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Highlights

- Avitzur's upper-bound solution for uniaxial compression test of discs is assessed for its validity range.
- Via kinematics of the deforming sample of Avitzur model, different approaches were adopted to solve the barreling parameter of the model.
- The model was investigated within two frameworks: static and dynamic.
- The results show that the model is valid as long as the side-surface folding is negligible, which occurred at $m=0.1$ in this work.

ACCEPTED MANUSCRIPT

Evaluation of barreling and friction in uniaxial compression test: A kinematic analysis

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ABSTRACT

Barreling compression test is an important tool for characterizing the mechanical responses of deformed material. This test also serve for studying the interfacial friction between the tool and workpiece, based on the only available method of Avitzur's "limit analysis of disc and strip forging". This method was found to be satisfactory for low friction conditions; however, it has remained to be examined in depth. In the present work, Avitzur model is thoroughly investigated, and its applicability and limitations are discussed. Along these lines, the kinematics of Avitzur model is presented to be used in describing both the barreling parameter (b) of the model and the profile of the deformed sample. Following an exhaustive examination on the barreling parameters of various deformed samples, one of the methods is identified to be valid for evaluating friction factor up to $m \leq 0.5$. Moreover, the results clearly shows that the model has its highest accuracy as long as the side-surface folding phenomenon is negligible ($\leq 0.25\%$).

1. Introduction

It is crucial to develop accurate models for describing metal flow within samples of compression tests, where the workpiece would barrel due to the surface friction between the tool and the workpiece [1, 2, 3]. While numerical solutions, e.g. finite difference, finite element and matrix methods, are widely used to study metal forming processes, their reliability depend on the accuracy of the provided data, including friction factor and material's flow curve. On the other hand, theoretical models could be used as characterizing tools for representing the deformation load as flow stress data¹. The most common way to obtain stress-strain curves from uniaxial compression tests is to assume a uniform deformation within the sample; see e.g. [6] for descriptive and practical notes on this model. Theoretically, this model is argued to be valid for proper lubrication conditions and limited deformations; however, it still cannot provide any information about the friction between the tool and workpiece. Another method, namely slab, can be used for prediction of the stress field and the stresses on tools [7]; however, this method does not account for shearing deformation within the sample, i.e. no barreling prediction. This problem could be overcome by applying another well-known method of analysis, called upper bound. The value of friction factor, however, is required in advance for both of these methods.

In a pioneering work, Ebrahimi and Najafizadeh [8] proposed a method based on Avitzur's limit analysis [9], which

could be used for estimating the friction factor; they derived the friction factor only as a function of the cylindrical sample's geometries before and after a compression test. From a slightly different approach, Solhjoo derived another formula for the friction factor of Avitzur model [1]. These approaches toward Avitzur model were employed numerous times for different purposes, including the evaluation of friction factors, and some were successful, e.g. for isotropic polymers [10]. However, the validating attempts showed that that the model is useful only for small strains and low frictions [1, 11, 5]. Khoddam and Hodgson discussed the possible reasons for poorly behavior of Avitzur model [5]: (1) the use of a simplified geometry to carry the upper bound integration, and (2) the poor estimation of the barreling parameter of the model. Yet, there is another discrepancy between Avitzur model and the evaluation attempts: Avitzur model assumes the sample's material to be a rigid-perfectly plastic one, while in both works of Solhjoo [1, 11] and Khoddam and Hodgson [5], the deformed material had work hardening in their plastic deformation regime. More importantly, Avitzur model does not account for side-surface folding, which is present at any practical conditions [12]. Avitzur and Kohser revised the original model, in order to incorporate folding phenomenon [13, 4]; however, its formulation was for problems with known friction factors.

The focus of the current investigation is on identifying the validity domains and examining the application ranges of the original Avitzur model. In this work, Avitzur's original model is briefly described, and its assumptions are clearly pointed out. Then, its solutions for the friction factor is investigated in order to explore their validity domains. Moreover, the kinematics of deformed sample in Avitzur model was described, resulting in a number of approximations of the optimum barreling parameter of the model. Moreover,

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¹The importance of developing theoretical models for deformation processes is not the focus of the current study, and the related discussions can be found elsewhere, see, e.g. [4, 5].

Figure 1: The schematic of the barreling phenomenon during the compression test. ① shows the initial state of a cylindrical sample with an initial radius R_0 and thickness T_0 . The sample is placed between two flat dies, which move toward each other along the z direction with constant velocities of $-U/2$ (top) and $+U/2$ (bottom). ② shows the deformed state of the barreled sample, with a thickness of T and a profile $R(z)$ changing between a mid-plane radius R_M and top plane radius R_T .

two different static (non-incremental) and dynamic (incremental) schemes were applied for solving the kinematics equations. With comparing the results with finite element reference solutions, it was found that the model can serve for estimating the friction factor with a high accuracy, as long as the side-surface folding has very small values.

2. A summary of Avitzur model

Figure 1 illustrates a schematic of the compression test problem, and its related parameters used in Avitzur model. In here it is assumed that the sample only shows barreling during the test. Avitzur modelled the problem in a cylindrical coordinate system, and treated only a cross-section of half of the sample by assuming homogeneity of workability within the sample. Therefore, the angular velocity was ruled out, and the height was bounded between 0 and $T/2$.

Avitzur proposed the following velocity field for the problem in the range of $z \in [0, T/2]$ [9]:

$$U_r(r, z) = BU \frac{r}{T} e^{-b_A z/T} \quad (1a)$$

$$U_z(z) = -\frac{U}{2} \frac{1 - e^{-b_A z/T}}{1 - e^{-b_A/2}} \quad (1b)$$

$$U_\theta = 0 \quad (1c)$$

with B to be found as [13]:

$$B = \frac{1}{2} \frac{b_A/2}{1 - e^{-b_A/2}}. \quad (2)$$

In this model b_A is an arbitrary coefficient representing the barreling of the deformed sample, known as the "barreling

parameter". In this model, it was assumed that the compressed material was rigid-perfectly plastic, i.e. the flow stress was constant and equal to a yield stress Y . Avitzur worked this velocity field in the upper-bound theorem, and proposed a solution for the average forging pressure p_{ave} as:

$$\frac{p_{ave}}{Y} = 8 \frac{b_A R_E}{T} \left(\left(\frac{1}{12} + \left(\frac{T}{b_A R_E} \right)^2 \right)^{3/2} - \left(\frac{T}{b_A R_E} \right)^3 - \frac{m}{24\sqrt{3}} \frac{1}{1 - e^{b_A/2}} \right), \quad (3)$$

where R_E is an effective radius of the deformed sample, defined as $R_E = R_0 \sqrt{T_0/T}$ based on the volume constancy of the sample. In the work of Avitzur, the value of m was assumed to be known, and the goal was to find the upper-bound solution for the average deformation pressure via minimizing p_{ave} ; this required an optimum value for the barreling parameter. Avitzur estimated the optimum b_A by vanishing derivative of Eq.3 with respect to b_A . By doing so, an implicit equation was obtained, which showed no advantage over Eq.3 for defining b_A [14]; though, with the assumption that the barreling parameter is in hand, a rearrangement of that derivative could be used to formulate the friction factor as:

$$m^* = 24\sqrt{3} \frac{(e^{b_A/2} - 1)^2}{b_A^3 (1 + e^{b_A/2}(b_A/2 - 1))} \left(\sqrt{\frac{b_A^2}{12} + \left(\frac{T}{R_E} \right)^2} \left(\frac{b_A^2}{12} - 2 \left(\frac{T}{R_E} \right)^2 \right) + 2 \left(\frac{T}{R_E} \right)^3 \right). \quad (4)$$

The star symbol indicates that no further assumptions besides the effective radius are made for deriving this equation. In order to find b_A , Avitzur solved the abovementioned derivative by the use of some proper series expansions and making the following assumptions: (1) $b_A < 2\sqrt{3}T/R_E$, and (2) $b_A \gg b_A^n$ for $n \geq 2$, such that $b_A^n \cong 0$. With these, Avitzur derived the following solution for b_A , in which the b_A is a function of the sample's geometry (R_E and T) and the known friction factor.

$$b_A = \frac{6}{1 + \frac{3\sqrt{3}}{2} \frac{R_E}{mT}}. \quad (5)$$

A rearrangement of this formula was later used by Ebrahimi and Najafizadeh [8] to identify the friction factor. Their simplified solution (m_{simp}) can be presented as [1]:

$$m_{simp} = \frac{3\sqrt{3}}{2} \frac{R_E}{T} \frac{b_A}{6 - b_A}. \quad (6)$$

Before ending this section, it is crucial to point out that Eq.3 was derived with an assumption that the radius of the deformed sample can simply be represented by an effective radius R_E , which is in contradiction with the aim of the solution, i.e. modelling a barreled sample. This can be noticed from the calculation of the internal power $\dot{w}_i$, in which

a constant effective radius was assumed for a given height. Moreover, the friction loss $\dot{w}_f$ was calculated for the contact between the tool and the sample using R_E instead of R_T .

3. Barreling parameter

Based on Avitzur model, the friction factor could be found from either Eq.4 or 6, which are functions of the deformed sample and the barreling parameter. While the geometry of the sample before and after deformation could be measured in experiments, the value of b_A is not measurable. In the literature, there are two approximations of b_A [8, 1], which are not fully validated yet. In an effort, Solhjoo compared Avitzur model with a series of finite element simulations [1], showing Avitzur model was satisfactory only at very small deformations. In here, based on the kinematic descriptions of deforming samples of Avitzur velocity field, we explore different approaches toward approximating b_A .

With an assumption that any point in the $r - z$ space moves with a constant velocity for a certain time range of $\Delta\tau = \tau_{i+1} - \tau_i$, the general description of the deformed sample can be written in two different sets of *forward* kinematics:

$$r^{(i+1)} = r^{(i)} + \Delta\tau U_r^{(i)}(r, z), \quad (7a)$$

$$z^{(i+1)} = z^{(i)} + \Delta\tau U_z^{(i)}(z), \quad (7b)$$

or *backward* kinematics:

$$r^{(i+1)} = r^{(i)} + \Delta\tau U_r^{(i+1)}(r, z), \quad (8a)$$

$$z^{(i+1)} = z^{(i)} + \Delta\tau U_z^{(i+1)}(z), \quad (8b)$$

where the superscripts $(i+1)$ and (i) indicate the corresponding time steps of τ_{i+1} and τ_i , respectively, at which the geometrical variables, i.e. T , R_M and R_T , were collected. It should be noted that all components of the acceleration field corresponding to Avitzur velocity field contain a b_A^2 term, which was assumed to be zero; this complies with the described kinematics.

3.1. Forward Kinematics

Using the forward kinematics description, the profile of the sample, $R(z)$, as well as its height can be written as:

$$R(z)^{(i+1)} = R(z)^{(i)} + \Delta\tau U_r^{(i)}(R(z), z), \quad (9a)$$

$$T^{(i+1)} = T^{(i)} - \Delta\tau U. \quad (9b)$$

Therefore, the mid-plane, top-plane, and the equivalent radii can be written as:

$$R_M^{(i+1)} = R_M^{(i)} + \Delta\tau U_r^{(i)}(R_M, 0), \quad (10a)$$

$$R_T^{(i+1)} = R_T^{(i)} + \Delta\tau U_r^{(i)}(R_T, T/2), \quad (10b)$$

$$R_E^{(i+1)} = R_E^{(i)} + \Delta\tau U_r^{(i)}(R_E, Z(R_E)), \quad (10c)$$

respectively. Accordingly, we can adopt different paths to derive different approximations of b_A .

3.1.1. Ebrahimi and Najafzadeh's approach

The first attempt of an analytical approximation on b_A was carried out by Ebrahimi and Najafzadeh, with the assumption that $R_M = R_T = R_E$ at any given height [8], which was in accord with the assumption made by Avitzur himself in formulating $\dot{w}_i$. This approach could be followed in here by rewriting the difference between eqs.10a and 10b as:

$$\begin{aligned} \Delta R^{(i+1)} - \Delta R^{(i)} &= \Delta\tau (U_{R_M}^{(i)} - U_{R_T}^{(i)}) \\ &= \Delta\tau B U \frac{R_M^{(i)} - R_T^{(i)} e^{-b_A/2}}{T^{(i)}}, \end{aligned} \quad (11)$$

where $\Delta R = R_M - R_T$. Using the formulation of B (Eq.2), assuming $R_M = R_T = R_E$, and replacing $\Delta\tau U$ with $T^{(i)} - T^{(i+1)}$ (from Eq.9b), Eq.11 can be rewritten, and rearranged to approximate the barreling parameter as:

$$b_{EN} = 4 \frac{T^{(i)} \Delta R^{(i+1)} - \Delta R^{(i)}}{R_E^{(i)} (T^{(i)} - T^{(i+1)})}. \quad (12)$$

3.1.2. Solhjoo's approach

In another attempt for approximating b_A , Solhjoo employed the EN's approach, but avoided the assumption of an effective R_E . Instead, he applied the other assumption of Avitzur model on the small values of b_A [1, 11]. Based on this approach, one can insert the formulation of B (Eq.2) and Eq.9b into Eq.11, to obtain:

$$\begin{aligned} 2(\Delta R^{(i+1)} - \Delta R^{(i)}) \left(\frac{T^{(i)}}{T^{(i)} - T^{(i+1)}} \right) = \\ \frac{b/2}{1 - e^{-b/2}} (R_M^{(i)} - R_T^{(i)} e^{-b_A/2}). \end{aligned} \quad (13)$$

Following Avitzur's method for deriving Eq.5, the Taylor series expansion could be employed for the functions $\frac{b_A/2}{1 - e^{-b_A/2}}$ and $e^{-b_A/2}$ at $b_A/2 = 0$. Omitting all squared and greater terms of b_A results in:

$$e^{-b_A/2} = 1 - b_A/2, \text{ and} \quad (14a)$$

$$\frac{b_A/2}{1 - e^{-b_A/2}} = 1 + b_A/4. \quad (14b)$$

Inserting Eq.14 into Eq.13, and after a rearrangement, the barreling parameter can be approximated as:

$$b_S = \frac{4}{\Sigma R^{(i)}} \times \left(2 \frac{T^{(i)}}{T^{(i)} - T^{(i+1)}} (\Delta R^{(i+1)} - \Delta R^{(i)}) - \Delta R^{(i)} \right), \quad (15)$$

where $\Sigma R = R_M + R_T$.

3.1.3. A pure kinematic approach

In here, we propose a new approximation of b_A by the use of Eq.10, which can be written as:

$$\frac{R_M^{(i+1)} - R_M^{(i)}}{R_T^{(i+1)} - R_T^{(i)}} = \frac{\Delta\tau U_r^{(i)}(R_M, 0)}{\Delta\tau U_r^{(i)}(R_T, T/2)} = \frac{R_M^{(i)}}{R_T^{(i)}} e^{b_A/2}. \quad (16)$$

With a rearrangement, the kinematic approximation of b_A can be found as:

$$b_K = 2 \ln \left(\frac{R_T^{(i)} R_M^{(i+1)} - R_M^{(i)}}{R_M^{(i)} R_T^{(i+1)} - R_T^{(i)}} \right). \quad (17)$$

Obviously, none of the assumptions of the previous approximations is imposed in the derivation of b_K .

3.1.4. Imposing Avitzur's assumptions on b_K

Although Eq.17 was approximated without any assumption except the validity of the forward kinematics (Eq.7), one can argue that an approximation of b_A should be found by imposing the same assumptions resulted in Eq.5. To that end, we suggest the following. Using Eqs.9b and 10c, one can find:

$$2 \frac{T^{(i)} R_E^{(i+1)} - R_E^{(i)}}{R_E^{(i)} T^{(i)} - T^{(i+1)}} = \frac{b_A/2}{1 - e^{-b_A/2}} e^{-b_A Z(R_E^{(i)})/T^{(i)}}. \quad (18)$$

Dividing Eq.10c by Eq.10a results in:

$$e^{-b_A Z(R_E^{(i)})/T^{(i)}} = \frac{R_M^{(i)} R_E^{(i+1)} - R_E^{(i)}}{R_E^{(i)} R_M^{(i+1)} - R_M^{(i)}}. \quad (19)$$

Inserting Eqs.14b and 19 into Eq.18, the new approximation can be found as:

$$b_{K2A} = 4 \left(2 \frac{T^{(i)} R_M^{(i+1)} - R_M^{(i)}}{R_M^{(i)} T^{(i)} - T^{(i+1)}} - 1 \right). \quad (20)$$

This new approximation is based on the forward kinematics of the problem; however, two assumptions are incorporated in it: (1) a single equivalent radius R_E defines the radius of the deformed sample, and (2) $b_A^n \cong 0$ for $n \geq 2$.

3.2. Backward Kinematics

The four discussed approaches can be followed via the backward kinematics (Eq.8), resulting in:

$$b_{EN} = 4 \frac{T^{(i+1)} \Delta R^{(i+1)} - \Delta R^{(i)}}{R_E^{(i+1)} T^{(i)} - T^{(i+1)}}, \quad (21)$$

$$b_S = 4 \frac{\Delta R^{(i+1)}}{\Sigma R^{(i+1)}} \left(2 \frac{T^{(i+1)}}{T^{(i)} - T^{(i+1)}} - 1 \right), \quad (22)$$

$$b_K = 2 \ln \left(\frac{R_T^{(i+1)} R_M^{(i+1)} - R_M^{(i)}}{R_M^{(i+1)} R_T^{(i+1)} - R_T^{(i)}} \right), \text{ and} \quad (23)$$

$$b_{K2A} = 4 \left(2 \frac{T^{(i+1)} R_M^{(i+1)} - R_M^{(i)}}{R_M^{(i+1)} T^{(i)} - T^{(i+1)}} - 1 \right). \quad (24)$$

4. Methodology: dynamic vs. static

With a survey in the literature regarding the estimation of the barreling parameter (see, e.g. [15, 16]), it is obvious that the barreling parameter is often treated as a static value,

not a dynamic one; in other words, b_A is estimated using the geometries of the sample only at two stages: before and after the deformation. This static scheme implies that the motion at a constant velocity is assumed to be valid for the whole course of the deformation process. However, one can argue that due to the dynamic nature of the compression test, a corresponding incremental methodology should be applied for estimating b_A [3, 5]. In other words, the sample deforms to some intermediate states before it reaches the final one.

In order to compare these two schemes, the approximations on b_A can be simplified by setting $\tau_i = 0$, that is the time that the sample is at its initial state, i.e. $T^{(i)} = T_0$ and $R_M^{(i)} = R_T^{(i)} = R_E^{(i)} = R_0$. Doing so, the formulae could be rewritten in their generic form, i.e. with dropping the notions of time. The simplified equations could be found in appendix A; comparing the approximations of appendix A with the early approximations of EN [8] and S [1] reveal that they were based on the backward kinematics within the static framework.

5. Materials and methods

In order to examine the applicability of Avitzur model through the discussed approximations of b_A , two methods were employed based on (1) Avitzur velocity field and (2) finite element (FE); using these methods, a number of reference models were prepared. Construction of the reference models and their sample was described in [1], and will be briefly summarized here. The initial geometry of the sample was selected to be $T_0 = 16\text{mm}$ and $R_0 = 5\text{mm}$, which was compressed between two rigid flat dies to its final thickness of 10 mm. With assuming a fixed value of friction factor for throughout the deformation, three values of $m = [0.1, 0.3, 0.5]$ were selected to perform the simulations. Moreover, in order to have a comparable case with Avitzur model, the sample was modelled as a perfect plastic with a yield stress of $Y = 100\text{MPa}$.

The samples' geometry were collected at different time steps of the deformation process, which were used for the approximation of b_A with the various discussed methods; table 1 summarizes the formulae that were worked in this study.

Table 1

A summary of the investigated approximations of b_A . The numbers refer to the corresponding equations.

Approximation	Forward	Backward
EN [8]	12	21
S [1]	15	22
Purely Kinematic (K)	17	23
K2A	20	24

6. Results and discussions

In this section, the solutions of m in its original (Eq.4) and simplified (Eq.6) form are compared first, in order to identify the validity domain of the simplified version. Then,

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Figure 2: The ratio of $H_{\text{simp}}^* = m^*/m_{\text{simp}}$ for $0 \leq b_A \leq 2$ and $0 \leq R_E/T \leq 20$. The values of H_{simp}^* are shown as the continuous and dotted lines in Zone I and Zone II, respectively. The dashed line indicates the first assumption on b_A , i.e. $b_A < 2\sqrt{3}T/R_E$.

the profile of a sample deformed under a friction factor of $m = 0.3$ is analyzed; upon this discussion, a correction to the measured R_T is devised for compensating the inaccuracy of Avitzur model. In the third part, the barreling parameters of different tests are estimated and compared with the optimum solution of b_A . Finally, the possible effects of side-surface folding phenomenon are discussed.

6.1. Comparisons between m^* and m_{simp}

In order to investigate the applicability of m_{simp} (Eq.6), a ratio $H_{\text{simp}}^* = m^*/m_{\text{simp}}$ was introduced, which is a function of R_E, T and b_A . For $H_{\text{simp}}^* = 1$, the simplified solution results in the same value as the original one; however, as H_{simp}^* deviates from 1, the discrepancies between the solutions increases, indicating the unreliability of m_{simp} . Figure 2 shows a comparison between these two solutions in terms of H_{simp}^* .

As the results show, as long as the first condition ($b_A < 2\sqrt{3}T/R_E$) is satisfied, the ratio H_{simp}^* is equal to or larger than 0.8 (Zone I), indicating a good match between m^* and m_{simp} . On the other hand, if this assumption is violated, H_{simp}^* becomes smaller and smaller (Zone II), and m_{simp} no longer would be valid.

The main finding of this comparison is that if the first assumption ($b_A < 2\sqrt{3}T/R_E$) is satisfied, then $H_{\text{simp}}^* \geq 0.8$, even if the second assumption ($b_A^n \cong 0$ for $n \geq 2$) is violated. Furthermore, figure 2 suggests that as the value of R_E/T becomes smaller, higher values of b_A would be valid for employing the simplified solution.

6.2. Profiles of the deformed samples

In order to compare the profile of the reference FE samples with Avitzur model, Eq.9 was used in the following

Figure 3: (a) The profile of the deformed sample with $m = 0.3$, and (b) the differences between the deformed sample's profile calculated via FEM and Avitzur model.

rewritten form of:

$$R(z)^{(i+1)} = R(z)^{(i)} \left(1 + B \frac{T^{(i)} - T^{(i+1)}}{T^{(i)}} e^{-b_A z / T^{(i)}} \right), \quad (25)$$

and was solved with both the dynamic and static schemes. It should be noted that Eq.25 was solved for known values of m ; therefore, the value of b_A could be obtained directly from Eq.5, and without using any of the approximations. Figure 3 compares the profile of the deformed reference samples and the corresponding predictions of Avitzur model for $m = 0.3$. As can be seen, the static scheme resulted in a volume inconsistency problem: the volume of the original sample and of the FEM solution were $\sim 1256.6 \text{ mm}^3$, while the volume of the deformed sample within the static scheme was found to be $\sim 1109.2 \text{ mm}^3$. On the other hand, the deformed material within the dynamic scheme showed a much lower volume inconsistency. It should be noted that the results of the dynamic scheme highly depends on the selected step size; e.g. solving the problem for compression steps (ΔT) of 0.1 mm and 0.4 mm resulted in deformed samples with approximate volumes of 1257.8 mm^3 and 1247.4 mm^3 , respectively.

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Figure 4: Comparison between b_A and its approximations calculated within the static framework, for both forward (top row) and backward (bottom row) kinematics. The deformations were modeled via Avitzur velocity field (Eq.1).

Using the dynamic framework, Avitzur model showed no volume constancy problem; however, the results show that Avitzur model have incorrect estimations on both mid-plane and top-plane radii. This indicates that the measured values of R_M and R_T cannot be directly used in the model for a correct description of the barreling parameter. In addition, with an unknown value of m , which is the case for experiments, there is no method to convert the measured profile into some compatible values with Avitzur model. Therefore, some modifications are required. Ebrahimi and Najafizadeh proposed an estimation of the top-radius based on an assumption of $2R_M^2 + R_T^2 = 3R_E^2$ [8]; however, this assumption does not necessarily follow the volume constancy of the problem.

In here, we suggest a remedy by using the measured R_M and calculating a top-radius ($R_{T,C}$) corrected for the volume constancy of the problem. $R_{T,C}$ can be estimated via different methods, depending on the approximation of the sample's profile; in the simplest form, one can assume the profile of the deformed sample between the top- and mid-planes is a straight line, which is not a bad approximation for Avitzur model (see figure 3 and [5]). This approximation results in:

$$R_{T,C} = R_M - \frac{R_M - R_T}{\sqrt{2}}. \quad (26)$$

It should be noted that the reason for correcting the top-plane radius, and not the mid-plane one, is that Avitzur model could be safely assumed to be valid for describing the deformation mechanics in the middle of the sample, while the phenomena dictating the enlargement of R_T are not fully incorporated in Avitzur model; see Effects of the side surface folding for further discussion.

6.3. Approximations of b_A

In here, we compare the approximations of b_A with their optimum values for two sets of deformations based on (1) Avitzur model and (2) FEM solutions. The corresponding formulae could be found in table 1.

6.3.1. Deforming by Avitzur velocity field

Figure 4 shows the results of the static solutions for both forward and backward kinematics. The results show that the forward kinematics approximations were close to b_A only at the very beginning of the deformation, and diverged from the expected values as the deformation proceeded. Moreover, it can be seen that both EN and S approaches resulted in the same values. On the other hand, the backward kinematics resulted in closer approximations, specially for the EN approach, which was indistinguishable from b_A . In addition, b_{K2A} approximations were far from the expected values, regardless of the working kinematics.

Figure 5 shows the results of the forward and backward kinematics within the dynamic framework. The result show that for low values of m , all approximations were close to the expected values of b_A ; however, for harsher friction conditions the EN and K2A approximations revealed their deviations with a difference in their behavior: b_{EN} diverged from b_A for larger deformations, while b_{K2A} converged toward the expected values. Furthermore, b_S and b_K showed some negligible deviations for the backward kinematics, and were indistinguishable from b_A for the forward kinematics.

A comparison between the static and dynamic methodologies show that the dynamic one resulted in lower discrepancies between b_A and its approximations.

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Figure 5: Comparison between b_A and its approximations for the samples deformed by Avitzur velocity field (Eq.1) using (a) forward and (b) backward kinematics within a dynamic framework.

Figure 6: The approximations of b_A within the static scheme using the backward kinematics equations. The deformation was performed using FEM.

6.3.2. FEM solutions

The FEM solutions were investigated by collecting the values of R_M and R_T at different steps of the process; moreover, the corresponding corrected top-radii ($R_{T,C}$) were calculated using Eq.26. The results of all approximations can be found in the supplementary data. In here, we only present the results of the deformed samples at $m = 0.3$ and for the static scheme; see figure 6. This configuration was selected as it was the only one showing comparable results with b_A at $m = 0.3$; also, no close comparison was found for $m = 0.5$.

One immediate noticeable difference between the approximations of b_A calculated from FEM solutions and the ones from Avitzur velocity field is their general behavior: while b_A and its approximations changed linearly for deformations based on Avitzur velocity field, the geometry of samples deformed by FEM resulted in a non-linear behavior. This deviation indicates the inaccuracy of describing the material's

flow in Avitzur model. Consequently, the newly developed purely kinematic K approach failed to accurately approximate the barreling parameter. On the other hand, the integrated assumptions within the EN and S approaches made them more flexible for approximating the barreling parameter.

The results showed that for $m = 0.1$, all of the dynamic solutions were close to b_A ; see supplementary data. However, with increasing the friction factor, all of the methods ended up with underestimations. Moreover, the approximations based on the forward kinematics within the static framework, overestimated the expected value of b_A at low frictions, and underestimated it at high frictions. On the other hand, the static scheme of the backward kinematics (for EN, S, and K approaches) resulted in close comparison with the expected values of b_A up to $m = 0.3$, as is shown in figure 6.

6.4. Effects of the side surface folding

The non-linear behavior of the approximations of b_A in figure 6 showed that the metal flow in the reference samples modeled with FEM were different from Avitzur model. While various reasons could result in deviations in real experiments, e.g. non-homogeneity of the material, the main reason for this behavior in FE simulations was the side surface folding phenomenon. This means that, during a compression test, the top-plane enlarges not only due to the slipping, but also by folding of some material originally placed on the free surface of the sample over the tool. This inevitable phenomenon was neglected in the original version of Avitzur model (Eq.1), while it occurred for all of the compression tests performed with FEM, resulting in a deformation path beyond the capabilities of Avitzur model. For the FEM deformed samples, the folding percentage was calculated using $\frac{R_T - R_S}{R_T} \times 100$, where R_T was the measured top radius, and R_S was the radius of the slipping part of the top-plane. The results are reported in figure 7. It was found

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Figure 7: The folding percentage of the samples at different friction conditions.

that the maximum folding percentage was 0.25%, 5.71%, and 10.00% for friction factors of 0.1, 0.3, and 0.5, respectively. Comparing the folding percentage and the approximations of b_A , it can be deduced that with negligible folding, the dynamic scheme results in close values regardless of the selected kinematics; however, as can be seen in figure 6, most of the approximations based on the backward kinematics within a static framework could estimate the optimum values of b_A for conditions with folding of $\sim 6\%$

6.4.1. On the use of R_S

This finding raises the question of using R_S in the Avitzur model for approximating b_A . This idea was examined for all approximations, and a complete set of results are available in the supplementary data. The results showed that the closest values to the expected values of b_A could be found using the backward kinematics of b_S within a static framework (Eq.A.5). Figure 8 shows the estimated friction factors using this approach.

7. Conclusions

The presented results of this paper demonstrate that the upper bound method undoubtedly is an important tool for the evaluation of friction via uniaxial compression tests. In spite of frequent usage of Avitzur model as one of the few analytical tools for estimating friction, the applicability of the model was yet remained to be assessed properly to show how its limiting assumptions affect the evaluated friction factor, and what approach toward solving the model should be employed to obtain the best results. Such an assessment was the point of focus in the current study. In this paper, the model was investigated for a number of compression tests on a perfectly plastic material, to make it comparable with the model.

One missing data in the literature regarding Avitzur model was the identification of validity domain of m_{simp} . In this investigation, we discussed as long as the condition $b_A <$

Figure 8: The estimated values of m_{simp} following S approach with the backward kinematics within the static scheme. The values of $b_{S,BK}$ were calculated using the slipped radius R_S of the deformed samples.

$2\sqrt{3}T/R_E$ is satisfied, m_{simp} (Eq.6) is up to 80% of its original value m^* (Eq.4).

Moreover, two sets of forward and backward kinematics were employed for four different approaches (including two newly developed ones) toward solving Avitzur model for its barreling parameter. Table 1 summarizes the obtained formulae. Moreover, the effects of static and dynamic framework on the calculation were discussed, showing that the static scheme results in some volume inconsistency problem. Comparing the approximations of b_A with the expected values using the various mentioned methods, different behaviors were found. The detailed discussions are provided in sections 6.3.1 and 6.3.2. In summary, noticing some inaccuracies of Avitzur model, some modifications on the top-plane radius was devised to assure the volume constancy of the problem. Moreover, it was found that all of the approaches could be applied within a dynamic framework, as long as the friction is very low, which was $m = 0.1$ in this work. This limited applicability of the model was explained by an ignored phenomenon in the Avitzur model, i.e. the inevitable side-surface folding. In other words, the model works as long as the side-surface folding is exceptionally small, which was 0.25% in this study. Moreover, the possible benefit of working with the slipped radius R_S instead of the top-plane radius R_T was discussed; using R_S in b_S for the backward kinematics (Eq.22) within a static framework resulted in accurate approximations of the barreling parameter and friction factor for all of the tested samples.

The results of the current detailed assessment showed that there are some certain limitations for describing uniaxial compression test via Avitzur model: the model should be employed with care for evaluation of friction or correcting the stress-strain curves obtained from (hot or cold) compression tests, specifically in the presence of side-surface folding higher than 0.25%. In order to correctly describe the metal

flow within a compressed sample, not only the barreling, but also the folding phenomenon (as was expressed in details by Avitzur and Kohser [13, 4]), and even the presence of the dead zone (see, e.g. [17]) are required to be considered. Thus, a more profound and deliberate, yet easily applicable model is required to be developed for a correct description of the metal flow in a barreling compression test, to be used as an analysis tool.

A. Approximations of b_A within a static framework

The time frames of different approximations formulae could be dropped within a static framework. This was done as was described in Methodology: dynamic vs. static. The followings are the rewritten forms of the approximations for forward and backward kinematics. It should be noted that in case of employing the slipped radius R_S in these formulae, $\Delta R = R_M - R_S$ and $\Sigma R = R_M + R_S$; see On the use of R_S for further discussion on the possible benefits of using R_S .

A.1. forward kinematics

$$b_{EN} = b_S = 4 \frac{T_0}{R_0} \frac{\Delta R}{\Delta T}, \quad (A.1)$$

$$b_K = 2 \ln \left(\frac{R_M - R_0}{R_T - R_0} \right), \text{ and} \quad (A.2)$$

$$b_{K2A} = 4 \left(2 \frac{T_0}{R_0} \frac{R_M - R_0}{\Delta T} - 1 \right), \quad (A.3)$$

where $\Delta T = T_0 - T$. It is interesting that both of EN and S approaches have the same form within the static framework.

A.2. backward kinematics

$$b_{EN} = 4 \frac{T}{R_E} \frac{\Delta R}{\Delta T}, \quad (A.4)$$

$$b_S = 4 \frac{\Delta R}{\Sigma R} \left(2 \frac{T}{\Delta T} - 1 \right), \quad (A.5)$$

$$b_K = 2 \ln \left(\frac{R_T}{R_M} \frac{R_M - R_0}{R_T - R_0} \right), \text{ and} \quad (A.6)$$

$$b_{K2A} = 4 \left(2 \frac{T}{R_M} \frac{R_M - R_0}{\Delta T} - 1 \right). \quad (A.7)$$

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