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ABSTRACT

The sensitivity of nonequilibrium Casimir forces on material optical properties can have strong impact on the actuation of devices. For this purpose, we considered nonequilibrium Casimir interactions between good and poor conductors, for example, gold (Au) and highly doped silicon carbide (SiC), respectively. Indeed, for autonomous conservative systems, the bifurcation and phase portrait analysis have shown that the nonequilibrium Casimir forces can have significant impact on the stable and unstable operating regimes depending on the material optical properties. At a few micrometer separations, for systems with high conductivity materials, an increasing temperature difference between the actuating components can enhance the stable operation range due to the reduction of the Casimir force, while for the poor conductive materials, the opposite takes place. For periodically driven dissipative systems, the Melnikov function and Poincaré portrait analysis have shown that for poor conductive systems, the nonequilibrium Casimir forces lead to an increased possibility for chaotic behavior and stiction with an increasing temperature difference between the actuating components. However, for good conducting systems, the thermal contribution to Casimir forces reduces the possibility for chaotic behavior with increasing temperature, as comparison with systems without thermal fluctuations shows. Nevertheless, the positive benefit of good conductors toward increased actuation stability and reduced the chaotic behavior under nonequilibrium conditions can be easily compromised by any voltage application. Therefore, thermal, nonequilibrium Casimir forces can influence the actuation of devices toward unstable and chaotic behavior in strong correlation with their optical properties, and associated conduction state, as well as applied electrostatic potentials.

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Micro- and nanoelectromechanical systems have attracted widespread attention from the scientific and technology point of view for sensor technologies, accelerometers, microswitches, etc. Because Casimir forces are omnipresent, they could always influence the actuation of devices, and under certain conditions, they can draw moving elements together into permanent adhesion, which is a phenomenon termed as stiction. As a result, the design of devices can be challenging due to the occurrence of chaotic behavior driven by the Casimir forces, which can cause abrupt changes in their dynamical behavior leading to possible stiction. Therefore, detailed knowledge about the phenomena of stiction due to Casimir interactions and its relation to chaotic motion during actuation is necessary in order to improve the long term performance of devices. It is shown here that the nonequilibrium Casimir forces, which are the most general case when the interacting components have different temperatures, can

influence the actuation of devices toward unstable and chaotic behavior in strong correlation with their optical properties, and associated conduction state, as well as applied electrostatic potentials.

I. INTRODUCTION

Micro- and nanoelectromechanical systems (MEMS/NEMS) have attracted widespread attention from the scientific and technology point of view for applications in industry related to sensor technologies, accelerometers, microswitches, etc. By decreasing the length scales into the micrometer and submicrometer regimes, new areas open not only for consideration but also for applications of Casimir forces.^{1–7} Because the electrostatic forces can be switched off when no potential is applied, while the Casimir forces

are omnipresent, the latter could always influence, in principle, the actuation dynamics of micro- and nanodevices. This is because these systems have surface areas large enough but gaps small enough for the Casimir force to play a significant role, and under certain conditions to be strong enough to draw moving mechanical elements together into permanent adhesion. The latter is a phenomenon known as stiction, and it can obscure the actuation dynamics of devices.^{2,8,9}

Casimir forces between two objects arise due to the perturbation of quantum fluctuations of the electromagnetic (EM) field, as it was predicted by Casimir in 1948,¹⁰ assuming two perfectly reflecting parallel plates. Lifshitz and co-workers in the 1950s¹¹ considered the general case of real dielectric plates by exploiting the fluctuation-dissipation theorem at thermal equilibrium, which relates the dissipative properties of the plates due to optical absorption by many microscopic dipoles and the resulting EM fluctuations. This theory describes the attractive interaction due to quantum fluctuations at all separations covering both the Casimir (long-range) and van der Waals (short-range) regimes.^{1–4,10–13} Several works so far have shown the strong dependence of the Casimir force on the material optical properties,^{14–20} which is an important attribute that can be utilized to tune the actuation dynamics of devices.^{21,22}

Here, we have considered the effect of Casimir forces in microswitches, which are essential MEMS/NEMS components. These systems are typically constructed from two electrodes of which one is fixed and the other is suspended by a mechanical spring governed by Hooke's law.²³ The application of a bias voltage between the electrodes actuates them toward each other, but it is also possible for the moving component to become unstable and collapse (pulls-in) onto the other.^{24,25} The design of MEMS could always be quite challenging due to the occurrence of chaotic behavior, which causes abrupt changes in their dynamical behaviors, and as a result, device malfunctions related to possible stiction of moving components. Obviously, more detailed knowledge about the phenomena of stiction and its relation to chaos is necessary to improve the long term performance and the design of MEMS devices operating under thermal equilibrium and/or nonequilibrium conditions.^{26–30}

Despite of several studies, which have been performed on the effect of optical properties on the actuation dynamic of microswitches,^{26–28} it still remains unexplored how significant is the influence of thermal nonequilibrium conditions on the performance of microdevices in view also of recent advances to measure thermal effects with MEMS.²⁹ Indeed, besides the optical properties of material coatings on device components, when MEMS operate under ambient conditions, additional factors like thermal nonequilibrium conditions, due to temperature differences between the interacting bodies, can influence the Casimir forces.³⁰ Consequently, the dynamical behavior of actuating components can also be affected with unknown long term effects. Therefore, here we aim to explore to what extent thermal nonequilibrium conditions influence the actuation dynamics of microswitches toward chaos, which prohibits the long term prediction of stable device operation. This will be accomplished by taking also into account the optical properties of different materials ranging from poor to good conductors in order to compare how significant is the choice of materials and any applied electrostatic potentials that lead to additional attractive interactions.

II. OPTICAL PROPERTIES OF MATERIALS AND DEVICE ACTUATION

In order to make realistic the results of this research in understanding the thermal effects on the actuation of a wide range of microdevices, we considered the effect of optical properties of gold (Au) as a good conductor and highly nitrogen-doped silicon carbide (SiC) as a poor conductor. In fact, Au as a good conductor is widely used for Casimir force measurements, while highly nitrogen (N)-doped conductive SiC is a promising material for devices operating under harsh environments, allowing also integration with Si based technologies. Both materials were optically characterized in the past with the same equipment at J. A. Woollam Co., Inc. using the ellipsometers VUV-VASE (0.5–9.34 eV) and IR-VASE (0.03–0.5 eV).^{19,29} Figure 1 shows the real and the imaginary parts of the measured dielectric functions of these materials. For comparison, after analysis of the optical data, the static conductivity ratio ω_p^2/ω_τ (in terms of the Drude model with ω_p the plasma frequency and ω_τ the relaxation frequency, see Appendix A) obtains for these materials the values $\omega_p^2/\omega_\tau|_{\text{SiC}} = 0.4 \text{ eV}^{29}$ and $\omega_p^2/\omega_\tau|_{\text{Au}} \approx 1600 \text{ eV}^{19}$. The latter indicates a conductivity contrast of almost four orders of

FIG. 1. Imaginary (a) and real (b) parts of the frequency dependent dielectric functions for both the Au and SiC systems measured with ellipsometry. The inset in (a) shows the schematic of the MEMS under consideration.

FIG. 2. Comparison between F_0 (zero-point fluctuations contribution) and the F_{Cas} ($= F_0 + F_{\text{thermal}}$) for both the Au-Au and SiC-SiC systems. The calculations for the thermal contribution F_{thermal} were performed under nonequilibrium conditions for $T_1 = 300$ K and $T_2 = 400$ K.

magnitude to justify these materials as suitable candidates for our analysis.

The equation of motion for the electromechanical system [inset Fig. 1(a)], where both the fixed and movable plates are considered to be coated with same material (Au or SiC), is given by

$$M \frac{d^2 z}{dt^2} + \left(\frac{M\omega_0}{Q} \right) \frac{dz}{dt} = -F_{\text{res}} + F_{\text{elec}} + F_{\text{Cas}} + \varepsilon F_0 \cos(\omega t), \quad (1)$$

FIG. 3. $\alpha_{\text{thermal}} = F_{\text{Cas}}/F_0$ vs z for nonequilibrium conditions for both Au-Au and SiC-SiC systems, where for the calculations we considered one plate at $T_1 = 300$ K.

FIG. 4. $\alpha_{\text{thermal}} (= F_{\text{Cas}}/F_0)$ vs T_2 (K) for both equilibrium ($T_1 = T_2$) and nonequilibrium ($T_1 = 300$ K) situations. The separation distance between plates is $z = 1 \mu\text{m}$, while that in the inset is $z = 2 \mu\text{m}$.

where M is the mass of the movable plate, and $(M\omega_0/Q)(dz/dt)$ is the intrinsic energy dissipation in the actuating system associated with a quality factor Q . The frequency ω_0 is assumed to be close to frequencies of dynamic mode atomic force microscopy (AFM) cantilevers and MEMS (typically 300 kHz). The parameter ε was introduced to distinguish between the conservative (frictionless) autonomous operation of the actuating system ($\varepsilon = 0$) and the nonconservative driven system by an external force ($\varepsilon = 1$) in the presence of dissipation (friction) having a finite quality factor Q . In fact, for a conservative system, we consider MEMS with a high quality factor $Q > 10^4$,³¹ so that we can neglect any dissipation effects. Moreover, for the study of nonequilibrium effects, we considered initial separations d (where any elastic restoring force is zero) between the actuating plates in the range of 850 nm–2.5 μm . We also assumed plates with flat surfaces having lateral dimensions $L_x = L_y = 10 \mu\text{m}$, though any nanoscale roughness only gives significant contributions at short separations below 100 nm.¹⁷

In Eq. (1), the Casimir force F_{Cas} is opposed by the elastic restoring force $F_{\text{res}} = -K(d - z)$, where K is the spring constant and F_{elec} is the electrostatic force acting between two plates,

$$F_{\text{elec}}(z) = \frac{1}{2} \frac{\varepsilon_0 A V^2}{z^2}, \quad (2)$$

where V is the applied voltage between the two plates with respect to the conduct potential difference due to differences in the work function of the material coatings of the plates, and ε_0 is the permittivity of vacuum. The Casimir force F_{Cas} is given by the expression

$$F_{\text{Cas}}(T_1, T_2, z) = F_0(z) + F_{\text{th}}^{\text{neq/eq}}(T_1, T_2, z), \quad (3)$$

where the contribution of the zero-point fluctuations $F_0(z)$ (at $T = 0$) is separated from the contribution due to thermal fluctuations

FIG. 5. Bifurcation diagrams δ_{Cas} vs $\lambda (= z/d)$ for nonequilibrium conditions ($T_1 = 300 \text{ K}$, $T_2 = 1 \text{ K}$ and 400 K), $\delta_v = 0$, and different initial actuation distances d : (a) and (b) $d = 1 \mu\text{m}$; (c) $d = 850 \text{ nm}$; (d) $d = 2.5 \mu\text{m}$. The solid and dashed lines represent the stable and unstable equilibrium points, respectively.

$F_{\text{thermal}} = F_{\text{th}}^{\text{neq/eq}}(T_1, T_2, z)$. $F_0(z)$ has been calculated with the Lifshitz theory, where we used as input the dielectric response function at imaginary frequencies $\epsilon(i\xi)$ after extrapolation at low optical frequencies with the Drude model (see [Appendix A](#)). The chosen materials show significant optical contrast for the dielectric function at imaginary frequencies $\epsilon(i\xi)$, which is the necessary input for the calculation of the Casimir force via the Lifshitz theory (see [Appendix A](#)).

According to Ref. [30](#), the thermal force between two bodies in both configurations for thermal equilibrium (eq) and nonequilibrium (neq) conditions is given by (see [Appendix B](#))

$$F_{\text{th}}^{\text{neq/eq}}(T_1, T_2, z) = F_{\text{th}}^{\text{neq/eq}}(T_1, 0, z) + F_{\text{th}}^{\text{neq/eq}}(0, T_2, z). \quad (4)$$

The first and second terms at the right side of Eq. (4) describe the thermal equilibrium/nonequilibrium of the two bodies at different temperatures T_1 and T_2 , respectively. The thermal part of the force

$F_{\text{th}}^{\text{neq/eq}}(T, z)$ for each single body can be split into two contributions having the form

$$F_{\text{th}}^{\text{neq/eq}}(T, z) = F_{\text{th}}^{\text{neq/eq, PW}}(T, z) + F_{\text{th}}^{\text{neq/eq, EW}}(T, z), \quad (5)$$

where the indices PW and EW denote propagating and evanescent waves, respectively. The thermal parts in Eqs. (3)–(5) can be calculated using the Lifshitz theory in the real frequency representation considering the PW and EW contributions. The PWs satisfy the condition $ck_{\perp} < \omega$ (with $k_{\perp}$ being the in-plane wave vector), which is valid for real photons that propagate both in the vacuum gap and inside the bodies. EWs satisfy the condition $\omega \leq ck_{\perp}$, and they exist only along the boundary planes, as well as their electromagnetic field decreases exponentially within the gap separating the bodies. [Figure 2](#) shows calculations of the Casimir force, where comparisons are made between the contribution of zero-point fluctuations F_0 and the total force $F_{\text{Cas}} (= F_0 + F_{\text{thermal}})$ for both the

FIG. 6. Contour plots $d\lambda/dt$ vs λ for the conservative system. The elliptical in shape homoclinic orbit contains the initial conditions that lead to stable oscillations. Au-Au system: (a) $\delta_{Cas} = 0.07$, $\delta_v = 0$, and $d = 850$ nm; (b) $\delta_{Cas} = 0.06$, $\delta_v = 0$, and $d = 2.5$ μ m. SiC-SiC system: (c) $\delta_{Cas} = 0.068$, $\delta_v = 0$, and $d = 850$ nm. (d) $\delta_{Cas} = 0.05$, $\delta_v = 0$, and $d = 2.5$ μ m. For the calculations, we used 150×150 initial conditions $(\lambda, d\lambda/dt)$ and considered $T_1 = 300$ K and T_2 as indicated.

Au-Au and SiC-SiC systems. Clearly, the thermal effects appear to influence more the poor conducting system leading to an increasing force with surface separation and increasing temperature differences. As a result, one can anticipate that for the poor conducting SiC-SiC system, the nonequilibrium effects vs separation will likely play more important role on actuation properties than those of the good conductors.

III. RESULTS AND DISCUSSION

A. Conservative actuating system ($\epsilon = 0$)

In order to investigate the thermal effect on the actuation of the microsystem, we introduced the bifurcation parameter $\delta_{Cas} = F_{Cas}^m/kd$ that represents the ratio of the minimum Casimir force $F_{Cas}^m = F_{Cas}(z = d)$ to the maximum restoring force $F_{res}^M = kd$.

FIG. 7. Contour plots $d\lambda/dt$ vs λ for the conservative system. The elliptical in shape homoclinic orbit contains the initial conditions that lead to stable oscillations. Au-Au system: (a) $\delta_{\text{Cas}} = 0.07$, $\delta_v = 0.02$, and $d = 850$ nm; (b) $\delta_{\text{Cas}} = 0.07$, $\delta_v = 0.02$, and $d = 2.5$ μm . SiC-SiC system: (c) $\delta_{\text{Cas}} = 0.05$, $\delta_v = 0.02$, and $d = 850$ nm. (d) $\delta_{\text{Cas}} = 0.05$, $\delta_v = 0.02$, and $d = 2.5$ μm . For the calculations, we used 150×150 initial conditions $(\lambda, d\lambda/dt)$ and considered $T_1 = 300$ K and T_2 as indicated.

The parameter δ_{Cas} is useful to determine when there is a stable periodic solution for the system, which corresponds to a sufficient restoring force to prevent stiction of the plates.²⁶ Using δ_{Cas} , Eq. (1) assumes the more convenient form for $\varepsilon = 0$

$$\frac{d^2\lambda}{dT^2} + \left(\frac{1}{Q}\right) \frac{d\lambda}{dT} = -(1 - \lambda) + \delta_v \frac{F_{\text{elec}}}{F_{\text{Cas}}^{\text{m}}} + \delta_{\text{Cas}} \frac{F_{\text{Cas}}}{F_{\text{Cas}}^{\text{m}}}, \quad (6)$$

with $\lambda = z/d$, $T = \omega_0 t$, and $\delta_v = F_{\text{elec}}^{\text{m}}/F_{\text{res}}^{\text{m}} = \varepsilon_0 A V^2 / 2kd^3$ the corresponding electrostatic bifurcation parameter.^{21,32}

Figures 3 and 4 compare the strength of thermal effect between equilibrium and nonequilibrium conditions for both poor (SiC) and good (Au) conducting materials. Indeed, for the good conductive system Au-Au, as Figs. 2–4 show the thermal correction F_{thermal} that has significant contribution at large separations (> 4 μm), while at shorter separations it decreases the Casimir force.³³ As Fig. 4 shows,

FIG. 8. Threshold curve $\alpha(=\gamma\omega_0 d/F_0)$ vs driving frequency ω/ω_0 (with ω_0 the natural frequency of the system) comparing the influence of zero-point fluctuation and thermal effects under nonequilibrium conditions with $T_1 = 300$ K and $T_2 = 1$ K or 400 K. (a) Au-Au system with $\delta_{\text{Cas}} = 0.075$ and (b) SiC-SiC system with $\delta_{\text{Cas}} = 0.058$. The initial distance was l in both cases $d = 1 \mu\text{m}$.

for the Au-Au system, the thermal correction vs T decreases up to 300 K and then it starts to increase at larger separations with the nonequilibrium effects contributing more to the force. By contrast, for the poor conductive system SiC-SiC, the nonequilibrium state has stronger effect before 300 K, and after this temperature the influence of thermal equilibrium increases. Nevertheless, for the SiC-SiC system, the thermal effects show significantly different contribution for both equilibrium and nonequilibrium situations.

The bifurcation diagrams in Fig. 5 illustrate how the thermal effect can influence the stable and unstable regions. If the restoring force is strong enough ($\delta_{\text{Cas}} < \delta_{\text{Cas}}^{\text{Max}}$), then there are two equilibria

points for the system. The stationary points closest to the initial separation d are stable centers with periodic solutions around them. While the ones closer to the fixed plate are unstable saddle points, and the motion around them will lead to stiction on the fixed plate due to the stronger Casimir force. These plots illustrate that the thermal contributions are likely to make significant change on the actuation and the performance of MEMS. The width of the stable area changes not only by considering the thermal contributions but also by changing the temperature between the interacting bodies. Clearly, the thermal effects are more prominent for the case of the poor conductive system (SiC-SiC). The thermal effect is also clearly demonstrated if one considers different initial separations d and compares the bifurcation diagrams in Figs. 5(c) and 5(d).

Besides the bifurcation analysis, the sensitive dependence of the actuation dynamics on the thermal effect is reflected by the Poincaré maps in Fig. 6. The homoclinic orbit separates unstable motion (leading to stiction within one period for the autonomous system) from the periodic closed orbits around the stable center point. When the thermal effects decrease the Casimir force, as shown in Fig. 6, they produce a wider stable region (red elliptic in shape central area). In fact, for the Au-Au system, the stable area increases by increasing the temperature for both small and large separations, indicating a decreasing Casimir force due to thermal effects. The situation is the reverse for the SiC-SiC system where the stable area decreases by increasing temperature for both small and large separations indicating an increasing Casimir force due to thermal effects. Nevertheless, the positive benefit of good conductors toward an increased actuation stability due to thermal nonequilibrium effects on Casimir forces can be compromised even by a weak voltage application (or due to imperfect compensation of contact potentials) that leads to additional attractive forces. This behavior is depicted in Fig. 7, which shows that any application of voltage strongly reduces the size of the region enclosed by the homoclinic orbit and, consequently, the range of initial conditions that favor stable motion as the material conductivity increases.

B. Nonconservative actuating systems ($\epsilon = 1$)

Here we performed calculations to investigate the existence of the chaotic behavior in dissipative microsystems undergoing forced oscillation via an applied external force $F_0 \cos(\omega t)$.³⁴ The chaotic behavior occurs if the separatrix (homoclinic orbit) of the conservative system splits. The latter can be addressed by the so-called Melnikov function and Poincaré map analysis.^{34,35} In the driven system, the unstable equilibrium turns into an unstable periodic orbit. If we define the homoclinic solution of the conservative system as $\varphi_{\text{hom}}^{\text{C}}(T)$, then the Melnikov function for the torsional system ($\epsilon = 1$) is given by^{34,35}

$$M(T_0) = \frac{1}{\varphi} \int_{-\infty}^{+\infty} \left(\frac{d\varphi_{\text{hom}}^{\text{C}}(T)}{dT} \right)^2 dT + \frac{\tau_0}{\tau_{\text{res}}^{\text{MAX}}} \int_{-\infty}^{+\infty} \frac{d\varphi_{\text{hom}}^{\text{C}}(T)}{dT} \cos \left[\frac{\omega}{\omega_0} (T + T_0) \right] dT. \quad (7)$$

The separatrix splits if the Melnikov function has simple zeros so that $M(T_0) = 0$ and $M'(T_0) \neq 0$. If $M(T_0)$ has no zeros, the motion will not be chaotic. The conditions of nonsimple zeros, $M(T_0) = 0$

FIG. 9. Contour plot of the transient times to stiction using Poincare phase maps $d\lambda/dt$ vs λ for the nonconservative system. (a) Left column is for the Au-Au system with $\delta_{\text{Cas}} = 0.075$; (b) right column is for the SiC-SiC system with $\delta_{\text{Cas}} = 0.058$. For the other parameters, we used $\delta_v = 0$, $d = 1 \mu\text{m}$, $\alpha = 0.35$, and $\omega/\omega_0 = 0.5$. For both systems, we considered also the effect of zero-point fluctuations (F_0), and for the nonequilibrium effects, $T_1 = 300\text{K}$ and T_2 as indicated. For the calculations, we used 150×150 initial conditions ($\lambda, d\lambda/dt$). The central elliptical in shape (red) area shows the initial condition for which the device continues to follow stable motion after 100 oscillations.

and $M'(T_0) = 0$, give the threshold condition for chaotic motion.^{34,35} If we define

$$\mu_{\text{hom}}^c = \int_{-\infty}^{+\infty} \left(\frac{d\varphi_{\text{hom}}^c(T)}{dT} \right)^2 dT \text{ and } \beta(\omega) = \left| H \left[\text{Re} \left(F \left\{ \frac{d\varphi_{\text{hom}}^c(T)}{dT} \right\} \right) \right] \right|, \quad (8)$$

then the threshold condition for chaotic motion $\alpha = \beta(\omega)/\mu_{\text{hom}}^c$ with $\alpha = (1/Q)(F_0/F_{\text{res}}^{\text{MAX}})^{-1} = \gamma\omega_0 d/F_0$ obtains the form

$$\alpha = \frac{\gamma\omega_0 d}{F_0} = \left| H \left[\text{Re} \left(F \left\{ \frac{d\varphi_{\text{hom}}^c(T)}{dT} \right\} \right) \right] \right| / \int_{-\infty}^{+\infty} \left(\frac{d\varphi_{\text{hom}}^c(T)}{dT} \right)^2 dT, \quad (9)$$

with $\gamma = I\omega_0/Q$, and $H[\dots]$ denoting the Hilbert transform.^{34,35}

Figures 8 shows the threshold curves $\alpha = \gamma\omega_0 d/F_0$ vs the driving frequency ratio ω/ω_0 . For large values of α (above the curve), the dissipation dominates the driving force ($\alpha \sim \gamma/F_0$) leading to a

regular motion, which approaches asymptotically the stable periodic orbit of the conservative system. However, for parameter values below the curve, the splitting of the separatrix leads possibly to chaotic motion. Clearly, for the good conductive system Au-Au, the nonequilibrium thermal effects lead to the reduction of the area below the threshold curve, suggesting a reduction of the possibility for chaotic motion with increasing temperature. However, the inverse behavior takes place for the poor conducting SiC-SiC, where the area below the threshold curve increases due to the increment of the Casimir force. The latter indicates enhanced possibility for chaotic motion that increases with increasing temperature and/or strengthening of the nonequilibrium state. By comparing Figs. 8(a) and 8(b), it becomes obvious that by weakening the dielectric function the thermal part becomes stronger to change stable actuation to chaotic motion and, consequently, limit the long term device predictability.

Since we study the occurrence of chaotic motion in terms of the sensitive dependence of the motion on its initial conditions, we present in Fig. 9 the Poincare maps to show how the thermal correction can change the dynamical prediction toward chaos for both good and poor conductive systems. When chaotic motion occurs (with decreasing value of $\alpha \sim \gamma/F_0$), there is a region of initial conditions where the distinction between qualitatively different solutions is

FIG. 10. Contour plot of the transient times to stiction using Poincaré phase maps $d\lambda/dt$ vs λ for the nonconservative system. (a) Left column is for the Au-Au system with $\delta_{\text{Cas}} = 0.07$; (b) right column is for the SiC-SiC system with $\delta_{\text{Cas}} = 0.05$. For other parameters, we used $\delta_v = 0.02$, $d = 1 \mu\text{m}$, $\alpha = 0.75$, and $\omega/\omega_0 = 0.5$. For both systems, we considered also the effect of zero-point fluctuations (F_0), and for the nonequilibrium effects, $T_1 = 300 \text{ K}$ and T_2 as indicated. For the calculations, we used 150×150 initial conditions $(\lambda, d\lambda/dt)$. The central elliptical in shape (red) area shows the initial condition for which the device continues to follow stable motion after 100 oscillations.

unclear. For chaotic motion, there is no longer a simple smooth boundary between the elliptical-like stable area (red) and the outer area with unstable motion (blue region) toward stiction on the fixed plate. As a result, if the motion is chaotic then stiction could take place after several periods prohibiting the long term prediction of the actuation state for the device. As a result, the chaotic behavior introduces significant risk for stiction, and this is more prominent to occur when the Casimir force tends to increase due to the thermal effects.

As Fig. 9(a) shows for the Au-Au system, with increasing temperature the thermal contribution clearly suppresses chaotic motion leading to stiction of the moving plate on the fixed one, since the central elliptical-like (red) area increases in size. On the other hand, the inverse takes place for the SiC, where the increasing Casimir force with the thermal corrections leads to strong reduction of the stable actuation area and, therefore, to enhancement of the chaotic behavior and possibility to stiction. Therefore, any consideration of thermal effects on the actuation of Casimir oscillators must be performed in close synergy with the material optical properties. Nevertheless, the influence of additional attractive electrostatic forces by voltage application has to be carefully considered also for the driven dissipative systems. In fact, Fig. 10 demonstrates the sensitive dependence of chaotic motion on the applied electrostatic potential for both the Au and SiC systems. Again it is confirmed that any voltage application

will strongly influence the chaotic behavior of the system having a dramatic effect for the higher conductivity materials, where any positive effects due to the thermal contributions on the Casimir forces are highly compromised.

IV. CONCLUSIONS

In conclusion, we investigated thermal nonequilibrium effects on the actuation of an autonomous and a driven microswitch under the influence of Casimir forces between interacting materials. For the latter, we considered Casimir interactions between good and poor conductors. In both cases, the Drude model was used to extrapolate at low optical far-infrared frequencies, where measured optical data were not available. Furthermore, for the frictionless autonomous systems, the bifurcation and phase portrait analysis shows that the nonequilibrium conditions can have significant impact on the stable and unstable operating regimes depending on the material optical properties. Indeed, at separation below $4 \mu\text{m}$, for systems with the high conductivity materials, an increasing temperature difference between the actuating components can enhance the stable operating region. This is due to the reduction of the thermal effect on the Casimir force, while for the poor conductive materials the opposite takes place.

For a dissipative but periodically driven system, the Melnikov function and Poincaré portrait analysis have shown that for the poor conductive systems (e.g., SiC) the thermal nonequilibrium effects, which will increase the Casimir forces, lead to an increased possibility for chaotic motion and possible stiction with increasing temperature differences between the actuating components. Therefore, in this case, the thermal nonequilibrium effects could make impossible the long term prediction of the actuation state of a device with increasing temperature differences. However, for good conducting systems (e.g., Au-Au), the thermal fluctuations reduce the possibility of the chaotic behavior with increasing temperature in comparison to a system without thermal fluctuations. Nevertheless, the positive benefit of good conductors toward increased actuation stability and reduced chaotic behavior due to nonequilibrium Casimir forces can be easily compromised by any voltage application. Therefore, thermal nonequilibrium Casimir forces can influence the actuation of devices toward an unstable and chaotic behavior in strong correlation with their optical properties, and associated conduction state, as well as applied electrostatic potentials.

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APPENDIX A: BRIEF LIFSHITZ THEORY AND DIELECTRIC FUNCTION OF MATERIALS WITH EXTRAPOLATIONS

The equilibrium part of the Casimir force F_0 ($T = 0$ K) in Eq. (3) was calculated using the integral expression for the Lifshitz theory,¹¹

$$F_{\text{Cas}}(d) = \frac{\hbar}{2\pi^2} \int_0^\infty d\xi \int_0^\infty dk_\perp k_\perp k_0 \sum_{v=\text{TE, TM}} \frac{r_v^{(1)} r_v^{(2)} \exp(-2k_0 d)}{1 - r_v^{(1)} r_v^{(2)} \exp(-2k_0 d)}. \quad (\text{A1})$$

The Fresnel reflection coefficients are given by $r_{\text{TE}}^{(i)} = (k_0 - k_i)/(k_0 + k_i)$ and $r_{\text{TM}}^{(i)} = (\epsilon_i k_0 - \epsilon_0 k_i)/(\epsilon_i k_0 + \epsilon_0 k_i)$ for the transverse electric (TE) and magnetic (TM) field polarizations, respectively. k_i ($i = 0, 1, 2$) = $\sqrt{\epsilon_i(i\xi_i)(\xi^2/c^2) + k_\perp^2}$ represents the out-of-plane wave vector in the gap between the interacting plates (k_0) and in each of the interacting plates ($k_{i=(1,2)}$). $k_\perp$ is the in-plane wave vector.

Moreover, the function $\epsilon(i\xi)$ is the dielectric function evaluated at imaginary frequencies (ξ), which is the necessary input for calculating the Casimir force between real materials using the Lifshitz theory. The latter is given by¹¹

$$\epsilon(i\xi) = 1 + \frac{2}{\pi} \int_0^\infty \frac{\omega \epsilon''(\omega)}{\omega^2 + \xi^2} d\omega. \quad (\text{A2})$$

For the calculation of the integral in Eq. (A2), one needs the measured data for the imaginary part $\epsilon''(\omega) = \text{Im}[\epsilon(\omega)]$ of the frequency dependent dielectric function $\epsilon(\omega)$. The materials were optically characterized by ellipsometry over a wide range of frequencies at J. A. Woollam Co. using the VUV-VASE (0.5–9.34 eV) and IR-VASE (0.03–0.5 eV).^{17,26–28} In any case, the experimental data for the imaginary part $\epsilon''(\omega)$ of the dielectric function cover only a limiting range of frequencies $\omega_1 (= 0.03 \text{ eV}) < \omega < \omega_2 (= 8.9 \text{ eV})$. Therefore, for low optical frequencies ($\omega < \omega_1$), we extrapolated using the imaginary part of the Drude model,^{17,26–28}

$$\epsilon_L''(\omega) = \frac{\omega_p^2 \omega_\tau}{\omega (\omega^2 + \omega_\tau^2)}, \quad (\text{A3})$$

where ω_p is the plasma frequency, and ω_τ is the relaxation frequency. Furthermore, for the high optical frequencies ($\omega > \omega_2$), we extrapolated using^{17,26–28}

$$\epsilon_H''(\omega) = \frac{A}{\omega^3}. \quad (\text{A4})$$

Finally, using Eqs. (A2)–(A4), $\epsilon(i\xi)$ is given by

$$\epsilon(i\xi)_D = 1 + \frac{2}{\pi} \int_{\omega_1}^{\omega_2} \frac{\omega \epsilon''_{\text{exp}}(\omega)}{\omega^2 + \xi^2} d\omega + \Delta_L \epsilon(i\xi) + \Delta_H \epsilon(i\xi) \quad (\text{A5})$$

with

$$\begin{aligned} \Delta_L \epsilon(i\xi) &= \frac{2}{\pi} \int_0^{\omega_1} \frac{\omega \epsilon_L''(\omega)}{\omega^2 + \xi^2} d\omega \text{ and } \Delta_H \epsilon(i\xi) \\ &= \frac{2}{\pi} \int_{\omega_2}^\infty \frac{\omega \epsilon_H''(\omega)}{\omega^2 + \xi^2} d\omega. \end{aligned} \quad (\text{A6})$$

APPENDIX B: BRIEF THEORY OF THE NONEQUILIBRIUM CASIMIR FORCE

As it has been shown in Ref. 30, when both components of the actuating device are made from the same material, the nonequilibrium thermal force between them is given by

$$\begin{aligned} F_{\text{th}}^{\text{neq}}(T_1, T_2, z) &= \frac{1}{2} [F_{\text{th}}^{\text{eq,pw}}(T_1, 0, z) + F_{\text{th}}^{\text{eq,ew}}(T_1, 0, z)] \\ &+ \frac{1}{2} [F_{\text{th}}^{\text{eq,pw}}(0, T_2, z) + F_{\text{th}}^{\text{eq,ew}}(0, T_2, z)]. \end{aligned} \quad (\text{B1})$$

The contribution of the propagating waves in thermal equilibrium can be written as

$$\begin{aligned} F^{\text{eq,pw}}(T, z) &= \frac{-\hbar}{\pi^2} \int_0^\infty d\omega \frac{1}{\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1} \int_0^K dk_\perp k_\perp k_0 \\ &\times \sum_{v=s,p} \frac{\text{Re}(r_1^v r_2^v \exp(2id k_0)) - |r_1^v r_2^v|^2}{|D_v|^2}, \end{aligned} \quad (\text{B2})$$

while for the evanescent component, we have

$$\begin{aligned} F^{\text{eq,ew}}(T, z) &= \frac{\hbar}{\pi^2} \int_0^\infty d\omega \frac{1}{\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1} \int_K^\infty dk_\perp k_\perp \text{Im}(k_0) \\ &\times \exp(-2d \text{Im}(k_0)) \sum_{v=\text{TE,T}} \frac{\text{Im}(r_1^v r_2^v)}{|D_v|^2}. \end{aligned} \quad (\text{B3})$$

Equations (B1)–(B3) describe the force per unit area, with ω the real frequency, and $D_v = 1 - r_1^v r_2^v \exp(2ik_0 z)$. Both coefficients r_1^v and r_2^v are defined in Appendix A with $k_i (i = 0, 1, 2) = \sqrt{\epsilon_i(\omega)(\omega^2/c^2) + k_{\perp}^2}$ and $\epsilon(\omega) = \text{Re}[\epsilon(\omega)] + i\text{Im}[\epsilon(\omega)]$.

REFERENCES

- ¹A. W. Rodriguez, F. Capasso, and S. G. Johnson, "The Casimir effect in microstructured geometries," *Nat. Photonics* **5**, 211 (2011).
- ²F. Capasso, J. N. Munday, D. Iannuzzi, and H. B. Chan, "Casimir forces and quantum electrodynamical torques: Physics and nanomechanics," *IEEE J. Sel. Top. Quant. Electron.* **13**, 400 (2007).
- ³M. Bordag, G. L. Klimchitskaya, U. Mohideen, and V. M. Mostepanenko, *Advances in the Casimir Effect* (Oxford University Press, New York, 2009).
- ⁴R. S. Decca, D. López, E. Fischbach, G. L. Klimchitskaya, D. E. Krause, and V. M. Mostepanenko, "Precise comparison of theory and new experiment for the Casimir force leads to stronger constraints on thermal quantum effects and long-range interactions," *Ann. Phys.* **318**, 37 (2005); R. S. Decca, D. López, E. Fischbach, G. L. Klimchitskaya, D. E. Krause, and V. M. Mostepanenko "Tests of new physics from precise measurements of the Casimir pressure between two gold-coated plates," *Phys. Rev. D* **75**, 077101 (2007).
- ⁵A. Ashourvan, M. F. Miri, and R. Golestanian, "Noncontact rack and pinion powered by the lateral Casimir force," *Phys. Rev. Lett.* **98**, 140801 (2007).
- ⁶M. F. Miri and R. Golestanian, "A frustrated nanomechanical device powered by the lateral Casimir force," *Appl. Phys. Lett.* **92**, 113103 (2008).
- ⁷A. Ashourvan, M. F. Miri, and R. Golestanian, "Rectification of the lateral Casimir force in a vibrating noncontact rack and pinion," *Phys. Rev. E* **75**, 040103 (2007).
- ⁸F. M. Serry, D. Walliser, and G. J. Maclay, "The role of the casimir effect in the static deflection and stiction of membrane strips in microelectromechanical systems (MEMS)," *J. Appl. Phys.* **84**, 2501 (1998); F. M. Serry, D. Walliser, and G. J. Maclay, "The role of the Casimir effect in the static deflection and stiction of membrane strips in microelectromechanical systems (MEMS)," *J. Microelectromech. Syst.* **4**, 193 (1995); G. Palasantzas and J. T. M. DeHosson, "Phase maps of microelectromechanical switches in the presence of electrostatic and Casimir forces," *Phys. Rev. B* **72**, 121409 (2005); G. Palasantzas and J. T. M. DeHosson, "Pull-in characteristics of electromechanical switches in the presence of Casimir forces: Influence of self-affine surface roughness," *Phys. Rev. B* **72**, 115426 (2005).
- ⁹F. W. DelRio, M. P. de Boer, J. A. Knapp, E. D. Reedy, Jr., P. J. Clews, and M. L. Dunn, "The role of van der Waals forces in adhesion of micromachined surfaces," *Nat. Mater.* **4**, 629 (2005).
- ¹⁰H. B. G. Casimir, "Zero point energy effects on quantum electrodynamics," *Proc. K. Ned. Akad. Wet.* **51**, 793 (1948).
- ¹¹E. M. Lifshitz, "The theory of molecular attractive forces between solids," *Sov. Phys. JETP* **2**, 73 (1956); I. E. Dzyaloshinskii, E. M. Lifshitz, and L. P. Pitaevskii, "General theory of van der Waals forces," *Sov. Phys. Usp.* **4**, 153 (1961).
- ¹²P. Ball, "Fundamental physics: Feel the force," *Nature* **447**, 77 (2007).
- ¹³H. G. Craighead, "Nanoelectromechanical systems," *Science* **290**, 1532 (2000).
- ¹⁴F. Chen, G. L. Klimchitskaya, V. M. Mostepanenko, and U. Mohideen, "Demonstration of optically modulated dispersion forces," *Opt. Express* **15**, 4823 (2007); G. Torricelli, I. Pirozhenko, S. Thornton, A. Lambrecht, and C. Binns, "Casimir force between a metal and a semimetal," *Europhys. Lett.* **93**, 51001 (2011).
- ¹⁵S. de Man, K. Heeck, R. J. Wijngaarden, and D. Iannuzzi, "Halving the Casimir force with conductive oxides," *Phys. Rev. Lett.* **103**, 040402 (2009).
- ¹⁶G. Torricelli, P. J. van Zwol, O. Shpak, C. Binns, G. Palasantzas, B. J. Kooi, V. B. Svetovoy, and M. Wuttig, "Switching Casimir forces with phase-change materials," *Phys. Rev. A* **82**, 010101 (R) (2010).
- ¹⁷G. Torricelli, P. J. van Zwol, O. Shpak, G. Palasantzas, V. B. Svetovoy, C. Binns, B. J. Kooi, P. Jost, and M. Wuttig, "Casimir force contrast between amorphous and crystalline phases of AIST," *Adv. Funct. Mater.* **22**, 3729 (2012).
- ¹⁸C.-C. Chang, A. A. Banishev, G. L. Klimchitskaya, V. M. Mostepanenko, and U. Mohideen, "Reduction of the Casimir force from indium tin oxide film by UV treatment," *Phys. Rev. Lett.* **107**, 090403 (2011).
- ¹⁹V. B. Svetovoy, P. J. van Zwol, G. Palasantzas, and J. T. M. DeHosson, "Optical properties of gold films and the Casimir force," *Phys. Rev. B* **77**, 035439 (2008); G. Bimonte, "Making precise predictions of the Casimir force between metallic plates via a weighted Kramers-Kronig transform," *Phys. Rev. A* **83**, 042109 (2011).
- ²⁰A. Canaguier-Durand, P. A. Maia Neto, A. Lambrecht, and S. Reynaud, "Thermal Casimir effect for drude metals in the plane-sphere geometry," *Phys. Rev. A* **82**, 012511 (2010).
- ²¹F. Tajik, M. Sedighi, M. Khorrami, A. A. Masoudi, and G. Palasantzas, "Chaotic behavior in Casimir oscillators: A case study for phase-change materials," *Phys. Rev. E* **96**, 042215 (2017); F. Tajik, M. Sedighi, and G. Palasantzas, "Sensitivity on materials optical properties of single beam torsional Casimir actuation," *J. Appl. Phys.* **121**, 174302 (2017).
- ²²F. Tajik, M. Sedighi, M. Khorrami, A. A. Masoudi, H. Waalkens, and G. Palasantzas, "Dependence of chaotic behavior on optical properties and electrostatic effects in double-beam torsional Casimir actuation," *Phys. Rev. E* **98**, 02210 (2018).
- ²³J. A. Pelesko and D. H. Bernstein, *Modeling MEMS and NEMS* (Chapman & Hall/CRC, Boca Raton, FL, 2003).
- ²⁴P. M. Osterberg, "Electrostatically actuated microelectromechanical test structures for material property measurement," Ph.D. thesis (MIT, 1995).
- ²⁵O. Bochobza-Degani and Y. Nemirowsky, "Modeling the pull-in parameters of electrostatic actuators with a novel lumped two degrees of freedom pull-in model," *Sens. Actuators A* **97**, 569 (2002).
- ²⁶M. Sedighi, W. H. Broer, G. Palasantzas, and B. J. Kooi, "Sensitivity of micromechanical actuation on amorphous to crystalline phase transformations under the influence of Casimir forces," *Phys. Rev. B* **88**, 165423 (2013).
- ²⁷M. Sedighi and G. Palasantzas, "Casimir and hydrodynamic force influence on microelectromechanical system actuation in ambient conditions," *Appl. Phys. Lett.* **104**, 074108 (2014).
- ²⁸M. Sedighi and G. Palasantzas, "Influence of low optical frequencies on actuation dynamics of microelectromechanical systems via Casimir forces," *J. Appl. Phys.* **117**, 144901 (2015).
- ²⁹M. Sedighi, V. B. Svetovoy, W. H. Broer, and G. Palasantzas, "Casimir forces from conductive silicon carbide surfaces," *Phys. Rev. B* **89**, 195440 (2014).
- ³⁰G. L. Klimchitskaya, V. M. Mostepanenko, and R. I. P. Sedmik, "Casimir pressure between metallic plates out of thermal equilibrium: Proposed test for the relaxation properties of free electrons," *Phys. Rev. A* **100**, 022511 (2019).
- ³¹M. Antezza, L. P. Pitaevskii, S. Stringari, and V. B. Svetovoy, "Casimir-Lifshitz force out of thermal equilibrium," *Phys. Rev. A* **77**, 022901 (2008).
- ³²R. Garcia and R. Perez, "Dynamic atomic force microscopy methods," *Surf. Sci. Rep.* **47**, 197 (2002); M. Li, H. X. Tang, and M. L. Roukes, "Ultra-sensitive NEMS-based cantilevers for sensing, scanned probe and very high-frequency applications," *Nat. Nanotechnol.* **2**, 114 (2007); D. Rugar, R. Budakian, H. J. Mamin, and B. W. Chui, "Single spin detection by magnetic resonance force microscopy," *Nature* **430**, 329 (2004).
- ³³O. Degani and Y. Nemirowsky, "Design considerations of rectangular electrostatic torsion actuators with rectangular plates based on analytical pull-in expressions," *J. Microelectromech. Syst.* **11**, 20 (2002).
- ³⁴R. S. Decca, D. López, H. B. Chan, E. Fischbach, D. E. Krause, and C. R. Jamell, "Constraining new forces in the Casimir regime using the isoelectronic technique," *Phys. Rev. Lett.* **94**, 240401 (2005); R. S. Decca, D. López, E. Fischbach, G. L. Klimchitskaya, D. E. Krause, and V. M. Mostepanenko, "Novel constraints on light elementary particles and extra-dimensional physics from the Casimir effect," *Eur. Phys. J. C* **51**, 963 (2007); V. A. Yampol'skii, S. Savel'ev, Z. A. Mayselis, S. S. Apostolov, and S. Nori, "Anomalous temperature dependence of the Casimir force for thin metal films," *Phys. Rev. Lett.* **101**, 096803 (2008); I. Brevik, S. A. Ellingsen, and K. A. Milton, "Thermal corrections to the Casimir effect," *New J. Phys.* **8**, 236 (2006).
- ³⁵W. Broer, H. Waalkens, V. B. Svetovoy, J. Knoester, and G. Palasantzas, "Nonlinear actuation dynamics of driven Casimir oscillators with rough surfaces," *Phys. Rev. Appl.* **4**, 054016 (2015).
- ³⁶M. W. Hirsch, S. Smale, and R. L. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos* (Elsevier Academic Press, San Diego, CA, 2004).