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Stabilization of a class of cyclo-passive systems using alternate storage functions

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Abstract—This note is devoted to the stabilization of a particular class of nonlinear *cyclo-passive* systems, namely, gradient-like systems. In order to accomplish the control task, we explore alternate representations of those systems with the aim of identifying (new) storage functions. Then, those storage functions are used to design a passivity-based controller that addresses the regulation problem without the necessity of solving partial differential equations.

Index Terms—Passivity, stabilization, storage function, nonlinear systems.

I. INTRODUCTION

The basic idea of Passivity-based control (PBC) is to design a controller that accomplishes the control task by rendering the system *passive* with respect to a desired *energy function* and by injecting damping into the closed-loop system [1]. In general, PBC techniques can be classified in two main groups, namely,

- The techniques that select the structure of the closed-loop energy function a priori, and subsequently design a controller that renders this function non-increasing with respect to the equilibrium point. Within this class we find the proportional Proportional-Integral-Derivative (PID) PBC [2], [3] and the energy-balancing (EB) controllers [1].
- The techniques that select a desired closed-loop structure, where the family of possible energy functions is characterized by a partial differential equation (PDE). In this approach we find the interconnection and damping assignment (IDA) [1], [4], [5] and the control by interconnection (CbI) [6]–[8].

In particular, the first family of PBC techniques provides a constructive methodology to assign a desired shape to the storage function of the closed-loop system. Moreover, this approach has been proved to be effective to stabilize complex physical systems, e.g., [9]–[12]. Nonetheless, the applicability of these PBC techniques hinges on the identification of suitable storage functions and their corresponding passive outputs. Whilst this seems to be an important drawback, it should be pointed out that, the storage function of a passive system is not unique [13]. Moreover, in some cases the identification of alternate storage functions is necessary to design the controller [3], and in others turns into a powerful tool

to simplify the control design or the stability analysis [14]–[17].

The main contribution of this note is the use of alternate storage functions to extend the applicability of the PID-PBCs reported in [18]–[20] to a more general class of systems, namely, gradient-like cyclo-passive systems. Towards this end, we revisit a methodology to find alternate representations of this kind of systems, reported in [21], which is an extension of the seminal work of Brayton and Moser [14]. Additionally, based on the so-called *power shaping* approach [17], we propose a new methodology to identify alternative storage functions for the aforementioned class of nonlinear systems.

The outline of this work is as follows. First, we provide the preliminaries and the problem formulation in Section II. Section III is devoted to the identification of new storage functions for cyclo-passive systems. In Section IV, we present the PBC technique and we carry out the stability analysis. An example that illustrates the results of the note is presented in Section V. Finally, we wrap-up this work with some concluding remarks in Section VI.

Notation. I_n is the $n \times n$ identity matrix and $\mathbf{0}_{n \times s}$ is a $n \times s$ matrix of zeros. For $x \in \mathbb{R}^n$, $K \in \mathbb{R}^{n \times n}$, $K = K^\top > 0$, we denote the Euclidean norm $|x|^2 := x^\top x$, and the weighted-norm $\|x\|_K^2 := x^\top K x$. Given a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ we define the differential operator $\nabla_x f := \left(\frac{\partial f}{\partial x}\right)^\top$ and $\nabla_x^2 f := \frac{\partial^2 f}{\partial x^2}$. For a mapping $F : \mathbb{R}^n \rightarrow \mathbb{R}^m$ we define the ij -th element of its $n \times m$ Jacobian matrix as $(\nabla_x F)_{ij} := \frac{\partial F_j}{\partial x_i}$. When clear from the context the subindex in ∇ is omitted. For any mapping $T : \mathbb{R}^n \rightarrow \mathbb{R}^{p \times q}$ and the distinguished element $x_* \in \mathbb{R}^n$ we define the constant matrix $T_* := T(x_*)$. For $A \in \mathbb{R}^{n \times n}$, $\text{sym}\{A\} := \frac{1}{2}(A + A^\top)$ and $\text{skew}\{A\} := \frac{1}{2}(A - A^\top)$. We denote the i -th vector of the n -dimensional Euclidean basis as $e_i \in \mathbb{R}^n$. All mappings are supposed smooth enough.

II. PRELIMINARIES AND PROBLEM FORMULATION

Consider a general affine nonlinear system of the form

$$\begin{aligned}\dot{x} &= f(x) + g(x)u \\ y &= h(x) + j(x)u\end{aligned}\tag{1}$$

where $x \in \mathcal{X} \subseteq \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the input vector, with $m < n$. Moreover, $f : \mathcal{X} \rightarrow \mathbb{R}^n$ denotes

the so-called *drift* vector field, $g : \mathcal{X} \rightarrow \mathbb{R}^{n \times m}$ is the input matrix, which is *full rank*, and $y \in \mathbb{R}^m$ is the output of the system defined through the mappings $h : \mathcal{X} \rightarrow \mathbb{R}^m$, $j : \mathcal{X} \rightarrow \mathbb{R}^{m \times m}$.

The controllers developed in this work are designed for cyclo-passive systems. Accordingly, for the sake of completeness, we recall Hill-Moylan's Theorem [22], which provides the necessary and sufficient conditions under which a nonlinear system of the form (1) is cyclo-passive.

Theorem 1: The system (1) is cyclo-passive with storage function $S(x)$ ¹ if and only if, for some $q \in \mathbb{N}$, there exist mappings $l : \mathcal{X} \rightarrow \mathbb{R}^q$ and $w : \mathcal{X} \rightarrow \mathbb{R}^{q \times m}$ such that

$$[\nabla S(x)]^\top f(x) = -l^\top(x)l(x) \quad (2)$$

$$h(x) = g^\top(x)\nabla S(x) + 2w^\top(x)l(x) \quad (3)$$

$$\text{sym}\{j(x)\} = w^\top(x)w(x). \quad (4)$$

We underscore that, while in some cases $S(x)$ is given by the energy of the system, e.g., port-Hamiltonian (pH) systems [23], [24], storage functions have no necessarily a physical interpretation [22].

Problem formulation. Given a cyclo-passive system of the form (1), provide a methodology to design a control law that ensures the closed-loop system has an (*asymptotically*) *stable* equilibrium at a desired point $x_* \in \mathcal{X}$, which belongs to the set of assignable equilibria²

$$\mathcal{E} := \{x \in \mathcal{X} \mid g^\perp(x)f(x) = \mathbf{0}_{n-m}\}. \quad (5)$$

In order to solve proposed the control problem, we proceed in two steps:

- (i) The identification of storage functions $S(x)$ for system (1), such that the condition (2) is verified. Furthermore, once the storage function is identified, the passive outputs are characterized by Theorem 1.
- (ii) The application of a PBC approach that provides a systematic way to shape $S(x)$ in a desired way, assign the desired equilibrium and inject damping into the closed-loop system. Furthermore, this PBC technique obviates the necessity of solving PDE's.

III. EQUIVALENT REPRESENTATION OF THE DRIFT VECTOR FIELD AND IDENTIFICATION OF STORAGE FUNCTIONS

Whilst Theorem 1 characterizes the cyclo-passive systems of the form (1), the identification of the storage function is not always straightforward. Accordingly, in this section we explore two methodologies to identify those storage functions, where we restrict our attention to systems that satisfy the following assumption.

¹In contrast to passivity, for cyclo-passivity is not required that $S(x) > 0$. For a detailed discussion about the differences between passivity and cyclo-passivity, we refer the reader to [22], [23].

²where $g^\perp : \mathcal{X} \rightarrow \mathbb{R}^{(n-m) \times n}$ is a *full rank*, left-annihilator of $g(x)$, that is, $g^\perp(x)g(x) = \mathbf{0}_{(n-m) \times m}$.

Assumption 1: There exist a *full rank* matrix $Q : \mathcal{X} \rightarrow \mathbb{R}^{n \times n}$ and a *potential* function $P : \mathcal{X} \rightarrow \mathbb{R}$ such that the drift vector field can be expressed as follows

$$f(x) = Q^{-1}(x)\nabla P(x). \quad (6)$$

Note that, several classes of physical systems satisfy Assumption 1, e.g., pH systems with *full rank* interconnection and damping matrix, gradient systems [25] or the Brayton-Moser's representation of electrical circuits [14]. Nonetheless, in some of these representations the storage function is not explicitly given or in some cases is not suitable for control design purposes, and thus, it is necessary to identify a new storage function. In order to solve these problems, we explore alternate representations of the drift vector field (6), i.e., we look for new pairs $\tilde{Q} : \mathcal{X} \rightarrow \mathbb{R}^{n \times n}$, $\tilde{P} : \mathcal{X} \rightarrow \mathbb{R}$ such that

$$f(x) = \tilde{Q}^{-1}(x)\nabla \tilde{P}(x). \quad (7)$$

These new representations of the drift vector field are instrumental in the identification of new storage functions. Whence, with the objective of finding pairs $\tilde{Q}(x)$, $\tilde{P}(x)$, we present the following propositions.

Proposition 1: Select an arbitrary constant $\lambda \in \mathbb{R}$ and a symmetric matrix $\Lambda : \mathcal{X} \rightarrow \mathbb{R}^{n \times n}$ such that the matrix $\Theta : \mathcal{X} \rightarrow \mathbb{R}^{n \times n}$, defined as

$$\Theta(x) := \lambda I_n + \nabla^2 P(x)\Lambda(x) + \frac{1}{2} \sum_{i=1}^n e_i [\nabla P(x)]^\top \frac{\partial \Lambda(x)}{\partial x_i}, \quad (8)$$

has *full rank*. Accordingly, the drift vector field (6) admits a representation of the form (7), with

$$\tilde{Q}(x) := \Theta(x)Q(x) \quad (9)$$

$$\tilde{P}(x) := \lambda P(x) + \frac{1}{2} [\nabla P(x)]^\top \Lambda(x) \nabla P(x). \quad (10)$$

The proof of Proposition 1 can be found [21] and is omitted for brevity. On the other hand, the next proposition establishes a new procedure to construct the pair $\tilde{Q}(x)$, $\tilde{P}(x)$.

Proposition 2: Select a *full rank* matrix $N : \mathcal{X} \rightarrow \mathbb{R}^{n \times n}$ such that the matrix $\Gamma : \mathcal{X} \rightarrow \mathbb{R}^{n \times n}$, defined as

$$\Gamma(x) := N(x)\nabla^2 P(x) + \sum_{i=1}^n \frac{\partial N(x)}{\partial x_i} \nabla P(x) e_i^\top, \quad (11)$$

verifies

$$\Gamma(x) = \Gamma^\top(x). \quad (12)$$

Then, there exists a potential function $\tilde{P} : \mathcal{X} \rightarrow \mathbb{R}$ such that the drift vector field (6) can be expressed as (7), with

$$\tilde{Q}(x) := N(x)Q(x) \quad (13)$$

$$\nabla \tilde{P}(x) = N(x)\nabla P(x). \quad (14)$$

Proof: To establish the proof, first, note that

$$\nabla[\nabla\tilde{P}(x)] = \Gamma(x),$$

with $\Gamma(x)$ defined in (11) and $\nabla\tilde{P}(x)$ given in (14). Hence, from Poincaré's Lemma, see Exercise 4.5 in [26], it follows that (12) is equivalent to the existence of $\tilde{P}(x)$ such that (14) holds. Furthermore,

$$\tilde{P}(x) = \int_0^1 [N(sx)\nabla P(sx)]^\top x ds + \tilde{P}(0). \quad (15)$$

The proof is completed replacing (13) and (14) in (7) to obtain (6). $\blacksquare$

We underscore that, the pairs $\tilde{Q}(x), \tilde{P}(x)$ provided in Propositions 1 and 2, have degrees of freedom in their design, namely, λ , $\Lambda(x)$ and $N(x)$. Moreover, the proposition below establishes that, if those degrees of freedom can be selected in such a way that the drift vector field (7) has a desired structure, then the identification of a storage function turns into a trivial task.

Proposition 3: Assume that $\text{sym}\{\tilde{Q}(x)\} \leq 0$ and that there exist s functions $\beta : \mathcal{X} \rightarrow \mathbb{R}$, with $s < n$, verifying

$$[\nabla\beta_i(x)]^\top f(x) = 0. \quad (16)$$

Then, system (1) defines a cyclo-passive mapping with storage function

$$S(x) = \tilde{P}(x) + \sum_{i=1}^s \beta_i(x), \quad i = 1, \dots, s. \quad (17)$$

Proof: Note that

$$\begin{aligned} (\nabla S)^\top f &= \sum_{i=1}^s (\nabla\beta_i)^\top f + (\nabla\tilde{P})^\top \tilde{Q}^{-1} \nabla\tilde{P} \\ &= (\nabla\tilde{P})^\top \tilde{Q}^{-1} \nabla\tilde{P} \end{aligned}$$

where we used (7) and (16). Moreover,

$$\text{sym}\{\tilde{Q}(x)\} \leq 0 \iff \text{sym}\{\tilde{Q}^{-1}(x)\} = -\phi^\top(x)\phi(x) \leq 0,$$

where $\phi : \mathcal{X} \rightarrow \mathbb{R}^{q \times n}$, with $q \geq \text{rank}\{\text{sym}\{\tilde{Q}^{-1}(x)\}\}$. Hence, the expression (2) holds true for

$$l(x) = \phi(x)\nabla\tilde{P}(x) \quad (18)$$

and the design parameter $w(x)$ determines the structure of feedthrough term $j(x)$. Furthermore, the time derivative of $S(x)$ is given by

$$\dot{S} = -|\phi(x)\nabla\tilde{P}(x) + w(x)u|^2 + y^\top u$$

where we used (3), (4) and (18). $\blacksquare$

IV. CONTROL DESIGN AND STABILITY ANALYSIS

In this section we design a PBC that renders *stable* the equilibrium point x_* . Additionally, we carry out the stability analysis of the closed-loop system, where both, the controller and the Lyapunov function depend on the storage functions identified in the previous section. Moreover, in contrast to the power shaping control approach [17] where the use of alternate representations of cyclo-passive systems is also exploited, the proposed control law is designed without the necessity of solving PDE's.

A. Equilibrium assignment

The first step to address the regulation problem for system (1) is to ensure that the closed-loop system has an equilibrium at the desired point. Accordingly, the following proposition provides a preliminary control law that assigns the desired equilibrium to the closed-loop system and contributes to the *energy shaping* process.

Proposition 4: Assume that system (1) is passive with storage function $S(x)$ and the PDE

$$[\nabla\gamma(x)]^\top \begin{bmatrix} f(x) \\ g(x) \end{bmatrix} = \begin{bmatrix} g^\top(x)\nabla S(x) + 2w^\top(x)l(x) \\ j(x) \end{bmatrix} \quad (19)$$

admits a solution $\gamma : \mathcal{X} \rightarrow \mathbb{R}^m$. Define

$$S_a(x) := S(x) + \Phi(\gamma(x)), \quad (20)$$

fix an equilibrium point $x_* \in \mathcal{E}$ and define the control law

$$u := -\nabla_\gamma\Phi(\gamma(x)) + v \quad (21)$$

where the mapping $\Phi : \mathbb{R}^m \rightarrow \mathbb{R}$ verifies

$$(\nabla_\gamma\Phi)_* = (g_*^\top g_*)^{-1} g_*^\top f_*. \quad (22)$$

Hence, the following statements hold true.

- (i) Fix $v = \mathbf{0}_m$. Then, the closed-loop system has an equilibrium at x_* .
- (ii) The mapping $v \mapsto y_a$, where

$$y_a := g^\top(x)\nabla S_a(x), \quad (23)$$

is passive with storage function $S_a(x)$.

Proof: The selection (22) is explained as follows, the point $x_* \in \mathcal{E}$ is an equilibrium of system (1) if and only if the following equivalence holds

$$f_* + g_* u_* = \mathbf{0}_n \iff u_* = -(g_*^\top g_*)^{-1} g_*^\top f_*. \quad (24)$$

Therefore, the identity (22) is obtained by substituting the right-hand expression of (24) in (21). To prove item (ii), we compute the derivative of the storage function $S_a(x)$, given by

$$\dot{S}_a = -|l - w\nabla_\gamma\Phi|^2 + y_a^\top v, \quad (25)$$

where we used (19) and (4). This completes the proof. $\blacksquare$

One of the main tasks of the term $\nabla_\gamma\Phi(\gamma(x))$ is to assign the equilibrium, nonetheless, in presence of pervasive dissipation, this is only possible if the feedthrough term

$j(x)$ is different from zero. This important condition is shown in Proposition 5.

Proposition 5: If $x_* \in \mathcal{E}$ is an equilibrium of system (1) in closed-loop with (21), then the following identity is satisfied

$$l_* = -w_* u_*. \quad (26)$$

Proof: Note that if the PDE (19) admits a solution $\gamma(x)$, then

$$\dot{\gamma} = y. \quad (27)$$

Furthermore, the latter identity implies that

$$y_* = \mathbf{0}_m. \quad (28)$$

On the other hand, we have the following chain of implications

$$\begin{aligned} x_* \in \mathcal{E} &\implies f_* + g_* u_* = \mathbf{0}_n \implies \\ (\nabla S)_*^\top f_* + (\nabla S)_*^\top g_* u_* &= 0 \implies \\ l_*^\top l + 2w_*^\top l_* u_* + u_*^\top j_* u_* &= 0 \implies \\ |l_* + w_* u_*|^2 = 0 &\iff l_* + w_* u_* = \mathbf{0}_q \end{aligned}$$

where we used (2), (19) and (28) to obtain the third implication. $\blacksquare$

Remark 1: From Proposition 5 it is clear that when system (1) exhibits dissipation at x_* , the desired equilibrium cannot be assigned with a control law designed around a passive output with *relative degree* one. This phenomenon is known as *dissipation obstacle*, for further details we refer the reader to [27].

While the task of solving PDE's is in general difficult, as reported in [18], a sufficient condition for the existence of solutions to PDE (19) is straightforward verifiable. This condition is characterized in the following remark.

Remark 2: Since the matrix $\tilde{Q}(x)$ is full rank, the particular selection

$$j(x) = -g^\top(x) \tilde{Q}^\top(x) g(x)$$

yields

$$y = -g^\top(x) \tilde{Q}^\top(x) \dot{x}.$$

Therefore, the PDE (19) admits a solution $\gamma(x)$ if

$$\nabla(\tilde{Q}(x) g_i(x)) = [\nabla(\tilde{Q}(x) g_i(x))]^\top, \quad i = 1, \dots, m.$$

B. Energy shaping and damping injection

In this subsection we present the last part of the control design, where the energy shaping is completed and damping is injected into the closed-loop system. Additionally, the conditions under which x_* is an (*asymptotically*) *stable* equilibrium of the closed-loop system are provided in the following proposition.

Proposition 6: Select the positive semi-definite gain $K_D \in \mathbb{R}^{m \times m}$ such that the matrix $K : \mathcal{X} \rightarrow \mathbb{R}^{m \times m}$, defined as

$$K(x) := I_m + K_D(\nabla y_a)^\top g(x), \quad (29)$$

has *full rank*. Fix an equilibrium $x_* \in \mathcal{E}$ and define

$$v := -K^{-1}(x) \{K_P y_a + K_D(\nabla y_a)^\top f(x)\}, \quad (30)$$

where the gain $K_P \in \mathbb{R}^{m \times m}$ is positive semi-definite. Assume that

$$x_* = \arg \min \{S_d(x)\}, \quad (31)$$

where

$$S_d(x) := S_a(x) + \frac{1}{2} \|y_a\|_{K_D}^2 \quad (32)$$

with $S_a(x)$ defined in (20).

- (i) The system (1) in closed-loop with the controller (21), (30) has a *stable* equilibrium at x_* with Lyapunov function $S_d(x)$.
- (ii) The equilibrium is *asymptotically stable* if the following implication holds true

$$\dot{S}_d = 0 \iff x = x_*.$$

- (iii) The stability properties are *global* if the storage function $S_d(x)$ is radially unbounded.

Proof: The first step to establish the proof is to rewrite the expression (30) as

$$v = -K_P y_a - K_D \dot{y}_a. \quad (33)$$

Therefore,

$$\dot{S}_d = -|l - w \nabla_\gamma \Phi|^2 - \|y_a\|_{K_P}^2 \quad (34)$$

where we used (25) and (33). Note that, (34) implies that the storage function $S_d(x)$ is *non-increasing*. Moreover, from (31) follows that $S_d(x)$ is positive definite with respect to x_* , and thus, the stability of the equilibrium is proved invoking Lyapunov Theory, see [26]. The items (ii) and (iii) follows directly from the application of the theorems of Barbashin and Krasovskii, see Corollaries 4.1 and 4.2 in [26]. $\blacksquare$

TABLE I

THE ROLE OF EACH PID-PBC COMPONENT.

	$K_P y_a$	$\nabla_\gamma \Phi$	$K_D \dot{y}_a$
Energy shaping		✓	✓
Equilibrium assignment		✓	
Damping injection	✓		

The control law (21), (30) has a PID structure³, where the feedback signal is the passive output, see [19], [20]. Moreover, Table I provides an insight of the relevance of each control term in the stabilization process. Note that, if $S_a(x) > 0$ and

$$x_* = \arg \min \{S_a(x)\}, \quad (35)$$

³Where $\nabla_\gamma \Phi$ is understood as a nonlinear integral-like action.

the term $K_D \dot{y}_a$ is not necessary and the control law

$$u = -\nabla_\gamma \Phi(\gamma(x)) - K_P y_a$$

solves the stabilization problem.

V. EXAMPLE

Fig. 1. RLC circuit with a nonlinear load

Consider the circuit depicted in Figure 1, whose Brayton-Moser representation is given by

$$\begin{bmatrix} -L & 0 \\ 0 & C \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} Rx_1 + x_2 \\ x_1 - \varphi(x_2) \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} u, \quad (36)$$

where the constant parameters L, C, R denotes the capacitance, inductance and resistance, respectively. The current in the inductor is represented by x_1 , x_2 denotes the voltage in the capacitor and the behavior of the current in the nonlinear load is described by

$$\varphi(x_2) := a \tanh(x_2), \quad (37)$$

where $a > 0$ is a constant parameter. Note that, the system (36) admits an affine representation of the form (1), where the drift vector field is given by (6) with

$$\begin{aligned} Q &= \text{diag}\{-L, C\} \\ P(x) &= \frac{1}{2}Rx_1^2 + x_1x_2 - a \ln(\cosh(x_2)), \end{aligned}$$

and input matrix $g = [\frac{1}{L} \ 0]^\top$. Furthermore, the set of assignable equilibria, given by

$$\mathcal{E} = \{x \in \mathbb{R}^2 \mid x_1 = \varphi(x_2)\};$$

and the control objective is to stabilize the voltage of the capacitor at a desired constant value $x_{2*} > 0$.

Note that $Q(x)$ has no definite sign and the potential function $P(x)$ does not qualify as storage function since the condition (2) is not satisfied. Therefore, we propose an alternate representation of the drift vector field, that is, we fix $\lambda = 0$ and $\Lambda = \text{diag}\{\frac{1}{L}, \frac{1}{C}\}$ in Proposition 1. Hence,

$$\tilde{Q} = \nabla^2 P \Lambda Q = \begin{bmatrix} -R & 1 \\ -1 & -\varphi' \end{bmatrix}, \quad (38)$$

where $\varphi' = a \text{sech}^2(x_2)$. Alternatively, the pair $\tilde{Q}(x)$, $\tilde{P}(x)$ given in (38), (39) can be obtained by choosing

$$N = \begin{bmatrix} \frac{R}{L} & \frac{1}{C} \\ \frac{1}{L} & -\frac{\varphi'}{C} \end{bmatrix}$$

in Proposition 2. Moreover, $\text{sym}\{\tilde{Q}\} < 0$ and

$$\begin{aligned} S &= \tilde{P} = \frac{1}{2L}(Rx_1 + x_2)^2 + \frac{1}{2C}(x_1 - \varphi)^2 \quad (39) \\ \nabla S &= \begin{bmatrix} \frac{R}{L}(Rx_1 + x_2) + \frac{1}{C}(x_1 - \varphi) \\ \frac{1}{L}(Rx_1 + x_2) - \frac{1}{C}\varphi'(x_1 - \varphi) \end{bmatrix} \\ \nabla^2 S &= \frac{1}{C} \begin{bmatrix} 1 & -\varphi' \\ -\varphi' & -\varphi''(x_1 - \varphi) + (\varphi')^2 \end{bmatrix} \\ &\quad + \frac{1}{L} \begin{bmatrix} R^2 & R \\ R & 1 \end{bmatrix}, \end{aligned}$$

where $\varphi'' = -2a \tanh(x) \text{sech}^2(x)$. Hence, the dissipative term l is given by

$$l = \begin{bmatrix} \frac{\sqrt{R}}{L}(Rx_1 + x_2) \\ \frac{1}{C}\sqrt{\varphi'}(x_1 - \varphi) \end{bmatrix}.$$

Moreover, we propose $j = \frac{R}{L^2}$ and $w = [\frac{\sqrt{R}}{L} \ 0]^\top$. Whence, the passive output is given by

$$y = \frac{R}{L^2}(u - Rx_1 - x_2) + \frac{1}{CL}(x_1 - \varphi) = \frac{1}{L}(R\dot{x}_1 + \dot{x}_2),$$

and thus, the function

$$\gamma = \frac{1}{L}(Rx_1 + x_2)$$

solves the PDE (19). Furthermore, we propose

$$\Phi = \kappa\gamma + \frac{1}{2}K_i(\gamma - \gamma_*)^2,$$

with $K_i > 0$ and $\kappa = -(Rx_{1*} + x_{2*})$. Therefore

$$\begin{aligned} (\nabla S_a)_* &= \frac{1}{L}(Rx_{1*} + x_{2*} + \kappa) \begin{bmatrix} R \\ 1 \end{bmatrix} = \mathbf{0}_n \\ (\nabla^2 S_a)_* &= \frac{1}{C} \begin{bmatrix} 1 & -\varphi'_* \\ -\varphi'_* & (\varphi')_*^2 \end{bmatrix} + \frac{1}{L} \begin{bmatrix} R^2 & R \\ R & LK_i \end{bmatrix} > 0. \end{aligned}$$

Note that (35) holds and the controller is given by

$$\begin{aligned} u &= -\kappa - K_i(\gamma - \gamma_*) - K_P y_a \\ y_a &= \frac{1}{CL}(x_1 - \varphi) + \frac{R}{L^2}[x_1 + \frac{1}{R}x_2 + \kappa + K_i(\gamma - \gamma_*)]. \end{aligned} \quad (40)$$

Therefore, the control law (40) and the parameters l, w satisfy the conditions (22) and (26). Furthermore, the following chain of implications holds true

$$\begin{aligned} \dot{S}_a = 0 &\implies l - w[\kappa + K_i(\gamma - \gamma_*)] = 0 \implies \\ \sqrt{\varphi'}(x_1 - \varphi) &= 0 \implies x_1 = \varphi, \end{aligned}$$

which implies that x_* is an asymptotically stable equilibrium point of the closed-loop system. Figure 2 shows the simulation of the closed-loop system with $R = 10[\Omega]$, $L = 1[mH]$, $C = 220[\mu F]$, $a = 0.5$, $x_{2*} = 5[V]$, $K_i = 0.1L$, $K_P = 0.01L^2$ and initial conditions $x(0) = \mathbf{0}_2$. From Figure 2 it is clear that the voltage of the capacitor converges to the desired value.

Fig. 2. Simulation of the voltage in the capacitor and the current in the inductor.

VI. CONCLUDING REMARKS

In this note we addressed the stabilization problem of a class of cyclo-passive nonlinear systems that encompasses many physical systems. Towards this end, we revisited the Brayton-Moser framework, and additionally, we propose a new method to obtain alternate representations of the drift vector field. Moreover, these alternate representations were exploited to propose storage functions that are suitable for control design purposes. Finally, we construct a PBC which is a direct extension of the controllers reported in [19], [20].

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