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Cost benefits of postponing time-based maintenance under lifetime distribution uncertainty

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ABSTRACT

We consider the problem of scheduling time-based preventive maintenance under uncertainty in the lifetime distribution of a unit, with the understanding that every time a maintenance action is carried out, additional information on the lifetime distribution becomes available. Under such circumstances, typically either point estimates for the unknown parameters are used, or expected costs are minimized taking the uncertainty in the parameters into account. Both approaches, however, ignore that the uncertainty is reduced much faster if preventive maintenance actions are postponed. Although this initially leads to higher costs due to a higher risk of breakdowns, the obtained additional information can be exploited thereafter as it enables better maintenance decisions going forward. We assess the long-term benefits of initially postponing preventive maintenance, and perform a numerical study to identify under what circumstances these benefits are largest. This study is the first to recognize that the choice of a maintenance strategy influences the information that becomes available, and aims to initiate follow-up research in the area of maintenance planning.

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1. Introduction

In many industries, a substantial part of the total costs and the total workforce is related to maintenance, indicating the importance of this area. As an illustration, over a quarter of the total workforce in the process industry, and up to 30% in the chemical industry, deal with maintenance operations [33,43]. Moreover, the amount of money spent on maintaining engineering and infrastructures is continuously increasing [42].

Maintenance involves both repairing failed systems (corrective maintenance) and preventing breakdowns (preventive maintenance). Preventive maintenance is generally preferred since breakdowns occur at unexpected moments and can have severe consequences. Two types of preventive maintenance can be distinguished: time-based maintenance and condition-based maintenance. Time-based maintenance is easier to plan, while condition-based maintenance, on the other hand, leads to more effectively planned maintenance actions because it takes the condition of the maintainable unit into account. Drawbacks of the latter type are that condition monitoring needs to be technically feasible and that monitoring equipment is required. In this paper we consider time-based preventive maintenance. Other recent studies on time-based maintenance include Chang [8],

Cheng et al. [9], Faccio et al. [16], Gustavsson et al. [19], and Xia et al. [45]. We refer to Ahmad and Kamaruddin [2] for an overview on condition-based maintenance.

A common assumption in many models and studies on time-based preventive maintenance planning is that the lifetime distribution, i.e. the distribution of the time until breakdown, is known. For example, the basic age-based maintenance model of Barlow and Hunter [4], which is included in many textbooks, minimizes the mean cost per unit time given a known lifetime distribution. Other examples of studies that assume a known lifetime distribution are Jiang et al. [23], Kijima et al. [25], Makis and Jardine [31], and Yeh and Lo [46].

In practice, however, it is usually not the case that the lifetime distribution is known with certainty. Reasons include incorrectly recorded or unrecorded failure codes, a lack of adequate descriptions of what was wrong and what repairs were performed [15,32], heavily right-censored data because of preventive maintenance in the past [6], and an insufficient amount of data to determine accurate estimates for model parameters [13]. In this paper, we explicitly take into account uncertainty in the lifetime distribution when studying the problem of scheduling time-based preventive maintenance.

Existing studies on this problem typically consider strategies that minimize the expected costs based on current information and update these strategies when more data becomes available [5]. For example, Gertsbakh [17] describes how the optimal maintenance age should be determined if the parameter

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Nomenclature

Roman symbols

c	normalized cost of a preventive maintenance action
k	shape parameter of the Weibull distribution
λ	scale parameter of the Weibull distribution
λ_i	scale parameter of unit type i , $i = 1$ (weak), 2 (strong)
p	initial probability that the unit is 'weak'
$\hat{p}$	Bayesian estimate of the probability that the unit is 'weak'
S	length of the lifespan of a unit
$f(t; \lambda, k)$	density function of the Weibull distribution with parameters λ and k

$F(t; \lambda, k)$	distribution function of the Weibull distribution with parameters λ and k
L_j	likelihood that $\lambda = \lambda_j$, $j = 1, 2$
t_i	length of duration i
z_i	type of duration i , $z_i = 1$ (event), $z_i = 0$ (censored)
t	preventive maintenance age
$\eta(T; \lambda, k)$	cost rate as a function of t and Weibull parameters λ and k
$\eta^E(T; \hat{p})$	expected cost rate as a function of t and $\hat{p}$
T_i	optimal maintenance age if the unit is of type i , $i = 1$ (weak), 2 (strong)
T^*	optimal maintenance age for the myopic policy
π	threshold level of the threshold policy
π^*	optimal threshold level

uncertainty is modelled by a discrete distribution, De Jonge et al. [14] consider the effect of uncertainty in the scale parameter of the lifetime distribution on the optimal maintenance age, and Silver and Fiechter [40] consider a unit that either fails at time 1 or time 2 with respective probabilities that are updated in a Bayesian way. They extend this setting to one with a discrete lifetime distribution, and they consider a heuristic maintenance policy that is based on the current estimated probabilities [41].

Another study using Bayesian updating is Mazzuchi [34], which determines the optimal age-based maintenance policy when the parameters of the Weibull lifetime distribution are uncertain. Juang and Anderson [24] extend this model with five possible maintenance actions and random failure costs. Moreover, Laggoun et al. [28] use the bootstrap technique to obtain distributions that model the uncertainties in the parameters of the Weibull distribution, and Coolen-Schrijner and Coolen [11,12] consider an adaptive maintenance strategy that is based on a nonparametric estimator of the lifetime distribution. This adaptive maintenance strategy determines, at the start of each maintenance cycle, a maintenance age based on the data available at that moment in time.

All the abovementioned research ignores that the choice of a maintenance age influences the information on the lifetime distribution that becomes available. That is why we propose to postpone preventive maintenance actions at the start of the lifespan of a unit. Although this will increase the expected costs during this first phase, it also leads to reduced uncertainty in the lifetime distribution for future decisions. As a consequence, preventive maintenance can be scheduled more effectively during the remaining lifespan of a unit. The aim of this paper is to investigate the potential cost benefits over the entire lifespan of a unit.

Before we investigate these potential cost benefits, we first analyze a more traditional so-called myopic policy, which determines an optimal maintenance age based on the information that is currently available. It turns out that under particular circumstances this myopic policy selects a very conservative, i.e. small, maintenance age so that little information on the lifetime distribution becomes available. Based on this observation we propose to use a threshold policy, which postpones preventive maintenance actions during the initial phase of the lifespan of a unit. Using a numerical study we show that, indeed, the benefits of postponing preventive maintenance actions can be substantial.

The remainder of this paper is organized as follows. In Section 2 we introduce the model we are considering. Next, in Section 3 we describe the myopic policy for scheduling preventive maintenance actions and we analyze its performance. In Section 4 we introduce the threshold policy and we investigate its potential cost benefits using a numerical study. We end with conclusions and directions for future research in Section 5.

2. Model formulation

We consider preventive maintenance planning for a single maintainable unit with a finite lifespan, where the lifetime distribution of the unit is not known with certainty. Instead, the unit is either 'weak' or 'strong' with corresponding lifetime distributions. When the unit breaks down, corrective maintenance has to be performed. A preventive maintenance action, on the other hand, can be scheduled at any moment in time at a lower cost. However, if preventive maintenance is performed, then the lifetime, i.e. the time until breakdown, is not observed and less information on the type of the unit is obtained. The main contribution of this paper is that we explicitly take this information aspect into account in the planning of preventive maintenance actions. In the remainder of this section the assumptions of our model are explained in more detail.

The lifetime of the unit is assumed to follow a Weibull distribution. This is the most commonly used distribution to model lifetimes of industrial machines and components and provides a good description for many types of lifetimes. The Weibull distribution has a shape parameter k and a scale parameter λ . We assume that the shape parameter k is known and that there is uncertainty in the scale parameter λ . This is a realistic assumption since the value of the shape parameter can often be estimated very accurately based on the failure mode of a unit [1]. It is, on the other hand, very likely that there is substantial uncertainty in the value of the scale parameter [48]. Other studies that consider this setting include Canavos and Tsokos [7], De Jonge et al. [14], Kwon [27], and Papadopoulos and Tsokos [37].

Because this is the first study on the benefits of postponing preventive maintenance actions, we consider a simple setting with two unit types. With probability p , the unit is 'weak' and has a scale parameter with value $\lambda=1$; and with probability $1-p$, the unit is 'strong' and has a scale parameter with value $\lambda=2$. The value of k is the same for both unit types. In practice, such a setting with two unit types occurs if components are selected from a stockpile that consists of weak and strong components [38], if a population of specific items comes from different suppliers or is produced by different manufacturing lines with varying quality [22], if a dealer sells items under his own brand label after buying them from two different manufacturers [35], or after a possible poor installation of a new component [38].

Both corrective and preventive maintenance are assumed to make the unit as-good-as-new. Furthermore, preventive maintenance is assumed to be less expensive than corrective maintenance because preventive maintenance can be planned in advance whereas breakdowns occur unexpectedly and are likely to have severe consequences. Both assumptions are also made in the

original paper on time-based maintenance of Barlow and Hunter [4], and in many subsequent studies, see, for example [14,22,39,47]. We normalize the cost of a corrective maintenance action to 1, and we denote the cost of a preventive maintenance action by $c < 1$. The cost of corrective maintenance comprises costs for all direct and indirect consequences of a breakdown. Because the cost of preventive maintenance is lower than the cost of corrective maintenance, performing preventive maintenance might be beneficial if it is scheduled effectively.

In this paper we assume that the unit has a finite lifespan with length S . This is a natural assumption since machines and components are often designed or purchased to function for a fixed time period [9,18,44]. Furthermore, the trade-off we consider is only relevant if the considered time horizon is finite, since initial costs of ascertaining the true unit type are negligible over an infinite time horizon, irrespective of the maintenance strategy used.

We conclude this section with an explanation of the updating process of the estimated probability $\hat{p}$ that the unit is weak ($\lambda = 1$). This probability is updated in a Bayesian way every time a maintenance action is carried out. Two types of durations are distinguished; the time until a breakdown is called an event duration and the time until a preventive maintenance action is called a censored duration. We differentiate between these two types of durations since a censored duration with length t implies that the lifetime is at least t , whereas an event duration with length t implies that the lifetime is exactly t . If we have observed n durations, then for every $i = 1, \dots, n$, the time length of duration i is denoted by t_i , and we let $z_i = 1$ if duration i is an event duration, and $z_i = 0$ if it is a censored duration. The Bayesian prior probability that the unit is weak is the initial probability p . We let $f(t; \lambda, k)$ denote the density function and $F(t; \lambda, k)$ the distribution function of the Weibull distribution with parameters λ and k . The likelihood L_j that $\lambda = \lambda_j$, $j = 1, 2$, with $\lambda_1 = 1$ and $\lambda_2 = 2$, is then given by

$$L_j = \prod_{i=1}^n [f(t_i; \lambda_j, k)]^{z_i} [1 - F(t_i; \lambda_j, k)]^{1-z_i}, \quad j = 1, 2,$$

from which it follows that the Bayesian posterior probability that the unit is weak equals

$$\hat{p} = \frac{pL_1}{pL_1 + (1-p)L_2}.$$

This Bayesian posterior probability serves as the estimated probability $\hat{p}$ that the unit is weak. We refer to Hamada et al. [20] for a more detailed explanation of Bayesian updating.

3. Myopic policy

Given the uncertainty in the lifetime distribution, a natural strategy is to select the maintenance age that minimizes the long-run cost rate under the current estimated probability $\hat{p}$. This maintenance age is updated every time a new duration is observed. We refer to this policy as the myopic policy since it ignores, when determining a maintenance age, that the estimated probability $\hat{p}$ will be updated in the future. A similar myopic approach is used by various authors, as outlined in the introduction. We note that this approach does not take into account that the time horizon is finite. However, it is used because determining optimal maintenance strategies for finite time horizons, which is not the essence of our study, is complex [10,21,30,36]. Furthermore, the error of minimizing the cost rate is acceptable if the length S of the time horizon is large compared to the mean time until a breakdown [29].

In this section we analyze the performance of the myopic policy and show under what circumstances postponing preventive

maintenance is most promising compared to the myopic policy. First, however, we explain how the optimal maintenance age under the myopic policy is determined based on the expected cost rate.

If the parameters λ and k of the Weibull lifetime distribution are known with certainty, it is well known that the cost rate (as a function of the maintenance age T) equals

$$\eta(T; \lambda, k) = \frac{F(T; \lambda, k) + c(1 - F(T; \lambda, k))}{\int_0^T (1 - F(x; \lambda, k)) dx}. \quad (1)$$

In our setting where $\lambda = \lambda_1$ with estimated probability $\hat{p}$, and where $\lambda = \lambda_2$ with estimated probability $1 - \hat{p}$, the expected cost rate (again as a function of the maintenance age T) is equal to

$$\eta^E(T; \hat{p}) = \hat{p}\eta(T; \lambda_1, k) + (1 - \hat{p})\eta(T; \lambda_2, k).$$

After each maintenance action, the estimated probability $\hat{p}$ is updated, and the myopic policy selects the maintenance age T^* that minimizes the expected cost rate $\eta^E(T; \hat{p})$ under the then current estimated probability $\hat{p}$ that the unit is weak.

3.1. Bounds on the expected cost rate

To assess the performance of the myopic policy, we compare it with a lower as well as an upper bound for the expected cost rate.

An upper bound is obtained by assuming that the prior probability p is not updated as more durations become available. This policy might be used in practice if gathering and maintaining the required information is too expensive, or even practically impossible. The expected cost rate of this no-update policy will be larger than that of the myopic policy, since the preventive maintenance decisions of the latter policy will be better in the long run because it updates the estimated probability $\hat{p}$.

A lower bound, on the other hand, is obtained by assuming that we know with certainty whether the unit is weak or strong. The optimal preventive maintenance age under this perfect information can be obtained by minimizing $\eta(T; \lambda, k)$ in (1) for $\lambda = 1$ or $\lambda = 2$, respectively. The resulting expected cost rate is a lower bound for the expected cost rate of any policy, including the optimal one. Therefore, if the cost difference between the myopic policy and the policy with perfect information is small, then the potential benefit of postponing preventive maintenance during the initial phase of the lifespan of a unit is also small. A large difference, on the other hand, indicates that a substantial improvement over the myopic policy might be achieved, for example by postponing preventive maintenance.

3.2. Performance myopic policy

We evaluate the performance of the myopic policy by comparing its expected cost rate with the bounds discussed in Section 3.1 for various values of the parameters S , k , p , and c . For each parameter, we study the effect of changing its value while keeping the other parameter values constant at $S = 40$, $k = 10$, $p = 0.5$, and $c = 0.1$, respectively. The expected cost rates of the myopic policy and of its bounds are computed using simulations with 10,000 runs, where for each run the costs consist of all preventive and corrective maintenance costs during the simulated lifespan, with additional cost rate $\eta^E(T; \hat{p})$ for the time period between the last maintenance action and the end of the lifespan. These additional costs are incurred to mitigate cut-off effects of the finite lifespan on the results. In addition, common random numbers are used for the different policies and for different parameter settings.

Fig. 1(a) shows the expected cost rate of the myopic policy and its bounds for a varying length S of the lifespan. The cost rates of the no-update policy and the policy under perfect information do not depend on S . If the lifespan of the unit is short, only a few

Fig. 1. Cost rate of the myopic policy and upper and lower bounds on this cost rate.

maintenance actions will be performed, so that the myopic policy is unable to learn whether the unit is weak or strong. As a result, the performance of the myopic policy is almost equal to that of the no-update policy. If the lifespan becomes longer, then there is more time to learn the true unit type, resulting in an improvement of the myopic policy over the no-update policy. Ultimately, if S becomes very large, the expected cost rate will converge to that under perfect information, since in the long run the myopic policy will learn the true type of the unit, and the additional costs of initial suboptimal decisions are negligible over a long time horizon.

The effects of the shape parameter k are shown in Fig. 1(b). We only consider shape parameters $k > 1$ because $k < 1$ corresponds to a decreasing failure rate, implying that preventive maintenance will never be beneficial. If, in addition, k is small, the Weibull distribution has a high variance, which makes preventive maintenance planning difficult [3]. Uncertainty in the scale parameter is a second source of variation in the lifetime distribution, but the effect of this additional variation is smaller if the individual Weibull distributions already have a large variance. Thus, a small k results in a small difference between the performance of the no-update policy and the performance under perfect information. If k increases, the variances of the individual lifetime distributions become smaller, and maintenance can be scheduled more effectively if the unit type is known. Therefore, the benefit of knowing the unit type becomes greater, resulting in an increasing difference between the two bounds. A second effect of an increasing k is that the myopic policy selects a maintenance age that is a more likely to prevent failures from both unit types. The consequence, however, is that only censored durations that provide little information on the underlying unit type are observed, and that the uncertainty in the lifetime distribution remains. The performance of the myopic policy therefore converges to that of the no-update policy as the shape parameter k increases.

Fig. 1(c) shows the effects of the probability p that the unit is weak. All policies are obviously equivalent if there is no uncertainty in the lifetime distribution, i.e. if $p=0$ or $p=1$. For $0 < p < 1$,

a weak performance of the no-update policy, compared with that under perfect information, is mainly caused by the fact that the no-update policy schedules preventive maintenance too early, even if the unit is actually strong. This is most pronounced if it is likely that the unit is strong, i.e. if p is small, resulting in a large difference between the bounds for such small values of p . Another notable observation is that the performance of the myopic policy virtually coincides with that of the no-update policy for large values of p . Similar as in the case that k is large, the myopic policy selects a very conservative, i.e. low, preventive maintenance age in such cases, so that there remains uncertainty in the lifetime distribution.

The effects of the preventive maintenance cost c are shown in Fig. 1(d). The performances of all policies are again equal in the two extreme cases $c=0$ and $c=1$, and there is a clear difference between the performances of the two bounds in between. If c is small, the myopic policy selects a very low preventive maintenance age. Consequently, only little information on the underlying unit type is obtained, and the performance is almost equal to that of the no-update policy. If c becomes larger, then a higher maintenance age is selected leading to longer censored durations and more event durations. In fact, if $c=1$, it is optimal not to perform preventive maintenance at all, so that only event durations are observed. In these cases the myopic policy is able to identify the type of the unit quickly, explaining why its performance approaches that under perfect information.

Summarizing, the performance of the myopic policy strongly depends on the various parameter values. In cases where there is a large gap between the performance of the myopic policy and the performance under perfect information, there is a large potential for improvement by postponing preventive maintenance. This is the case if S is small, if k is large, or if c is small. The probability p has a smaller influence on the potential improvement of postponing preventive maintenance, as long as it is not extremely small or large. In the next section we use these insights to select of number of cases for which we will assess the performance of the proposed threshold policy compared with the myopic policy.

Table 1
Experiment results.

Exp	S	k	p	c	NP	MP	TP	PI	MP–TP (%)	π^*
1	20	5	0.25	0.05	0.1146	0.1063	0.1049	0.0928	1.30	0.25
2	20	5	0.25	0.10	0.1967	0.1756	0.1734	0.1603	1.30	0.30
3	20	5	0.25	0.20	0.3335	0.2888	0.2861	0.2734	0.95	0.30
4	20	5	0.50	0.05	0.1299	0.1281	0.1281	0.1114	0.00	0.00
5	20	5	0.50	0.10	0.2248	0.2146	0.2133	0.1923	0.58	0.40
6	20	5	0.50	0.20	0.3812	0.3507	0.3481	0.3283	0.76	0.45
7	20	5	0.75	0.05	0.1406	0.1403	0.1403	0.1299	0.00	0.00
8	20	5	0.75	0.10	0.2425	0.2410	0.2410	0.2243	0.00	0.00
9	20	5	0.75	0.20	0.4138	0.4034	0.4027	0.3831	0.17	0.35
10	20	10	0.25	0.05	0.0808	0.0794	0.0695	0.0580	12.47	0.35
11	20	10	0.25	0.10	0.1504	0.1381	0.1184	0.1070	14.31	0.35
12	20	10	0.25	0.20	0.2765	0.2227	0.2081	0.1989	6.52	0.55
13	20	10	0.50	0.05	0.0864	0.0862	0.0862	0.0695	0.00	0.00
14	20	10	0.50	0.10	0.1610	0.1597	0.1503	0.1291	5.89	0.60
15	20	10	0.50	0.20	0.2970	0.2846	0.2565	0.2386	9.89	0.55
16	20	10	0.75	0.05	0.0900	0.0900	0.0900	0.0811	0.00	0.00
17	20	10	0.75	0.10	0.1672	0.1673	0.1673	0.1505	0.00	0.00
18	20	10	0.75	0.20	0.3093	0.3083	0.3048	0.2784	1.16	0.80
19	40	5	0.25	0.05	0.1145	0.1012	0.1000	0.0928	1.23	0.30
20	40	5	0.25	0.10	0.1972	0.1692	0.1678	0.1602	0.83	0.30
21	40	5	0.25	0.20	0.3341	0.2816	0.2803	0.2736	0.46	0.50
22	40	5	0.50	0.05	0.1300	0.1237	0.1228	0.1114	0.69	0.35
23	40	5	0.50	0.10	0.2244	0.2053	0.2042	0.1923	0.51	0.50
24	40	5	0.50	0.20	0.3820	0.3407	0.3389	0.3282	0.51	0.50
25	40	5	0.75	0.05	0.1406	0.1394	0.1394	0.1299	0.00	0.00
26	40	5	0.75	0.10	0.2422	0.2365	0.2360	0.2243	0.20	0.40
27	40	5	0.75	0.20	0.4136	0.3947	0.3940	0.3828	0.17	0.45
28	40	10	0.25	0.05	0.0808	0.0751	0.0638	0.0579	15.07	0.35
29	40	10	0.25	0.10	0.1503	0.1230	0.1131	0.1076	8.08	0.45
30	40	10	0.25	0.20	0.2764	0.2109	0.2037	0.1988	3.42	0.70
31	40	10	0.50	0.05	0.0866	0.0860	0.0811	0.0696	5.77	0.60
32	40	10	0.50	0.10	0.1609	0.1570	0.1398	0.1291	10.94	0.55
33	40	10	0.50	0.20	0.2969	0.2619	0.2478	0.2386	5.38	0.60
34	40	10	0.75	0.05	0.0902	0.0900	0.0900	0.0812	0.00	0.00
35	40	10	0.75	0.10	0.1674	0.1670	0.1666	0.1507	0.27	0.80
36	40	10	0.75	0.20	0.3092	0.3038	0.2918	0.2784	3.94	0.80

4. Postponing preventive maintenance

The advantage of postponing preventive maintenance actions is that we obtain more valuable information on the lifetime distribution, resulting in better maintenance decisions in the long run. Postponing such actions, however, is only profitable if these long-run benefits outweigh the additional expected corrective maintenance costs due to the corresponding increased risk of breakdowns. We now introduce a threshold policy that postpones preventive maintenance actions, taking this trade-off into account. Thereafter, we perform a numerical study to assess the benefits of this policy compared to the myopic policy.

4.1. The threshold policy

The main drawback of the myopic policy is that it might schedule preventive maintenance very early, and, as a consequence, never or only very late ascertains that the unit actually is strong. That is why the proposed threshold policy postpones preventive maintenance actions. However, it only does so if the probability that the unit is strong is large enough; otherwise, the initial additional expected corrective maintenance costs may be too high. The postponed preventive maintenance actions are performed if the optimal maintenance age T_2 of a strong unit is reached; T_2 is thus the optimum of (1) for $\lambda = 2$. More formally, we select the maintenance age T_2 instead of the optimal maintenance age T^* of the myopic policy if and only if $\hat{p} < \pi$. The threshold level π is a parameter of the threshold policy, and optimization over π yields the optimal threshold policy.

We remark that the performance of the optimal threshold policy is always at least as good as the performance of the myopic policy. In cases where postponing preventive maintenance is not beneficial, the optimal threshold level π^* is equal to 0, and both policies are equivalent.

4.2. Numerical results

We continue with a numerical study to assess the benefits of the threshold policy compared with the myopic policy for various values of the parameters S , k , p , and c . The considered values of the parameters are based on the insights on the cost difference between the myopic policy and its lower bound, see Section 3.2. We consider lifespans with lengths $S=20$ and 40 , as these allow for the largest potential cost savings compared to the myopic policy. The considered levels for the shape parameter of the Weibull lifetime distribution are $k=5$ and 10 . Smaller values of k offer little room for improvement, whereas larger values are less realistic in practice. The potential cost benefit of the threshold policy may be substantial as long as the probability that the unit is weak is not too close to 0 or 1; we therefore consider probabilities $p=0.25$, 0.5 , and 0.75 that the unit is weak. Finally, we consider relative preventive maintenance costs $c=0.05$, 0.1 , and 0.2 , since in these cases the difference in cost rates between the myopic policy and its perfect information bound is largest.

We consider each combination of the suggested parameter values, which leads to 36 cases. For each of these cases, we simulate 10,000 lifespans for the no-update policy (NP), the myopic policy (MP), the threshold policy (TP) for $\pi = 0.05, 0.1, \dots$, and 1 , and under perfect information (PI). The cost rates of these experiments are shown in Table 1. The optimal threshold levels π^* and corresponding cost rates are also shown. Furthermore, the column MP–TP shows the percentage cost benefits of the threshold policy compared to the myopic policy.

We observe that this percentage varies from 0% to 15%. In cases where no benefits can be obtained by postponing preventive maintenance, we either have that $S=20$, $c=0.05$, or $p=0.75$. If $S=20$, then there might not be enough time to benefit from the additional knowledge. If $c=0.05$, the consequences of a breakdown are relatively large. Finally, if $p=0.75$ the probability of a breakdown due to postponed preventive maintenance is relatively high. In all these cases, it is likely that the benefits of postponing preventive maintenance do not outweigh their additional initial expected corrective maintenance costs.

The results of the experiments are in contrast with those obtained in Section 3, where we identified the circumstances under which the potential improvements over the myopic policy are largest. Indeed, if for example c is small, this difference is large because the myopic policy selects a very conservative maintenance age and is thus unable to identify the type of the unit. However, postponing preventive maintenance is very expensive in this situation. Therefore, selecting a conservative maintenance age, as the myopic policy does, may actually be close to optimal. The large difference between the performance of the myopic policy and under perfect information is thus not caused by a bad performance of the myopic policy, but by a weak lower bound based on perfect information.

Many values of the parameters do actually result in substantial cost reductions over the entire lifespan. The obtained results provide insights into the trade-off between the additional costs due to the initially postponed maintenance actions and the benefits of the improved maintenance decisions on the long run. The optimal threshold level π^* is for example increasing if S increases from 20 to 40 (all else unchanged). A longer lifespan implies more time to benefit from more effectively scheduled preventive maintenance. As a result, postponing preventive

Fig. 2. Sample paths of the estimated probability $\hat{p}$ for the myopic policy (a) and the threshold policy (b).

maintenance already becomes beneficial for smaller likelihoods that the unit is strong, i.e. greater likelihoods that the unit is weak, and the optimal threshold level π^* is thus increasing. Moreover, for a small S and c we only postpone preventive maintenance actions if p is small. For such a high breakdown cost, only when $p=0.25$ the risk of having a breakdown is small enough to make postponing preventive maintenance profitable. For larger S and c , considerable cost reductions are also obtained when $p=0.5$.

In all cases considered in Table 1, the benefit of postponing preventive maintenance actions becomes larger if k increases from 5 to 10. Moreover, the optimal threshold level increases in k . This is in line with the results obtained in Section 3 that reveal a cost difference between the myopic policy and the policy under perfect information that is also increasing in k . For large values of k , the myopic policy selects a maintenance age that is likely to prevent failures of both unit types. As a consequence, it takes a long time until the policy eventually identifies (with near certainty) whether the unit is weak or strong. This is illustrated by Fig. 2(a) where the estimated probability $\hat{p}$ that the unit is weak is shown over time for 1000 runs of the myopic policy with parameter values $S=40$, $k=10$, $p=0.5$, and $c=0.1$, and for the case that the unit considered is actually strong. In almost all runs $\hat{p}$ decreases slowly to zero. Sudden jumps of $\hat{p}$ to values close to 1 are caused by early breakdowns which suggest that the unit is actually weak. Fig. 2 (b) shows the behavior of $\hat{p}$ for the optimal threshold policy in the same situation. The effect of postponing preventive maintenance actions is clear: in almost all cases the type of the unit is already identified correctly after the first maintenance action. Although there is a small probability that $\hat{p}$ first increases due to an early breakdown, the threshold policy yields a substantial improvement over the myopic policy on average. Moreover, we note that $\hat{p}$ may increase above the optimal threshold level $\pi^*=0.55$. In this case, the same conservative maintenance age as under the myopic policy is selected so that $\hat{p}$ only slowly decreases. Once $\hat{p}$ drops below π^* , preventive maintenance is postponed and the unit is identified as strong.

We conclude that substantial cost benefits may be obtained by postponing preventive maintenance actions; in particular if the myopic policy selects a conservative maintenance age and if the additional expected costs incurred by postponing preventive maintenance are not too large. This is for example the case if c is not too small or if p is not too large. By postponing preventive maintenance actions we are able to identify the type of the unit much faster than the myopic policy does.

5. Conclusions and future research directions

We have considered the problem of time-based preventive maintenance planning under uncertainty in the lifetime distribution of a unit, and we have investigated the benefits of postponing these preventive maintenance actions during the first phase of the lifespan of this unit. Although these postponed maintenance actions initially lead to higher costs, the additional information which becomes available also enables more effectively planned maintenance actions during the remainder of the lifespan.

Because this is the first study to recognize that the choice of a maintenance age influences the information that becomes available, we have considered a simple setting with two possible unit types (weak and strong). A threshold policy is proposed that postpones preventive maintenance as long as the estimated probability that the unit is strong exceeds a certain threshold. This policy is compared with a myopic policy that repeatedly selects a maintenance age that minimizes the cost rate under the then current estimated probabilities of the unit types.

The threshold policy offers substantial cost savings compared to the myopic policy. This is especially the case if the variances of the lifetime distributions of the separate unit types are small, and if the relative consequences of failures are severe. In such situations, the myopic policy selects a conservative maintenance age that is likely to prevent failures arising from both unit types, implying that almost no information about the true unit type becomes available. Meanwhile, by postponing preventive maintenance, it is likely that we are able to identify the unit type in such cases, resulting in substantial cost benefits over the entire lifespan. This is most pronounced if the risk of a breakdown is not too large as a consequence of the postponed maintenance actions. Furthermore, the lifespan of the unit should be sufficiently long to ensure that the benefits outweigh the additional costs.

Although these insights are obtained for a simple model under several assumptions, they are also relevant for more general preventive maintenance scheduling problems with uncertainty in the lifetime distribution. In particular, if the current maintenance policy is such that little information on the lifetime distribution becomes available, substantial cost benefits may be obtained by postponing preventive maintenance actions. Specific examples include block-based preventive maintenance policies with very frequent maintenance activities [4] and opportunistic maintenance policies for multi-unit systems with a high degree of clustering [26].

Moreover, the results of this paper are also relevant for condition-based maintenance problems. Existing studies on condition-based maintenance generally assume that the behavior of the deterioration process and the distribution of the deterioration level at which failure occurs are known. In many practical situations, however, these assumptions are questionable. If the behavior of the deterioration process is uncertain, then cost savings over the entire lifespan might be obtained by initially performing more frequent inspections. In cases where there is uncertainty in the distribution of the failure deterioration level, valuable information might be obtained by first observing the deterioration level at which failure occurs a few times. It is evident

that there are ample research opportunities in the area of condition-based maintenance related to this problem.

Two natural extensions of our model are considering more than two unit types or continuous uncertainty in the parameters of the lifetime distribution. If the relative difference between the possible lifetime distributions is large, then the benefit of initially postponing preventive maintenance is expected to be substantial. However, addressing these extensions is not straightforward. If there are multiple unit types, the expected probabilities for each of the types can no longer be represented by a single variable. As a consequence, the specification of a threshold policy is not straightforward in this setting. Furthermore, with only two unit types, postponing preventive maintenance actions to the optimal maintenance age of the strong unit type is a natural choice. However, selecting the maintenance age of a postponed maintenance action becomes more complicated if there are multiple unit types. Recapitulating, the proposed extensions allow for various interesting opportunities for future research, both regarding the decision when and the decision how far preventive maintenance actions should be postponed.

Another remark we would like to make is that the proposed threshold policy is not the optimal policy that minimizes the total expected costs over the lifespan of a unit. Thus, instead of focusing on more complex situations, future research could also aim to improve the threshold policy. Insights into the directions in which such improvements can be found can be obtained by further simplifying the structure of the model, for example by discretizing time.

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