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Kruse, Yves

Published in:
Economics Letters

DOI:
[10.1016/j.econlet.2015.07.019](https://doi.org/10.1016/j.econlet.2015.07.019)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2015

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Kruse, Y. (2015). A modified test against spurious long memory. *Economics Letters*, 135(10), 34-38.
<https://doi.org/10.1016/j.econlet.2015.07.019>

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A modified test against spurious long memory^{*}

Robinson Kruse^{*}

*Rijksuniversiteit Groningen, Faculty of Economics and Business, University of Groningen, PO Box 800, 9700 AV Groningen, The Netherlands
CREATES, Aarhus University, Department of Economics and Business, Fuglesangs Allé 4, DK-8210 Aarhus V, Denmark*

HIGHLIGHTS

- Detection of spurious long memory can be complicated when the data contains also a truly fractionally integrated component.
- A recent test against spurious long memory by Qu (2011) is modified to improve its finite-sample power properties in this case.
- The modified test builds on prior fractional differencing and offers gains in power while maintaining similar size properties.
- The same critical values can be used as the modified statistic has the same limit distribution as the original test by Qu (2011).

ARTICLE INFO

Article history:

Received 28 April 2015

Received in revised form

29 June 2015

Accepted 20 July 2015

Available online 31 July 2015

JEL classification:

C12

C22

Keywords:

Long memory

Structural breaks

Fractional differencing

ABSTRACT

This paper considers the situation where a time series is composed of a fractionally integrated component with long memory parameter $d \in (-1/2; 1/2)$ and some contamination in the form of level shifts or trends. The test against spurious long memory by Qu (2011) is consistent in this case as the standard local Whittle estimator for unknown d is upward biased. As demonstrated in this work, the power can be improved by removing the fractional component from the series prior to application of the test. This task can be accomplished by using the modified local Whittle approach by Hou and Perron (2014). This estimator is robust against contaminations and yields nearly unbiased point estimates of d , irrespective of whether contaminations are present or not. The suggested testing procedure has similar size properties as the original test and is often more powerful.

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1. Introduction

Testing for true versus spurious long memory is an active field of econometric research nowadays. Spurious long memory processes are characterized by structural breaks or (smooth) trends and also lead to hyperbolically decaying autocorrelations, see Diebold and Inoue (2001). Recent contributions for tests against true long memory (in the sense of fractional integration) include Ohanissian et al. (2008), Perron and Qu (2010), Qu (2011) and Hassler et al. (2014) inter alia.

^{*} The author would like to thank an anonymous referee for helpful comments. Suggestions from Matei Demetrescu and Uwe Hassler are greatly acknowledged. Ly Hoang Thi Mai provided excellent proof reading. Robinson Kruse gratefully acknowledges support from CREATES—Center for Research in Econometric Analysis of Time Series (DNRF78), funded by the Danish National Research Foundation. R codes are available upon request.

^{*} Correspondence to: CREATES, Aarhus University, Department of Economics and Business, Fuglesangs Allé 4, DK-8210 Aarhus V, Denmark.

E-mail addresses: y.r.kruse@rug.nl, rkruse@creates.au.dk.

Qu (2011) proposes a test in the frequency domain based on the profiled local Whittle likelihood function. While true long memory processes are characterized by a uniform behaviour of the periodogram, structural break processes (as random level shifts) behave non-uniformly around certain frequencies. The suggested test statistic therein exploits this distinct behaviour for discrimination. The test turns out to be powerful when the series is solely generated by random level shifts, non-monotonic trends or Markov regime switching.¹ But recent work find evidence for the presence of both features, see Varneskov and Perron (2015) and Grassi and Santucci di Magistris (2014). This work extends the scope towards mixed processes where both, a fractionally integrated component and some form of contamination are possibly present and thereby follows McCloskey and Perron (2013) and Hou and Perron (2014).

The test by Qu (2011) is still consistent also when the data generating process (DGP) consists of fractional integration and some

¹ See also Leccadito et al. (2015) for a recent comparison of competing tests. Their results indicate that the Qu (2011) test is highly competitive.

contamination. It turns out that it can be improved in terms of power (while maintaining similar size properties) in such situations by transforming the data according to a GLS-type approach. In fact, it is suggested to take a fractional difference of the series prior to testing. By doing so, the fractionally integrated component is removed. When the true DGP does not contain a fractionally integrated component, the loss in power is small and outweighed by the potential power gains when it is actually present. Even for rather small values of d , power gains are already achievable.

The suggested procedure works as follows: In a first step, the fractional component is filtered from the series. As the true long memory parameter is unknown, some consistent estimator shall be used. The standard local Whittle estimator is known to be strongly upward biased and thus not of much use in this respect. On the contrary, the recently proposed modified local Whittle estimator by [Hou and Perron \(2014\)](#) is robust and can cope with several forms of contaminations, including random level shifts. The simulation results in [Hou and Perron \(2014\)](#) are encouraging and demonstrate that the suggested modification yields nearly unbiased point estimates of d , irrespective of whether the contamination is actually present or not. Thus, the fractionally integrated component can be removed consistently by fractional differencing. As typical for such a GLS-type approach, the contamination is subject to fractional differencing as well. The consequences for the behaviour of the periodogram are discussed. The modified test is consistent as well and it turns out that it is often more powerful. While the original test is introduced in Section 2, suggested modification is presented in Section 3 and further investigated via Monte Carlo simulations in Section 4 where also size and power experiments are conducted.

2. Testing against spurious long memory

This section briefly reviews the [Qu \(2011\)](#) test.² Under the null hypothesis, the series y_t ($t = 1, 2, \dots, T$) is stationary and has spectral density $f(\lambda) \simeq G\lambda^{-2d}$ as $\lambda \rightarrow 0_+$. The long memory parameter d is restricted to lie in the interval $(-1/2; 1/2)$ and G is a positive, finite constant. This situation covers for instance the class of ARFIMA processes. The local Whittle likelihood is given by ($m < T$)

$$Q(G, d) = \frac{1}{m} \sum_{j=1}^m \left(\log G\lambda_j^{-2d} + \frac{I_{y,j}}{G\lambda_j^{-2d}} \right)$$

where $\lambda_j = 2\pi j/T$ are frequencies and $I_{y,j}$ denotes the periodogram of y_t , i.e.

$$I_{y,j} = \frac{1}{2\pi T} \left| \sum_{t=1}^T y_t \exp(i\lambda_j t) \right|^2.$$

Under the validity of the null hypothesis, the expected ratio of the periodogram to the spectral density tends to unity for all values of j , given that $j \rightarrow \infty$ when $T \rightarrow \infty$ and $j/T \rightarrow 0$. The profiled likelihood (after concentrating G out of Q) reads

$$R(d) = \log G(d) - \frac{2}{m} d \sum_{j=1}^m \log \lambda_j$$

with $G(d) = \frac{1}{m} \sum_{j=1}^m \lambda_j^{2d} I_{y,j}$. The test statistic by [Qu \(2011\)](#) is based on the first derivative of R with respect to d and is related to the Lagrange Multiplier test by [Robinson \(1994\)](#). It is given by

$$W = \sup_{r \in [\varepsilon, 1]} \frac{1}{\sum_{j=1}^m v_j^2} \left| \sum_{j=1}^m v_j \left(\frac{I_{y,j}}{G(\widehat{d})\lambda_j^{-2\widehat{d}}} - 1 \right) \right| \quad (1)$$

with $v_j = \log \lambda_j - \frac{1}{m} \sum_{j=1}^m \log \lambda_j$. Here, $\widehat{d}$ signifies the standard local Whittle estimator and ε is a trimming parameter.³ The limiting distribution of W is non-standard and critical values are provided in [Qu \(2011\)](#). Under the alternative, the series y_t is short memory, but subject to some form of contamination. The probably most prominent example is the case of random level shifts, see for instance [Qu and Perron \(2013\)](#). In this case, $y_t = x_t + u_t$ with x_t being short memory and $u_t = \sum_{i=1}^t \delta_i = \sum_{i=1}^t \pi_i \eta_i$ with $\pi_t \sim i.i.d. B(p/T, 1)$ and $\eta_t \sim i.i.d. (0, \sigma_\eta^2)$; π_t and η_t are mutually independent. [Perron and Qu \(2010\)](#) show that the periodogram of y_t can be decomposed as follows:

$$I_{y,j} = \frac{1}{2\pi T} \left| \sum_{t=1}^T x_t \exp(i\lambda_j t) \right|^2 + \frac{1}{2\pi T} \left| \sum_{t=1}^T u_t \exp(i\lambda_j t) \right|^2 + \frac{2}{2\pi T} \sum_{t=1}^T \sum_{s=1}^T x_t u_s \cos(\lambda_j(t-s)).$$

For $\lambda_j = o(1)$, they derive the following behaviour

$$I_{y,j} = O_p(1) + O_p\left(\frac{1}{T\lambda_j^2}\right) + O_p\left(\frac{1}{\sqrt{T}\lambda_j}\right).$$

The second term dominates $I_{y,j}$ for $j = o(\sqrt{T})$, while the first term is dominant for $j/\sqrt{T} \rightarrow \infty$. The consistency of the W test hinges on the upward bias of the standard local Whittle estimator $\widehat{d}$ (Theorem 1 [Qu, 2011](#)). Even if the series is composed by both types (stationary fractional integration with parameter $d \in (-1/2; 1/2)$ and contamination), the test retains its consistency. An important, but mild requirement is that the standard local Whittle estimator is still upward biased, i.e. $P(\widehat{d} - d > \epsilon) \rightarrow 1$ for $\epsilon > 0$ (Corollary 1 [Qu, 2011](#)) and that a variance condition holds. Following [McCloskey and Perron \(2013\)](#), the periodogram behaves as

$$I_{y,j} = O_p\left(\frac{1}{\lambda_j^{2d}}\right) + O_p\left(\frac{1}{T\lambda_j^2}\right) + O_p\left(\frac{1}{\sqrt{T}\lambda_j^{1+d}}\right).$$

The second component dominates for $j = o(T^{(1-2d)/(2-2d)})$, while the first one dominates for $jT^{(2d-1)/(2-2d)} \rightarrow \infty$. For $d = 0$, the former type of behaviour emerges as a special case, see [Perron and Qu \(2010\)](#).

3. Modified test

3.1. Infeasible version

This section introduces the suggested modification. First, an infeasible version is presented to illustrate the idea. Below, the implementation of a feasible test procedure is described. There are two steps to be considered. Let L denote the lag operator.

1. Build the fractional difference $z_t = (1-L)^d y_t$ for the observable time series y_t .
2. Apply the [Qu \(2011\)](#) test statistic to z_t (instead of y_t). In particular, compute

$$W_z = \sup_{r \in [\varepsilon, 1]} \frac{1}{\sqrt{\sum_{j=1}^m v_j^2}} \left| \sum_{j=1}^m v_j \left(\frac{I_{z,j}}{G_z(\widehat{d}_z)\lambda_j^{-2\widehat{d}_z}} - 1 \right) \right| \quad (2)$$

where $\widehat{d}_z$ denotes the standard local Whittle estimator for d applied to the series z_t . Moreover, $G_z(\widehat{d}_z) = \frac{1}{m} \sum_{j=1}^m \lambda_j^{2\widehat{d}_z} I_{z,j}$.

² The reader is referred to the original contribution for a complete presentation.

³ [Qu \(2011\)](#) recommends $m = T^{0.7}$ and $\varepsilon = 0.02$ for $T \geq 500$.

Table 1Simulation results for size experiments. W denotes the original test by Qu (2011) and $\tilde{W}_z$ is the modified test.

	$T = 500$					$T = 3000$				
	$d = 0$	$d = 0.1$	$d = 0.2$	$d = 0.3$	$d = 0.4$	$d = 0$	$d = 0.1$	$d = 0.2$	$d = 0.3$	$d = 0.4$
W	0.024	0.020	0.022	0.020	0.017	0.032	0.029	0.028	0.026	0.037
$\tilde{W}_z$	0.024	0.020	0.022	0.018	0.015	0.032	0.028	0.026	0.023	0.032

Critical values are 1.118, 1.252 and 1.517 at the nominal significance levels of 10%, 5% and 1%, respectively, c.f. Qu (2011), Table 1.

The periodogram of the differenced series z_t behaves as follows:

$$I_{z,j} = O_p(1) + O_p\left(\frac{1}{T\lambda_j^{2-2d}}\right) + O_p\left(\frac{1}{\sqrt{T}\lambda_j^{1-d}}\right).$$

The first term is similar to the one for the original test under short memory. The second part measures the contribution of the fractionally differenced contamination. It can be seen that its influence is only slightly lowered by having a rate of $T\lambda_j^{2-2d}$ instead of $T\lambda_j^2$. Still, the second term dominates for $j = o(T^{(1-2d)/(2-2d)})$, while the first one dominates for $jT^{(2d-1)/(2-2d)} \rightarrow \infty$. This behaviour is important for establishing the limiting properties of W_z . Obviously, the modified test statistic has the same limiting distribution as the original statistic. Thus, exactly the same critical values can be used as for the original W statistic by Qu (2011). Under H_0 , z_t is $I(0)$ without contamination and thus satisfies all requirements stated in Qu (2011). Moreover, the modified test is consistent. Similar to the original test, it is required that the standard local Whittle estimator $\hat{d}_z$ applied to z_t is asymptotically upward biased, i.e. $P(\hat{d}_z > \epsilon) \rightarrow 1$ and that a variance condition holds.⁴

The idea behind the suggested modification relates to a GLS-type approach which has been successfully applied for instance by Iacone et al. (2014) and Xu and Perron (2013) in the context of testing for structural breaks under possible fractional integration. The data is accordingly transformed by applying a fractional filter to the series. As the simulation results (presented in the next section) demonstrate, the power of the original test lowers in the presence of a fractionally integrated component. Discrimination between true and spurious long memory is thus more complicated in situations where both features are present in the data. By transforming the data according to the suggested GLS-type approach, this behaviour is mostly alleviated yielding better power.

3.2. Feasibility

In order to obtain a feasible testing procedure, one needs to estimate the unknown value of d in the first step which consists of building a fractional difference of y_t , i.e. $z_t = (1 - L)^d y_t$. It is suggested to apply the modified local Whittle estimator by Hou and Perron (2014) labelled as $\tilde{d}$. The first feasible step is thus to compute the fractional difference of the observable time series with the estimate $\tilde{d}$, i.e. $\tilde{z}_t = (1 - L)^{\tilde{d}} y_t$. The second step of the procedure then uses $\tilde{z}_t$ instead of z_t and the resulting test statistic is correspondingly labelled as W_z (c.f. Section 3.1., Eq. (2)).

Regarding the Hou and Perron (2014) estimator, the corresponding profiled local Whittle likelihood is given by

$$J(d, \theta) = \log G(d, \theta) + \frac{1}{m} \sum_{j=1}^m \log(\lambda_j^{-2d} + (\theta/T)\lambda_j^{-2})$$

with $G(d, \theta) = \frac{1}{m} \sum_{j=1}^m \frac{I_{y,j}}{\lambda_j^{-2d} + (\theta/T)\lambda_j^{-2}}$. The modified estimator is defined as $(\tilde{d}, \tilde{\theta}) = \arg \min J(d, \theta)$. Here, θ controls the influence of contaminations at low frequencies. For $\theta = 0$, $J(d, 0)$ reduces to $R(d)$. Hou and Perron (2014) recommend to use a bandwidth of $m = T^{0.8}$. These authors also prove consistency and asymptotic normality (without assuming Gaussianity for the series). In the absence of contaminations, the modified estimator does not have an asymptotic efficiency loss. The simulations reported in Hou and Perron (2014) demonstrate that the efficiency loss is rather small in finite samples and that the estimator performs well in various settings.

4. Monte Carlo simulations

The following DGP (including fractional integration and random level shifts) is investigated.

$$y_t = x_t + u_t$$

with $(1 - L)^d x_t = v_t$, $v_t \sim i.i.d. N(0, 1)$ and $u_t = \sum_{i=1}^t \delta_i = \sum_{i=1}^t \pi_i \eta_i$ with $\pi_t \sim i.i.d. B(p/T, 1)$ and $\eta_t \sim i.i.d. N(0, \sigma_\eta^2)$; π_t and η_t are mutually independent. The parameters of interest are the long memory parameter $d = \{0, 0.1, 0.2, 0.3, 0.4\}$, the expected number of breaks $p = \{5, 10, 15, 20\}$ and the variance of the level shift component $\sigma_\eta^2 = \{0, 0.5, 1, 1.5, 2\}$. For $\sigma_\eta^2 = 0$ and $d > 0$, the persistence in the series y_t is purely generated by a fractionally integrated process x_t . This setting corresponds to size experiments. On the contrary, when $d = 0$ and $\sigma_\eta^2 > 0$, the persistence in y_t is driven by the random level shift component u_t . When $d > 0$ and $\sigma_\eta^2 > 0$, both features are simultaneously present. These situations are considered in power experiments.

The sample sizes are $T = \{500, 3000\}$ and 5000 Monte Carlo replications are used. Regarding bandwidth and trimming parameters, the suggestions by Qu (2011) and Hou and Perron (2014) are considered.⁵ The nominal significance level is 5%. The original test statistic W is compared to the feasible test statistic $\tilde{W}_z$. Simulation results for size experiments are presented in Table 1. The empirical size properties of the two tests are very similar. For larger d , the W_z test rejects slightly less frequently in comparison to the W test.

Power results are reported in Table 2.⁶ For the case of short memory ($d = 0$), the original test performs somewhat better as expected, while W_z only has a small loss in power. When $T = 3000$, both tests perform similar. In the case of $T = 500$, the original test offers some power gains for large p and large σ_η^2 . But when $p \leq 15$, the maximal power loss of the modified test equals five percentage points. When both features are present ($d > 0$), the modified test outperforms the original one in many cases (e.g. $\sigma_\eta^2 \geq 1$). The larger d , p and σ_η^2 , the larger possible gains in terms of power. For $T = 500$ and $T = 3000$, the maximal power gains are 21.6 and 44.3

⁵ Simulation runs with $\hat{d} > 0.5$ are excluded as stationarity is required.

⁶ Unreported simulations confirm that the standard local Whittle estimator is still upward biased when applied to z_t .

⁴ Proofs follow exactly the arguments in Qu (2011) and are omitted for space considerations.

Table 2

Simulation results for power experiments. Coloured entries indicate the highest rejection frequency for a given experiment. d is the long memory parameter, p is the expected number of level shifts and σ_η^2 is the variance of the level shift component η_t .

		$T = 500$				$T = 3000$			
		$\sigma_\eta^2 = 0.5$	$\sigma_\eta^2 = 1$	$\sigma_\eta^2 = 1.5$	$\sigma_\eta^2 = 2$	$\sigma_\eta^2 = 0.5$	$\sigma_\eta^2 = 1$	$\sigma_\eta^2 = 1.5$	$\sigma_\eta^2 = 2$
$p = 5$									
$d = 0$	W	0.544	0.679	0.668	0.644	0.937	0.970	0.963	0.963
	$\tilde{W}_z$	0.532	0.673	0.658	0.618	0.937	0.969	0.965	0.965
$d = 0.1$	W	0.356	0.512	0.552	0.517	0.846	0.948	0.960	0.937
	$\tilde{W}_z$	0.354	0.543	0.592	0.576	0.842	0.954	0.971	0.966
$d = 0.2$	W	0.196	0.323	0.325	0.326	0.611	0.836	0.890	0.874
	$\tilde{W}_z$	0.193	0.369	0.430	0.455	0.596	0.848	0.927	0.944
$d = 0.3$	W	0.089	0.153	0.191	0.149	0.289	0.606	0.680	0.674
	$\tilde{W}_z$	0.088	0.217	0.287	0.301	0.262	0.622	0.788	0.848
$d = 0.4$	W	0.037	0.070	0.082	0.089	0.107	0.234	0.290	0.281
	$\tilde{W}_z$	0.039	0.107	0.149	0.186	0.098	0.287	0.470	0.614
$p = 10$									
$d = 0$	W	0.691	0.694	0.670	0.573	0.991	0.998	0.996	0.995
	$\tilde{W}_z$	0.670	0.695	0.629	0.538	0.991	0.998	0.995	0.996
$d = 0.1$	W	0.504	0.570	0.535	0.480	0.956	0.993	0.995	0.977
	$\tilde{W}_z$	0.511	0.612	0.581	0.519	0.957	0.995	0.998	0.991
$d = 0.2$	W	0.293	0.411	0.354	0.304	0.807	0.946	0.955	0.971
	$\tilde{W}_z$	0.316	0.492	0.474	0.432	0.805	0.964	0.981	0.990
$d = 0.3$	W	0.128	0.205	0.185	0.157	0.466	0.761	0.809	0.846
	$\tilde{W}_z$	0.141	0.306	0.331	0.327	0.452	0.827	0.921	0.958
$d = 0.4$	W	0.051	0.093	0.083	0.102	0.166	0.304	0.388	0.335
	$\tilde{W}_z$	0.059	0.140	0.206	0.207	0.152	0.472	0.685	0.778
$p = 15$									
$d = 0$	W	0.744	0.692	0.594	0.464	0.997	1.000	0.991	1.000
	$\tilde{W}_z$	0.732	0.682	0.534	0.414	0.997	1.000	0.994	0.991
$d = 0.1$	W	0.549	0.519	0.432	0.431	0.985	0.995	1.000	1.000
	$\tilde{W}_z$	0.557	0.578	0.513	0.424	0.984	0.996	0.999	0.998
$d = 0.2$	W	0.357	0.378	0.364	0.270	0.895	0.978	0.958	0.974
	$\tilde{W}_z$	0.389	0.500	0.452	0.387	0.898	0.985	0.988	0.987
$d = 0.3$	W	0.167	0.225	0.144	0.244	0.604	0.812	0.835	0.855
	$\tilde{W}_z$	0.202	0.327	0.320	0.300	0.610	0.888	0.954	0.969
$d = 0.4$	W	0.071	0.097	0.084	0.107	0.219	0.379	0.333	0.400
	$\tilde{W}_z$	0.086	0.186	0.228	0.216	0.230	0.600	0.767	0.834
$p = 20$									
$d = 0$	W	0.749	0.631	0.483	0.538	1.000	1.000	1.000	1.000
	$\tilde{W}_z$	0.725	0.647	0.425	0.366	1.000	1.000	1.000	1.000
$d = 0.1$	W	0.562	0.486	0.400	0.188	0.995	0.998	0.991	1.000
	$\tilde{W}_z$	0.591	0.573	0.469	0.356	0.995	0.999	1.000	0.998
$d = 0.2$	W	0.374	0.347	0.268	0.091	0.932	0.969	0.974	1.000
	$\tilde{W}_z$	0.429	0.494	0.407	0.307	0.935	0.989	0.993	0.985
$d = 0.3$	W	0.192	0.197	0.136	0.286	0.670	0.831	0.802	0.938
	$\tilde{W}_z$	0.244	0.334	0.311	0.278	0.680	0.934	0.963	0.969
$d = 0.4$	W	0.086	0.103	0.105	0.107	0.278	0.419	0.438	0.429
	$\tilde{W}_z$	0.110	0.213	0.199	0.196	0.322	0.692	0.827	0.852

percentage points, respectively. Already for $d = 0.2$, considerable achievements can be made. For $T = 3000$, it can be seen that both tests perform similarly well (except for large d and/or large p , where the modified test is clearly better).

It is concluded that the modified test presents a viable alternative in many situations. The small power losses in the absence of true long memory are outweighed when the true DGP is driven by a fractionally integrated and a random level shift component.

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