (<http://www.labntech.com/>). When customers want to purchase the products, the dealer usually determines both the sale price and the promised delivery time (lead time) to customers. Because of diversified and uncertain customer requirements, the manufacturer only starts the production/assembly after customers finalize their orders. Therefore, to deliver the orders to customers on time, the capacity decision is very imperative. Another example is Internet retailing. When customers visit Amazon.com or BestBuy.com, they can observe the on-line information of the price and the delivery time

determined by the e-commerce firms. After customers confirm their orders, the e-retailers may ask UPS or the postal service to handle the delivery. Those logistics providers have to guarantee a sufficient capacity to meet the requirement, specially in the peak demand season. From the above examples, we observe that to maintain customer satisfaction and achieve competitive advantages, it is essential to integrate the capacity, pricing, and lead time decisions by aligning the interests of different companies, which is a fundamental step to achieve a triple-A supply chain (Lee, 2004). However, as Narayanan and Raman (2004) point out, most companies do not worry about their partners' strategy when building supply chains to deliver goods and services to customers, which results in poor supply chain performance.

From the perspective of supply chain integration, the purpose of this paper is to investigate the impact of integration of capacity, pricing, and lead time decisions in a decentralized supply chain (DSC) on the marketing strategies and profits of the firms and how to coordinate such a DSC in order to achieve a win-win outcome for all the firms. As a long term decision, suppliers usually make the capacity decision based on historical demand information but ignore the impact of the capacity decision on their supply chain partner, which may cause a negative effect on her own. Therefore, the supplier's capacity decision should be aligned with her partner's pricing and time decisions. Moreover, it is necessary to motivate the supplier to make the first-best capacity decision in order to yield high profits for all the participants in the chain.

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However, in most of the previous literature, the capacity decision is made independently of the pricing and lead time decisions. To the best of my knowledge, the coordination of capacity, pricing, and lead time decisions in the DSC has not been well studied yet. Hence, this paper addresses the following questions. What are the optimal capacity, pricing, and lead time decisions in the DSC? Can the incorporation of capacity decision significantly increase both the supplier's and the retailer's profits? Can it also reduce the profit loss caused by double marginalization? How can an incentive mechanism be designed in order to align the interests of the two individual firms and achieve channel coordination? We believe that the answers to the above questions can provide helpful guidelines and decision-making tools for managers.

We consider the DSC consisting of a supplier (or a manufacturer) and a retailer (or a distributor). We model the dynamic decision process between the supplier and the retailer by using a Stackelberg game. The supplier, as a leader, determines the capacity and wholesale decisions. Given the supplier's decisions, the retailer, as a follower, decides the sale price and promised delivery time (PDT). First, we develop the procedure to obtain the optimal decisions of capacity, pricing, and PDT for the DSC. Second, by using the model without capacity decision as a benchmark, we find that when the capacity decision is underestimated in the benchmark model, the incorporation of capacity decision can yield a shorter PDT, a higher retail price, and a larger market share. As a result, the profits of both firms increase greatly. However, when the capacity decision is overestimated, only the supplier's profit is increased while the retailer's profit is decreased. Third, we find that the integration of capacity, pricing and time decisions may significantly reduce the negative impact of double marginalization on the DSC. Finally, we demonstrate that the revenue-sharing and two-part tariff contracts cannot coordinate the DSC and propose a franchise contract with contingent rebate that can achieve channel coordination and a win-win outcome.

The remainder of the paper is organized as follows. In Section 2, we review the related literature. Section 3 formulates the decentralized decision models for the retailer and the supplier. In Section 4, we derive the equilibrium. Section 5 investigates the impact of the integration of capacity decision on the firms' profits and double marginalization. Next, we examine how to achieve channel coordination and a win-win outcome in Section 6. Section 7 summarizes the paper and points out some potential directions for further research.

2. Literature review

There are two streams of research related to our work: lead time quotation and capacity management, as well as coordination of the DSC. There exists an extensive study on the first stream. By assuming an exogenous capacity, Palaka et al. (1998) and So and Song (1998) study the optimal selection of price and delivery time (which is uniformly applied to all customers' orders) so as to maximize the profit of an $M/M/1$ -type Make-to-Order system. So (2000) extends the work to multiple firms and derives the equilibrium solution. Ray and Jewkes (2004) study the optimal delivery time strategy for a single firm with capacity investment and a service-level constraint by explicitly modelling the relationship between price and delivery times. Boyaci and Ray (2006) analyze the impact of capacity cost on the lead time quotation for two substitutable products. Shang and Liu (2011) consider a Nash game where multiple firms make their lead time and capacity decisions. In these papers, a static PDT is used as a decision variable that affects demands directly like the price. This is similar to our work. However, these papers consider either a single decision maker or multiple decision makers at the same stage of

a supply chain while we consider two independent decision makers at two stages in a DSC and provide a more in-depth study of the system performance under different decision scenarios. The dynamic PDT quotation and/or pricing problem has been studied by Duenyas and Hopp (1995), Plambeck (2004), Celik and Malgalaras (2008), and Feng et al. (2011).

Wang and Gerchak (2003) study capacity games in push and pull systems in which both the assembler and the component suppliers have to make capacity decisions before the uncertain demand is realized. As for the integration of pricing, lead time and capacity decisions, Boyaci and Ray (2003) examine the three decisions of two substitutable products for a centralized firm with price and time-sensitive demands. Ray et al. (2005) explore the optimal pricing and stocking decisions in a serial two-echelon decentralized supply chain with uncertain price-sensitive demand and random delivery time. Liu et al. (2007) consider pricing and leadtime decisions and show that the inefficiency in the DSC is strongly affected by market and operational factors. Those papers consider either pricing and capacity decisions or pricing and leadtime decisions while our paper studies the integration of pricing, lead time, and capacity decisions. Recently, Pekgün et al. (2008) study the capacity, pricing, and leadtime decisions for two decentralized departments in a firm and treat the marketing as the revenue center and the production as a cost center. The authors suggest a revenue-sharing contract with transfer price to coordinate the pricing and leadtime decisions. Different from this paper, we consider two individual players in a decentralized channel and each of them aims to maximize their own profit rates.

For the second stream, there is a significant body of literature on DSCs for channel coordination. Various incentive schemes have been proposed to achieve coordination, including two-part tariff by Zusman and Etgar (1981) and Moorthy (1987), buy-back contracts by Pasternack (1985), franchise arrangement by Lal (1990), quantity flexibility contracts by Tsay (1999), sales-rebate contracts by Taylor (2002), and revenue-sharing contract by Wang et al. (2004) and Cachon and Lariviere (2005). These papers focus on the coordination of pricing and replenishment quantity decisions in a DSC. Pekgün et al. (2008) consider the coordination of pricing and time decisions between manufacturing and marketing departments in a firm. Leng and Parlar (2009) show that a profit-sharing contract can coordinate the decision of lead time between a manufacturer and a retailer. Hu et al. (2011) analyze a coordination scheme with an internal price for the manufacturing and the sales departments. Xiao and Qi (2012) study a two-stage chain consisting of one supplier and one manufacturer. The manufacturer determines the sale price and quoted lead-time, and maintains a delivery time standard. Under their setting, the authors find that all-unit quantity discount mechanism can coordinate the decentralized channel for most cases. Different from these papers, we address the coordination of pricing, lead time, and capacity decisions. Interested readers can refer to a recent comprehensive review by Cachon (2003).

3. Decision models

We consider the DSC consisting of one supplier and one retailer. Upon receiving an order from the retailer, the supplier completes the finished product and delivers it to the retailer. Potential customers are sensitive to both price and PDT, which requires the retailer to offer an appropriate retail price and PDT. As independent decision makers, the supplier and the retailer make their own decisions in order to maximize their individual profit rates. The supplier controls her production facility. Naturally, the supplier should determine the production capacity (μ). Then, the supplier also chooses the wholesale price (p_s). In response to the

supplier's price and capacity decisions, the retailer determines the best retail price (p_r) and PDT (ℓ) to be quoted to customers so that the retailer's own profit rate is maximized. In making these decisions, the retailer will also be fully responsible for any late delivery penalties. Since the retail price together with the PDT will affect the level of demands and thus the supplier's operation cost and profit, the supplier must consider the reaction of the retailer when making her decisions. Thus, the two members of the supply chain interact in a Stackelberg game with the supplier as the leader and the retailer as the follower. We assume that all the information is the common knowledge for both players.

Suppose that the utility of individual customers follows a uniform distribution (Mendelson and Whang, 1990; Stidham, 1992). A customer will purchase only if his or her expected utility is greater than or equal to $p_r + c_w \ell$, where c_w is a standard waiting cost per unit of PDT. Customers play a role in the decision process in which their independent decisions collectively define the demand level corresponding to the quoted retail price and PDT. Then, we can derive the demand model suggested by Liu et al. (2007), i.e.,

$$\lambda(p_r, \ell) = \lambda_0 - \alpha p_r - \beta \ell, \quad (1)$$

where λ_0 is the size of the potential market, $\alpha > 0$ is the price sensitivity factor, and $\beta > 0$ is the PDT sensitivity factor. Further, we have $c_w = \beta/\alpha$.

Now, let us define the plays' objective functions. For the retailer, because the capacity of the supplier is finite and there may exist supply and demand uncertainties in the DSC, the order response time (ORT) may deviate from PDT, where the ORT is the time period from the epoch when an order is received to the epoch when the customer receives the order. One one hand, when late delivery occurs (the realized ORT is longer than the PDT), customers must be compensated. In practice, there are different ways to compensate affected customers. For instance, a discount or partial refund can be offered if a customer is willing to wait. Expedited delivery, e.g., airfreight instead of ocean shipping, can also be used without additional charge to the customer. Here, we assume that customers' additional waiting cost and inconvenience from late delivery will be covered exactly by the retailer with a generic penalty cost per order per unit time given by b , and thus the possible delivery delay and the associated compensation are known to the customer. On the other hand, when an early completion occurs (the realized ORT is shorter than the PDT), the firm may deliver the finished product to the customer or may hold it until the promised delivery time. In the former case, the firm may or may not incur an early delivery cost to compensate the customer for the early delivery. In the latter case, the firm incurs a finished-goods holding cost. In either case, we can model this scenario by charging the firm an early completion cost h per order per unit time. Let R_λ be the distribution function of the ORT for a given demand rate λ . Then, the retailer's operating cost is given by

$$C(\ell, R_\lambda) = h \int_0^\ell (\ell - t) dR_\lambda(t) + b \int_\ell^\infty (t - \ell) dR_\lambda(t), \quad (2)$$

where $C(\ell, R_\lambda)$ also represents the retailer's expected "leadtime cost". The retailer's optimization problem (ROP) is given by

$$\text{ROP: } \max_{p_r, \ell} \Pi_r(p_r, \ell, p_s, \mu) = [p_r - p_s - c_r - C(\ell, R_\lambda)] \lambda(p_r, \ell), \quad (3)$$

where c_r is the administrative expenses, arising from communication with customers, order handling and delivery. Note that Π_r represents the profit rate of the retailer that is defined as the multiplication of the profit per unit of demand and the demand rate. Further, μ affects the probability distribution of the order

response time, which has an effect on the retailer's operating cost. If the early delivery is allowed without penalty, the holding cost h can be set to zero and the above model is still completely valid and the corresponding results hold.

For the supplier, the profit structure is relatively simpler, and includes p_s as the unit wholesale price, c_s as variable cost per unit, and c_0 as cost per unit capacity. Thus, the supplier's optimization problem (SOP) is given by

$$\text{SOP: } \max_{\mu, p_s} \Pi_s(p_r, \ell, p_s, \mu) = (p_s - c_s) \lambda(p_r, \ell) - c_0 \mu, \quad (4)$$

where we assume that the capacity cost is a linear function in terms of the capacity like So and Song (1998) and Ray and Jewkes (2004). In fact, we extend this assumption to a convex function of μ and our main results can still hold. The list of notation is summarized in Table 1.

4. Stackelberg equilibrium

In this section, we characterize both the supplier's and retailer's optimal strategies. We first present the retailer's best response to the supplier's decisions. It is usually very difficult to derive the ORT distribution for a real supply chain. Even though for some simple systems we can derive the exact ORT distributions, they would be too complicated to be useful in our optimization. Since we focus on understanding the impact of the pricing and PDT decisions by the retailer and the capacity decision by the supplier on the realized delivery lead time through the demand rate, we assume that the supplier's production facility is modelled by an $M/M/1$ queue. The $M/M/1$ model has been often used in the supply chain literature, e.g., So and Song (1998) and So (2000). It provides a good approximation of light-tailed waiting times (Boyaci and Ray, 2003). From standard queueing results, we have

$$R_\lambda(t) = 1 - e^{-(\mu - \lambda)t}, \quad (5)$$

where $R_\lambda(t)$ represents the probability that the order response time is no longer than t . By substituting (5) into (2), we obtain

$$C(\ell, R_\lambda) = h\ell + \frac{(h+b)e^{-(\mu-\lambda)\ell} - h}{\mu - \lambda}.$$

Table 1
Notations.

Decision variables	
μ	The supplier's production capacity
p_s	The wholesale price per unit
p_r	The retail price per unit
ℓ	The promised delivery time quoted by the retailer
λ	The demand rate
Model parameters	
λ_0	The size of the potential market
α	The price sensitivity factor
β	The time sensitivity factor
c_w	The waiting cost per unit time
h	The inconvenience cost of early completing per order per unit time
b	The penalty cost per order per unit time
c_r	The administrative cost per unit time
c_0	Unit capacity cost
Other notations	
R_λ	The probability distribution of the order response time
$C(\ell, R_\lambda)$	The retailer's operating cost
Π_s	The supplier's profit rate
Π_r	The retailer's profit rate
*	Denotes optimality

4.1. Retailer's best response

Using (1) to write p_r in terms of λ and substituting (5) into (3), we can express the retailer's problem in terms of λ and ℓ , i.e.,

$$\max_{\lambda, \ell} \Pi_r(\lambda, \ell) = [p_r^{\max} - \lambda/\alpha - c_w \ell - C(\ell, R_\lambda) - c_r - p_s] \lambda, \quad (6)$$

where $p_r^{\max} = \lambda_0/\alpha$ is the maximum feasible retail price.

By (6), for any given λ , it is obvious that the retailer's best PDT is uniquely given by

$$\ell^* = \frac{1}{\mu - \lambda} \ln \frac{h+b}{h+c_w}. \quad (7)$$

Remark 1. From (7), if the retailer's penalty cost is less than customers' waiting cost, i.e., $b \leq c_w$, then it is optimal for the retailer to always quote a zero leadtime regardless of the demand rate. Thus, we assume $b > c_w$ to exclude this trivial solution.

We use a sequential solution procedure to solve (6); for a given λ , we first obtain $\ell^*(\lambda)$ as a function of λ ; we then substitute $\ell^*(\lambda)$ into Π_r and change the retailer's decision problem to a single-variable problem, i.e.,

$$\Pi_r(\lambda, \ell^*(\lambda)) = [p_r^{\max} - \lambda/\alpha - c_w \ell^*(\lambda) - C(\ell^*, R_\lambda) - c_r - p_s] \lambda. \quad (8)$$

Lemma 1. The retailer's profit rate (8) is concave in λ .

The proof of this lemma as well as all the other proofs in this paper are given in the appendix. The following proposition gives the retailer's best response to any given μ and p_s .

Proposition 4.1. For given μ and p_s , the retailer's best PDT and pricing strategies are given by (7) and

$$p_r^* = p_r^{\max} - \lambda/\alpha - c_w \ell^*, \quad (9)$$

where λ is uniquely determined by

$$\frac{\lambda_0 - 2\lambda}{\alpha} - p_s - c_r - \frac{\delta_0 \mu}{(\mu - \lambda)^2} = 0, \quad (10)$$

$$\text{and } \delta_0 = (h + c_w) \ln[(h + b)/(h + c_w)] + c_w.$$

4.2. Supplier's optimal strategy

Using (10) to write p_s in terms of λ and μ , we have

$$p_s = \frac{\lambda_0 - 2\lambda}{\alpha} - \frac{\delta_0 \mu}{(\mu - \lambda)^2} - c_r. \quad (11)$$

Then, substituting (11) into (4), we can express the supplier's problem in terms of λ and μ , i.e.,

$$\max_{\lambda, \mu} \Pi_s(\lambda, \mu) = \lambda \left[\frac{\lambda_0 - 2\lambda}{\alpha} - \frac{\delta_0 \mu}{(\mu - \lambda)^2} - c_r - c_s \right] - c_0 \mu. \quad (12)$$

Lemma 2. For a fixed λ , the supplier's best capacity choice is uniquely given by

$$\frac{\delta_0 \lambda (\mu + \lambda)}{(\mu - \lambda)^3} - c_0 = 0. \quad (13)$$

By (13), we can treat $\mu^*(\lambda)$ as a function of λ . Then, we can rewrite the supplier's decision problem to a single-variable problem, i.e.,

$$\Pi_s(\lambda, \mu^*(\lambda)) = \lambda \left[\frac{\lambda_0 - 2\lambda}{\alpha} - \frac{\delta_0 \mu^*(\lambda)}{(\mu^*(\lambda) - \lambda)^2} - c_r - c_s \right] - c_0 \mu^*(\lambda). \quad (14)$$

We have the following lemma.

Lemma 3. The supplier's profit rate (14) is unimodal in λ .

The following theorem characterizes the Stackelberg equilibrium of the game.

Theorem 1. The unique Stackelberg equilibrium for the decentralized problem is given by $(\ell^*, p_r^*, p_s^*, \mu^*)$, where ℓ^* from (7), p_r^* from (9), p_s^* from (11), and μ^* from (13). The optimal demand rate λ^* is the unique solution to

$$\lambda^* (\delta_1 - c_0 - 4\lambda^*/\alpha)^3 (\delta_1 + c_0 - 4\lambda^*/\alpha)^{-1} = c_0 \delta_0, \quad (15)$$

where $\delta_1 = \lambda_0/\alpha - c_r - c_s$.

The optimal profit rates of the retailer and the supplier are now given, respectively, by

$$\Pi_s^* = \lambda^* \left[\frac{2\lambda^*}{\alpha} - \frac{\delta_0 \mu^*}{(\mu^* - \lambda^*)^2} \right],$$

$$\Pi_r^* = \lambda^{*2} \left[\frac{1}{\alpha} + \frac{\delta_0}{(\mu^* - \lambda^*)^2} \right].$$

The following two corollaries specify the range of the demand rate and compare the supplier's and retailer's profit rates under different intervals.

Corollary 1. We have $\lambda_l < \lambda^* < \lambda_u$, where $\lambda_u = \alpha((\delta_1 - c_0)/4)$ and $\lambda_l = \alpha((5\delta_1 - c_0 + \sqrt{\delta_1^2 + c_0^2 + 14\delta_1 c_0})/24)$. λ_u and λ_l represent the upper and lower bound of the demand rate in the DSC, respectively.

Corollary 2. If $\lambda^* \in [\alpha((\delta_1 - c_0)/5), \lambda_u)$, $\Pi_s^* \geq \Pi_r^*$; if $\lambda^* \in (\lambda_l, \alpha((\delta_1 - c_0)/5))$, $\Pi_s^* < \Pi_r^*$.

Corollary 2 indicates that on one hand, because of the capacity investment, the supplier needs to have a sufficiently large market share to generate enough profit to cover the overhead cost. On the other hand, the retailer mainly tries to find the optimal trade-off between the revenue and the cost for the maximal profit. The retailer prefers not to have a high-congested/high-utilization system.

5. Impact of capacity decision

As a long term decision, the supplier usually makes the capacity decision based on historical demand information but ignores the impact of the capacity decision on her supply chain partner, which may eventually cause a negative effect on her own. To motivate the supplier to consider the appropriate capacity decision, we first investigate the profit gain by the integration of the capacity decision and comparing the current model (termed the CPL model) with the same model without a capacity decision (termed the PL model). Next, we demonstrate that the incorporation of the capacity decision may also significantly reduce the negative impact of double marginalization on the channel profit.

5.1. On players' profit

Here, we investigate the PL model with a pre-fixed capacity. In other words, the supplier does not treat the capacity as a decision. Therefore, we assume that the supplier only determines the wholesale price. To distinguish with the solutions to the CPL model, we use the symbol bar to indicate the corresponding solution to the PL model. For example, $\bar{\lambda}$ means the demand rate for the PL model.

Similar to **Theorem 1**, we can show that the unique Stackelberg equilibrium for the decentralized problem is given by $(\bar{\ell}^*, \bar{p}_r^*, \bar{p}_s^*)$, where $\bar{\ell}^*$ from (7), $\bar{p}_r^*$ from (9), $\bar{p}_s^*$ from (11). The optimal demand

Table 2
Impact of the capacity decision on the profit gap.

μ	Δ_s (%)	Δ_r (%)	μ	Δ_s (%)	Δ_r (%)
8	14.10	50.63	17	6.02	−11.83
9	7.59	32.58	18	8.62	−13.00
10	3.52	19.51	19	11.61	−13.91
11	1.19	10.01	20	14.96	−14.63
12	0.15	3.11	21	18.68	−15.22
13	0.07	−1.92	22	22.80	−15.69
14	0.75	−5.59	23	27.34	−16.09
15	2.03	−8.30	24	32.34	−16.42
16	3.80	−10.31	25	37.84	−16.70

Fig. 1. Impact of μ on p_r and p_s for the PL model.

rate $\bar{\lambda}^*$ is uniquely given by

$$\frac{\lambda_0 - 4\bar{\lambda}^*}{\alpha} - \frac{\delta_0 \mu (\mu + \bar{\lambda}^*)}{(\mu - \bar{\lambda}^*)^3} - c_r - c_s = 0. \quad (16)$$

The profit rates are given by

$$\bar{\Pi}_s^* = \bar{\lambda}^{*2} \left[\frac{2}{\alpha} + \frac{2\delta_0 \mu}{(\mu - \bar{\lambda}^*)^3} \right] - c_0 \mu,$$

$$\bar{\Pi}_r^* = \bar{\lambda}^{*2} \left[\frac{1}{\alpha} + \frac{\delta_0}{(\mu - \bar{\lambda}^*)^2} \right].$$

Proposition 5.1. $\bar{\lambda}^*$ is increasing in μ , $\bar{\ell}^*$ is decreasing in μ , $\bar{\Pi}_r^*$ is increasing in μ .

Proposition 5.1 indicates that when the supplier's capacity increases, the retailer offers a shorter PDT to expand its market share. As a result, the retailer's profit rate increases.

Next, we numerically explore the impact of the capacity decision on pricing decisions and profit rates. We try to address the following questions: How are the optimal pricing strategies affected by the capacity? Does the supplier's profit rate increase significantly by incorporating the capacity decision? Does the retailer also benefit from the incorporation of the capacity decision?

In the following numerical example, we set $\lambda_0 = 30$, $\alpha = 1$, $\beta = 4$, $c_s = 1.5$, $c_r = 0.8$, $h = 0.5$, $b = 6$, and $c_0 = 2$. Under this parameter setting, we first compute the optimal solution to the CPL model, i. e., $\mu^* = 12.58$, $\lambda^* = 5.85$, $\ell^* = 0.06$, $p_r^* = 23.93$, $p_s^* = 15.93$, $\bar{\Pi}_s^* = 59.25$, and $\bar{\Pi}_r^* = 38.49$. In Table 2, we compute the profit gaps of the supplier and the retailer between the CPL model and

the PL mode, where Δ_s and Δ_r are the profit gaps of the supplier and the retailer, respectively, where

$$\Delta_i = \frac{\bar{\Pi}_i^* - \bar{\Pi}_i}{\bar{\Pi}_i^*} \times 100\%$$

and $i = s, r$.

From Fig. 1, we find that for the PL model, both the retail price and the wholesale price are decreasing in μ . When the supplier's capacity is low, the retailer has to charge a high price and quote a long PDT to customers, which deteriorates competitiveness of the DSC. When the capacity becomes larger, the retailer can offer a lower price and a shorter PDT to attract more customers, which benefits both parties.

From Table 2, we observe the following phenomena: (i) the inappropriate capacity decision may cause the significant profit loss for the supplier. For example, when the capacity is wrongly set at 25, the supplier's profit could be decreased about 40%; (ii) the supplier's profit is more sensitive on the change of the capacity when the capacity is overestimated compared with the optimal value; (iii) the retailer's profit is more sensitive on the change of the capacity when the capacity is underestimated compared with the optimal value; (iv) when the pre-fixed capacity is smaller than the optimal capacity, the retailer's profit increases with the incorporation of capacity decision. However, when the capacity is larger than the optimal capacity, the retailer's profit decreases.

According to the above results, we believe that there does exist a strong incentive for the supplier as the leader to integrate the capacity as a decision in order to improve the profit. In particular, the supplier should avoid the overcapacity situation. Further, from the retailer's perspective, the larger the capacity is, the higher the retailer's profit is. Thus, the retailer only benefits from the capacity decision when the current capacity of the PL model is insufficient compared with the optimal capacity of the CPL model. However, the retailer suffers from the profit loss if the current capacity of the PL model is higher than the optimal one of the CPL model. In short, when the capacity is underestimated compared with the optimal value, the retailer's interest is aligned with that of the supplier in order to make the right capacity decision. However, when the capacity is overestimated compared with the optimal value, the retailer is not willing to participate in the coordination. Thus, it is necessary to design an appropriate incentive mechanism to make both parties pull in the same direction in order to ensure that the supply chain deliver goods and services quickly and cost-effectively, which will be addressed in Section 6.

5.2. On channel performance

It is well-known that the channel performance in the DSC is always deteriorated by double marginalization (e.g., Lee et al., 2000), which reflects the incentive conflict between the supplier and the retailer. To explore the impact of the incorporation of capacity decision on the channel performance, we first compare the performance of the DSC with that of a corresponding centralized supply chain (CSC). Second, we demonstrate the value of the incorporation of capacity decision for the reduction of double marginalization by numerical examples.

For the CPL model, let us consider the CSC with a single decision maker and the same cost structure as that of the DSC. The wholesale price, as an internal transfer in the CSC, is not considered. For convenience, we use a subscript c or d to indicate a centralized system or a decentralized system whenever necessary, for example, p_{rc} and ℓ_c are the retail price and PDT for the CSC.

Performing a similar linear transformation on the demand function, we have

$$p_{rc} = p_r^{max} - \lambda_c / \alpha - c_w \ell_c, \quad (17)$$

Fig. 2. Sensitivity analysis for α . Fig. 3. Sensitivity analysis for β . Fig. 4. Sensitivity analysis for λ_0 .
Table 3
 Profit rate gap of the CPL model

α	Δ	β	Δ	λ_0	Δ
0.4	0.360	0.05	0.367	12	0.363
0.6	0.378	0.2	0.369	16	0.366
0.8	0.397	0.35	0.361	18	0.361

and thus the system-wide expected profit rate is

$$\Pi_c(\lambda_c, \ell_c, \mu_c) = [p_r^{\max} - \lambda_c/\alpha - c_w \ell_c - c_r - c_s - C(\ell_c, R_{ic})] \lambda_c - c_0 \mu_c.$$

By performing the sequential optimization as Section 4.2, we can show that there exists a unique solution to maximizing $\Pi_c(\lambda_c, \ell_c, \mu_c)$. The solution can be found by the following procedure: compute λ_c^* from

$$\lambda_c^* (\delta_1 - c_0 - 2\lambda_c^*/\alpha)^2 = c_0 \delta_0, \quad (18)$$

then, compute μ_c^* by

$$\frac{\delta_0 \lambda_c^*}{(\mu_c^* - \lambda_c^*)^2} - c_0 = 0, \quad (19)$$

then compute p_{rc}^* by (17), and ℓ_c^* by

$$\ell_c^* = \frac{1}{\mu_c^* - \lambda_c^*} \ln \frac{h+b}{h+c_w},$$

finally the maximal profit rate is given by

$$\Pi_c^* = \lambda_c^* (3\lambda_c^*/\alpha - \delta_1 + c_0).$$

To examine the impact of decision inefficiency on the firms' decisions and performance, we compare the optimal decisions and the corresponding profit rates in the DSC with those in the CSC in the following theorem. Denote $\Pi_d^* = \Pi_s^* + \Pi_r^*$ as the maximal profit rate of the DSC.

Theorem 2. The following inequalities hold:

- (i) $\lambda_c^* > 2\lambda_d^*$;
- (ii) $\mu_c^* > \mu_d^*$. In particular, if $\delta_1 \leq 2c_0$, $\mu_c^* > 2\mu_d^*$;
- (iii) Denote $\rho = \lambda^*/\mu^*$. $\rho_c > \rho_d$;
- (iv) $\ell_c^* > \ell_d^*$ and $p_{rc}^* < p_{rd}^*$;
- (v) $\Pi_c^* > \Pi_d^*$ and $\Pi_c^* > 2\Pi_s^*$.

Theorem 2 shows that for a market with characteristics as defined here, the desirable strategy for a centralized supply chain is to use a lower price to capture a higher market share and a longer PDT to accommodate the higher demand rate while maintaining the same service level (the probability of on-time delivery). In a decentralized setting, independent players deviate from this strategy, resulting in a lower system-wide profit. Moreover, we find that if the decision inefficiency can be eliminated, then the market share can at least double in size. By choosing an appropriate capacity decision, the supplier can improve the system utilization and increase the profit, which strongly motivates the supplier to promote the channel coordination.

Next, we use the following example to illustrate the impact of the integration of capacity decision on the channel performance in terms of double marginalization. The base parameter setting is given by $\lambda_0 = 16$, $\alpha = 0.5$, $\beta = 0.2$, $c_s = 0.5$, $c_r = 0.2$, $h = 0.03$, $b = 0.3$, and $c_0 = 1.5$. We perform sensitivity analysis with respect to the potential market size (λ_0), price-sensitive factor (α), time-sensitive factor (β). We denote $\bar{\Delta}$ as the profit rate gap between the DSC and the CSC for the PL model and Δ as that for the CPL model, i.e., $\bar{\Delta} = (\Pi_c^* - \Pi_d^*)/\Pi_d^*$, $\Delta = (\Pi_c^* - \Pi_d^*)/\Pi_d^*$.

From Figs. 2–4, we observe that (i) $\bar{\Delta}$ is increasing in μ ; (ii) $\bar{\Delta}$ is increasing in α ; (iii) $\bar{\Delta}$ is robust with respect to the change of β ; (iv) $\bar{\Delta}$ is

decreasing in λ_0 . The profit rate gap of the CPL model is shown in Table 3 and the range of Δ is from 0.36 to 0.4. Comparing Δ with $\bar{\Delta}$, we find that the integration of capacity decision can significantly reduce the profit loss caused by double marginalization. In particular, when the market is very price sensitive ($\alpha = 0.8$), $\bar{\Delta}$ at $\mu = 28$ is 1.348 and about three times as large as $\Delta = 0.397$. The above finding indicates that in the DSC, before pursuing the channel coordination, the supplier should integrate the capacity decision into the decision process with her supply chain partner as a first step in order to gain more profit.

6. Coordination of the decentralized channel

The previous discussion shows that the integration of capacity decision may reduce the negative impact of the double marginalization on the channel profit. Table 3 shows that there may still exist a profit rate gap about 0.4 between the DSC and the CSC. Therefore, it is necessary to find an incentive mechanism to achieve channel coordination and a win-win outcome for both parties. Further, because of $\Pi_c^* > 2\Pi_s^*$, the supplier, as the leader in the DSC, should have a strong incentive to design the incentive mechanism to gain more profit. Different coordination mechanisms have been proposed in the supply chain literature for price and inventory decisions. In the following, we study and compare three well-known contracts: a two-part tariff, a revenue-sharing contract, and a franchise contract.

Let us start with the two-part tariff that is a price policy consisting of a fixed payment for a good or service payment as well as a per-unit charge (see Waldman, 2004). Under the two-part tariff, the supplier charges a wholesale price equal to c_s , and a fixed fee, F , which serves to allocate the profit between the supplier and the retailer. As pointed out by Zusman and Etgar (1981) and Moorthy (1987), the two-part tariff can achieve coordination by motivating the retailer to set the channel-profit maximizing price in a dyadic channel. However, we demonstrate that the two-part tariff cannot coordinate the DSC in our model. Since p_s is set to c_s , the supplier profit rate function can be written as $\Pi_s = -c_0\mu + F$. Because F is constant, the supplier will choose the minimal capacity setting in order to make the least investment. Thus, μ_c^* cannot be obtained. We find that the reason is that under the assumption by the above authors, the retailer can fully control the quantity sold by selecting the retail price. However, for our model, although the demand rate is directly controlled by both the retail price and the PDT determined by the retailer, the demand rate is restricted by the supplier's capacity since the demand rate cannot exceed the capacity available. In short, the two-part tariff works well when the supplier only needs to make the wholesale-price decision. If the supplier also needs to make other decisions, such as the capacity, the two-part tariff fails. From the retailer's perspective, since the retailer's profit rate is increasing in μ and the supplier sets the capacity at the minimal level, the two-part tariff causes the decrease of the retailer's profit rate. Therefore, the retailer will reject the two-part tariff.

Next, we consider a revenue-sharing contract by Cachon and Lariviere (2005). Under such a contract, the retailer pays a wholesale price per unit plus a percentage of the revenue ϕ , where p_s is set as $\phi(c_s + c_r) - c_r$, and ϕ is the retailer's share of revenue generated from each demand unit. Then, we can rewrite the supplier's profit rate function as

$$\Pi_s(\mu) = (p_s - c_s)\lambda - c_0\mu + (1 - \phi)\lambda \left(\frac{\lambda_0 - \lambda}{\alpha} - \frac{\delta_0}{\mu - \lambda} \right),$$

where λ is a function of μ and is given by

$$\frac{\lambda_0 - 2\lambda}{\alpha} - c_r - c_s - \frac{\delta_0\mu}{(\mu - \lambda)^2} = 0.$$

Since $\Pi_s(\mu)$ is concave in μ , the optimal capacity decision is given by the following first order condition:

$$(p_s - c_s) \frac{d\lambda}{d\mu} - c_0 + (1 - \phi) \left(\frac{\lambda_0 - 2\lambda}{\alpha} - \frac{\delta_0\mu}{(\mu - \lambda)^2} \right) \frac{d\lambda}{d\mu} + (1 - \phi)\lambda \frac{\delta_0}{(\mu - \lambda)^2} = 0.$$

By simplification, the above equation is rewritten as

$$(1 - \phi)\lambda \frac{\delta_0}{(\mu - \lambda)^2} - c_0 = 0. \quad (20)$$

To achieve μ_c^* , we require (20) identical to (19). Hence, ϕ must be equal to 0, which means that no revenue is allocated to the retailer. Clearly, the retailer is worse off and will not accept the contract. Some reader may propose a franchise contract combining revenue sharing with a two-part tariff. In other words, besides the revenue-sharing contract mentioned above, the supplier also pays a fixed fee F to the retailer. However, under the franchise contract, we can show that the channel coordination is still impossible. Based on the analysis, $\phi = 0$. Then, $\Pi_r = -(p_s + c_r)\lambda + F$. Clearly, the retailer prefers the smallest value of λ by choosing the retail price and PDT so that λ_c^* cannot be reached. As Cachon and Lariviere (2005) have pointed out that there exist several limitations for the revenue-sharing contract, we add one more. That is, even for the non-competitive chain, when the supplier needs to make multiple decisions, the revenue-sharing contract may not coordinate the DSC.

Finally, we consider one franchise contract that can coordinate the channel and achieve a win-win outcome. Based on the franchising arrangement described by Lal (1990), the supplier is the franchisor and the retailer is the franchisee. For the franchise contract, the supplier declares the wholesale price per unit, the percentage of revenue, and a contingent rebate paid to the retailer. In short, the contract combines revenue sharing with a contingent rebate given by the theorem below.

Theorem 3. Under a franchise contract with ϕ , where $\phi \in (0, 1)$,

$$p_s = -\frac{\phi\delta_0}{\mu} \left(\ln \frac{\lambda}{\mu - \lambda} + \frac{\mu}{\mu - \lambda} \right) + \phi(c_s + c_r) - c_r,$$

and a contingent rebate $\Theta(\lambda, \mu)$ given by

$$\Theta(\lambda, \mu) = -\frac{\phi\delta_0\lambda}{\mu} \ln \frac{\lambda}{\mu - \lambda}, \quad (21)$$

we have

- (i) The franchise contract achieves coordination;
- (ii) The resulting profits to the supplier and the retailer are

$$\begin{aligned} \Pi_{sc}^* &= (1 - \phi)\lambda_c^* \left[\frac{\lambda_c}{\alpha} + \frac{\delta_0\mu_c^*}{(\mu_c^* - \lambda_c^*)^2} \right] \\ &\quad - \frac{\delta_0\lambda_c^*(2\mu_c^* - \lambda_c^*)}{(\mu_c^* - \lambda_c^*)^2}, \\ \Pi_{rc}^* &= \phi\lambda_c^* \left[\frac{\lambda_c}{\alpha} + \frac{\delta_0\mu_c^*}{(\mu_c^* - \lambda_c^*)^2} \right]; \end{aligned}$$

- (iii) If $\phi \in (\Pi_c^* [(\lambda_c^{*2}/\alpha) + (\delta_0\lambda_c^*\mu_c^*/(\mu_c^* - \lambda_c^*)^2)]^{-1}, (\Pi_c^* - \Pi_s^*) [(\lambda_c^{*2}/\alpha) + (\delta_0\lambda_c^*\mu_c^*/(\mu_c^* - \lambda_c^*)^2)]^{-1})$, both parties are better off and a win-win outcome is achieved.

The key insight from Theorem 3 is that the contingent rebate depends on the revenue-sharing percentage, the retailer's cost parameters, λ , and μ . Note that λ integrates the pricing and PDT decisions at the retailer's side and μ represents the decision at the supplier's side. Moreover, we can rewrite (21) as

$$\Theta(\rho) = -\phi\delta_0\rho \ln \frac{\rho}{1 - \rho}.$$

We can show that $\theta(\rho)$ is strictly decreasing in ρ . In particular, for $\rho < 1/2$, $\theta(\rho) > 0$, which means that to induce the retailer to choose the first-best solution, the supplier has to offer a side payment to the retailer when the utilization is below 50%. The reason is that when the potential market size is relatively small ($\rho < 50\%$), the retailer has to charge a lower price in the DSC, which causes the decrease of the profit. To motivate the retailer to participate in the coordination, the supplier has to compensate the retailer's loss. For $\rho > 1/2$, $\theta(\rho) < 0$, which means that the supplier has to be compensated for the capacity investment when the utilization is above 50%. In short, the contingent rebate suggests an allocation mechanism of the profit gain in order to coordinate the DSC.

7. Conclusions and future research

This paper studies the integration of three critical decisions (capacity, price, and lead time) in the DSC with one supplier and one retailer. The dynamic decision process is modelled by the Stackelberg game, where the supplier is the leader who determines the capacity decision and the retailer is a follower who determines the price and lead time decision. We first obtain the optimal strategies for both the supplier and the retailer. Then, we explore the impact of the incorporation of capacity decision on the players' optimal strategies and profits. Comparing with the benchmark model without capacity decision, we find that when the capacity decision is underestimated in the benchmark model, the incorporation of capacity decision can yield a shorter PDT, a higher retail price, and a larger market share. As a result, the profits of both firms increase greatly. However, when the capacity decision is overestimated, only the supplier's profit is increased while the retailer's profit is decreased. Further, we demonstrate that the incorporation of capacity decision can significantly reduce the profit loss caused by double marginalization. Finally, to achieve channel coordination and a win-win outcome, we propose a franchise contract with revenue-sharing and contingent rebate to align the players' interests so that the decision inefficiency disappears.

There exist several extensions for the current model. First, we assume that the ORT distribution is given by the sojourn time distribution of the $M/M/1$ queue. It is natural to extend to a general ORT distribution. However, due to the complexity of the ORT distribution, the model may not be tractable. One possible way is to use the combination model by Liu et al. (2007) as an approximation. Second, it may be interesting to investigate the model in which the retailer acts a leader and the supplier acts as a follower. Can the incorporation of capacity decision still result in a significant benefit? Third, if the information of system parameters is not shared between the supplier and the retailer, how should the supplier and retailer make their decisions under the scenario with asymmetric information? According to the knowledge of micro-economic theory, the mechanism of signaling and screening could be applied.

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Appendix

Proof of Lemma 1. Differentiating $\Pi_r(\lambda, \ell^*(\lambda))$ with respect to λ twice, we have

$$\frac{d^2 \Pi_r(\lambda, \ell^*(\lambda))}{d\lambda^2} = -\frac{2\delta_0\mu}{(\mu-\lambda)^3}.$$

Clearly, Π_r is concave in λ since the second derivative is negative. $\square$

Proof of Proposition 4.1. By Lemma 1, because of the concavity, λ is uniquely determined by the first order condition, i.e., (10). Then, substituting λ into (7), we obtain ℓ^* . By (1), (9) is derived. $\square$

Proof of Lemma 2. For a fixed λ , by (12), we can show that Π_s is concave in μ . Thus, the supplier's best capacity decision is given by the first-order condition (13). $\square$

Proof of Lemma 3. Differentiating Π_s with respect to λ , we have

$$\begin{aligned} \frac{d\Pi_s(\lambda, \mu^*(\lambda))}{d\lambda} &= \frac{\lambda_0 - 4\lambda}{\alpha} - \frac{\delta_0\mu^*(\lambda)(\mu^*(\lambda) + \lambda)}{(\mu^*(\lambda) - \lambda)^3} - c_r - c_s, \\ &= \frac{\lambda_0 - 4\lambda}{\alpha} - \frac{c_0\mu^*(\lambda)}{\lambda} - c_r - c_s, \end{aligned} \quad (22)$$

where the second equality is obtained by substituting (13) into the first equality.

Denote $f(\lambda) = ((\lambda_0 - 4\lambda)/\alpha) - (c_0\mu^*(\lambda)/\lambda) - c_r - c_s$. Differentiating $f(\lambda)$ twice with respect to λ , we have

$$\begin{aligned} \frac{df(\lambda)}{d\lambda} &= -\frac{4}{\alpha} + c_0 \frac{\mu^{*2} - \lambda^2}{2\lambda^2(\mu^* + 2\lambda)}, \\ \frac{df^2(\lambda)}{d\lambda^2} &= -3c_0 \frac{\mu^{*4} + 4\mu^{*3}\lambda + 2\mu^{*2}\lambda^2 - 4\mu^*\lambda^3 - 3\lambda^4}{4\lambda^3(\mu^*(\lambda) + 2\lambda)^3}. \end{aligned}$$

Since $\mu^* > \lambda$, $df^2(\lambda)/d\lambda^2 < 0$, which indicates that $f(\lambda)$ is concave in λ .

By (22), we have $\lambda < \alpha(\delta_1 - c_0)/4$. If $\lambda \geq \alpha(\delta_1 - c_0)/4$, then the optimal demand rate becomes zero, which makes the problem trivial and uninteresting. Thus, we have $0 < \lambda < \alpha(\delta_1 - c_0)/4$. Moreover, because of the concavity of $f(\lambda)$, $\lim_{\lambda \rightarrow 0} f'(\lambda) > 0$, and $\lim_{\lambda \rightarrow \alpha(\delta_1 - c_0)/4} f'(\lambda) < 0$ (note that $\mu - \lambda$ when $\lambda \rightarrow \alpha(\delta_1 - c_0)/4$), we obtain that $\Pi_s(\lambda, \mu(\lambda))$ is unimodal in λ . $\square$

Proof of Theorem 1. By Lemma 3, there exists a unique global optimal λ^* that satisfies the following first order condition:

$$\frac{\lambda_0 - 4\lambda^*}{\alpha} - \frac{\delta_0\mu^*(\lambda^*)(\mu^*(\lambda^*) + \lambda^*)}{(\mu^*(\lambda^*) - \lambda^*)^3} - c_r - c_s = 0. \quad (23)$$

Substituting (13) into (23), we have $(\lambda_0 - 4\lambda^*)/\alpha - (c_0\mu^*(\lambda^*)/\lambda^*) - c_r - c_s = 0$. We can rewrite it as

$$\mu^*(\lambda^*) = \frac{\lambda}{c_0} \left(\frac{\lambda_0 - 4\lambda^*}{\alpha} - c_r - c_s \right). \quad (24)$$

Substituting (24) into (23), we obtain (15).

By Lemma 2, μ^* is uniquely determined by λ^* . By (7), since ℓ^* is increasing in λ , ℓ^* is uniquely determined by λ^* . Furthermore, by (11), p_s is decreasing in λ , hence the optimal p_s^* is uniquely determined by λ^* . Finally, by (9), p_r^* is also uniquely determined by λ^* . It is obvious that changing the solution $(p_r^*, \ell^*, p_s^*, \mu^*)$ by any firm unilaterally will reduce the profit rate of that firm. Thus, it is indeed the unique equilibrium of the game. $\square$

Proof of Corollary 1. By Lemma 3, we have $\lambda < \lambda_u$ to make sure that $\lambda^* > 0$.

To show that $\lambda_l < \lambda^*$, let us consider Π_s . To participate the game, it is clear that $\Pi_s^* > 0$, which requires

$$\frac{2\lambda^*}{\alpha} - \frac{\delta_0\mu^*}{(\mu^* - \lambda^*)^2} > 0 \Leftrightarrow \frac{2\lambda^*}{\alpha} - \frac{(\delta_1 - c_0 - 4\lambda^*/\alpha)(\delta_1 - 4\lambda^*/\alpha)}{\delta_1 + c_0 - 4\lambda^*/\alpha} > 0,$$

where the second inequality is obtained by substituting

$$\mu^* = \frac{\lambda^*}{c_0}(\delta_1 - 4\lambda^*/\alpha),$$

$$\mu^* - \lambda^* = \frac{\lambda^*}{c_0}(\delta_1 - c_0 - 4\lambda^*/\alpha).$$

By the above second inequality, $\lambda_l < \lambda^*$. $\square$

Proof of Corollary 2. By comparing Π_s^* and Π_r^* , we have

$$\begin{aligned} \Pi_s^* - \Pi_r^* &= \lambda^2 \left[\frac{1}{\alpha} - \frac{\delta_0}{(\mu^* - \lambda^*)^2} \left(\frac{\mu^*}{\lambda^*} + 1 \right) \right] \\ &\sim \frac{1}{\alpha} - \frac{\delta_0}{(\mu^* - \lambda^*)^2} \frac{\delta_1 + c_\mu - 4\lambda^*/\alpha}{c_\mu} \\ &= \frac{1}{\alpha} - \frac{c_\mu \delta_0}{\lambda^2 (\delta_1 - c_\mu - 4\lambda^*/\alpha)^2} \\ &= \frac{1}{\alpha} - \frac{\delta_1 - c_\mu - 4\lambda^*/\alpha}{\lambda^*}. \quad \square \end{aligned}$$

Proof of Proposition 5.1. Differentiating (16), (7), and $\bar{\Pi}_r^*$ with respect to μ , we obtain

$$\begin{aligned} \frac{d\bar{\lambda}^*}{d\mu} &= \frac{\alpha\delta_0(\mu^2 + \bar{\lambda}^{*2} + 4\mu\bar{\lambda}^*)}{4(\mu - \bar{\lambda}^*)^4 + \alpha\delta_0\mu(4\mu + 2\bar{\lambda}^*)}, \\ \frac{d\bar{\ell}^*}{d\mu} &= -\delta_0 \frac{1 - d\bar{\lambda}^*/d\mu}{(\mu - \bar{\lambda}^*)^2}, \\ \frac{d\bar{\Pi}_r^*}{d\mu} &= 2\bar{\lambda}^* \left[\frac{1}{\alpha} + \frac{\delta_0\mu}{(\mu - \bar{\lambda}^*)^3} \right] \frac{d\bar{\lambda}^*}{d\mu} - \frac{2\delta_0\bar{\lambda}^{*2}}{(\mu - \bar{\lambda}^*)^3}. \end{aligned}$$

It is clear that $d\bar{\lambda}^*/d\mu > 0$. Substituting $d\bar{\lambda}^*/d\mu$ into the expressions of $d\bar{\ell}^*/d\mu$ and $d\bar{\Pi}_r^*/d\mu$, we have $d\bar{\ell}^*/d\mu < 0$, and $d\bar{\Pi}_r^*/d\mu > 0$. $\square$

Proof of Theorem 2. For (i), by (15) and (18), the relationship between λ_c^* and λ_d^* must be either $\lambda_c^* > 2\lambda_d^*$ or $\lambda_c^* < \lambda_d^*$. Now, we show that $\lambda_c^* < \lambda_d^*$ is impossible. The logic is as follows. To guarantee that $\Pi_c^* > 0$, it is clear that $\lambda_c^* > (\alpha(\delta_1 - c_0))/3$. If $\lambda_c^* < \lambda_d^*$, by Corollary 1, we have $\lambda_c^* < \alpha(\delta_1 - c_0)/4$, which makes $\Pi_c^* < 0$. Thus, $\lambda_c^* > 2\lambda_d^*$.

For (ii), we prove it by contradiction. We have

$$\mu_d^* = \frac{\lambda_d^*}{c_0} \left(\frac{\lambda_0 - 4\lambda_d^*}{\alpha} - c_r - c_s \right), \quad (25)$$

$$\mu_c^* = \frac{\lambda_c^*}{c_0} \left(\frac{\lambda_0 - 2\lambda_c^*}{\alpha} - c_r - c_s \right). \quad (26)$$

By (25), $\mu_d^* \leq \alpha\delta_1^2/(16c_0)$. Suppose that $\mu_c^* < \mu_d^*$. Then, $\mu_c^* < \alpha\delta_1^2/(16c_0)$ for all the possible values of λ_c^* . Denote $f(x) = x(\delta_1 - 2x/\alpha)/c_0$. We can show that $f(\alpha(\delta_1 - c_0)/3) > \alpha\delta_1^2/(16c_0)$. To guarantee a positive Π_c^* , we require $\lambda_c^* > \alpha(\delta_1 - c_0)/3$. Because of the continuity of $f(x)$, we can always find a λ_c^* so that $\mu_c^* > \alpha\delta_1^2/(16c_0)$, which contradicts with the assumption. Thus, $\mu_c^* > \mu_d^*$. Further, if $\delta_1 \leq 2c_0$, by (25) and (26), we can write λ_d^* and λ_c^* as a function of μ_d^* and μ_c^* , respectively. That is

$$\lambda_c^* = \frac{\alpha\delta_1 - \sqrt{\alpha^2\delta_1^2 - 8\alpha c_0\mu_c^*}}{4},$$

$$\lambda_d^* = \frac{\alpha\delta_1 - \sqrt{\alpha^2\delta_1^2 - 16\alpha c_0\mu_d^*}}{8}.$$

Since $\lambda_c^* > 2\lambda_d^*$, it is obvious that $2\mu_d^* < \mu_c^*$.

For (iii), based on the first order conditions, we have

$$\begin{aligned} \delta_1 - 4\lambda_d^*/\alpha - c_0\mu_d^*/\lambda_d^* &= 0, \\ \delta_1 - 2\lambda_c^*/\alpha - c_0\mu_c^*/\lambda_c^* &= 0. \end{aligned}$$

Since $\lambda_c^* > 2\lambda_d^*$, it is clear that $\mu_c^*/\lambda_c^* < \mu_d^*/\lambda_d^*$. By definition of ρ , we have $\rho_c > \rho_d$.

For (iv), we have

$$\begin{aligned} \mu_c^* - \lambda_c^* &= \frac{\lambda_c^*}{c_0}(\delta_1 - c_0 - 2\lambda_c^*/\alpha) = \frac{\delta_0}{\delta_1 - c_0 - 2\lambda_c^*/\alpha} < \frac{\delta_0}{\delta_1 - c_0 - 4\lambda_d^*/\alpha} \\ &< \frac{\delta_0(\delta_1 - c_0 + 4\lambda_d^*/\alpha)}{(\delta_1 - c_0 - 4\lambda_d^*/\alpha)^2} = \mu_d^* - \lambda_d^*. \end{aligned}$$

By (7), since $\mu_d^* - \lambda_d^* > \mu_c^* - \lambda_c^*$, $\ell_c^* > \ell_d^*$. By (1), since $\lambda_c^* > 2\lambda_d^*$ and $\ell_c^* > \ell_d^*$, $p_c^* < p_d^*$.

For (v), it is clear that $\Pi_c^* > \Pi_d^*$ since the CSC simultaneously optimizes all the decisions. By comparing Π_c^* with Π_s^* , we have

$$\begin{aligned} \Pi_c^* &= \frac{\lambda_c^{*2}}{\alpha} - \frac{\delta_0\lambda_c^*}{\mu_c^* - \lambda_c^*}, \\ \Pi_s^* &= \frac{2\lambda_d^{*2}}{\alpha} - \frac{\delta_0\mu_d^*\lambda_d^*}{(\mu_d^* - \lambda_d^*)^2}, \\ \Pi_c^* - 2\Pi_s^* &= \frac{\lambda_c^{*2} - 4\lambda_d^{*2}}{\alpha} + \frac{2\delta_0\mu_d^*\lambda_d^*}{(\mu_d^* - \lambda_d^*)^2} - \frac{\delta_0\lambda_c^*}{\mu_c^* - \lambda_c^*} > \frac{2\delta_0\mu_d^*\lambda_d^*}{(\mu_d^* - \lambda_d^*)^2} - \frac{\delta_0\lambda_c^*}{\mu_c^* - \lambda_c^*} \\ &= \frac{2c_0\mu_d^*(\mu_d^* - \lambda_d^*)}{\mu_d^* + \lambda_d^*} - c_0(\mu_c^* - \lambda_c^*). \end{aligned}$$

Since $\mu_d^* - \lambda_d^* > \mu_c^* - \lambda_c^*$ and $\mu_d^* > \lambda_d^*$, we have $\Pi_c^* - 2\Pi_s^* > 0$. $\square$

Proof of Theorem 3. For (i), to achieve the coordination, the key is how to induce the retailer to make the first-best decision. under this franchise contract, the retailer's profit rate function is given by

$$\Pi_r(p_r, \ell) = \{ \phi[p_r - C(\ell, R_\lambda)] - p_s - c_r \} \lambda(p_r, \ell) + \theta(\lambda, \mu).$$

By Proposition 4.1, the optimal demand rate is given by

$$\phi \left[\frac{\lambda_0 - 2\lambda}{\alpha} - \frac{\delta_0\mu}{(\mu - \lambda)^2} \right] - p_s - c_r + \frac{\partial\theta(\lambda, \mu)}{\partial\lambda} = 0.$$

To make λ equal to λ_c^* , we require that $p_s = \partial\theta(\lambda, \mu)/\partial\lambda + \phi(c_s + c_r) - c_r$. Substituting p_s into the supplier's profit rate function, we have

$$\Pi_s(\mu) = \lambda \left[\frac{\partial\theta(\lambda, \mu)}{\partial\lambda} + \phi(c_s + c_r) - c_s - c_r \right] + (1 - \phi)\lambda[p_r - C(\ell, R_\lambda)] - c_0\mu - \theta(\lambda, \mu).$$

To make μ equal to μ_c^* , we find the expression of $\theta(\lambda, \mu)$ given by (21).

For (ii), it is straightforward to show the results by substituting the optimal solutions into the profit rate functions.

For (iii), to achieve a win-win outcome, we require that $\Pi_{sc}^* > \Pi_s^*$ and $\Pi_{rc}^* > \Pi_r^*$. From (ii), the results immediately follow. $\square$

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