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Published in:
Applied Ocean Research

DOI:
[10.1016/j.apor.2019.05.009](https://doi.org/10.1016/j.apor.2019.05.009)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Final author's version (accepted by publisher, after peer review)

Publication date:
2019

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Wei, Y., Bechlenberg, A., van Rooij, M., Jayawardhana, B., & Vakis, A. I. (2019). Modelling of a wave energy converter array with a nonlinear power take-off system in the frequency domain. *Applied Ocean Research*, 90, [101824]. <https://doi.org/10.1016/j.apor.2019.05.009>

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Modelling of a wave energy converter array with a nonlinear power take-off system in the frequency domain

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Abstract

This paper presents a nonlinear frequency domain model and uses this to assess the performance of a wave energy converter (WEC) array with a nonlinear power take-off (PTO). In this model, the nonlinear PTO forces are approximated by a truncated Fourier series, while the dynamics of the WEC array are described by a set of linear motion equations in the frequency domain, and the hydrodynamic coefficients are obtained with the boundary element method. A single heave absorber is firstly investigated to establish the accuracy of the new model in capturing the nonlinear behavior of the pumping system. Subsequently, simulations of a 2D array with 18 WECs and a pillar in the center (representing the tower of a wind turbine) are carried out to understand wave interference effects. Several optimization strategies are proposed to improve the overall performance of the WEC array. These results demonstrate a computationally effective method for accounting for nonlinear effects in large WEC arrays. The proposed approach may potentially be applied for developing control algorithms for the adaptability of a 2D array to incoming wave excitation.

Keywords: Ocean Grazer, nonlinear power take-off, wave energy converter arrays, Fourier approximation approach

1. Introduction

Ocean wave energy is a sustainable and abundant energy source, which has the potential to make a significant contribution to the growing energy demand of the world. However, it remains a great challenge to harvest utility-scale energy from ocean waves. Various approaches have been explored to improve wave energy converter (WEC) efficiency. These

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include the optimization of the geometry and inertia of the buoy (Flocard and Finnigan, 2012; Goggins and Finnegan, 2014), the development of novel PTO systems (Guerinel et al., 2017; Younesian and Alam, 2017; Zhang et al., 2018), and the application of active control to the device (Zhang et al., 2014; Son and Yeung, 2017; Wu et al., 2018). Grouping WECs in an array, which can directly increase energy production, possibly reduce the capital and operating costs and provide more stable electricity supply has therefore attracted significant attention from researchers. Several dense array concepts have been proposed in the last decade, which generally consist of a cluster of heave point absorbers sharing a common platform. Examples are Wave Star (Marquis et al., 2010), the Manchester Bobber (Weller et al., 2010) and FO³ (De Backer et al., 2010).

Wave interactions are very significant for WECs and have been investigated with various approaches. In the early days of wave energy research, analytical models based on the “point absorber approximation”, which assumed that oscillators were small compared to the incident wavelength and thus neglected wave scattering in the array, were widely used to investigate hydrodynamic interactions (Budal, 1977; Evans, 1980; Falnes, 1980). Later, the “plane-wave approximation”, which simplifies the local radiated and scattered waves into plane waves, was employed in studies by Simon (1982) and McIver and Evans (1984). However, these theoretical models cannot be applied to dense arrays because the wide spacing assumption is not fulfilled, as stated by Borgarino et al. (2012). To deal with dense arrays of small buoys, Garnaud and Mei (2009) derived a theory for dense arrays of small WECs with a multiple scales method. In the last two decades, numerical models using the boundary element method (BEM), which is able to describe the arbitrary geometry of floating bodies and consider the wave interactions between them, has become the most popular approach (Babarit, 2010; Borgarino et al., 2012). These numerical models are generally frequency-domain models, which omit the nonlinear terms. More recently, with the advancement of computational capabilities, computational fluid dynamics (CFD)-based approaches have also been employed to investigate nonlinear wave interactions (Devolder et al., 2018), but their application has been limited due to their heavy computational load. A comprehensive review on wave interactions can be found in Babarit (2013).

Following advances in hydrodynamic modelling, a number of spatial optimization studies on array configurations have been performed. The objective function is commonly chosen to maximize the overall power output. Such optimization problems can be implemented with various methods, e.g., the sequential quadratic programming method (Sinha et al., 2016) and machine learning approaches (mainly genetic algorithms) (Child and Venugopal, 2010; Sarkar et al., 2016). In order to obtain more realistic results, additional constraints, e.g., slamming, stroke and force restrictions, were also introduced into optimization problems (De Backer et al., 2010; Sinha et al., 2016). For some layouts, the wave directionality may be of concern to the power output; this was investigated by Wolgamot et al. (2012). While the majority of layout optimization has addressed arrays of relatively large spacing, some researchers have focused on the layout of dense arrays. Various arrangements of small size arrays (2–4 absorbers) (de Andrs et al., 2014; Chen et al., 2016) and moderate size arrays (12 absorbers) (Sinha et al., 2016) have been examined, aimed at achieving constructive effects from wave interaction. A general finding from these research studies was that the wave interaction became significant in a dense array and the optimal layout appeared to be crucial in order to take advantage of constructive effects.

The power take-off (PTO) configuration is also an important factor that will influence the power output. For sparse arrays, it is common to choose a uniform PTO configuration over an array, whereby the PTO damping coefficient could be set to equal the optimal damping of an isolated device (Babarit, 2010) or the one that can maximize the total power output (Renzi et al., 2014). For dense arrays, it was suggested by De Backer et al. (2010) that the absorbers should be configured individually in order to achieve higher power absorption. Note that all these studies employed frequency domain models in the investigation, hence only considered linear PTOs. Nonlinear PTOs using, e.g., Coulomb damping, have been found to outperform linear PTOs from the hydrodynamic and structural perspectives (Lamont-Kane et al., 2017). However, at the time of writing, no report is known to the authors that has investigated a large array with nonlinear PTOs. The reason may be because of the high computational demands of the conventional time domain models for a large array.

Although the frequency domain approach has been widely accepted in WEC array investigations due to its high efficiency, its linearization assumption has limited its application scope. The scope has been expanded by Korde and Ringwood (2016) by introducing a Fourier-Galerkin direct transcription method to describe the dynamics of WEC. In such an approach, the displacement and PTO force are approximated by a truncated Fourier series; thus, the numerical integration of the convolution integral can be avoided when solving the motion equations, which significantly reduces the computational cost. Merigaud and Ringwood (2017c) have applied this so-called “nonlinear frequency-domain (NLFD)” approach to investigate WECs with various nonlinear terms. Their results indicate that such an approach is superior for applications involving a large number of simulations, such as WEC parametric optimization. This approach also has potential to be applied in the development of control strategies for WECs, as stated by Merigaud and Ringwood (2017a).

Recently, a novel hybrid renewable energy device termed the Ocean Grazer (OG) 3.0 was proposed, which combines adaptable WEC technology with on-site energy storage and wind turbines to harvest renewable energy offshore (see the detailed description in section 2). The present work is devoted to investigating the OG-WEC device, which consists of a large array of point absorbers, each connected to a piston pumping unit. To this end, a cost-effective numerical model that can handle dense WEC arrays with nonlinear PTOs is proposed based on the NLFD approach. The paper is organized as follows: the OG 3.0 concept is briefly introduced in section 2; next, the numerical model and its solution are described in section 3; later, the results about a single device and an array are discussed and the preliminary optimization of the PTO configuration is performed in section 4; finally, conclusions are drawn in section 5.

2. Ocean Grazer 3.0

The OG 3.0, which is a revised version of OG 1.0 described in Vakis and Anagnostopoulos (2016), is a bottom-fixed wave and wind hybrid device combining an offshore wind turbine in its center with a closely spaced WEC array around it and featuring an on-site hydropower-type storage system. An artist’s impression of OG 3.0 is shown in Fig. 1, while more details can be found in Fig. 2. The integration of technologies and infrastructure leads to a cost-

Figure 1: Artist's impression of OG 3.0 (developed by Ocean Grazer B.V., <http://www.oceangrazer.com>)

efficient solution for harvesting multi-source offshore renewable energy. The on-site storage system smoothens out the intermittent power output of wind and wave energy converters, offering energy buffering capabilities to satisfy energy market demand and maximize revenue; see Barradas-Berglind et al. (2018) as an example.

The OG-WEC is designed to react to the high variability of waves by its adaptability, which is realized by using a novel method, termed the MP²PTO (multi-piston, multi-pump power take-off) system with a highly variable load control. The advantage of the adaptability for wave energy extraction has been demonstrated in Wei et al. (2017). The storage system is housed in a gravity-based concrete structure which enables the device to store finite amounts of potential energy, similarly to the concept of hydropower. Conditioned working fluid is pumped within a closed system from a concrete rigid reservoir at atmospheric pressure to an outside flexible bladder. Working fluid that is being pumped into this bladder structure reaches the same hydrostatic pressure as the surrounding ocean water, thereby creating pressure difference between the outer flexible bladder and the rigid concrete reservoir. Subsequently, multiple hydro-turbines can convert this pressure difference into electricity on demand. This results in having on-site storage and an energy supply that is decoupled from the demand.

3. Nonlinear frequency domain model

3.1. Governing equations

Referring to Fig. 2, consider an OG-WEC system comprising a finite number of buoys in an ocean of finite depth D . A honeycomb arrangement for the array of 18 buoys with a stationary pillar in the centre is considered, as shown in Fig. 2(a). The inner buoys are placed in a circle with a radius of 15 m, and all buoys are separated by the same distance between.

They are connected to the pillar via trusses to maintain their configuration and spacing (not sketched). All buoys have a uniform geometry as described in Fig. 2(b). Fig. 2(c) sketches the PTO system of the OG-WEC. Each buoy is linked to a controllable transmission system via a cable. The transmission system (currently in development and, hence, not detailed in the sketch but represented with a red dashed line in Fig. 2(c)) converts the heaving motion of the buoys into the driven force which pumps the working fluid from the inner rigid reservoir into the flexible bladder. When the stored potential energy is transformed into electricity by means of a hydraulic turbine, the rigid reservoir is refilled.

Figure 2: Sketch of the OG-WEC system: honeycomb arrangement of an array (a), geometry of a cone-cylinder buoy (b) and schematic of the power take-off system (c). Pressure in the pillar and rigid reservoir is atmospheric (ATM).

The mechanical system of one buoy with an individual piston pump can be simplified into a lumped-element model. The buoy and piston are considered as two mass bodies, and the connecting cable is regarded as a spring-damper element. The transmission system is assumed to be a lossless system with an adaptable transmission ratio α , which is defined as the ratio of the output force to the input force:

$$\alpha = \frac{F_{out}}{F_{in}} = \frac{\dot{z}_{in}}{\dot{z}_{out}} \quad (1)$$

Thus, the complete buoy-piston system is governed by the following motion equations:

$$\mathbf{M}_b \ddot{\mathbf{X}}_b = \mathbf{F}_e + \mathbf{F}_r + \mathbf{F}_{hs} + \mathbf{F}_c \quad (2)$$

$$m_p (\ddot{z}_p + g) = -F_p - F_f - \alpha F_{c,3} \quad (3)$$

where $\mathbf{M}_b \in \mathbb{R}^{6 \times 6}$ is a diagonal matrix of masses and moments of inertia for the general case of a buoy with 6 DoFs, $\mathbf{X}_b \in \mathbb{R}^6$ is the displacement vector of the buoy, and $\mathbf{F}_e$, $\mathbf{F}_r$, $\mathbf{F}_{hs}$ and $\mathbf{F}_c \in \mathbb{R}^6$ represent the vectors of excitation, radiation, hydrostatic restoring and cable forces, respectively. In addition, m_p is the mass of the piston, z_p is its displacement, F_p is the pumping force due to the pressure difference between two ends of the piston, F_f is the friction force between the piston and cylinder, and $F_{c,3} = [0 \ 0 \ 1 \ 0 \ 0 \ 0] \mathbf{F}_c$ is the cable force in the heaving direction.

In Eq. (3), the friction force is about four orders of magnitude smaller than the pumping force, according to calculations by Vakis and Anagnostopoulos (2016). Therefore, friction is neglected in the present model. The pumping force depends on both the dynamics of the check valve and the piston. It can be very large during the upstroke, but becomes zero during the downstroke, which introduces a nonlinearity into the WEC system. Due to the variation of the working fluid levels in the reservoir being very small and the hydrostatic pressure in the bladder being roughly constant over a short period, the pressure difference between the reservoir and the bladder can be assumed to be constant during the simulation time. Thus, the pumping force can be described as:

$$F_p = \rho g (D - h_r) A_t + \rho l_p \ddot{z}_p A_t + \rho \dot{z}_p^2 A_t, \quad (4)$$

where A_t is the time-varying open area of the check valve, l_p is the length of the piping between the reservoir and the bladder, h_r is the depth of the reservoir and ρ is the density of the working fluid (assumed to be the same as sea water density); refer to Fig. 2(c). The first term in Eq. (4) represents the hydrostatic force due to the hydraulic head, which behaves similarly to a Coulomb damper; the second term accounts for the inertia of the working fluid in the pipes; and the third term describes the dynamic pressure of the working fluid. The check valve will be open during the upstroke and closes passively during the downstroke. The switching between the upstroke and downstroke is non-smooth and can be problematic in numerical modelling. The sigmoidal function is commonly employed to deal with such discontinuities. For example, Merigaud and Ringwood (2017b) used a hyperbolic tangent function to approximate a Coulomb damping force. In this work, we model the switching by assuming that the open area of the check valve is a logistic function of the piston velocity, namely

$$A_t = \frac{A_p}{1 + e^{-\beta(\dot{z}_p - \dot{z}_{p0})}}, \quad (5)$$

130 where A_p is the piston cross-sectional area, $\dot{z}_{p0}$ is the threshold piston velocity that makes
 131 the check valve half open, and β is the steepness coefficient. The parameters in Eq. (5) can
 132 be empirically determined to match the results of the single piston pump experiment of Wei
 133 et al. (2018).

Due to the presence of the non-linear terms, Eqs. (2) and (3) cannot be simplified into the usual linear frequency-domain equations. Instead, we consider the steady-state motion of a WEC array, the free water surface elevation, as well as the displacements of buoy and piston approximated through a truncated Fourier series as follows:

$$\eta(t) = A_0 + \sum_{n=1}^{N_h} \text{Re} \left\{ \hat{A}_{\omega,n} e^{in\omega t} \right\} \quad (6)$$

$$\mathbf{X}_b(t) = \mathbf{X}_{b,0} + \sum_{n=1}^{N_h} \text{Re} \left\{ \hat{\mathbf{X}}_{b,n} e^{in\omega t} \right\} \quad (7)$$

$$z_p(t) = z_{p,0} + \sum_{n=1}^{N_h} \text{Re} \left\{ \hat{z}_{p,n} e^{in\omega t} \right\} \quad (8)$$

where ω is the frequency step for the signal with period $T = \frac{2\pi}{\omega}$, and N_h is the number of the harmonic frequencies. For a frequency $n\omega$, where $\hat{A}_{\omega,n}$ represents the complex wave amplitude components obtained from a wave spectrum or a designed wave train, $\hat{\mathbf{X}}_{b,n}$ corresponds

to the amplitude of the buoy displacement, and $\hat{z}_{p,n}$ is the amplitude of the piston displacement. The $n = 0$ component indicates the mean water level or displacements. Consequently, the forces in Eqs. (2) and (3) can be written as:

$$\mathbf{F}_e(t) = \sum_{n=1}^{N_h} \text{Re} \left\{ \hat{A}_{w,n} \hat{\mathbf{F}}_{e,n} e^{in\omega t} \right\} \quad (9)$$

$$\mathbf{F}_r(t) = \sum_{n=1}^{N_h} \text{Re} \left\{ (n^2 \omega^2 \mathbf{A}_{r,n} - in\omega \mathbf{B}_{r,n}) \hat{\mathbf{X}}_{b,n} e^{in\omega t} \right\} \quad (10)$$

$$\mathbf{F}_{hs}(t) = -\mathbf{K}_{hs} \mathbf{X}_{b,0} + \sum_{n=1}^{N_h} \text{Re} \left\{ -\mathbf{K}_{hs} \hat{\mathbf{X}}_{b,n} e^{in\omega t} \right\} \quad (11)$$

$$\mathbf{F}_c(t) = \mathbf{K}_c (\mathbf{X}_{b,0} - \alpha \mathbf{z}_{p,0}) + \sum_{n=1}^{N_h} \text{Re} \left\{ (-in\omega \mathbf{C}_c - \mathbf{K}_c) (\hat{\mathbf{X}}_{b,n} - \alpha \hat{\mathbf{z}}_{p,n}) e^{in\omega t} \right\} \quad (12)$$

$$F_p(t) = F_{p,0} + \sum_{n=1}^{N_h} \text{Re} \left\{ \hat{F}_{p,n} e^{in\omega t} \right\} \quad (13)$$

where $\hat{\mathbf{F}}_{e,n}$, $\mathbf{A}_{r,n}$ and $\mathbf{B}_{r,n}$ are the excitation, added mass and radiation damping at the n th harmonic frequency, respectively, and $\mathbf{K}_{hs}$ contains the hydrostatic stiffness coefficients. The former can be obtained numerically with a BEM solver such as NEMOH (Babartit and Delhommeau, 2015). $\mathbf{K}_c$ and $\mathbf{C}_c$ are the stiffness and damping coefficients of the cable. The n th harmonic pumping force coefficients $\hat{F}_{p,n}$ can be obtained via Fourier transform, by performing the numerical integration:

$$F_{p,0} = \frac{1}{T} \int_0^T F_p(\dot{z}_p, \ddot{z}_p) dt \quad \text{when } n = 0 \quad (14)$$

$$\hat{F}_{p,n} = \frac{2}{T} \int_0^T F_p(\dot{z}_p, \ddot{z}_p) e^{-in\omega t} dt \quad \text{when } n \geq 1 \quad (15)$$

Substituting Eqs. (9) and (13) into Eqs. (2) and (3) and sorting out the terms with the same frequency, the system of equations (2)-(3) can be converted into a set of linearized equations over each harmonic, i.e., the n th harmonic motion equations for the i th WEC are expressed as:

$$\begin{aligned} & \left[-n^2 \omega^2 \left(\mathbf{M}_{b,n}^{(i)} + \mathbf{A}_{r,n}^{(i,i)} \right) + in\omega \left(\mathbf{B}_{r,n}^{(i,i)} + \mathbf{C}_c^{(i)} \right) + \left(\mathbf{K}_{hs,n}^{(i)} + \mathbf{K}_c^{(i)} \right) \right] \hat{\mathbf{X}}_{b,n}^{(i)} \\ & + \sum_{j=1, j \neq i}^{N_b} \left(-n^2 \omega^2 \mathbf{A}_{r,n}^{(i,j)} + in\omega \mathbf{B}_{r,n}^{(i,j)} \right) \hat{\mathbf{X}}_{b,n}^{(j)} - \alpha_i \left(in\omega \mathbf{C}_c^{(i)} + \mathbf{K}_c^{(i)} \right) \hat{\mathbf{z}}_{p,n}^{(i)} = \hat{\mathbf{F}}_{e,n}^{(i)}, \end{aligned} \quad (16)$$

$$- \alpha \left(in\omega \mathbf{C}_c^{(i)} + \mathbf{K}_c^{(i)} \right) \hat{\mathbf{X}}_{b,3}^{(i)} + \left[-n^2 \omega^2 m_{p,n}^{(i)} + \alpha_i^2 \left(in\omega \mathbf{C}_c^{(i)} + \mathbf{K}_c^{(i)} \right) \right] \hat{\mathbf{z}}_{p,n}^{(i)} = -\hat{F}_{p,n}^{(i)}, \quad (17)$$

134 where N_b is the number of the buoy.

The pumping power is used to assess the performance of the OG-WEC. The mean pumping power of the i th WEC is calculated by:

$$\overline{P_p^{(i)}} = \frac{1}{T} \int_0^T F_p^{(i)}(t) \dot{z}_p^{(i)}(t) dt. \quad (18)$$

For the single device, the capture factor is used to assess its performance, which is defined as the ratio between the mean pumping power and the power of the incident wave per unit width of the device, i.e.,

$$CF = \frac{P_p}{\frac{1}{8} \rho g H^2 c_g D_w}, \quad (19)$$

135 where D_w is the width of the floater, and c_g is the group velocity.

The constructive or destructive interaction of a WEC array can be quantified by the q -factor (Babarit, 2010). The q -factor expresses the average capture power of an array compared to the capture power of an isolated device P_{iso} ,

$$q = \frac{\sum_{i=1}^{N_b} \overline{P_p^{(i)}}}{N_b P_{iso}}. \quad (20)$$

136 3.2. Numerical solution

One can obtain the dynamics of the array by solving the motion equations of Eqs. (16)-(17) in all the harmonic frequencies for given incident waves. These are linearized equations that can be solved easily in a conventional numerical computing environment such as MATLAB. Note that $\hat{F}_{p,n}$ in the equations is unknown and is a function of piston displacement, as expressed in Eq. (15). Hence, an iterative procedure is required to solve for it since Eq. (15) is a non-analytic complex function (Nevanlinna and Paatero, 1969), whose real and imaginary parts must be solved separately. Eqs. (14) and (15) can be expressed as a nonlinear vector equation of the form

$$\mathbf{G} = \mathbf{F}^0 - \mathbf{F}^1(\mathbf{Z}^0) = 0, \quad (21)$$

where the superscripts of 0 and 1 indicate the results of the current and next iteration, and $\mathbf{Z}$ and $\mathbf{F}$ are the displacement and pumping force coefficients, respectively, as follows:

$$\begin{aligned} \mathbf{Z} = & \left[z_{p,0}^{(1)}, \text{Re} \left\{ \hat{z}_{p,1}^{(1)}, \dots, \hat{z}_{p,N_h}^{(1)} \right\}, \text{Im} \left\{ \hat{z}_{p,1}^{(1)}, \dots, \hat{z}_{p,N_h}^{(1)} \right\}, \dots, \right. \\ & \left. z_{p,0}^{(N_b)}, \text{Re} \left\{ \hat{z}_{p,1}^{(N_b)}, \dots, \hat{z}_{p,N_h}^{(N_b)} \right\}, \text{Im} \left\{ \hat{z}_{p,1}^{(N_b)}, \dots, \hat{z}_{p,N_h}^{(N_b)} \right\} \right]^T \in \mathbb{R}^{(2N_h+1)N_b}, \end{aligned} \quad (22)$$

$$\begin{aligned} \mathbf{F} = & \left[F_{p,0}^{(1)}, \text{Re} \left\{ \hat{F}_{p,1}^{(1)}, \dots, \hat{F}_{p,N_h}^{(1)} \right\}, \text{Im} \left\{ \hat{F}_{p,1}^{(1)}, \dots, \hat{F}_{p,N_h}^{(1)} \right\}, \dots, \right. \\ & \left. F_{p,0}^{(N_b)}, \text{Re} \left\{ \hat{F}_{p,1}^{(N_b)}, \dots, \hat{F}_{p,N_h}^{(N_b)} \right\}, \text{Im} \left\{ \hat{F}_{p,1}^{(N_b)}, \dots, \hat{F}_{p,N_h}^{(N_b)} \right\} \right]^T \in \mathbb{R}^{(2N_h+1)N_b}. \end{aligned} \quad (23)$$

Thus, the total number of the nonlinear equations to be solved is $(2N_h + 1)N_b$. The convergence results of $\hat{\mathbf{X}}_{b,n}$, $\hat{z}_{p,n}$ and $\hat{F}_{p,n}$ are obtained until $\|\mathbf{G}\|^2$ is smaller than a given tolerance.

To solve such a nonlinear algebraic system, the Jacobian matrix $\mathbf{J}_G(\mathbf{F}^0)$ should be provided,

$$J_{ij} = \frac{\partial G_i}{\partial F_j^0} = \delta_{ij} - \frac{\partial F_i^1}{\partial F_j^0}, \quad (24)$$

where δ_{ij} is the Kronecker delta. Calculating the Jacobian matrix through the finite difference method can be quite time consuming, particularly for large scale problems, i.e., a large number of buoys, and/or many orders of harmonic motion considered, because each iteration implements a lot of numerical integration operations. To improve the computational performance, Merigaud and Ringwood (2017a) applied an analytic Jacobian by approximating the nonlinearities in a cubic polynomial form, as detailed in Failla et al. (2003). Such an approximation may not be suitable for the square-wave-like pumping force. Hence, we use a revised expression of a numerical Jacobian matrix instead:

$$J_{ij} = \delta_{ij} - \sum_{k=1}^{(2N_h+1)N_b} \frac{\partial F_i^1}{\partial Z_k^0} \frac{\partial Z_k^0}{\partial F_j^0}. \quad (25)$$

According to Eqs. (14) and (15), $\frac{\partial F_i^1}{\partial Z_k^0} = 0$ if F_i and Z_k are the coefficients from different buoys. Consequently, Eq. (25) is more computationally efficient than Eq. (24), because the total amount of numerical integration operations is reduced from $(2N_h + 1)^2 N_b^2$ to $(2N_h + 1)^2 N_b$. Tests we conducted show that using the two Jacobian expressions can derive the same results, while the computational time with the revised Jacobian is less than $1/10^{th}$ of the original, for the case of an array with 18 WECs.

4. Results and discussion

In this section, the simulation results of single WEC and WEC arrays will be presented. As this constitutes a preliminary study of the OG-WEC aiming to show the applicability of the NLFD method to this system, further simplifications are made in the present simulation: (1) since we do not yet have a mature design of the truss configuration that restricts the motion of the buoys, only one DoF in heave is considered for the buoys in the simulation to represent the idealized behavior of a mechanically interconnected floater array; (2) the dynamics of the cable are not investigated and very large stiffness and zero damping coefficient are chosen for the cable. The parameters of OG-WEC are summarized in Table 1 and are used in all simulations presented in this paper.

The proposed numerical model has been validated by comparing the simulation results with the results from the single piston pump experiment presented in Wei et al. (2018). One of the significant differences of the system dynamics between using the piston pump versus a linear PTO is the incidence of the sticking phenomenon. Specifically, in one wave cycle, the wave excitation force may not overcome the pumping force and, as a result, the piston will stall ($\dot{z}_p = 0$); this is referred to as sticking. A convergence study by Wei et al. (2018)

Table 1: Parameters used in the simulations

Description	Symbol	Value	Unit
Water depth	D	60	m
Buoy mass	m_b	15000	kg
Stiffness of cable	K_c	1×10^9	m/N
Damping of cable	C_c	0	m/(sN)
Piston mass	m_p	1000	kg
Piston diameter	d_p	1.0	m
Working fluid density	ρ	1025	kg/m ³
Length of pipe	l_p	30	m
Threshold piston velocity	$\dot{z}_p$	0	m/s
Steepness of check valve	β	50	s/m
Number of buoys	N_b	18	–
Number of harmonics	N_h	5	–

161 suggested that 3rd order harmonics are essential in order to describe such a phenomenon. In
162 this paper, all the simulations are carried out with 5 harmonic frequencies.

163 4.1. Single WEC analysis

164 OG-WEC adaptability is essentially achieved by varying the transmission ratio to achieve the
165 maximum power output, as demonstrated in Fig. 3(a), where a definite peak of the capture
166 factor can be found at point C for $\alpha = 1.02$. Applying a low transmission ratio, e.g., at point
167 A, means that the output force for driving the piston is scaled down, according to Eq. (1).
168 Thus, this force may be not high enough to overcome the resistance due to the hydrostatic
169 pressure difference at the two sides of the piston. Consequently, the displacement of the
170 buoy is very small, as can be observed in Fig. 3(b). As a result, the extracted energy is low;
171 this can be described with pumping force–piston displacement (F–D) diagram in Fig. 3(c).
172 Physically, the extracted energy over one wave cycle is equal to the area enclosed by the F–D
173 curve in the diagram, which is analogous to a pressure–volume diagram. The curve A is an
174 ellipse with the piston hardly oscillating, as indicated by the ellipse’s small horizontal width.
175 By increasing the transmission ratio, the capture factor increases rapidly, as can be observed
176 by comparing the enclosed areas of curves A, B and C in Fig. 3(c). However, increasing the
177 transmission ratio does not further increase the power output although the displacement of
178 the buoy slightly increases, as shown in curves D and E in Fig. 3(b). The dynamics of the
179 buoy in these cases suggest that it tends to behave as a free-floating body.

A single WEC with a piston pump is analogous to the mechanical system of a harmoni-
cally excited oscillator incorporating Coulomb-type friction, which has been investigated by
Csernák and Stépán (2006) who analyzed the dynamic patterns of the oscillator in non-
dimensional form. In the current paper, the impact of the transmission ratio on the system
can also be understood by the dynamic patterns of the piston about its equilibrium position,
as shown in Fig. 4. Analogous to Csernák and Stépán (2006), the time variation of the piston

Figure 3: Variation of capture power for a single WEC with transmission ratio α (a), phase portrait of the buoy (b) and pumping force–piston displacement diagram (c). The dashed line in (b) is the phase portrait of a water particle on the free surface in wave.

Figure 4: Dynamic patterns of the piston in which A, C and E refer to the points in Fig. 3.

displacement is presented in non-dimensional form, which is defined as:

$$\gamma = \frac{\rho g (D - h_r) A_p}{2\alpha |\hat{F}_e(\omega)|} \quad (26)$$

$$t^* = t \frac{\omega}{2\pi} \quad (27)$$

$$z_p^* = \frac{\alpha z_p K_{hs}}{|\hat{F}_e(\omega)|} \quad (28)$$

where γ is the ratio between half of the hydrostatic force (the first term of the pumping force) and the wave excitation (scaled by the transmission ratio), which indicates the capacity for the excitation to overcome the resistance of the system. The upper and lower bounds of the shaded zone in Fig. 4 are calculated by:

$$z_{\pm}^* = \frac{\text{Re} \left\{ \hat{F}_e(\omega) e^{i\omega t} \right\}}{|\hat{F}_e(\omega)|} \pm \gamma. \quad (29)$$

180 A piston displacement within the shaded zone implies that the wave excitation is smaller
 181 than the resistance of the piston pump. The sticking phenomenon occurs when the piston
 182 switches its moving direction in the shaded zone. As a result, the piston remains stationary
 183 until the excitation force can overcome the pumping force. It is clear in Fig. 4 that applying
 184 a low transmission ratio (A) leads to a wide shaded zone, which translates into a higher
 185 probability of sticking, whereas a large transmission ratio (E) results in a narrow shaded
 186 zone, for which the duration of the sticking becomes shorter or vanishes.

In order to understand the optimal configuration of a single WEC, a series of simulations with various wave conditions is carried out, with results presented in Fig. 5. For a simulation

Figure 5: Comparison of the maximum capture factor between WECs with a linear PTO and an adaptable pumping unit.

with a specific wave condition, the built-in single-variable minimizer with lower and upper bounds “fminbnd” in Matlab is used to identify the optimal transmission ratio that can lead to maximum overall power production. The maximum capture factor obtained from a linear PTO with a theoretical optimal damping coefficient (Falnes, 1980) is also plotted for the comparison. It is found that the maximum capture factors under low amplitude waves are in good agreement with those of a linear PTO. However, significant deviation occurs under larger amplitude waves. The capture factor in these cases is slightly lower than that

with a linear PTO under low frequency waves and becomes significantly larger under high frequency waves. This finding is inconsistent with that of Lamont-Kane et al. (2017), where the influence of the damping profile on WEC performance was investigated and the authors stated that the optimal power output by a Coulomb damper was less than approximately 2% across over 95% of the sea states (mean wave frequency between 0.43rad/s - 1.80rad/s), compared to that by a linear damper. The natural frequency of a floating buoy is high, about 2.66 rad/s in the present configuration, which is beyond most of the wave frequencies occurring in the ocean. When the buoy drives the piston to move upward, the fluid in the pipe flows synchronously with the piston. Since the mass of the moving water column is in the same order of the mass of the buoy, the fluid inertia effects have significant impact on the WEC system dynamics, which is equivalent to introducing a supplementary mass to the motion equation during the upstroke. Substituting Eq. (4) into Eq. (3), and combining Eqs. (2) and (3), one can obtain this supplementary mass which is expressed as:

$$m_{sup} = \frac{1}{\alpha^2} \rho l_p A_t. \quad (30)$$

Consequently, the system's resonant frequency is lowered and the pumping unit can theoretically extract more energy with an optimal configuration. In addition, because the supplementary mass is inversely proportional to the square of the transmission ratio, and the optimum transmission ratio decreases with the wave amplitude as shown in Fig. 6(b), the pumping unit performs better under large amplitude waves. Finally, the spikes observed at $\omega \approx 1.1$ rad/s in Fig. 5 are attributed to the sharp variation of the sub-harmonic hydrodynamic coefficients, as discussed in Wei et al. (2018).

Figure 6: Contours of maximum capture factor of a single WEC (a), optimum transmission ratio (b), RAO of the buoy (c) and ratio between hydraulic force and scaled excitation force (d) in various wave conditions.

Fig. 6 presents the comprehensive results of a single WEC in various wave conditions. It is common knowledge that the capture factor is independent of wave amplitude for a linear WEC system. However, this is not true for a WEC with a nonlinear PTO. The maximum

capture power in Fig. 6(a) shows that the performance of the single WEC is not sensitive to the wave amplitude under low frequency waves, while it is superior under large amplitude and high frequency waves because of the fluid inertia effects discussed previously.

The corresponding contour of the optimal transmission ratio is shown in Fig. 6(b). It can be seen that the optimal transmission ratio increases with wave frequency and decreases with wave amplitude. There is a definite inverse correlation between the optimal transmission ratio and the power of the incident waves. Essentially, the more powerful are the incident waves, the stronger is the required piston pump capacity, as would be expected. In addition, the optimal transmission ratio varies over a wide range, roughly 0.6–6.7, which implies that the adaptability of the transmission system will be very significant when a WEC extracts energy under realistic sea states; this implication will be investigated in future work.

The response amplitude operator (RAO) of a single WEC with nonlinear displacement in Fig. 6(c) is defined as the ratio of the stroke of the buoy and the incident wave height. The variation of the RAO is complex over the wave conditions of interest and may be caused by the occurrence of the sub-harmonic resonance as mentioned in Csernák et al. (2007). The spikes are observed at the same frequency as that in Fig. 5, which can therefore be attributed to the same reason stated previously.

Interestingly, the ratio of the pumping force to the scaled excitation force γ in Fig. 6(d) is approximately constant at 0.65 across a wide range of wave conditions. This finding may be very useful for the preliminary design of the PTO system for the OG-WEC. Specifically, a general rule of thumb for an optimized design of the pumping system is to make the pumping force roughly equal to two thirds of the scaled excitation force.

It must be emphasized that the parameters in Table 1 have not been optimized yet. Careful design of the piston pump and buoy, e.g., by changing the length of the pipe to vary the inertia of the system, or modifying the geometry and density of the buoy, can potentially improve its performance. This is not the scope of the present work but will be investigated for the OG-WEC device in the future.

4.2. WEC array analysis

This section discusses the results of a WEC array where each buoy is connected to a pumping unit with a uniform transmission ratio. Similarly to the single buoy optimization presented in the previous section, a single-variable minimizer is employed to obtain the optimal uniform transmission ratio.

Fig. 7(a) shows the q -factor of an array with the corresponding optimal uniform transmission ratio in Fig. 7(b). The q -factor is always less than 1, indicating that the wave interference has destructive effects on the output power of the array with the current honeycomb layout. These effects become more destructive under high frequency waves. This is not a surprise, since the scattering effect plays a more important role under high frequency wave conditions that may diminish the overall power absorption of the array. Nevertheless, the q -factor is greater than 0.8 in the majority of the wave conditions and destructive effects of such

Figure 7: Contours of q -factor (a) and optimal transmission ratio (b) of an array with uniform PTO configuration.

236 magnitude can be acceptable for a dense array. In addition, due to the complex (sub-)
 237 harmonic oscillations in the array, there is no clear correlation between the q -factor and
 238 wave amplitude found in Fig. 7(a). The contour of the optimal transmission ratio of an
 239 array (Fig. 7(b)) looks similar to that of a single WEC in Fig. 6(b), except under high
 240 frequency waves. The differences may be explained by the reduction of the excitation force
 241 as evidenced in Fig. 8(a), where a significant drop of the mean excitation force can be
 242 observed for $\omega > 1.4$ rad/s.

243 The interaction in the array can be analyzed by sorting the WECs into different groups
 244 roughly by rows: GR1 (9, 15, 10, 16, 11), GR2 (14, 3, 4, 5, 17), GR3 (8, 2, 6, 12), GR4
 245 (13, 1, 18, 7); and columns: GC1 (9, 14, 8), GC2 (15, 3, 2, 13), GC3 (10, 4, 1, 7), with
 246 the numbers referring to Fig. 2(a). The interactions in the array are very complex. The
 247 excitation force on buoys is significantly influenced by their neighbours for a wide range of
 248 wave frequencies. Either enhancement or suppression may occur on each row, as can be
 249 seen in Fig. 8(b). The first buoys interacting with the incident waves (GR1) may not always
 250 suffer the strongest wave excitation. For example, the mean excitation force of the WECs
 251 in GR1 is maximum at low frequencies but becomes almost the lowest at high frequencies,
 252 while that of GR4 appears to have the reverse trend. The groups in column (Fig. 8(c)) only
 253 show small discrepancies that may be attributed to the honeycomb arrangement of the array
 that results in less masking effects for the columns.

Figure 8: Comparison of mean excitation force of the array and that of a single WEC (a), mean excitation force of each row (b) and mean excitation force of each column (c).

Fig. 9 shows the variation of the power production with wave frequency. Fig. 9(a) offers an explicit comparison of the performance of a single WEC and an array. The destructive effects increase with wave frequency; this has also been observed in Fig. 6(a). The great difference of the power production by each row can be found in Fig. 9(b). The 1st and 2nd rows (GR1 and GR2) of the WECs feature higher power production relative than other rows, as they are rarely masked by their neighbors and benefit from waves being radiated by the two other rows. GR1 yields smaller averaged power than that of GR2 in the frequency range of $0.7 - 1.2$ rad/s because the mean excitation force of GR2 is greater than that of GR1, as shown in Fig. 8(b). The 4th row (GR4) has constant negative interaction and features the lowest power production due to both masking effects and out of phase waves radiated by neighbouring WECs. The power production by each column does not show significant differences in Fig. 7(c). A general trend is for WECs placed close to the pillar (GC1) to produce less power than those placed farther away (GC2&3).

Figure 9: Comparison of mean capture power of the WECs in the array and that of a single WEC (a), mean capture power of the WECs each row (b) and mean capture power of each column (c). The wave amplitude for the simulation in this figure is $A_w = 1.5$ m.

In order to understand the power production characteristic of an array, the results of WECs for three wave frequencies are presented in Fig. 10. Based on the uniform PTO configuration, the WECs which encounter larger excitation force can generally produce more power (larger q -factor), but a few exceptions can be found under higher frequency waves. For example, in the case of $\omega = 1.0$ rad/s, the excitation force on WEC 4 is the largest, but its q -factor is smaller than those of its surrounding WECs (3, 5 and 10); this may be because the presence of the pillar has positive influence on its neighbors at this frequency; in the case of $\omega = 1.5$ rad/s, the largest excitation force occurs for WEC 7 which is located at the right of the array, while the largest q -factor is found at the left (WEC 10). This implies that, as expected, the radiated waves play a significant role in the performance of WECs in dense arrays.

Figure 10: Results of individual excitation force (top) and q -factor (bottom) in an array under various frequency waves. Wave propagation is from left to right.

4.3. PTO configuration optimization

In the previous section, a uniform transmission ratio was used in the simulations. However, this may be not an efficient way to absorb energy due to the strong interactions in an array that may require for the transmission ratio of each WEC to be tuned individually. In this section, two other optimization strategies, namely excitation force-based optimization (EO) and individual optimization (IO), are proposed and compared with the uniform optimization (UO) discussed in the previous section.

According to Fig. 6(b) and (d), the maximum power output of a single WEC can be achieved by adjusting the ratio of the pumping force to the scaled excitation force to a uniform constant value. The EO strategy aims to maximize the possible power output of each WEC by applying the transmission ratio optimized for a single WEC to all individual WECs in the array. Based on the excitation force on each WEC, the transmission ratio is calculated with a lookup table from the results in Fig. 6(b). Note that this strategy only takes into account the excitation force and neglects the radiation by other WECs.

The IO searches the maximum overall power production of an array by varying the transmission ratio of each WEC. It adopts a similar numerical approach as that of UO, but uses a multivariate minimizer function “fmincon”, instead of a single-variable one. Since the function is based on a sequential quadratic programming algorithm which might converge to a local minimum depending on the initial condition, the results by the EO are used as the guess value. The IO is time consuming because the NLFD model has to be implemented at every interval.

Figure 11: Results of individual q -factor (top) and corresponding optimal transmission ratio (bottom) in an array under various frequency waves, using the EO. Wave propagation is from left to right.

Figure 12: Results of individual q -factor (top) and corresponding optimal transmission ratio (bottom) in an array under various frequency waves, using the IO. Wave propagation is from left to right.

Table 2: Mean transmission ratio and q -factor by various optimization strategies

Strategies	$\bar{\alpha}$			$\bar{q}$		
	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
UO	1.103	1.486	2.969	0.932	0.805	0.573
EO	1.130	1.403	4.030	0.934	0.813	0.535
IO	1.111	1.464	2.954	0.938	0.828	0.618

Figs. 11 and 12 show the results of the q -factor and corresponding optimal transmission ratio with the EO and IO under three wave frequencies. From the results of the three optimization strategies in Figs. 10, 11 and 12, it can be seen that the distribution of the q -factors behaves similarly under various frequency waves, although their values present noticeable differences on some WECs, particularly under high frequency waves. Table 2 summarizes the results of the various optimization strategies. It can be found that applying a uniform PTO configuration (UO) cannot achieve the maximum power production in general. Although no noticeable improvement is made with the proposed optimization strategies under low frequency waves, applying an appropriate PTO configuration in the array can potentially increase power production under higher frequency waves by a few percentage points. The EO strategy can enhance the power production slightly under moderate frequency waves without a heavy computational load, while it may not be suitable for high frequency waves because it ignores the radiation by the neighbouring WECs and thus overestimates the transmission ratio under high frequency waves, i.e., $\bar{\alpha}|_{\omega=1.5\text{rad/s}} = 4.030$ that is significantly larger than that of other two strategies. The IO strategy outperforms UO and EO in all simulations. However, it is not feasible to carry out such intensive simulations for real-time control due to their high computational demand. It is the aim of the authors to develop more efficient strategies in future research.

5. Conclusions

In the present work, an NLFD model which is based on the Fourier approximation method is developed. This model has been applied to investigate the OG-WEC device which consists of a large WEC array with a cluster of highly nonlinear pumping units. A series of simulations is carried out to analyze the dynamics, power production characteristics and optimal configurations of the OG-WEC device. The following conclusions can be drawn from the present study:

- The NLFD model can be employed to investigate the nonlinear dynamics of a large WEC array with high computational efficiency and acceptable accuracy. The latter can be improved significantly with the provision of the revised Jacobian matrix.
- Theoretically, a WEC with an MP²PTO system outperforms one with a linear PTO due to the fluid inertia effects in the pumping system.
- The optimal transmission ratio α of a single WEC increases with the wave frequency and decreases with wave amplitude, whereas the ratio of the pumping force to the

scaled excitation force γ is approximately constant at 0.65. This finding can provide a general rule of thumb for designing the piston pump of an OG-WEC device.

- Grouping the WECs into a large dense array leads to destructive effects to the overall energy extraction in general, with such effects becoming more negative under high frequency waves. It is recommended to perform WEC geometry and array layout optimization for the OG-WEC, in order to obtain constructive effects.
- Using an optimized non-uniform PTO configuration over an WEC array can improve the overall performance of the device. A computationally cost effective optimization should be developed in advance.

This work investigated only the dynamics and power production of the OG-WEC in regular waves, although the proposed model can deal with irregular waves as well. Modelling the response of an OG-WEC array under irregular waves requires a large number of harmonics for a good approximation of the solution, which, on the other hand, will be significantly more time consuming (and may become inefficient compared to the time-domain model). Further simplification of the NLFD model may be required to balance the accuracy and efficiency. It is possible to apply the NLFD model (or the modification thereof with much shorter computational time) in real-time control. The real-time control principle will follow a similar approach to model predictive control (MPC), where our NLFD model will serve as the predictive model in the MPC based on the incoming waves and the receding horizon optimization can be related to the maximization of wave energy conversion with the number of wave cycles as the time horizon. This is on-going work. Only the honeycomb arrangement of the array with one wave direction is considered in the present work, while other WEC layouts and their directionality should also be investigated in the future.

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