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Modeling for control of a kinematic wobble-yoke Stirling engine[☆]

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ABSTRACT

In this paper we derive the dynamical model of a four-cylinder double-acting wobble-yoke Stirling engine. In addition to the classical thermodynamics methods that dominate the literature of Stirling mechanisms, we present a control systems viewpoint to analyze the dynamic properties of the engine. We show that the Stirling engine can be viewed as a closed-loop system, in which the pressure variations in the cylinders behave as the feedback control law.

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Energy savings and concern for the environment and climate are major issues nowadays within our society. Due to the high costs of extraction and processing of fossil fuels—which have made their utilization increasingly expensive [1–3], not to mention the adverse effects to the environment—, sustainable energies such as wind and solar energy are becoming popular around the world [4–8]. Moreover, in recent decades there has been an enormous interest in the application of heat engines for converting different types of heat sources into electrical energy [9,10].

One of the most promising applications is micro-combined heat and power (CHP) generation, or in other words, the simultaneous production of heat and power at a small-scale [11]. A micro-CHP consists of a gas engine which drives an electrical generator. Among the advantages of using micro-CHP systems we can mention: cutting the power transmission losses, because the waste heat can be captured and used locally; and generating electricity that can be either used in the house or exported to the grid in order to be consumed by the neighbors [11]. Supplying electricity back to the grid raises important economical and research/scientific challenges [10] which are not within the scope of this paper (see for instance [12] and references therein).

Micro-CHP systems can attain a similar conversion efficiency from gas to useful heat as a conventional boiler, typically around

80%. However, in addition, around 10–15% can be converted to electricity. Among the technologies that have been proposed for micro-CHP applications we can mention fuel cells, internal combustion engines and Stirling engines [11,13]. The Stirling engine is an external combustion reciprocating engine invented by Robert Stirling in 1816. Theoretically, Stirling engines seem to be the most efficient device for converting heat into mechanical work, with high efficiencies, requiring high-temperatures [14]. Stirling engines are generally externally heated engines. Therefore, most sources of heat can be used to drive them.

Because of the Stirling engine inherent complexity, providing modeling and simulation tools for improving its design has raised important research challenges for the scientific community during the last decades. Studies that rely on thermodynamics methods and intuitive design techniques can be found in Refs. [15–19]. In Refs. [20–23], the application of computational fluid dynamics modeling to improve the design of Stirling engines is discussed. There exist, however, few literature on the application of systems and control methods to investigate their stability and dynamic properties, see for instance [24–28] and the recent works [29,30].

In this work we present a dynamic systems and control perspective to analyze the complex behavior of the Stirling engine. To this end, we take as a case study a kinematic wobble-yoke Stirling engine [31–33], consisting of a four-cylinder double-acting Stirling mechanism whose design is based on the classical spherical four-bar linkage. Our contributions are threefold. First, we present the complete nonlinear dynamical model of the kinematic wobble-yoke Stirling engine (originally developed by the authors in Ref. [34]). Second, we show that the Stirling engine can be viewed as a

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closed-loop feedback system, in which the pressure variation inside the cylinders behaves as the state feedback control law. Third, following a similar approach from Refs. [29,35], we investigate the dynamic and stability properties of the Stirling engine.

The paper is organized as follows. Section 1 describes the working principle of the kinematic wobble-yoke Stirling engine. Section 2 introduces the dynamic modeling of the engine. In Section 3 the linear dynamics of the engine is analyzed. Section 4 presents the application of linear control tools to study the behavior of the Stirling engine. Simulations results are given in Section 5 and finally Section 6 outlines some concluding remarks.

1. Description of the system

Fig. 1 shows the schematic representation of the four-cylinder double-acting Stirling engine. The four cylinders are phased at 90° from each other with respect to ϕ . The links connecting the cylinders form the wobble-yoke mechanism whose function is to translate the reciprocating motion in vertical direction of the cylinders into the rotational motion through the shaft angle ϕ . The design of the wobble yoke mechanism is based on the classical spherical four-bar linkage [31]. These kind of linkages, which are well known in robotics, have the property that every link in the system rotates about the same fixed point [36,37]. Hence, as indicated by its name, the trajectories of the points at the end of each link lie on concentric spheres. In robotics, only the revolute joint is compatible with this rotational movement and its axis must pass through the fixed point. The wobble yoke is indeed a particular class of the spherical linkage known as *spherical crank rocker* [31]. In this case, the revolute joints are replaced by the spherical bearings located at points b_1, b_3, c_1 and d (cf. Fig. 2). The axis of the aforementioned bearings must intersect the sphere center O .

The working principle of this mechanism can be explained by referring to Fig. 2. The mechanism is based on a beam which pivots about its center O in one plane (e_2e_3 for beam 1, and e_1e_3 for beam 2). Each beam is attached to pistons via bearings a_1 and a_3 . An eccentric bearing c_1 is attached to the drive shaft and it is connected to the beam via two bearings b_1 and b_3 . The eccentric bearing c_1 is the rotating part of the mechanism. When the engine is working, the reciprocating motion in vertical direction of the pistons inside the cylinders (not shown in Fig. 2), induces a rotational movement on bearing c_1 . Due to the geometrical and physical configuration of the mechanism, bearing c_1 describes a circle of radius l_{c_1d} . The axis

Fig. 2. Schematic picture of beam 1. θ_1 is the angle between the beam and the axis e_2 . The angle ϕ is measured in the counterclockwise direction from the positive axis e_1 .

of bearings b_1, b_3, c_1 and d must intersect the center O , so that the kinematic constraints of the spherical crank rocker [36,37] are satisfied. We also notice that the axis $l_{O c_1}$ of bearing c_1 is perpendicular to the beam, i.e., $l_{O c_1} \perp l_{b_1 b_3}$. An analogous discussion applies to the second beam. We refer the reader to [31,32] for more details about the wobble-yoke Stirling engine.

2. Modeling for control

In this section we derive the equations of motion for the wobble-yoke Stirling engine [34]. The definition of the parameters as well as their nominal values are summarized in Table 1. We make the following fundamental assumption:

A1. *Small motion:* Let $-15^\circ < \theta_j < 15^\circ$, then $\cos \theta_j \approx 1$, $\sin \theta_j \approx \theta_j$, $\dot{\theta}_j^2 \approx 0$.

Fig. 1. Schematic representation and cylinders configuration of the wobble-yoke Stirling engine.

Table 1
System parameters (nominal values shown).

Symb.	Value	Description
a_i		Connecting rod bearing center,
A_p	0.0014	Piston area [m ²]
A_r	2.4053×10^{-4}	Piston rod area [m ²]
b_i		Wobble yoke-beam bearing center,
b_{p_i}	10	Damping coefficient [Ns/m],
c_j		Nutating bearing center,
d		Crankshaft bearing center,
e_n		Axes of the fixed reference frame,
$F(\cdot)$		Force [N],
g	9.81	Acceleration due to gravity [m/s ²],
h_{dc}	0.0006	Height of dead volume in compression space [m]
h_{de}	0.00263	Height of dead volume in expansion space [m]
h_s	0.025	Stroke of the piston [m]
I	0.0135	Mass moment of inertia about the pivot O [kgm ²],
k_{p_i}	450	Piston spring constant [N/m],
$l(\cdot)$		Distance, [m]
l_{Oa_i}	0.0705	Distance [m],
m	0.4384	Piston assembly mass incl. the connecting rod [kg],
m_{c_i}		Mass of the working gas in the compression space [kg]
m_{e_i}		Mass of the working gas in the expansion space [kg]
m_h		Mass of the working gas in the heater [kg]
m_k		Mass of the working gas in the cooler [kg]
m_T	0.0015	Total mass of the working gas [kg],
M_i		Angular momentum with respect to the axis e_i [Nm],
O		Center of the fixed reference frame, main pivot center,
p_{c_i}		Pressure in compression space [N/m ²],
p_{cc}		Crankcase pressure [N/m ²],
p_{e_i}		Pressure in expansion space [N/m ²],
p_m	2.5×10^6	Mean pressure in the working space [N/m ²],
R	8.3144×10^{15}	Gas constant [J/(K·mol)],
T_h	975	Hot end temperature [K],
T_k	360	Cold end temperature [K],
T_r	617.2632	Regenerator effective temperature [K],
V_{c_i}		Compression space volume of the i -th cylinder, [m ³]
$V_{c_{max}}$		Maximum compression space volume, [m ³]
V_{e_i}		Expansion space volume of the i -th cylinder, [m ³]
V_{dc}	6.7512×10^{-7}	Dead volume in compression space [m ³],
V_{de}	3.5918×10^{-6}	Dead volume in expansion space [m ³],
$V_{e_{max}}$		Maximum expansion space volume, [m ³],
V_h	9.1800×10^{-6}	Heater volume [m ³],
V_k	8.2687×10^{-6}	Cooler volume [m ³],
V_{swc}	2.8130×10^{-5}	Swept volume in compression space [m ³],
V_{swe}	3.4143×10^{-5}	Swept volume in expansion space [m ³],
z_i		Vertical displacement [m],
$z_{i_{eq}}$		Equilibrium length of the i -th piston spring [m],
z_{max}	0.0125	Maximum piston displacement = $h_s/2$ [m],
$\dot{z}_i$		Velocity [m/s],
$\ddot{z}_i$		Acceleration [m/s ²],
α_n		Constant
β_n		Constant
γ_i		Constant
θ_j		Beam angle [rad],
$\dot{\theta}_j$		Beam angular velocity [rad/s],
$\ddot{\theta}_j$		Beam angular acceleration [rad/s ²],
$\theta_{j_{max}}$	0.1782	Maximum beam angle [rad],
ϕ		Crankshaft angle [rad],
τ		Shaft torque [Nm].

During operation of the engine, the beam angle θ_1 (respectively θ_2)—see Fig. 2—between the beam and the horizontal axis e_2 (respectively e_1) varies between its maximum $\theta_{1_{max}}$ and its minimum $-\theta_{1_{max}}$ (respectively $\theta_{2_{max}}$ and $-\theta_{2_{max}}$). Due to physical constraints of the engine, the maximum beam angle is approximately 10.21°, thus, assumption A1 is physically correct.

2.1. Kinematics

Consider the schematic representation of beam 1 shown in Fig. 2. We define the reference frame e_n , $n = 1, \dots, 3$, which is fixed at the pivot center of the beam O . As was explained in Section 1, the vertical motion of the pistons (not shown in Fig. 2) inside the cylinders, leads to a rotation of the beam around O . This rotation is represented by the instantaneous value of θ_1 . The variation on θ_1 causes as well a rotational movement around the axis e_3 , which is represented by the crank angle ϕ . Then, the kinematic problem for the beam 1 consists in finding the equations of the angular displacements θ_j , $j = 1, 2$, and the vertical displacements z_i , $i = 1, \dots, 4$, in terms of the crank angle ϕ .

To solve the problem we proceed as follows (see also [31]). Referring to Fig. 2, let the vectors $\mathbf{b}_1 = [b_1, b_2, b_3]^T$ and $\mathbf{c}_1 = [c_1, c_2, c_3]^T$ denote the coordinates in the reference frame e_n , $n = 1, \dots, 3$, of the wobble yoke-beam bearing center b_1 and the nutating bearing center c_1 of the beam 1, respectively. Then the bearing center distance $l_{b_1c_1}$ is given by.

$$l_{b_1c_1}^2 = (c_1 - b_1)^2 + (c_2 - b_2)^2 + (c_3 - b_3)^2, \quad (1)$$

where $b_1 = 0$, $b_2 = l_{Ob_1} \cos \theta_1$, $b_3 = l_{Ob_1} \sin \theta_1$, $c_1 = l_{c_1d} \cos \phi$, $c_2 = l_{c_1d} \sin \phi$, and $c_3 = -l_{Od}$. Taking into account that $l_{Oc_1}^2 = l_{c_1d}^2 + l_{Od}^2$, and $l_{b_1c_1}^2 = l_{Ob_1}^2 + l_{Oc_1}^2 = 2l_{Ob_1}^2$ (Fig. 2), we have that equation (1), after expanding and further simplifying, becomes

$$l_{Od} \sin \theta_1 - l_{c_1d} \cos \theta_1 \sin \phi = 0. \quad (2)$$

Solving (2) for θ_1 we obtain.

$$\theta_1 = \tan^{-1}(\kappa \sin \phi), \quad (3)$$

where we have used $l_{c_1d} = l_{Od} \tan \theta_{1_{max}}$ and $\kappa = \tan \theta_{1_{max}}$. Equation (3) gives the angular displacement θ_1 of beam 1 for a particular crank angle ϕ . By taking the first and second time derivative of (3), we get, after some straightforward but cumbersome computations, the angular velocity and angular acceleration as

$$\dot{\theta}_1 = \kappa \dot{\phi} \cos \phi, \quad (4)$$

$$\ddot{\theta}_1 = \kappa \ddot{\phi} \cos \phi - \kappa (\sin \phi + 2\theta_1 \cos^2 \phi) \dot{\phi}^2, \quad (5)$$

where we have used Assumption A1 to further simplify the above equations.

Let $\mathbf{a}_1 = [a_1, a_2, a_3]^T$ be the coordinates of the connecting rod bearing center a_1 , where $a_1 = 0$, $a_2 = l_{Oa_1} \cos \theta_1$, $a_3 = l_{Oa_1} \sin \theta_1$ (see Fig. 2). By Assumption A1, $\mathbf{a}_1 = [0, l_{Oa_1}, l_{Oa_1} \theta_1]^T$. Therefore, for small motions there is only a vertical displacement of bearing a_1 in the direction of the e_3 axis. Hence, the angular beam displacement is translated to the vertical displacement of bearing a_1 as a function of the crank angle by.

$$z_1 = a_3 = l_{Oa_1} \theta_1 = l_{Oa_1} \tan^{-1}(\kappa \sin \phi). \quad (6)$$

Taking the first and second derivatives with respect to time of (6), and substituting (4) and (5), the vertical velocity and acceleration of bearing a_1 can be recast into.

$$\dot{z}_1 = l_{Oa_1} \dot{\theta}_1 = l_{Oa_1} \kappa \dot{\phi} \cos \phi, \quad (7)$$

$$\ddot{z}_1 = l_{Oa_1} \ddot{\theta}_1 = l_{Oa_1} \kappa [\ddot{\phi} \cos \phi - (\sin \phi + 2\theta_1 \cos^2 \phi) \dot{\phi}^2]. \quad (8)$$

Following similar geometric arguments for the connecting rod bearing center a_3 , we obtain.

$$z_3 = a_3 = -z_1, \quad \dot{z}_2 = -\dot{z}_1, \quad \ddot{z}_2 = -\ddot{z}_1. \quad (9)$$

Due to the symmetry of the beams and to their phase difference of $\pi/2$ with respect to ϕ (see Fig. 1), the previous calculations remain valid for beam 2 by taking into account $\theta_{1\max} = \theta_{2\max}$, and by replacing $\sin(\phi)$ by $\sin(\phi + \pi/2)$, $\cos\phi$ by $\cos(\phi + \pi/2)$, θ_1 by θ_2 , z_1 by z_2 , and z_3 by z_4 in equations (3)–(9), respectively.

Summarizing, the kinematic model of the wobble-yoke Stirling engine is given by.

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \tan^{-1}(\kappa \sin \phi) \\ \tan^{-1}(\kappa \cos \phi) \end{bmatrix}, \quad \dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} \kappa \dot{\phi} \cos \phi \\ -\kappa \dot{\phi} \sin \phi \end{bmatrix}, \quad (10)$$

$$\ddot{\theta} = \begin{bmatrix} \kappa \ddot{\phi} \cos \phi - \kappa \dot{\phi}^2 (\sin \phi + 2 \tan^{-1}(\kappa \sin \phi) \cos^2 \phi) \\ -\kappa \ddot{\phi} \sin \phi - \kappa \dot{\phi}^2 (\cos \phi + 2 \tan^{-1}(\kappa \cos \phi) \sin^2 \phi) \end{bmatrix}, \quad (11)$$

where we have used $\sin(\phi + \pi/2) = \cos\phi$ and $\cos(\phi + \pi/2) = -\sin\phi$. For the vertical displacements z_i , $i = 1, \dots, 4$, we only have two independent equations, namely, z_1 and z_2 , therefore, we define the vector of vertical displacements $\mathbf{z}$ as $\mathbf{z} = [z_1, z_2]^T$, with $\mathbf{z} = l_{Oa_1} \theta$, $\dot{\mathbf{z}} = l_{Oa_1} \dot{\theta}$, $\ddot{\mathbf{z}} = l_{Oa_1} \ddot{\theta}$.

2.2. Dynamics of the pistons motion

In this section we borrow some inspiration from Ref. [29] to derive the motion equation of the piston depicted in Fig. 3. The dynamical equation of this system is given by.

$$m\ddot{z}_i = (A_p - A_r)p_{c_i} - A_p p_{e_i} + A_r p_{cc} - k_{p_i}(z_i - z_{ieq}) - b_{p_i}\dot{z}_i - mg, \quad (12)$$

where the terms $(A_p - A_r)p_{c_i}$, $A_p p_{e_i}$, and $A_r p_{cc}$ are the force in the compression space, the force in the expansion space and the force in the crank space of the i -th cylinder, respectively.

Fig. 3. Free-body diagram for the pistons of the wobble-yoke Stirling engine.

Assume the initial condition z_{i0} to be the point of vertical static equilibrium, i.e., when the engine is not yet running, in other words, $\dot{z}_i = 0$, $\ddot{z}_i = 0$. Then the equation for the equilibrium length of the piston spring z_{ieq} can be obtained from (12) as follows:

$$k_{p_i} z_{ieq} = -(A_p - A_r)p_{c_{i0}} + A_p p_{e_{i0}} - A_r p_{cc} + k_{p_i} z_{i0} + mg, \quad (13)$$

where $p_{c_{i0}}$ and $p_{e_{i0}}$ are the initial pressures in the compression and expansion space of the i -th cylinder, respectively. Thus, the length of z_{ieq} depends on the force due to the gravity and on the initial pressure difference in the cylinder. This pressure difference exists if the engine is pressurized prior to operation [29]. Substituting (13) in (12), we finally obtain the equation for the vertical motion of the i -th piston as

$$m\ddot{z}_i = (A_p - A_r)(p_{c_i} - p_{c_{i0}}) - A_p(p_{e_i} - p_{e_{i0}}) - k_{p_i}(z_i - z_{i0}) - b_{p_i}\dot{z}_i. \quad (14)$$

2.3. Thermodynamics

To complete the piston motion equation (14), we incorporate now the pressure dynamics into equation (14). To this end, we apply the classical Schmidt analysis [38] with the following thermodynamic assumptions:

- A2. The working gases are ideal.
- A3. The working spaces and heat exchangers are isothermal at any instance and location.
- A4. The mass of the working gas is constant.

Remark 1. Although the Schmidt analysis is one of the most widely used methods to analyze Stirling engines for educational purposes, it is conservative and subject to certain limitations. For instance, Assumption A3 implies that the heat exchangers, including the regenerator, are perfectly effective [38,39]. Moreover, the Schmidt analysis does not accurately predict the performance of the engine. Thus, when designing Stirling engines, more advanced modeling techniques such as computational fluid dynamic (CFD) [20–23] or combined dynamic and thermodynamic modeling [40], among others are needed.

In the thermodynamic analysis of the Stirling engine, the engine consist of five serially-connected components, namely, a compression space, cooler, regenerator, heater and expansion space. These five components form an equivalent of a single internal gas circuit. In the case of the wobble-yoke Stirling engine, the engine is composed of four internal gas circuits (cf. Fig. 4). Each internal gas circuit consist of the compression space of cylinder i -th, the expansion space of cylinder $(i + 1)$ -th and the connecting cooler, regenerator and heater between cylinders i -th and $(i + 1)$ -th.

Since the isothermal analysis does not account for pressure gradients, following assumption A3 we assume no pressure drop across the cooler, regenerator and heater, and thus, the pressure in the compression space of the i -th cylinder equals the pressure in the expansion space of the adjacent cylinder $(i + 1)$ -th (cf. Fig. 4), i.e.,

$$p_{e_1} = p_{c_4}, \quad p_{e_2} = p_{c_1}, \quad p_{e_3} = p_{c_2}, \quad p_{e_4} = p_{c_3}. \quad (15)$$

Recalling from Subsection 2.1 that the equations for z_1 and z_2 completely describe the piston displacements ($z_3 = -z_1$ and $z_4 = -z_2$), the dynamic equations of the piston motion (14), after substituting (15) becomes.

Fig. 4. Cylinder order viewed from the burner end and cylinder configuration.

$$m\ddot{z}_1 = (A_p - A_r)(p_{c1} - p_{c10}) - A_p(p_{c4} - p_{c40}) - k_{p1}(z_1 - z_{10}) - b_{p1}\dot{z}_1, \quad (16)$$

$$m\ddot{z}_2 = (A_p - A_r)(p_{c2} - p_{c20}) - A_p(p_{c1} - p_{c10}) - k_{p2}(z_2 - z_{20}) - b_{p2}\dot{z}_2. \quad (17)$$

By using the ideal gas law (cf. Assumption A2) and Assumption A4, it can be shown that the pressure variation in the compression spaces of each thermodynamic cycle is given by Ref. [38]

$$p_{c_i} = m_T R \left(\frac{V_{c_i}}{T_k} + \frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} + \frac{V_{e_{i+1}}}{T_h} \right)^{-1} \quad (18)$$

where $m_T = m_{c_i} + m_k + m_r + m_h + m_{e_{i+1}}$ and $T_r = (T_h - T_k) / \ln(T_h / T_k)$ is the effective temperature of the ideal regenerator assuming a linear temperature distribution.

Remark 2. As noticed by Refs. [38], it might be more accurate to assume that equation (18) describes some intermediate pressure between the compression and expansion space. However, assuming (18) to be the pressure variation in the compression space simplifies the analysis to a great extent.

From (18), we observe that for a given geometry, gas type and temperature distribution of the working gas, the pressure is only function of the volume variations of the compression and expansion spaces V_{c_i} and $V_{e_{i+1}}$. The following equations give the volume variation in the compression space (Fig. 5-left)

$$V_{dc} = (A_p - A_r)h_{dc}, \quad V_{swc} = (A_p - A_r)\frac{h_s}{2}, \quad (19)$$

$$V_{c_{min}} = V_{dc}, \quad V_{c_{max}} = V_{dc} + V_{swc}. \quad (20)$$

Similarly, for the expansion space we have.

$$V_{de} = A_p h_{de}, \quad V_{swe} = A_p \frac{h_s}{2}, \quad (21)$$

$$V_{e_{min}} = V_{de}, \quad V_{e_{max}} = V_{de} + V_{swe}. \quad (22)$$

Then, from Fig. 5-right, and equations (19)–(22), we obtain that the volume variation in the compression and expansion spaces as a function of the piston position are respectively given by.

$$V_{c_i} = \frac{A_p - A_r}{2} z_i + \frac{V_{swc}}{2} + V_{dc}, \quad (23)$$

$$V_{e_i} = -\frac{A_p}{2} z_i + \frac{V_{swe}}{2} + V_{de}. \quad (24)$$

Fig. 5. Volume variations in the compression and expansion spaces.

Fig. 6. Forces and momenta acting on the beams.

Substituting (23) and (24) into the pressure equation (18) and after grouping the constant terms in β_1 , we have that the instantaneous pressure in the compression spaces is given by.

$$p_{c_i} = p_m \left(1 + \frac{A_p - A_r}{2\beta_1 T_k} z_i - \frac{A_p}{2\beta_1 T_h} z_{i+1} \right)^{-1} \quad (25)$$

where $p_m = MR/\beta_1$ is the mean pressure in the working spaces, and β_1 is a constant defined in Appendix A (equation (A.1)).

2.4. Rotational movement

As mentioned in Section 1, the wobble-yoke mechanism translates the vertical motion of the piston into rotational motion through the shaft angle. To compute the shaft torque equation we apply a slightly modification of the approach followed by Refs. [31]—the main differences being the computation of the forces and the output torque—The net forces acting at the connecting rod bearing a_i are defined as.

$$F_i = m\ddot{z}_i - (A_p - A_r)(p_{c_i} - p_{c_{i0}}) + A_p(p_{e_i} - p_{e_{i0}}) + k_{p_i}(z_i - z_{i0}) + b_{p_i}\dot{z}_i, \quad (26)$$

with the pressure variables p_{e_i} , $p_{e_{i0}}$, p_{c_i} , and $p_{c_{i0}}$, given by equations (15) and (25). The difference between the forces F_1 and F_3 acting at the beam ends a_1 and a_3 (cf. Fig. 6) produces an angular momentum¹ in the pivot center O pointing in the e_1 axis direction

$$M_1 = l_{Oa_1}(F_1 - F_3) - I\ddot{\theta}_1.$$

Similarly, for the second beam, we have an angular momentum in the pivot center O pointing in the e_2 axis direction.

$$M_2 = l_{Oa_1}(F_2 - F_4) - I\ddot{\theta}_2.$$

In order to obtain the equation of the output-shaft torque τ in the e_3 direction, we observe that the angular momenta M_1 and M_2 can be translated into equivalent forces F_{1eq} and F_{2eq} acting at the points c_1 and c_2 , for beams 1 and 2 respectively. If we constraint the forces F_{1eq} and F_{2eq} to lie in the plane e_1e_2 (cf. Fig. 7), it can be shown

that $F_{1eq} = [0, f_{12}, 0]^T$ and $F_{2eq} = [f_{21}, 0, 0]^T$, with $f_{12} = M_1/l_{Od}$ and $f_{21} = -M_2/l_{Od}$. The output-shaft torque τ is obtained as (cf. Fig. 7)

$$\tau = \kappa(M_1 + M_2)\cos\phi = (M_1 + M_2)\tan\theta_2, \quad (27)$$

where we have used $l_{c_1d} = l_{Od}\kappa$ and (10). By Assumption A1, $\tan\theta_2 \approx \theta_2 \approx z_2/l_{Oa_1}$. Thus, (27) finally becomes

$$\tau = \frac{M_1 + M_2}{l_{Oa_1}} z_2. \quad (28)$$

3. Linear dynamics

Because of the pressure dynamics (25), the piston motion equations (16) and (17) are nonlinear. However, since the working gas behaves like a linear spring [41], the system can be studied via linear analysis methods. For convenience, we linearize equation (25) around the initial piston positions $z_i = z_{i0}$ and $z_{i+1} = z_{(i+1)0}$ as follows²

$$p_{c_i} = p_{c_{i0}} + \left. \frac{\partial p_{c_i}}{\partial z_i} \right|_{z_i = z_{i0}} (z_i - z_{i0}) + \left. \frac{\partial p_{c_i}}{\partial z_{i+1}} \right|_{z_{i+1} = z_{(i+1)0}} (z_{i+1} - z_{(i+1)0}). \quad (29)$$

After some simplifications, equation (29) becomes.

$$p_{c_i} = \frac{p_m}{\gamma_i} \left(1 - \frac{A_p - A_r}{2\beta_1 \gamma_i T_k} (z_i - z_{i0}) + \frac{A_p}{2\beta_1 \gamma_i T_h} (z_{i+1} - z_{(i+1)0}) \right) \quad (30)$$

with the constant term γ_i given by

$$\gamma_i = 1 + \frac{A_p - A_r}{2\beta_1 T_k} z_{i0} - \frac{A_p}{2\beta_1 T_h} z_{(i+1)0}.$$

Define $\tilde{z}_i = z_i - z_{i0}$ and $\tilde{p}_{c_i} = p_{c_i} - p_{c_{i0}}$. Then equations (16), (17) and (30) can be represented in state-space as:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}, \quad (31)$$

$$\mathbf{y} = \mathbf{Cx}, \quad (32)$$

where $\mathbf{x} = [\tilde{z}_1, \tilde{z}_2, \dot{\tilde{z}}_1, \dot{\tilde{z}}_2]^T \in \mathbb{R}^4$ is the state-space vector. The output vector $\mathbf{y} \in \mathbb{R}^p$ contains the variable we are interested in controlling, which are determined from the structure of the output matrix $\mathbf{C} \in \mathbb{R}^{p \times 4}$. The state matrix $\mathbf{A} \in \mathbb{R}^{4 \times 4}$ and the input matrix $\mathbf{B} \in \mathbb{R}^{4 \times 3}$ are respectively given by

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_{p_1}}{m} & 0 & -\frac{b_{p_1}}{m} & 0 \\ 0 & -\frac{k_{p_2}}{m} & 0 & -\frac{b_{p_2}}{m} \end{bmatrix},$$

¹ Throughout the document, we follow the right-hand rule for rotational movement, i.e., a counterclockwise rotation produces a positive momentum about the rotational axis.

² Although the binomial expansion, or equivalently, the linearization around zero initial piston positions, has been widely used in free-piston Stirling engines to linearize the pressure equation (25), in the kinematic wobble-yoke Stirling engine, the choice $z_i = z_{i+1} = 0$ is not possible due to the physical constraints of the engine.

Fig. 7. Equivalent forces and output-shaft-torque computation.

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{(A_p - A_r)}{m} & 0 & \frac{A_p}{m} \\ \frac{A_p}{m} & \frac{(A_p - A_r)}{m} & 0 \end{bmatrix}.$$

Using the linearized pressure equation (30), the input term $\mathbf{u} = \tilde{\mathbf{p}} = [\tilde{p}_{c1}, \tilde{p}_{c2}, \tilde{p}_{c4}]^T \in \mathbb{R}^3$ can be written as the state feedback equation.

$$\mathbf{u} = -\mathbf{K}\mathbf{x}, \quad (33)$$

with constant gain matrix $\mathbf{K} \in \mathbb{R}^{3 \times 4}$

$$\mathbf{K} = \begin{bmatrix} \beta_2 & \beta_3 & 0 & 0 \\ \beta_4 & \beta_5 & 0 & 0 \\ \beta_8 & \beta_9 & 0 & 0 \end{bmatrix},$$

and constant terms β_i , $i = 2, \dots, 9$ defined in Appendix A, equations (A.2)–(A.6).

Remark 3. Fig. 8 depicts the block diagram of the linearized system (31)–(33). It is worth noticing that the system (31) is self-excited via the input term $\mathbf{u}$ (33), which depends on the initial pressure and temperature conditions, so as to induce the engine operation. Therefore, as already pointed out by Refs. [29,35,42], the Stirling engine can be viewed as a dynamical system subject to a state-feedback law (the pressure), which can be altered either by adding or taking out the working space some of the working fluid, or by manipulating the parameters of the system.

Remark 4. Fig. 9 shows the root locus of the closed-loop system (31)–(33) as the viscous friction parameter b_{p_i} varies. Taking $b_{p_i} = 10$ Ns/m as in Refs. [17,29], the closed-loop poles are located at

Fig. 8. Block diagram of the closed-loop system.

$133.15 \pm 311.89j$ and $-155.96 \pm 311.89j$. Due to the positive real part of two poles, the system is unstable and the piston displacement z_i will oscillate with increasing amplitude.

Remark 5. Theoretically, the wobble-yoke Stirling engine works in a stable cyclic steady-state, when the characteristic polynomial has two imaginary roots and two roots with a negative real part [43]. This situation occurs if $b_{p_i} \approx 143$ Ns/m. Unfortunately, at the moment of our study, not all the technical specifications of the wobble-yoke Stirling engine have been fully disclosed.

4. Pre-compensator and observer design

For the scope of this work, in this section we illustrate how linear control tools can be used to modify the behavior of the Stirling engine.

Fig. 9. Root-locus trajectories for varying values of b_{p_i} . The closed-loop poles when $b_{p_i} = 10$ N s/m are located at $133.15 \pm 311.89j$ and $-155.96 \pm 311.89j$.

4.1. Pre-compensator

Suppose it is possible to incorporate in the Stirling engine a pre-compensator $\mathbf{v}(\mathbf{x})$ that modifies the system's input $\mathbf{u}$ (Fig. 8). Then our control objective is to design a state feedback $\mathbf{v}(\mathbf{x})$ so that the new input.

$$\mathbf{u}(\mathbf{x}) = -\mathbf{K}\mathbf{x} + \mathbf{v}(\mathbf{x}) \quad (34)$$

ensures a stable piston oscillation at 30 Hz with a maximal amplitude of $z_{\max} = h_s/2$. To this end, we assume that all components of the vector state $\mathbf{x}$ are measured, and set the control input as

$$\mathbf{u} = \mathbf{u}_r - \bar{\mathbf{K}}(\mathbf{x} - \mathbf{x}_r), \quad (35)$$

where $\mathbf{x}_r$ is the desired trajectory, $\mathbf{u}_r$ is a feedforward control input, and $\bar{\mathbf{K}}$ is an appropriate gain matrix, which can be chosen by pole placement. Solving (34) and (35) for $\mathbf{v}$ yields

$$\mathbf{v} = \mathbf{u}_r - \bar{\mathbf{K}}(\mathbf{x} - \mathbf{x}_r) + \mathbf{K}\mathbf{x}. \quad (36)$$

The new closed-loop system, after substituting (36) and (34) into (31) results in $\dot{\tilde{\mathbf{x}}} = (\mathbf{A} - \mathbf{B}\bar{\mathbf{K}})\tilde{\mathbf{x}} + \mathbf{B}\bar{\mathbf{K}}\mathbf{x}_r + \mathbf{B}\mathbf{u}_r$. Define the tracking error $\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_r$, then the error dynamics are obtained as.

$$\dot{\tilde{\mathbf{x}}} = (\mathbf{A} - \mathbf{B}\bar{\mathbf{K}})\tilde{\mathbf{x}}, \quad (37)$$

where we have used

$$\dot{\mathbf{x}}_r = \mathbf{A}\mathbf{x}_r + \mathbf{B}\mathbf{u}_r \quad (38)$$

for the dynamics of the reference trajectory and feedforward generator. Equation (37) clearly shows that the tracking error will converge to zero provided a proper choice of the gain matrix $\bar{\mathbf{K}}$, done by pole placement. The block diagram representation in Fig. 10 depicts the pre-compensator system as proposed so far.

4.2. Trajectory and feedforward generator

Since the pressure variation $\tilde{p}_{c_3}$ does not appear explicitly in the system equation (31), the feedforward term $\mathbf{u}_r$ cannot be directly computed from (38) by using the pseudo-inverse of $\mathbf{B}$. To take into account the effect of $\tilde{p}_{c_3}$, equation (14) is evaluated for the four pistons at the reference trajectories $\mathbf{x}_r$, resulting in.

$$\Delta_{u_{r_f}} \mathbf{u}_{r_f} = \Delta_{\dot{\mathbf{x}}_r} \dot{\mathbf{x}}_r + \Delta_{\mathbf{x}_r} \mathbf{x}_r, \quad (39)$$

where $\mathbf{u}_{r_f} = [\tilde{p}_{c_{1r}}, \tilde{p}_{c_{2r}}, \tilde{p}_{c_{3r}}, \tilde{p}_{c_{4r}}]^T$, and

Fig. 10. Pre-compensator block diagram representation for the wobble-yoke Stirling engine.

$$\Delta_{u_{r_f}} = \begin{bmatrix} A_p - A_r & 0 & 0 & -A_p \\ -A_p & A_p - A_r & 0 & 0 \\ 0 & -A_p & A_p - A_r & 0 \\ 0 & 0 & -A_p & A_p - A_r \end{bmatrix},$$

$$\Delta_{\dot{\mathbf{x}}_r} = \begin{bmatrix} bp_1 & 0 & m & 0 \\ 0 & bp_2 & 0 & m \\ -bp_1 & 0 & -m & 0 \\ 0 & -bp_2 & 0 & -m \end{bmatrix},$$

$$\Delta_{\mathbf{x}_r} = \begin{bmatrix} kp_1 & 0 & 0 & 0 \\ 0 & kp_2 & 0 & 0 \\ -kp_1 & 0 & 0 & 0 \\ 0 & -kp_2 & 0 & 0 \end{bmatrix}.$$

Solving (39) for $\mathbf{u}_{r_f}$ yields $\mathbf{u}_{r_f} = \Delta_{u_{r_f}}^{-1}(\Delta_{\dot{\mathbf{x}}_r} \dot{\mathbf{x}}_r + \Delta_{\mathbf{x}_r} \mathbf{x}_r)$, from which the feedforward term $\mathbf{u}_r = [\tilde{p}_{c_{1r}}, \tilde{p}_{c_{2r}}, \tilde{p}_{c_{4r}}]^T$ can be obtained.

4.3. Observer design

The synthesis of the state-feedback pre-compensator presented in Subsection 4.1 considered that the state $\mathbf{x}$ was fully measured. However, from a practical viewpoint, the piston velocities cannot be directly measured, and thus, a reduced-order estimator is required.

Assume the state can be partitioned as $\mathbf{x} = [\mathbf{x}_a, \mathbf{x}_b]^T$, where $\mathbf{x}_a = [z_1, z_2]^T$ can be directly measured and $\mathbf{x}_b = [\dot{z}_1, \dot{z}_2]^T$ has to be estimated. According to this partition, equations 31 and 32 can be written as.

$$\begin{bmatrix} \dot{\mathbf{x}}_a \\ \dot{\mathbf{x}}_b \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{aa} & \mathbf{A}_{ab} \\ \mathbf{A}_{ba} & \mathbf{A}_{bb} \end{bmatrix} \begin{bmatrix} \mathbf{x}_a \\ \mathbf{x}_b \end{bmatrix} + \begin{bmatrix} \mathbf{B}_a \\ \mathbf{B}_b \end{bmatrix} \mathbf{u}, \quad (40)$$

$$\mathbf{y} = \mathbf{x}_a, \quad (41)$$

where $\mathbf{A}_{aa} = \mathbf{0}_{2 \times 2}$, $\mathbf{A}_{ab} = \mathbf{I}_{2 \times 2}$, $\mathbf{B}_a = \mathbf{0}_{2 \times 3}$ and

$$\mathbf{A}_{ba} = \begin{bmatrix} \frac{\beta_8 A_p - \beta_2 (A_p - A_r) - kp_1}{m} & \frac{\beta_9 A_p - \beta_3 (A_p - A_r)}{m} \\ \frac{\beta_2 A_p - \beta_4 (A_p - A_r)}{m} & \frac{\beta_3 A_p - \beta_5 (A_p - A_r) - kp_2}{m} \end{bmatrix},$$

$$\mathbf{A}_{bb} = \begin{bmatrix} \frac{bp_1}{m} & 0 \\ 0 & \frac{bp_2}{m} \end{bmatrix}, \mathbf{B}_b = \begin{bmatrix} \frac{A_p - A_r}{m} & 0 & \frac{A_p}{m} \\ \frac{A_p}{m} & \frac{A_p - A_r}{m} & 0 \end{bmatrix}.$$

Consider the following estimator [44]

Fig. 11. Pre-compensator-estimator block diagram representation for the wobble-yoke Stirling engine.

Fig. 12. Piston displacements: reference trajectories (continuous line) and simulated piston displacements (dashed line).

Fig. 13. Output shaft torque.

$$\dot{\hat{\mathbf{x}}}_b = (\mathbf{A}_{bb} - \mathbf{L}\mathbf{A}_{ab})\hat{\mathbf{x}}_b + \mathbf{A}_{ba}\mathbf{x}_a + \mathbf{B}_b\mathbf{u} + \mathbf{L}\dot{\mathbf{y}}, \quad (42)$$

where $\mathbf{L} \in \mathbb{R}^{2 \times 2}$ is the estimator gain matrix. Define the new estimator state $\hat{\mathbf{x}}_c = \hat{\mathbf{x}}_b - \mathbf{L}\mathbf{y}$. Then, the dynamics of the reduced-order estimator (42) can be written as

$$\dot{\hat{\mathbf{x}}}_c = (\mathbf{A}_{bb} - \mathbf{L}\mathbf{A}_{ab})\hat{\mathbf{x}}_b + \mathbf{A}_{ba}\mathbf{x}_a + \mathbf{B}_b\mathbf{u}.$$

By defining the estimator error $\tilde{\mathbf{x}}_b = \mathbf{x}_b - \hat{\mathbf{x}}_b$, it can be shown that $\dot{\tilde{\mathbf{x}}}_b = (\mathbf{A}_{bb} - \mathbf{L}\mathbf{A}_{ab})\tilde{\mathbf{x}}_b$. Therefore, given an adequate choice of the gains in matrix $\mathbf{L}$, done by pole placement, the estimator error will converge to zero. A block-diagram representation of the reduced-order estimator is shown in Fig. 11.

5. Simulation results

In order to illustrate the pre-compensator and observer designed for the Stirling engine, numerical simulations have

been carried out by using the linear controller/observer in the original nonlinear model. The desired pistons displacements are sinusoids with phase difference of $\pi/2$, frequency of 30 Hz and amplitude of $h_s/2 = 0.0125$. The initial crankshaft angle is $\phi(0) = \pi/3$, resulting in the initial piston positions and velocities $\mathbf{x}(0) = [0.0109, 0.0063, 0, 0]^T$. On the other hand, the observer is initialized at $\hat{\mathbf{x}}_b = [0, 0]^T$. The desired closed-loop poles for the controller are located at $[-300, -300, -200, -200]$, whereas those of the estimator are located at $[-600, -400]$. The controller and observer gain matrices were chosen considering the following two requirements. First, a fast response without overshoot from the controller, in order to constrain the piston motion to their encasement while converging to their reference trajectories. Second, an even faster response from the observer so that the estimated estates emulate the system states in a very short period.

The plots in Fig. 12 show the desired trajectory for the piston displacements (continuous line) as well as the resulting piston displacements (dashed line) during time $t \in [4, 4.1]$ s, while the resulting shaft torque is shown in Fig. 13 for the same period.

6. Conclusions and future work

We have presented a control systems approach for the analysis and control of a kinematic wobble-yoke Stirling engine mechanism. We have shown that the Stirling engine can be viewed as a closed-loop system, in which the pressure variations in the cylinders behave as the feedback control law. Moreover, we have illustrated how linear control tools can be used to modify the behavior of the Stirling engine such as making the piston's displacements follow a reference trajectory. Current research is underway to accurately identify the parameters of the system in order to experimentally validate the proposed controller in the future.

Other issues that will be explored in the future include:

- the robustness of the controllers in the presence of parametric uncertainties in the model,
- the energy performance and efficiency of the engine in terms of the mean recoverable mechanical power, and
- the modeling of the regenerator and heat exchangers so that pressure drops between the compression and expansion spaces are taken into account.

Appendix A. Parameters of the model

$$\beta_1 = \frac{1}{T_k} \left(V_{dc} + \frac{V_{swc}}{2} \right) + \frac{1}{T_h} \left(V_{de} + \frac{V_{swe}}{2} \right) + \frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h}. \quad (\text{A.1})$$

$$\beta_2 = \frac{8p_m T_h^2 \beta_1 (A_p - A_r)}{T_k (A_p h_s - 4T_h \beta_1)^2}, \quad \beta_3 = -\frac{8p_m T_h A_p \beta_1}{(A_p h_s - 4T_h \beta_1)^2}, \quad (\text{A.2})$$

$$\beta_4 = \frac{8p_m T_k^2 A_p \beta_1}{(4T_k \beta_1 + (A_p - A_r) h_s)^2 T_h}, \quad (\text{A.3})$$

$$\beta_5 = \frac{8p_m T_k \beta_1 (A_p - A_r)}{(4T_k \beta_1 + (A_p - A_r) h_s)^2}, \quad \beta_6 = -\frac{8p_m T_h^2 \beta_1 (A_p - A_r)}{(A_p h_s + 4T_h \beta_1)^2 T_k}, \quad (\text{A.4})$$

$$\beta_7 = \frac{8p_m T_h A_p \beta_1}{(A_p h_s + 4T_h \beta_1)^2}, \quad \beta_8 = \frac{-8p_m T_k^2 A_p \beta_1}{(-4T_k \beta_1 + (A_p - A_r) h_s)^2 T_h}, \quad (\text{A.5})$$

$$\beta_9 = -\frac{8p_m T_k \beta_1 (A_p - A_r)}{(-4T_k \beta_1 + (A_p - A_r) h_s)^2} \quad (\text{A.6})$$

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