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Optimal pricing for ride-sourcing platforms

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Title Page: Optimal pricing for ride-sourcing platforms

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Abstract

Online car hailing platforms are rapidly gaining popularity. Unlike most two-sided markets, these platforms have pricing power. The price for a specific customer ride request affects the number of interested drivers and the likelihood that a customer will accept a selected driver (and not opt for a regular taxi service). This study determines the optimal pricing strategy for online car hailing platforms, taking both ride details and driver location into account, and assuming that drivers and customers maximize utility. We do so for two types of driver selection: first to respond or closest to the customer. Under selection of the first driver to respond, we find that the platform price consists of a ride length based fare (set relative to the competing regular taxi fare) and a rush hour congestion fee, and increases with the customer waiting cost. Furthermore, the platform price is below the regular taxi fare if traffic conditions are good, drivers have low profit expectations, and the platform commission is low. We also discuss the effects on price and profit if the platform switches from first-to-respond to selecting the closest driver, which the popular Didi Chuxing platform has recently done in many Chinese cities. A numerical study based on the Beijing market further illustrates the findings.

Keywords Transportation; Online car hailing platforms; Two-sided market; Sharing economy.

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1. Introduction

Two-sided markets for sharing economies/platforms are gaining popularity with the development of internet technology. This has also transformed the taxi market, with the development of taxi hailing platforms (only taxi drivers involved) and ride-sourcing platforms (any licensed car owner can participate). Online car hailing platforms such as Uber, Lyft, and Didi Chuxing can match customers to drivers quickly and are rapidly increasing their market shares (Agatz et al., 2012). In recent years, Didi Chuxing has started to change its operating mode in some cities, from acting purely as an information platform and selecting/ connecting the first driver to respond to a consumer request, to the mode of selecting/ assigning the closest driver. Doing so can obviously reduce waiting times and, as our research will show, allow a platform to charge higher prices and thereby increase profits for drivers as well as for the platform. However, the mode change has also led to complaints by drivers who see the first-to-respond process as fairer (Wang et al., 2016) and prefer the freedom

to choose their own ride direction and/or customer (Sina, 2017a; Sina, 2017b). In this research, we analyse the pricing strategy differences between the two modes and the corresponding profit differences.

Pricing is a more complex task for online car hailing platforms than for most other two-sided markets, since locations of customers and vacant drivers affect waiting/driving times and thereby the cost for customers and the profit for drivers. So far, this has been largely ignored in the literature on car hailing platform pricing. Most authors do not consider the spatial impacts of customers' and drivers' locations at all. They restrict their analysis to a ride of average distance in an average zone, as will be explained in further detail in the next section. Some authors do consider multiple zones and the effect of taxis moving between zones, but they still consider a ride of average distance per zone. In other words, research so far has considered average (zone related) pricing rather than the 'per-service' pricing rule used in real life that takes ride details into account. Related, the existing literature does not address how a driver's specific location affects the likelihood that a customer will reject an offer, which platforms such as Didi Chuxing do allow.

This study derives the optimal per-service pricing strategy for online car hailing platforms, under both discussed driver selection modes, by considering the locations of customers and drivers. We consider a ride-sourcing platform, which mimics the way that Didi Chuxing operates. A customer puts in a request that specifies the departure point and the destination. The platform then sets the price and selects a driver using either of two discussed modes. As the final step of the matching process, upon receiving the ride price and driver details, a customer decides whether or not to accept the deal. We remark that although we determine the price per-service, the resulting optimal pricing rule can in practice be communicated to drivers and customers in advance, as many platforms including Didi Chuxing indeed do.

A key finding is that, when selecting the first driver to respond, the optimal price is such that the maximum acceptable customer-driver distance is the same for both drivers and customers. Although intuitive, this is not straightforward. A higher price for a certain request would increase the potential profit, but it turns out that this comes at a too high risk of losing the customer. A sensitivity analysis reveals further insights into the effects of model parameters on the optimal price and profit for the platform. Moreover, we quantify the added profit from selecting the closest driver rather than the first to respond. A numerical study based on the Beijing market illustrates the findings. Furthermore, we compare the current pricing strategy of Didi Chuxing with the optimal strategy for our model. It turns out that they have the same core structure with a base fare based on the ride length and a rush hour congestion fee.

The remainder of this paper is organized as follows. In Section 2, we review the related literature. In Section 3, we present the model. In Section 4, we derive the platform's optimal pricing strategy when selecting the first driver to respond. A sensitivity analysis is conducted in Section 5 and we further provide the platform optimum pricing strategy considering competition with the regular taxi market. Section 6 considers the effects of selecting the closest driver rather than the first to respond. Finally, section 7 provides a conclusion.

2. Related literature

We first discuss important results on two-sided markets in general. Then we move to online car hailing platforms, focusing on matching supply and demand and on pricing strategies. Finally, we point out our key contributions.

2.1. Two-sided markets

In 2004, Rochet and Tirole first introduced clear definitions of two-sided market and identified differences compared with other markets. According to Rochet and Tirole (2004), two-sided markets refer to “any change in the price share between two groups affect the volume of transactions”. During the first few years, they were mostly studied in the credit card and telecom market, mainly focusing on: pricing policy (Rochet and Tirole, 2003; Armstrong, 2006; Rochet and Tirole, 2006), network effects (Caillaud and Jullien, 2003; Rochet and Tirole, 2003), and competition (Gabszewicz and Wauthy, 2004; Chakravorti and Roson, 2006). Two-sided market theory then expanded to different fields, including the media industry (Anderson and Gabszewicz, 2006; Kind and Stähler, 2010), e-commerce (Gaudeul and Jullien, 2007), online games (Parker and Alstyne, 2005), and transportation (Furuhata et al., 2013; Wang et al., 2016; Djavadian and Chow, 2017).

An important distinction in the literature on two-sided transport markets is that between taxi-hailing platforms (He and Shen, 2015; Wang et al., 2016; Qian and Ukkusuri, 2017a) and ride-sourcing platforms (Zha, Yin, and Yang, 2016; Zha, Yin, and Du, 2017). The key difference is that ride-sourcing platforms have pricing power, whilst taxi-hailing platforms do not (as cab companies decide on the price).

2.2. Matching function

Many contributions on online car hailing centre on matching functions. Such functions result from equilibrium models for supply and demand (Yang, Wong, and Wong, 2002). They were originally developed for the traditional taxi market (Yang and Yang, 2011), where drivers and customers encountering each other by chance and the matching function specifies how the matching rate (the number of meetings) depends on the number of vacant drivers and customers during a period of time (i.e., 1 hour). He and Shen (2015) were the first to explore matching functions for taxi-hailing platforms. They compared customer choice and taxi movements for e-hailing and offline hailing. Zha, Yin, and Yang (2016) extended the work of Yang and Yang (2011) to ride-sourcing platforms, showing that controlling the commission charged by the platform, rather than trip fare or fleet size, is crucial to achieving an efficient solution. Wang et al. (2016) further investigated the effects of commissions or subsidies (i.e., price reductions) on platform profit and social welfare. Zha, Yin, and Du (2017) studied the effects of surge pricing on ride-sourcing platforms under different behavioural assumptions of labour supply. It is important to stress, however, that this line of research does not consider the complete pricing strategy, because the equilibrium models take all rides as homogenous products and so do not differentiate based on the length of a ride or the driver's location. Related, the matching process was assumed to be completed with the selection of a driver, ignoring the possibility that a customer may reject a deal.

Many researchers modelled the matching process between drivers and customers as an unobservable queue (Banerjee, Johari, and Riquelme, 2015; Shi and Lian, 2016; Bai et al., 2018; Hu and Zhou, 2018b; Taylor, 2018), which also takes all rides as homogenous products and does not differentiate based on the length of a ride or the driver's location. Generally, they assumed that customers arrived in a Poisson process, and drivers can be viewed as servers in a queueing system.

Some researchers have looked at matching with heterogeneous demand, but they do not consider pricing strategies. For instance, Hu and Zhou (2018a) considered matching (for transportation but also more generally) during time periods, where the rewards for each potential matched pair are given and costs are incurred at the end of each period unmatched supply and demand. Feng, Kong, and Wang (2017) compared the matching efficiency in terms of average waiting time of online demand matching (nearest idle taxi) vs. street hailing (first taxi passing by), for a stylized system with one circular road.

2.3. Pricing strategy

We next discuss studies that are primarily focused on pricing, as our study is. Most of the literature so far has taken a purely time-based view in deriving a platform's optimal price (Banerjee, Johari, and Riquelme, 2015; Cachon, Daniels, and Lobel, 2017; Castillo, Knoepfle, and Weyl, 2017; Hu and Zhou, 2018b; Taylor, 2018; Bai et al., 2018; Gurvich, Lariviere, and Moreno, 2019). Some derived the per-service optimal price and wage by considering the impacts of customers and drivers behaviours, such as customer waiting-time sensitivity (Bai et al., 2018; Taylor, 2018) and driver idle-time sensitivity (Taylor, 2018). Hu and Zhou (2018b) investigated the impacts of a constant commission rate on the per-service optimal price and wage. Banerjee, Johari, and Riquelme (2015) found that no dynamic pricing policy can perform better than the optimal static pricing policy if platforms do not set different prices. Cachon, Daniels, and Lobel (2017) compared the impacts of static pricing and dynamic pricing on all participants (i.e., customers, drivers, and platforms), and found that surge pricing is relatively better for customers if the lack of dynamic prices and wages leads to poor service for customers in high demand periods.

Some studies have (also) considered the influence of space on price, exploring whether pricing can help balance supply and demand over time by influencing providing taxi drivers an incentive to move to zones where demand is larger than supply. Bimpikis et al. (2016) were the first to do so. They showed that differentiating the price based on customer location can indeed increase profits for drivers and the platform as well as the consumer surplus. Their main analysis assumes equidistant locations, implying that the ride distance is fixed. They also shortly discussed varying ride distances, but restrict the ride price to be proportional to the ride distance. Guda and Subramanian (2019) considered the strategic interaction amongst drivers in their decisions to move between zones, and found that surge pricing can be profitable even in a zone where the supply of drivers exceeds demand. However, they use a constant (regular or surge) price per zone, independent of the ride details. Zha, Yin, and Xu (2018) divided an urban region into different geographic zones and further investigated the optimal price in each zone under demand surges. They too do not consider ride details but rather focus on an average ride with a corresponding average fare.

The effects of competition and regulations in car hailing on pricing have also been considered. Zha, Yin, and Yang (2016) studied competition between multiple ride-sourcing platforms, and found that competition does not necessarily lower the price level or improve social welfare. Qian and Ukkusuri (2017a) modelled the taxi market as a multi-leader-follower game, and studied the equilibrium under competition between offline hailing and online hailing in the car hailing market. They found that the fleet size and pricing policy are related

to the competition level in the car hailing market, and have an impact on the total passenger cost, average waiting time, and fleet utilization. Posen (2015) concluded that regulators should take on flexible, experimental regulations for online car hailing platforms and focus on passenger safety and experience, rather than using already-existing regulations, until the market becomes more mature.

2.4. Contribution

As is transparent from our literature review, so far pricing strategies have mainly been developed for a ride of average length with the objective to match the supply and demand rate. Most of those studies provide insights into how price differentiation helps to balance demand and supply over time, and to a lesser extent it has been shown that zone based pricing can help redistribute taxis over a larger area.

We consider pricing from a different and arguably more realistic perspective. Rather than assuming that all rides are equal and focusing on balancing demand and supply over time, we derive the optimal price for a specific service request (at a specific time) based on the ride distance. We do not directly model competition, but do consider the platform pricing strategy relative to that of a regular taxi service based on a fixed fee and distance related component. Drivers and consumers only accept a ride if their net utility from using the platform is positive. By considering both selection of the first driver to respond and selection of the closest driver, we are able to derive comparative results.

This leads to a number of new insights into per-service platform pricing. First, under both considered selection rules and in line with practice, the platform price is linear in the ride length, contains a rush hour congestion fee, and increases with the offline driver profit expectations and the platform commission. Second, switching from selecting the first driver to respond to the driver closest to the customer, as Didi Chuxing has recently done, increases platform and driver profits, but ultimately hurts the consumer despite the reduced waiting cost. Third, based on data collected for the Beijing car hailing market, we find that the optimal prices resulting from our model are overall close to Didi Chuxing's current prices, and have the same core structure with a base fare based on the ride length and a rush hour congestion fee.

3. Model

The e-hailing process begins with a service request by a customer. After specifying the departure point and destination on the app, the customer is informed about the price. We remark that in practice, apps such as Didi Chuxing provide an estimated price based on route navigation calculations and a traffic condition forecast. The realized transaction price is based on the real ride length and real-time traffic condition. Our model does not consider the possible deviation between the estimated and realized.

After the driver is informed about the price, the platform selects a driver, and we consider two methods for doing so. Under the first to respond method, which we consider initially, the details of a customer request (departure point and destination) are sent out to nearby drivers through devices, mostly individual mobiles in practice. In turn, the details of the first driver to accept the request (position, plate and mobile number) are sent to the customer. The alternative method is for the platform to directly select the closest available driver, which requires that all driver locations and availability are known centrally.

We remark that Didi Chuxing has gradually switched from the former to the latter selection mode. Under either mode, after receiving the driver details, the customer either acquiesces and waits time until the driver arrives, or aborts the hailing process and the customer is lost (and hails a regular taxi). We do not consider a possibly earlier abortion when a customer is informed about the estimated price, since that is typically based on a distance and traffic dependent pricing system known to (regular) customers. The waiting time is the unknown factor that drives the accept/reject decision.

We do not fix the size of the considered area or the associated number of drivers up front. Indeed, as is intuitive and will appear from the analysis, the pricing strategy controls the size of the area where drivers are interested. To ensure the relevance of our analysis, we assume that (for the customer request considered) a match can indeed be made, i.e., that some price exists for which at least one driver is interested who is acceptable to the customer.

Behaviour of customers

In practice, besides offline and online taxi's, customers may have other alternatives for getting to their destination, i.e., by bus, tram, subway, bike-sharing, car-sharing. However, for most potential e-hailing customers the offline taxi service is the most 'natural' alternative, since consumers who consider taxi services typically prefer a 'door-to-door' service and want to avoid the typically larger waiting times of other services, despite the potential savings. In our analysis, we will therefore consider the offline taxi service as the only alternative for e-hailing, and determine e-hailing taxi fares relative to offline taxi fares. However, as we will indicate where relevant, a similar analysis applies to other alternative transport services. We opt not to consider multiple alternatives to e-hailing at the same time, since this would be more complex and less insightful.

The customer only accepts a price and driver combination if the associated total travel cost, including a waiting time cost, is at most equal to the expected total ride cost of the offline taxi service. We assume that the (expected) travel time is the same for both options and hence ignore it for comparing costs. After deriving the cost expressions for both options, we present the customer decision rule.

- E-hailing

The waiting time equals r_i/v , where r_i denotes the distance from driver i to the customer departure point and v the average speed of driver per hour. Thus, to the total e-hailing cost is

$$C_E = F + \tau \cdot \frac{r_i}{v}, \quad (1)$$

where F is the price for a ride; and τ is the waiting cost per time unit.

- Offline

Most regular taxi service charge a minimum (setup) cost and an additional cost per kilometre above a certain minimum (Qian and Ukkusuri, 2017b). Also including the waiting cost, we obtain the offline total ride cost as

$$C_O = [a + k \cdot (l - m, 0)^+] + \tau \cdot \alpha^T, \quad (2)$$

where a is starting taxi fare; k is additional regular taxi price per kilometre after the first m kilometres; l is length from departure point to destination; m is the maximum distance covered by the starting regular taxi fare; and α^T is the offline customer average waiting time. We remark that the cost parameters as well as the average waiting time α^T can vary based on e.g., the day of the week, time of the day, the customer departure location, and taxi driver availability. In the analysis that follows, we assume that the average offline

waiting time (α^T) is finite. This applies as long as some offline taxis are present in a considered area, which applies in most practical situations as a conventional and universal transport mode. As we focus on a specific ride request in our per-service analysis, all parameters in (2) are considered to be fixed. However, it is important to interpret the optimal platform pricing relative to offline pricing (under the same conditions), which we will do.

Note from (2) that the offline total ride cost tends to infinity if the offline waiting time α^T grows very large. Relating to the previous discussion on other alternatives, in such situations with very low availability of offline taxi's and correspondingly very large waiting times, another alternative such as the tram or subway will be preferable (if available). In such situations, by re-interpreting and re-estimating the parameters in (2) for the other transport mode, that equation and the analysis that will follow still apply.

Note also from (1) and (2) that we consider the waiting cost to increase linearly with the waiting time. Although this is a natural starting point for our exploratory analysis, there may also be situations in practice where customers become increasingly impatient as they wait for a taxi, which would imply that waiting costs are strictly convex rather than linear in the waiting time. In Appendix C, we indeed consider such situations by assuming a quadratic relation and we will refer to it when discussing the generality of our findings. We remark that more general relations of waiting cost and waiting time can also be considered, but this would complicate the analysis. Indeed, that is why we assume a linear relation in our main analysis, which allows for an insightful analysis and meaningful results.

From the above expressions and assuming that a rational customer will only switch to the offline service if $C_E \leq C_O$, we easily get that the customer will accept an e-hailing fare F if (and only if) the distance to the selected driver is at most

$$M_c = v \cdot \left\{ \frac{[a + k \cdot (l - m, 0)^+] - F}{\tau} + \alpha^T \right\}. \quad (3)$$

Behaviour of drivers

In line with Zha, Yin, and Xu (2018), we assume that drivers are interested in a ride if the profit per time unit is above some opportunity threshold. The value of p is likely to be influenced by many factors, including: a driver's historical average profit; and a driver's behaviour regarding working time, working length, frequency, etc. These factors are all exogenous to our model, and we therefore assume that p is given. In a later section, however, we will discuss how its value affects the optimal platform pricing strategy.

A driver indicated that he is willing to accept a ride if the expected profit per time unit is at least p . Letting f denotes the fraction of the ride price that is charged by the platform for matching the driver to a customer, a driver earns ride profit $(1 - f)F$ during $(r_i + l)/v$ time units, and so driver i is interested only if

$$(1 - f)Fv \geq p(r_i + l).$$

It easily follows that the maximum distance to a potential customer for a driver to accept a ride equals

$$M_d = \left\lceil v \cdot \frac{(1 - f)F}{p} \right\rceil - l. \quad (4)$$

Distribution of driver-customer distance

We assume that drivers are uniformly distributed over any considered area. For a circular area with radius R and a customer at the centre, this implies that the probability density function for the distance r of any driver to the customer is (Alouini and Goldsmith, 1999)

$$f_r(r) = \frac{2r}{R^2}, 0 \leq r \leq R, \quad (5)$$

and the corresponding probability distribution function of distance is

$$F_r(r) = \frac{r^2}{R^2}, 0 \leq r \leq R. \quad (6)$$

We remark that, as will also appear from the analysis, the distribution of drivers over the area affects the expected distance from the selected driver to the customer. Therefore, the exact distribution needs to be specified. However, for distributions other than uniform, the analysis presented would still serve as an approximation. Moreover, assuming that drivers are uniformly distributed and therefore randomly scattered over the considered area seems natural, and in practice it would be very difficult to estimate the exact distribution. Table 1 lists the notations that have been introduced in this section.

l	length from departure point to destination
v	average speed of driver per hour
r_i	the distance from driver i to the customer departure point
τ	the waiting cost per time unit
F	price for a ride
f	fraction of the price that is earned by the platform
p	minimum profit per time unit that a driver aims for
α^T	the offline customer average waiting time
a	starting taxi fare
m	the maximum distance covered by the starting regular taxi fare
k	additional regular taxi price per kilometre after the first m kilometres

Table 1. Notations.

4. Platform's optimal pricing strategy under selecting the first driver to respond

In this section, we derive the optimal pricing strategy when the platform selects the first driver to respond. In Section 4.1, we graphically present the relevant price range where both the customer and drivers are potentially interested. Then, in Section 4.2, we derive the expected profit function and the optimum price.

4.1. Relevant price range

Obviously, the platform does not make any profit if either M_d or M_c is negative, because otherwise either no driver would respond or no selection driver would be accepted by the customer. It is also clear and intuitive that M_d increases with the price and M_c decreases with the price.

From (4), we find that $M_d \geq 0$ if $F \geq \frac{lp}{(1-f)v}$. So, the minimum price to ensure that drivers are interested is $F_m = \frac{lp}{(1-f)v}$. From (3), we find that $M_c \geq 0$ if $F \leq [a + k \cdot (l - m, 0)^+] + \tau \cdot \alpha^T$. So, the maximum distance acceptable to a customer is $F_M = [a + k \cdot (l - m, 0)^+] + \tau \cdot \alpha^T$. Obviously, a customer is always lost and no profit can be made when $F_m > F_M$. Therefore, we can restrict our attention to the case where $F_m \leq F_M$, where a match between a driver and customer is possible, so that a profit can be made. In Figure 1, we display the maximum acceptable distances for both customers and drivers for that case. We denote the price at which these lines cross, i.e., where $M_c = M_d$, by F_R . Using (3) and (4) we find that

$$F_R = \frac{v \cdot \left\{ \frac{[a + k \cdot (l - m, 0)^+]}{\tau} + \alpha^T \right\} + l}{v \cdot \left[\frac{(1-f)}{p} + \frac{1}{\tau} \right]}. \quad (7)$$

Fig. 1. Maximum acceptable distance of customer and drivers as a function of ride price F .

As is apparent from Fig. 1, all interested drivers are acceptable for the customer for $F \in [F_m, F_R]$ and only a fraction are acceptable for $F \in [F_R, F_M]$. Recall from the basic model description in Section 3, that we assume a match can be made, i.e., that there is some price at which at least one driver is available who is acceptable for the customer. If that applies to some price in the range $[F_m, F_R]$, then it must also apply to the specific price F_R , as any driver interested in a price below F_R remains interested at price F_R and all interested drivers are acceptable to the customer over this range. Similarly, if some driver is interested and acceptable to the customer at some price in the range $[F_R, F_M]$, then this driver obviously remains acceptable to the customer at the lower price F_R and the driver also remains interested as all customers are acceptable over this range. Therefore, there is at least one interested driver at price F_R that is acceptable to the customer. Next, we show that this is also the optimal price.

4.2. Expected profit function and optimum price

For $F \in [F_m, F_R]$, all responding drivers are acceptable to the customer and so the expected platform profit function $g(F)$ is given by

$$g(F) = f \cdot F \cdot \Pr\{\text{at least one driver is interested at price } F\}.$$

Clearly, this function is increasing in the price and so $F < F_R$ cannot be optimal.

For $F \in [F_R, F_M]$, not all responding drivers are acceptable to the customer and so the analysis is more complex. The request can be fulfilled if (and only if) the responding driver is within customer's acceptable

distance. Since all drivers in the circular area around the customer with range M_d are interested and only those within distance M_c are acceptable to the customer, we can combine (3), (4) and (6) to write the expected platform profit function $g(F)$ as

$$g(F) = f \cdot F \cdot \frac{v^2 \cdot \left\{ \frac{[a + k \cdot (l - m, 0)^+] - F}{\tau} + \alpha^T \right\}^2}{\left(\left\lfloor \frac{v \cdot (1 - f)F}{p} \right\rfloor - l \right)^2}. \quad (8)$$

By taking the derivative of (8) with respect to F , we get

$$\frac{dg(F)}{dF} = \frac{f \cdot v^2 \left\{ \frac{[a + k \cdot (l - m, 0)^+] - F}{\tau} + \alpha^T \right\}^2}{\left(\left\lfloor \frac{v \cdot (f - 1)F}{p} \right\rfloor - l \right)^2} \cdot \frac{F_M F - 2F \cdot F_m + 2F^2 + F_M F_m}{F_M \cdot (F_m - F)} < 0.$$

So $F > F_R$ cannot be optimal.

Therefore, $F = F_R$ enables the platform to reach the highest profit. So the optimal price is the largest price for which all interested drivers are acceptable to a customer. As it turns out, the potentially added profit of setting a higher price does not outweigh the risk of losing a customer because of the increased price and increased maximum distance of the first driver to respond.

5. Sensitivity analysis

To evaluate how model parameters affect the optimal price and profit for platform, a partial-derivative based sensitivity analysis is conducted in this section. Table 2 presents the signs of the derivatives. The proofs are given in Appendix A.

parameter	first partial derivative of F_R	first partial derivative of $g(F_R)$	second partial derivative of F_R	second partial derivative of $g(F_R)$
l	+	+	0	0
p	+	+	−	−
v	−	−	+	+
α^T	+	+	0	0
τ	+*	+*	−*	−*
a	+	+	0	0
k	+	+	0	0
m	−	−	0	0
f	+	+	+	+

* If $[a + k \cdot (l - m, 0)^+] \cdot (1 - f) < p \left(\frac{l}{v} + \alpha^T \right)$.

Table 2. The results of sensitivity analysis.

Rather than discuss all analytical sensitivity results separately, we combine them into a theorem on how the price is affected by ride length, traffic conditions, cost of waiting, regular taxi fare, and platform commission.

Theorem 1. *On the sensitivities of price and platform profit, we have the following results.*

- a. (ride length).** *A longer ride length ($l \uparrow$) implies a higher optimal price and profit.*
- b. (traffic conditions).** *During rush hour, with higher expected driver's earnings per hour ($p \uparrow$), a lower driving speed ($v \downarrow$) and a longer offline average waiting time ($\alpha^T \uparrow$), the platform's optimal price and profit increase. The changes in price and profit increase with the expected earnings, decrease with the driving speed, and are constant in the average offline waiting time.*
- c. (cost of waiting).** *If $(a + k \cdot (l - m, 0)^+)(1 - f) < p \left(\frac{l}{v} + \alpha^T \right)$, then the optimal price and platform's profit increase in the waiting cost per time unit ($\tau \uparrow$). However, a higher waiting cost per time unit reduces the growth rate of price and profit.*
- d. (regular taxi fare).** *If a regular taxi ride becomes more expensive ($a \uparrow, k \uparrow, m \downarrow$), then the optimal price and platform's profit increase.*
- e. (platform commission)** *If the platform charges a higher commission ($f \uparrow$), then the optimal price and platform's profit increase. Moreover, these increases accelerate with a higher commission.*

The intuition for these findings is as follows.

a. (ride length). It is obviously expected that a longer ride is more expensive, and this indeed applies to the platform price as it does to the regular taxi fare.

b. (traffic conditions). During rush hour, drivers expect to earn more and customers are willing to pay a higher price as the expected waiting time for getting a regular taxi also increases. As a result, the platform charges a higher price and also makes more profit per ride. In other words, it is optimal for platforms to charge a congestion fee during rush hour as part of their pricing strategy. An absolute decrease of speed or increase of earnings has a relatively larger effect for smaller speed levels and smaller earnings levels, and so the increases in price and profit from a speed reduction or from higher expected earnings are largest under those conditions.

c. (cost of waiting). Note that the condition for this sensitivity result holds under rush hour conditions as described in Theorem 1(b) (low speed v , long waiting time α^T , high expected earnings p). If a passenger has a high cost of waiting, then she is willing to pay a higher price for an online taxi. So, the platform will charge a higher price and earn more profit. However, a higher price also attracts drivers from further away, which increases the average waiting time of e-hailing (compared to the offline taxi service). This explains why the growth rates of price and profit slow down with an increasing waiting cost per time unit.

The cost of waiting is also related to the level of emergency for the customer. If a customer request is more urgent, then she is willing to pay a higher price. This is in line with previous research on two-sided markets, where urgency is sometimes ‘exploited’ by offering a faster urgency response option at an additional emergency fee to customers. Food delivery platforms such as Meituan, for instance, offer such an option to their customers. In our model and in the real life transport platforms that we know of, such a strategy is not considered.

d. (regular taxi fare). As expected, the platforms’ price and thereby the profit increases with the competing regular taxi’s price.

e. (platform commission). If the platform’s percentage commission increases, then a higher price is needed to keep (enough) drivers interested. Therefore, the optimal price increases with the platform’s commission. Moreover, as the relative effect of an absolute increase in the commission (f) on the driver compensation ($1 - f$) increases with f , the increase in price is larger if the commission is (already) larger. It is important to keep in mind, though, that we do not consider (long term) effects of the commission (and related pricing decisions) on the number of customers and drivers that sign up to a network.

Reflecting on the sensitivity results and their interpretations, we find that the platform price consists of a base fare based on the ride length (a) and a rush hour congestion fee (b), increases with the customer (emergency) cost of waiting for a regular taxi (c), is set relative to the competing regular taxi fare (d), and increases with the platform commission (e).

5.1. Price comparison between online and offline

We know from the results so far that, as expected, the platform price increases with that of a regular taxi price. Indeed, the difference rather than the two separate prices affects the decision of a customer whether or not to accept a ride (beside other issues such as the selected driver's location). In this section, we explore the price difference further.

By comparing F_R with taxi fare $a + k \cdot (l - m, 0)^+$, we find that the difference can be expressed as:

$$\begin{aligned} F_R - [a + k \cdot (l - m)] &= \frac{v \cdot \left\{ \frac{[a + k \cdot (l - m, 0)^+]}{\tau} + \alpha^T \right\} + l}{v \cdot \left[\frac{(1-f)}{p} + \frac{1}{\tau} \right]} - [a + k \cdot (l - m, 0)^+] \\ &= \frac{\frac{l}{v} + \alpha^T - [a + k \cdot (l - m, 0)^+] \cdot \frac{(1-f)}{p}}{\frac{(1-f)}{p} + \frac{1}{\tau}}, \end{aligned} \quad (9)$$

which leads to the following result.

Theorem 2. *The platform charges more than the regular taxi fare if and only if $\alpha^T > [a + k \cdot (l - m, 0)^+] \cdot \frac{(1-f)}{p} - \frac{l}{v}$.*

Theorem 2 implies that online rides are relatively more expensive if traffic conditions are bad (a low driving speed, v , and/or a long time to find a regular taxi, α^T), regular taxi fares $[a + k \cdot (l - m, 0)^+]$ are low, drivers expect a high profit rate p , and the platform commission f is high.

5.2. The Beijing taxi market

We next apply our model to the Beijing car market, to get insight into how the Didi Chuxing platform, which part motivated this study, should price e-hailing rides compared to offline taxi rides. Table 3 lists all estimated model parameter values, where interested readers can refer to Appendix B for sources and explanations.

Parameter	Value
Platform commission	$f = 0.3$
Drivers' expected profit (in Yuan/h)	$p = 29.96$
Starting taxi fare for a regular taxi (in Yuan)	$a = 14$
Regular taxi price after first m kilometres (in Yuan/km)	$k = 2.3$
Distance covered by starting taxi fare (in km)	$m = 3$
Customers' waiting cost (in Yuan/h)	$\tau = 48.16$
Regular taxi waiting time (in hours)	$\alpha^T = 0.198$ (= 11.9/60)
Driver speed (in km/h)	$v = 25$
Ride length (in km)	$l = 8.1$

Table 3. Parameter values based on Beijing market.

Fig. 2(a)-(d) further illustrates how the online vs regular price difference depends on driving speed (v), ride length (l), average waiting time for a regular taxi (α^T), and platform commission (f).

Fig. 2. Effects of speed (v), ride length (l), average waiting time for a regular taxi (α^T), and platform commission (f) on the online vs regular price difference.

Note from Fig. 2(b) that under average conditions, the platform price is below the regular taxi fare for any requested ride (distance). The non-monotone price difference is related to the regular taxi fare structure; that difference is smallest when the distance is exactly equal to the maximum distance that can be travelled at the starting fare, which can be seen as the most economical distance from a customer perspective.

Also under most other considered conditions, the platform prices below the competing taxi fare. However, when traffic is very slow and customers experience a high cost from waiting, then the quick matching process allows the platform to charge more than the regular taxi fare. Moreover, when the platform commission is very large (above 40%), then a higher price is needed to keep drivers interested.

6. Effects of selecting the closest driver

Recall from Section 4.1 that at least one driver is available within distance $M_d(F_R)$ from the customer (under the assumption that a match can indeed be made). Let N denote the number of drivers distanced at most $M_d(F_R)$ from the customer, and arbitrarily number them from 1 to N . Recall also that driver locations are assumed to be distributed uniformly, and so the locations of these N drivers are distributed uniformly over the circular area with radius $M_d(F_R)$.

In Section 6.1, we will derive the expected price when the platform selects the closest driver rather than the first to respond, and discuss the effect on the platform profit. In Section 6.2, we explore the effect on the customer e-hailing cost.

6.1. Expected closest distance and optimum price

If the platform selects the closest driver, then a successful match occurs if (and only if) that driver is close enough to be accepted by the customer given the price. Obviously, the platform maximizes its profit by selecting the highest possible price that is acceptable, given the distance of the closest driver to the customer. So, in order to find the expected price, we must first determine the distribution of that distance.

Let y_i , $1 \leq i \leq N$, denote the distance to the customer for closest driver i . The probability distribution function of the distance for the closest driver then is

$$F_Y(y) = P(\min[y_1, y_2, \dots, y_N] \leq y) = 1 - \left(1 - \frac{y^2}{[M_d(F_R)]^2}\right)^N.$$

Differentiating $F_Y(y)$ with y , the probability density function of closest distance is obtained as

$$f_Y(y) = \frac{2Ny}{[M_d(F_R)]^2} \left(1 - \frac{y^2}{[M_d(F_R)]^2}\right)^{N-1}.$$

So the expected closest distance is

$$\begin{aligned} E(y) &= \int_0^{M_d(F_R)} y \cdot f_Y(y) dy = \int_0^{M_d(F_R)} y \cdot \frac{2Ny}{[M_d(F_R)]^2} \left(1 - \frac{y^2}{[M_d(F_R)]^2}\right)^{N-1} dy \\ &= \frac{\sqrt{\pi} \cdot N \cdot M_d(F_R) \cdot \Gamma(N)}{2\Gamma(N + \frac{3}{2})}. \end{aligned} \quad (10)$$

Following Davis (1959), using

$$\Gamma(N) = (N-1)!,$$

and

$$\Gamma\left(N + \frac{3}{2}\right) = \left(N + \frac{1}{2}\right) \Gamma\left(N + \frac{1}{2}\right) = \left(N + \frac{1}{2}\right) 2^{-N} \sqrt{\pi} (2N-1)!!,$$

where $n!!$ stands for the double factorial (Smarandache, 1991). We can rewrite (10) further as

$$\begin{aligned} E(y) &= \frac{\sqrt{\pi} \cdot N \cdot M_d(F_R) \cdot (N-1)!}{2\left(N + \frac{1}{2}\right) \cdot 2^{-N} \cdot \sqrt{\pi} \cdot (2N-1)!!} = \frac{2^N \cdot N!}{(2N+1)(2N-1)!!} M_d(F_R) \\ &= \frac{2^N \cdot N!}{(2N+1)!!} M_d(F_R) = \frac{(2N)!!}{(2N+1)!!} M_d(F_R). \end{aligned} \quad (11)$$

Combining this with (3) and the definition of F_M (see also Fig. 1), we find that the expected price (that the customer is willing to pay for the closest driver), denoted by F_C , is

$$E(F_C) = F_M - \frac{\tau}{v} \cdot E(y) = F_M - \frac{\tau}{v} \cdot M_d(F_R) \cdot \frac{(2N)!!}{(2N+1)!!}.$$

Using (4) and (7), we further get

$$E(F_C) = F_M \left(1 - \frac{(2N)!!}{(2N+1)!!}\right) + F_R \cdot \frac{(2N)!!}{(2N+1)!!}. \quad (12)$$

By comparing $E(F_C)$ with the optimal price when selecting the first driver, we find that the difference is

$$E(F_C) - F_R = (F_M - F_R) \cdot \left(1 - \frac{(2N)!!}{(2N+1)!!}\right) \geq 0. \quad (13)$$

Using (13), it can easily be seen that the price difference is positive and increasing in the number of drivers (N). Correspondingly, the platform profit increases with the number of drivers, and so does the expected utility of the selected driver.

6.2. Effect on the customer e-hailing cost

The effect on the customer utility is less obvious. On one hand the customer pays a higher price, but on the other hand she benefits from a reduced expected waiting time. The expected customer waiting cost of selecting the closest driver is

$$\tau \cdot \frac{E(y)}{v} = \frac{\tau}{v} \cdot M_d(F_R) \cdot \frac{(2N)!!}{(2N+1)!!}.$$

Using (5), we get that the comparable cost under selection of the first driver to respond is

$$\tau \cdot \frac{E(r)}{v} = \frac{\tau}{v} E(r) = \frac{\tau}{v} \int_0^{M_d(F_R)} r f_r(r) dr = \frac{2\tau M_d(F_R)}{3v}.$$

So the effect of selecting the closest driver on the customer waiting cost is

$$\tau \cdot \frac{E(y)}{v} - \tau \cdot \frac{E(r)}{v} = \frac{\tau}{v} \left(M_d(F_R) \frac{(2N)!!}{(2N+1)!!} - \frac{2}{3} M_d(F_R) \right) \leq 0. \quad (14)$$

From Equation (14), we find that the difference is negative and decreasing in the number of interested drivers (N). During rush hours or in hot spots, with more interested drivers nearby ($N \uparrow$), the expected distance of the closest driver is smaller and so the expected customer waiting time is reduced.

Fig. 3(a) and Fig. 3(b) compare the optimum price, customer waiting cost, and customer hailing cost under selecting the first driver to respond and selecting the closest driver, respectively, for the before discussed Beijing market – see Table 3 for model parameter estimates. Fig. 3(a) illustrates that the optimum price, customer waiting cost, and customer hailing cost are independent of number of drivers when selecting the first driver to respond. Fig. 3(b) shows that under selecting the closest driver, customers are slightly worse off despite the reduced waiting time, and the platform and drivers are considerably better off.

Fig. 3. Continued results for the Beijing market: price, waiting cost and hailing cost.

Next, we compare the numerical results with Didi Chuxing's actual pricing scheme in Beijing. This is done under both normal ($v = 25\text{km/h}$) and bad ($v = 10\text{km/h}$) traffic conditions in Fig. 4(a) and Fig. 4(b), respectively. We estimated the number of drivers (N) under different traffic conditions and ride lengths, depending on the total number of Didi Chuxing's drivers in Beijing, the non-mountainous area of Beijing, and the average working rate of drivers who work for online car hailing platforms in different periods per day (Guo et al., 2017). We remark that the reported Didi Chuxing prices calculated as the unweighted average over both different time period per day and different service classes without ride sharing (including Didi Express and Didi Premier). As shown in Fig. 4, the numerical results are overall close to Didi Chuxing's actual prices. They both increase with the ride length, which indicates that they have the same core structure with a base fare based on the ride length. Under bad traffic conditions in Fig. 4(b), both Didi Chuxing's prices and the numerical prices present a higher price comparing with Fig. 4(a), which implies that there exists a rush hour congestion fee. We remark that in Fig. 4(b), a rational customer switches to an offline taxi due to a lower cost if a ride is beyond 8 km (when driving speed v equals 10 km/h), and so the black dash lines in Fig. 4(b) are virtual.

Fig. 4. Price comparison between modelling results and Didi Chuxing.

7. Conclusion

We studied matching of supply and demand for ride-sourcing platforms that have pricing power. Contrary to existing equilibrium models that consider ‘average’ ride requests (in different zones), we considered the per service price determination for any specific ride request, allowing for possibility that a customer may cancel an order after receiving the price and driver details. This allowed us to derive the optimal pricing strategy that takes ride distance into account, besides the time and zone effects considered so far. Furthermore, by considering the competing regular taxi service, we were able to reflect on the optimal platform pricing relative to the traditional taxi market.

We first considered a platform that selects the first driver to respond to a customer request, arguably the fairest system to drivers. Key observations are as follows. The optimal platform price is that for which the maximum acceptable customer-driver distance is the same for both drivers and customers, thereby ensuring that the customer accepts the offered price and driver. It consists of three parts: (a) a base fare based on the ride length, (b) a rush hour congestion fee, and (c) an emergency fee. Moreover, the platform price is low relative to the regular taxi fare if traffic conditions are good, drivers have low profit expectations, and the platform commission is reasonably low.

We then considered selection of the driver closest to the customer. This obviously reduces the distance to the customer, who is therefore willing to pay a higher maximum price. By charging that higher price, the platform increases its profit. This is also advantageous to the (selected) driver, who further benefits from a reduced cost to reach the customer. The customer benefits from a reduced waiting cost too, but has to pay a higher price and is overall worse off in terms of utility as the platform is able to charge the maximum price that she is willing to pay.

Using publicly available sources, we collected data from the Beijing car hailing market. These showed that under selection of the first driver to respond, the optimal platform price is below the competing regular taxi fare under average road conditions and given the current platform commission. However, in rush hour and under emergency conditions, the platform can charge a higher price as waiting times for regular taxis are longer and more costly for customers. Furthermore, further price increases and profit gains can be achieved by switching from first-to-respond to selecting the closest driver, as Didi Chuxing has recently done in several cities. We also found that the optimal prices resulting from our model are overall close to Didi Chuxing’s actual prices, and have the same core structure consisting of a base fare based on the ride length and a rush hour congestion fee.

There are a number of limitations of our research and findings, linking to further research opportunities. First, some of our modelling assumptions can be altered/relaxed. We assumed the utility of drivers and customers to be linear in both price and waiting/driving time. In particular, in our main analysis we assumed that the customer waiting cost increases linearly with the waiting time. Reflecting that customers may grow increasingly impatient as they wait longer, we did also compare to the case with a quadratic relation of waiting cost and waiting time. However, other relations can be considered. Although the analysis may no longer be tractable, they could be studied numerically. One special and interesting case would be to assume a waiting time threshold with either a step increase (discontinuity) in the waiting cost function at that point or an increase in the slope (implying a non-differentiable but still continuous function).

We assumed homogenous drivers and customers, whereas they are heterogeneous in real life. We assumed that customers are rational and will switch to the offline service if it has lower cost (including the waiting cost). However, there may be customers who prefer using the app, even at a higher cost. Future research can consider customer cancelation behaviour in more detail.

Our analysis is deterministic, but uncertainties may play a role in practice. In particular, the driving time from a driver to a customer may depend on traffic conditions that can rapidly change. For our base model with a linear waiting cost, this uncertainty will not have much effect on the results, since the expected waiting cost only depends on the expected waiting time and not on the uncertainty. For other waiting cost functions, however, this is different. For the alternative quadratic cost function that we considered (in Appendix C), more uncertainty leads to a higher expected waiting cost. Of course, this would apply to both e-hailing and offline taxi services, and to the combined effect on the platform price and profit is not clear up front. Further research could delve into this, also taking stochasticity into account although this will obviously make the analysis harder.

Besides competition between ride-sourcing and offline services, one could also consider the competition between ride-sourcing and taxi e-hailing services. Future research in this direction could also take other customers' options into account.

We only looked at the short-term pricing strategy per ride request, without considering long-term effects on the number of customers and drivers in a system. Future studies could take a long-term perspective and focus, for instance, on whether platforms should offer subsidies to drivers and/or customers when they enter

the market. Related, further research can address pricing competition between multiple platforms and taxi companies, and how this evolves over time.

In pointing out the potential benefit of selecting the closest driver (instead of the first to respond), we did not address some of the potential complications. In particular, although it is reasonable to assume that the platform knows the locations of all drivers (using GPS tracking), it may not know who is actively looking for a customer and therefore willing to perform the ride. Indeed, where Didi Chuxing operates this system, they first enquire whether the closest driver is indeed willing, and if not go to the next closest one. To avoid delays, drivers are charged a penalty after they reject a certain number of rides. Future research could model the driver acceptance decision in more detail. Furthermore, some online car hailing apps, including Shouqi Limousine & Chauffeur and Yidao Yongche (Didi Chuxing also adopted it before), allow drivers to opt between staying in charge (first to respond) or letting the platform select (closest). Analysing such combined systems would involve modelling what option the driver (per shift) and platform (per ride) select.

Appendix A. Sensitivity analysis: derivatives

x	$\frac{dF_R}{dx}$	$\frac{dg(F_R)}{dx}$
a	$\frac{1}{\tau \cdot [\frac{(1-f)}{p} + \frac{1}{\tau}]} > 0$	$\frac{f}{\tau \cdot [\frac{(1-f)}{p} + \frac{1}{\tau}]} > 0$
f	$\frac{v \cdot \left\{ \frac{[a + k \cdot (l - m, 0)^+]}{\tau} + \alpha^T \right\} + l}{vp \cdot [\frac{(1-f)}{p} + \frac{1}{\tau}]^2} > 0$	$f \cdot \frac{v \cdot \left\{ \frac{[a + k \cdot (l - m, 0)^+]}{\tau} + \alpha^T \right\} + l}{vp \cdot [\frac{(1-f)}{p} + \frac{1}{\tau}]^2} + \frac{v \cdot \left\{ \frac{[a + k \cdot (l - m, 0)^+]}{\tau} + \alpha^T \right\} + l}{v \cdot [\frac{(1-f)}{p} + \frac{1}{\tau}]} > 0$
k	$\frac{1}{\tau \cdot [\frac{(1-f)}{p} + \frac{1}{\tau}]} > 0$	$\frac{f \cdot (l - m, 0)^+}{\tau \cdot [\frac{(1-f)}{p} + \frac{1}{\tau}]} > 0$
$l(l \geq m)$	$\frac{k}{\tau \cdot [\frac{(1-f)}{p} + \frac{1}{\tau}]} + \frac{1}{v \cdot [\frac{(1-f)}{p} + \frac{1}{\tau}]} > 0$	$\frac{f \cdot k}{\tau \cdot [\frac{(1-f)}{p} + \frac{1}{\tau}]} + \frac{f}{v \cdot [\frac{(1-f)}{p} + \frac{1}{\tau}]} > 0$
$m(l \geq m)$	$-\frac{k}{\tau \cdot [\frac{(1-f)}{p} + \frac{1}{\tau}]} < 0$	$-\frac{f \cdot k}{\tau \cdot [\frac{(1-f)}{p} + \frac{1}{\tau}]} < 0$
p	$\frac{v \cdot \left\{ \frac{[a + k \cdot (l - m, 0)^+]}{\tau} + \alpha^T \right\} + l}{v \cdot [\frac{(1-f)}{p} + \frac{1}{\tau}]^2} \cdot \frac{1-f}{p^2} > 0$	$f \cdot \frac{v \cdot \left\{ \frac{[a + k \cdot (l - m, 0)^+]}{\tau} + \alpha^T \right\} + l}{v \cdot [\frac{(1-f)}{p} + \frac{1}{\tau}]^2} \cdot \frac{1-f}{p^2} > 0$
v	$-\frac{l}{v^2 \cdot [\frac{(1-f)}{p} + \frac{1}{\tau}]} < 0$	$-\frac{f \cdot l}{v^2 \cdot [\frac{(1-f)}{p} + \frac{1}{\tau}]} < 0$
α^T	$\frac{1}{[\frac{(1-f)}{p} + \frac{1}{\tau}]} > 0$	$\frac{f}{[\frac{(1-f)}{p} + \frac{1}{\tau}]} > 0$
τ	$\frac{-\frac{1}{\tau^2} \frac{(1-f)}{p}}{[\frac{(1-f)}{p} + \frac{1}{\tau}]^2} \cdot \left[a + k \cdot (l - m, 0)^+ - \frac{p}{1-f} \left(\frac{l}{v} + \alpha^T \right) \right] > 0 *$	$f \cdot \frac{-\frac{1}{\tau^2} \frac{(1-f)}{p}}{[\frac{(1-f)}{p} + \frac{1}{\tau}]^2} \cdot \left[a + k \cdot (l - m, 0)^+ - \frac{p}{1-f} \left(\frac{l}{v} + \alpha^T \right) \right] > 0 *$

* If $[a + k \cdot (l - m, 0)^+] \cdot (1 - f) < p \left(\frac{l}{v} + \alpha^T \right)$.

x	$\frac{d^2 F_R}{dx^2}$	$\frac{d^2 g(F_R)}{dx^2}$
a	0	0
f	$\frac{2(l + v(\alpha^T + \frac{a + k(l - m, 0)^+}{\tau}))}{p^2 v(\frac{1-f}{p} + \frac{1}{\tau})^3} > 0$	$\frac{2p\tau(p + \tau)(av + k(l - m, 0)^+ v + (l + v\alpha^T)\tau)}{v(p + \tau - f\tau)^3} > 0$
k	0	0
$l(l \geq m)$	0	0
$m(l \geq m)$	0	0
p	$\frac{-2(1-f)\tau(av + k(l - m, 0)^+ v + (l + v\alpha^T)\tau)}{v(p + \tau(1-f))^3} < 0$	$\frac{-2(1-f)f\tau(av + k(l - m, 0)^+ v + (l + v\alpha^T)\tau)}{v(p + \tau - f\tau)^3} < 0$
v	$\frac{2lp\tau}{v^3(p + \tau - f\tau)} > 0$	$\frac{2fl}{v^3(\frac{1-f}{p} + \frac{1}{\tau})} > 0$
α^T	0	0
τ	$\frac{\frac{1}{\tau^4} \frac{(1-f)}{p}}{\left[\frac{(1-f)}{p} + \frac{1}{\tau}\right]^3} \cdot \left[a + k \cdot (l - m, 0)^+ - \frac{p}{1-f} \left(\frac{l}{v} + \alpha^T \right) \right]$ $\cdot \left[2f\tau \frac{(1-f)}{p} \right] < 0 *$	$f \cdot \frac{\frac{1}{\tau^4} \frac{(1-f)}{p}}{\left[\frac{(1-f)}{p} + \frac{1}{\tau}\right]^3} \cdot \left[a + k \cdot (l - m, 0)^+ - \frac{p}{1-f} \left(\frac{l}{v} + \alpha^T \right) \right] \cdot \left[2f\tau \frac{(1-f)}{p} \right]$ $< 0 *$

* If $[a + k \cdot (l - m, 0)^+] \cdot (1 - f) < p \left(\frac{l}{v} + \alpha^T \right)$.

Appendix B. Data (sources) for the Beijing market

Platform commission: We set the platform commission to $f = 0.3$, since Didi Chuxing's drivers receive about 70% per ride of the gross revenue in Beijing.

Drivers' expected profit: According to the investigation report on online hailing drivers in Beijing (<http://m.biaozhun007.com/>), the expected profit for a driver per hour is around 29.96 Yuan.

Taxi fare: According to the Beijing's taxi pricing rule, customers are charged at 13 Yuan up to 3 kilometres. In addition, there is a fuel surcharge of 1 Yuan per ride. Thus, the starting price is 14 Yuan. Per additional kilometre, 2.3 Yuan is charged.

Customer's valuation: According to the website of the Beijing Municipal Bureau of Statistics (<http://www.bjstats.gov.cn/>), the average salary of urban employees in Beijing is around 48.16 Yuan per hour. We take this as the waiting cost of customers per hour.

Average waiting time for a regular taxi: According to Roland Berger's 2016 report (<https://www.rolandberger.com/>), the average waiting time for a regular taxi α^T is about 11.9 minutes in Beijing.

Average driver speed: According to the "2017 China Traffic Report of Major Cities", the average driving speed in Beijing is 25km/h.

Average ride length: According to the 5th Beijing's transportation investigation from the Ministry of Transport of China, the average customers' ride length in 2014 was 8.1 km per ride.

Appendix C. Effects of impatient customers

In this appendix, the cost of waiting time is assumed to be quadratic, reflecting the customers' growing impatience with a longer waiting time. So, the customer's total cost is $C_E = F + \left(\tau \cdot \frac{r_i}{v}\right)^2$ for e-hailing and $C_O = [a + k \cdot (l - m, 0)^+] + \tau^2 \alpha^{T^2}$ for an offline taxi. The analysis is similar to that for a linear cost in the main text, and so we present it as concisely as possibly.

- Selecting the first driver to respond

Since $M_c' > 0$, the maximum acceptable distance for a customer is easily derived as

$$M_c' = \frac{v \sqrt{a - F + k(l - m, 0)^+ + \alpha^{T^2} \tau^2}}{\tau}. \quad (15)$$

From (4) and (15), we then find (in reversed order) that the minimum and maximum relevant prices to be considered are $F_m' = \frac{lp}{(1-f)v}$ and $F_M' = [a + k \cdot (l - m, 0)^+] + (\tau \cdot \alpha^T)^2$, respectively. We also easily get that $M_d = M_c'$ for price

$$F_R' = \frac{1}{2(-1+f)^2 v^2 \tau^2} \left[-pv(pv + 2(-1+f)l\tau^2) + \sqrt{p^2 v^3 (p^2 v + 4(-1+f)lp\tau^2 + 4(-1+f)^2 v\tau^2 (a + k(l - m, 0)^+ + \alpha^{T^2} \tau^2))} \right].$$

- Selecting the closest driver

Using (11), (15) and the definition of F_M' , we find that the expectation of the price F_C' (that the customer is willing to pay for the closest driver under the quadratic cost of waiting time) is

$$E(F_C') = F_M' - \left(\frac{\tau}{v} \cdot E(y)\right)^2 = F_M' - \left(\frac{\tau}{v} \cdot M_d(F_R') \cdot \frac{(2N)!!}{(2N+1)!!}\right)^2.$$

We next explore the effects of changing from a linear to a quadratic waiting cost on the (expected) price of a ride. Table 4 shows the (numerical) results for the settings as used in the main text and listed in Table 3. It is obvious that the effects are much more pronounced under selection of the closest driver. Using a quadratic waiting cost leads to somewhat smaller prices when selecting the first taxi to respond, but much higher prices when selecting the closest driver. This is related to the increased customer impatience under a quadratic waiting cost. Under selection of the first driver to respond, this provides more of an incentive to ensure that only nearby drivers respond by keeping the price low. To the opposite, when selecting the closest driver, it provides an

opportunity to increase the price substantially, since customers want to avoid the longer (average) offline waiting time.

Ride Length	Linear waiting cost		Quadratic waiting cost	
	First driver	Closest driver	First driver	Closest driver
5	17.8	26.2	17.1	109.5
10	27.7	37.4	25.8	120.8
15	37.7	48.6	34.5	132.2
20	47.6	59.8	43.2	144.1

Table 4. Optimal prices under selection of the first driver to respond and selection of the closest driver, for both linear and quadratic waiting costs per time unit.

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