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Distributed Formation with Diffusive Obstacle Avoidance Control in Coordinated Mobile Robots

Nelson P.K. Chan, Bayu Jayawardhana, and Jacquélien M.A. Scherpen

Abstract—We study a multitasking control problem in a group of networked mobile robots where they simultaneously have to reach a prescribed shape/formation and to move as a whole towards a desired position while avoiding any obstacles encountered during transition. We propose a modular control framework that comprises of a distributed controller for achieving the formation task and obstacle avoidance task and a centralized controller for solving the group motion task. In particular, we can independently design a controller for each individual task and obtain the unified distributed controller by a linear combination of these controllers. We prove that the unified controller solves the aforementioned multitasking control problem. An illustrative example is given to show the efficacy of our proposed approach.

I. INTRODUCTION

In recent decades, the field of networked multi-agent systems has attracted a lot of research attention in various domains. Compared to a single agent, a group of networked agents has the advantages of being flexible, redundant and fault tolerant [1] and as such can be employed to perform complex tasks [2] in a centralized or in a distributed manner. In particular, sensing and controlling the group of networked agents in a distributed manner is of interest and various application areas can be found, such as detection of events in wireless sensor networks or execution of search and rescue operations in disasters. Along this line, we consider the problem of steering a group of agents as a formation towards a final desired destination while avoiding obstacles along the course of motion. A possible application can be found in smart manufacturing systems where a group of mobile robots are required to work together to transport an object from position A to position B. In such smart manufacturing systems there may be barriers which the mobile robots individually and as a group should avoid while carrying out the requested task.

In literature, numerous references can be found for achieving each of the individual tasks (formation control, group motion control, obstacle avoidance) or combinations thereof. See, for example, the paper [3] for an overview of various approaches for achieving the formation control

task based on the interconnection topology and sensing capability of the agents. In [1], [4], the weighted centroid tracking problem is considered in which the formation centroid is required to track an assigned task function. In [5], a decentralized controller is constructed based on the notion of *navigation function*. This controller guarantees the convergence of the multi-agent system to the desired formation while maintaining network connectivity and avoiding obstacles. In [2], the multi-agent collision avoidance problem is introduced and formulated as a nonlinear differential game. Dynamic feedback strategies are constructed guaranteeing the avoidance of collisions with obstacles or other agents while the individual agents reach their target. In [6], Lyapunov-like barrier functions are introduced such that the objective of avoiding obstacles can be composed together with other control objectives of the multi-agent systems into a single function for every agent. Furthermore, avoiding obstacles can be regarded as part of the requirements for guaranteeing safety of a (non)linear system; other requirements being the state and input constraints on the system. In [7], [8], the problem of synthesizing controllers for safety critical systems is considered. In both papers, the control design is based on the unification of both Control Lyapunov functions and Control Barrier functions for satisfying respectively the performance/stability properties and safety objectives.

In this paper, we propose a new distributed control law for solving the obstacle avoidance task that can be combined linearly with standard formation control law and group motion control law. In our setting, each individual agent is assumed to be able to measure the relative position with respect to the obstacle and the agent is equipped with a potential-based distributed control law that can be used to compute local collision avoidance manoeuvre. Correspondingly, we introduce a distributed dynamic controller that allows the diffusion of such collision avoidance manoeuvre action to the neighbors and as a consequence to the rest of the network. In comparison to the approach proposed in [2], our obstacle avoidance approach is completely decentralized and we prove rigorously that our approach can be combined linearly with other distributed control laws for solving the multitasking problem simultaneously. By contrast, in [2], the agents are not required to achieve a network objective (such as, a desired formation or consensus) while moving to the individual target. As will be shown later in our simulation results, by diffusing the obstacle

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avoidance manoeuvre in the network through a dynamic controller, we are able to minimize the deformation in the formation shape that is mainly caused by the local collision avoidance movement. In contrast to the work in [5], we consider as well the movement of the formation as a whole towards a desired destination in the plane. Without loss of generality, we consider for now a centralized control law for achieving the group motion control.

The outline of this paper is as follows. In Section II-A, the problem of steering a group of agents towards a desired destination is formulated. The control laws for the individual tasks are given in Section III. The merging of these control laws for an agent is as well considered. We illustrate our approach with an example in Section IV.

Notation: Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}^n$ be the set of n -dimensional real vectors and $\mathbb{R}^{n \times m}$ define the set of $n \times m$ real matrices. The sets $\mathbb{R}_{\geq 0}$ and $\mathbb{R}_{> 0}$ are then subsets of $\mathbb{R}$ that contain only non-negative and positive real numbers, respectively. The cardinality of a set S is given by $|S|$. For a given vector $x \in \mathbb{R}^n$, x_i denotes its i -th entry; $x^\top$ is the transpose and $\|x\| = \sqrt{x^\top x}$ is the usual Euclidean norm. Given vectors $p, q \in \mathbb{R}^n$, the notation $\text{dist}(p, q) := \|p - q\|$. The vector $\mathbf{1}_n \in \mathbb{R}^n$ (respectively, the vector $\mathbf{0}_n \in \mathbb{R}^n$) denotes the vector with entries being all 1s (and respectively, all 0s). Consider a set of vectors $v_1, v_2, \dots \in \mathbb{R}^n$, the notation $\text{span}\{v_1, v_2, \dots\}$ denotes the set of all linear combinations of $v_1, v_2, \dots$. Let $A \in \mathbb{R}^{n \times m}$ be an $n \times m$ rectangular matrix. A_{ij} is the i -th row and j -th column entry of A . The symbols $\mathcal{N}(A) \subset \mathbb{R}^m$ and $\mathcal{R}(A) \subset \mathbb{R}^n$ denote respectively the null space and range space of A . For a symmetric matrix $P \in \mathbb{R}^{n \times n}$, $P > 0$ denotes a positive definite matrix and $P \geq 0$ denotes a positive semidefinite matrix. The $n \times n$ identity matrix is given by I_n while $\text{diag}(v)$ represents the diagonal matrix with entries of vector v on its main diagonal block and all other entries are zero. We denote the Kronecker product of P and I_n by $\tilde{P} := (P \otimes I_n)$.

II. PRELIMINARIES & PROBLEM FORMULATION

Let $\mathcal{G}(\mathcal{V}, \mathcal{E})$ represents a weighted undirected graph with $\mathcal{V} = \{1, \dots, N\}$ denoting the set of vertices and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denoting the set of edges. The direct neighbors of each vertex i is denoted by $\mathcal{N}_i = \{j \mid (i, j) \in \mathcal{E}\}$. The cardinality of $\mathcal{V}$ and $\mathcal{E}$ is $|\mathcal{V}| = N$ and $|\mathcal{E}| = M$. By arbitrarily assigning an orientation to each edge, we obtain an oriented graph $\mathcal{G}$. Let $B \in \mathbb{R}^{N \times M}$ be the (vertex edge) incidence matrix related to this graph where each row of B corresponds to a vertex and each column to an edge. The entries of B are such that $B_{nm} = -1$ if vertex n is the initial vertex of edge e_m and $B_{nm} = 1$ if vertex n is the terminal vertex of edge e_m ; all the other entries of the column m is zero. Let $W = \text{diag}([w_{e_1}, \dots, w_{e_M}]^\top)$ be the diagonal matrix with $w_{e_\bullet}$ denoting the weight of the edge $e_\bullet$. The Laplacian related to $\mathcal{G}$ is the matrix product $L := BWB^\top \in \mathbb{R}^{N \times N}$. For a connected graph, we have $\mathcal{N}(L) = \text{span}\{\mathbf{1}_N\}$. The other $N - 1$ eigenvalues of L

are positive and real and can be ordered as $\lambda_2 \leq \lambda_3 \leq \dots \leq \lambda_N$.

Lemma 2.1: Let $J = \frac{1}{N} \mathbf{1}_N \mathbf{1}_N^\top$ be the matrix of all ones scaled by a factor $\frac{1}{N}$ and define $K = L + J$ where L is the Laplacian related to the graph $\mathcal{G}$. Furthermore, assume $\mathcal{G}$ is connected, then the matrix K is a positive definite matrix.

A. Multitasking control problem formulation

We consider a group of N identical agents moving in the 2-dimensional plane. Each agent is modeled as a single integrator, i.e.,

$$\dot{p}_i = u_i, \quad i \in \mathcal{V}, \quad (1)$$

where $p_i \in \mathbb{R}^2$ and $u_i \in \mathbb{R}^2$ denote the position and controlled velocity of agent i , respectively. In the following, we will discuss three tasks which will be handled simultaneously by our proposed controller.

Obstacle Avoidance Control Problem

Without loss of generality, we consider obstacles that can be encapsulated by K circles in the plane where the centroid and radii of each circle $\mathcal{O}_k$, $k = 1, \dots, K$, is given by $p_k^{\text{obs}} \in \mathbb{R}^2$ and $R_k^{\text{obs}} \in \mathbb{R}_{> 0}$, respectively. In this case, the boundary of the k -th obstacle is defined by

$$\partial \mathcal{O}_k := \{x \in \mathbb{R}^2 \mid \|x - p_k^{\text{obs}}\| - R_k^{\text{obs}} = 0\}.$$

For our later description of distributed obstacle avoidance control law, we associate each agent i with a parameter $R_i^{\text{safe}} \in \mathbb{R}_{> 0}$ which defines the safe distance to the boundary of any obstacle. Roughly speaking, when the agent's distance to the boundary of an obstacle is less than R_i^{safe} , then the distributed obstacle avoidance control will set in for the i -th agent. Throughout this paper, we will consider the case of R_i^{safe} being constant for all i .

For the following definitions, let $\mathcal{P}_0$ denote the set of initial conditions which do not intersect $\mathcal{O}_k$ for all k .

Definition 2.1 (local obstacle avoidance): The i -th agent is said to *avoid collision with obstacle* $\mathcal{O}_k$ if for almost all $p_i(0) \in \mathcal{P}_0$ the trajectories $p_i(t)$ do not enter $\mathcal{O}_k$ for all $t \in \mathbb{R}_{> 0}$, i.e., $p_i(t) \notin \mathcal{O}_k$ for all $t \in \mathbb{R}_{> 0}$.

Definition 2.2 (obstacle avoidance task): The group of N agents is said to achieve *obstacle avoidance* if every i -th agent avoids collision with all obstacles $\mathcal{O}_k$, $k = 1, \dots, K$.

Formation Control Problem

The interaction between the agents is represented by a weighted, undirected, and connected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$. In the present work we assume that the agents can obtain relative position information from its neighbors, i.e., agent i has access to $p_j - p_i$, $\forall j \in \mathcal{N}_i$. The desired relative position between the agents is encoded in a vector $P^* \in \mathbb{R}^{2N}$. Note that P^* has to satisfy geometric constraints that define the shape of the formation. We refer to the exposition in [9], [10], [11] on the graph formalism of mobile robots' formation. For a given P^* , the group of N -agents is said to be in *the desired formation* if $p_j - p_i = p_j^* - p_i^*$, $\forall (i, j) \in \mathcal{E}$.

Throughout the paper, we always assume that the desired relative position P^* is well-posed, i.e., the formation shape is geometrically feasible.

Definition 2.3 (formation task): For a given P^* , the group of N -agents is said to achieve *formation w.r.t. P^** if all agents' trajectories asymptotically converge to the desired formation, i.e.,

$$\lim_{t \rightarrow \infty} (p_j(t) - p_i(t)) = p_j^* - p_i^*, \quad \forall (i, j) \in \mathcal{E}.$$

Group Stabilization Control Problem

In addition to the above distributed control tasks, we can add a group motion task where the group's centroid and orientation are controlled to achieve certain control behaviour, such as, following a given reference trajectory or converging to a desired point (e.g., group stabilization).

Definition 2.4 (group stabilization task): The group of N -agents is said to achieve *group stabilization* if the centroid of the group $p_{\text{cen}} := \frac{1}{N} \mathbf{1}_N^\top P$ converges asymptotically to the origin, i.e., $p_{\text{cen}}(t) \rightarrow 0$ as $t \rightarrow \infty$.

In the following section, we present our control design framework which can accomplish all three different control tasks. Both obstacle avoidance and formation tasks are solved by distributed control laws while group stabilization task is solved by a coordinated control law.

Assumption 2.1: During the transition, the formation centroid p_{cen} is allowed to cross the obstacles.

III. DISTRIBUTED OBSTACLE AVOIDANCE-FORMATION CONTROL DESIGN

The navigation function as used in [5] requires us to have apriori information of all tasks so that we can embed this information to the navigation function. In contrast to this approach, we will pursue a control design that is modular where one can add and remove control law for particular task directly without jeopardizing the completion of other tasks. Therefore we assume that the control law for each agent is a linear combination of control laws of different tasks, i.e.,

$$u_i = u_i^f + u_i^g + u_i^o, \quad \forall i \in \mathcal{V} \quad (2)$$

where u_i^f is the local control law for solving formation task of i -th agent, u_i^g is the local control law for solving group stabilization task of i -th agent and u_i^o is the local control law for solving obstacle avoidance task of i -th agent.

In the following sub-sections, we present a particular control law for each of the aforementioned tasks that will be used in our unified control framework. While our framework is not restricted to these control laws, we will focus mainly on these laws in this paper where we can demonstrate the applicability of our approach.

A. Distributed formation control law

For achieving formation task, we consider the following standard relative position-based formation control law

$$u_i^f = c_f \sum_{j \in \mathcal{N}_i} w_{ij} (p_j - p_i - (p_j^* - p_i^*)), \quad (3)$$

where $c_f \in \mathbb{R}_{>0}$ is the formation gain and $w_{ij} \in \mathbb{R}_{>0}$, $i, j \in \mathcal{V}$ the weight of the edge $(i, j) \in \mathcal{E}$. In compact form, the above law can be written as

$$U^f = c_f \tilde{L}(P^* - P) \quad (4)$$

where U^f is the stacked vector of u_i^f , $i \in \mathcal{V}$. The stability analysis of closed-loop system using only this formation control law can be found, among others, in [3].

B. Coordinated group stabilization control law

For stabilizing the group, we assume the existence of a central coordinator. The role of the central coordinator is to calculate the formation centroid p_{cen} based on the position of the agents and use it to compute the stabilizing control law for the group.

The design of the control law for group stabilization is based on the assumption that the desired formation has been reached and there is no obstacle. In this case, using (1), (2), (3) and $u_i^o = \mathbf{0}_2$, the dynamics of the centroid's position is given by

$$\dot{p}_{\text{cen}} = \frac{1}{N} \mathbf{1}_N^\top \dot{P} = u^g, \quad (5)$$

where we assume that the control law u^g will be communicated to all the agents and as such $u_1^g = u_2^g = \dots = u_N^g = u^g$.

We propose the following group motion control law:

$$u^g = -c_p p_{\text{cen}} + c_l \gamma, \quad \dot{\gamma} = -p_{\text{cen}}, \quad (6)$$

where $c_p \in \mathbb{R}_{>0}$ and $c_l \in \mathbb{R}_{>0}$ are the group proportional and integral gain, respectively.

In compact form, we can write the group stabilization control law for the whole group as

$$U^g = \mathbf{1}_N (-c_p p_{\text{cen}} + c_l \gamma), \quad (7)$$

where U^g is the stacked vector of u_i^g , $i \in \mathcal{V}$.

If we do not need to solve obstacle avoidance task then the group stabilization task can be solved simply by using a proportional controller. As it will be clear later, the integral action is needed in our control law for compensating the drift that is introduced by the distributed control law for the obstacle avoidance given in the next subsection. We also remark that as each agent gets u^g , the formation control is not affected by the group motion control. Therefore, both control laws are complementary to each other.

C. Distributed obstacle avoidance control law

While the agents are maintaining the formation shape and moving together towards the desired destination, all agents should locally avoid any obstacle that they encounter during the course of transition. Let us consider for the moment an agent i and an obstacle $\mathcal{O}_k$. As defined before, the agent i avoids obstacle $\mathcal{O}_k$ when $p_i(t) \notin \mathcal{O}_k$ for all $t \in \mathbb{R}_{\geq 0}$. Note that the distance from p_i to $\mathcal{O}_k$ is

$$d_k(p_i) := \min_{x \in \partial \mathcal{O}_k} \text{dist}(p_i, x). \quad (8)$$

while the point in $\mathcal{O}_k$ that is closest to p_i is denoted by

$$x_k^*(p_i) := \operatorname{argmin}_{x \in \partial \mathcal{O}_k} \operatorname{dist}(p_i, x). \quad (9)$$

Using these notations, agent i avoids obstacle $\mathcal{O}_k$ implies that $d_k(p_i(t)) > 0$ for all $t \in \mathbb{R}_{\geq 0}$. For constructing a local obstacle avoidance control law, we can consider a potential function which is a logarithmic barrier function given by

$$W_k(d_k(p_i)) = \left(\max \left(-c_\alpha \log \left(\frac{d_k(p_i)}{R^{\text{safe}}} \right), 0 \right) \right)^2. \quad (10)$$

It can be checked that as $p_i \rightarrow \partial \mathcal{O}_k$, $W_k(p_i) \rightarrow \infty$. Using such potential function, the following gradient-based control law for the agent i when it is within the vicinity to the obstacle $\mathcal{O}_k$ has been used for mobile robots to avoid obstacles (see, for instance, [12], [13]).

$$u_i^o = \alpha_i^k(p_i) \quad (11)$$

where $\alpha_i^k(p_i) = -\frac{\partial W_k(p_i)}{\partial p_i}^\top$. The constant $c_\alpha \in \mathbb{R}_{>0}$ in (10) defines the controller's gain and it can be checked that

$$\frac{\partial W_k(d_k(p_i))}{\partial p_i} = 2c_\alpha^2 \log \left(\frac{d_k(p_i)}{R^{\text{safe}}} \right) \frac{(p_i - x_k^*(p_i))^\top}{d_k^2(p_i)}$$

when $d_k(p_i) < R^{\text{safe}}$ while it is $\mathbf{0}_2$ otherwise based on the use of potential function W_k as in (10).

So when the relative distance between an agent i and the obstacle k is less than the threshold R^{safe} , agent i will activate the obstacle avoidance action. However, if we apply this obstacle avoidance control law locally on agent i then the rest of the agents will only be driven by the formation and group stabilization control laws (c.f. (2)). As a consequence, the unexpected obstacle avoidance manoeuvre by agent i that is not communicated with the others will introduce undesirable deformation to the formation shape. On the other hand, the real-time communication of the obstacle avoidance control action to all nodes should be prohibited as it will unnecessarily consume the communication channel and is not scalable.

In order to “diffuse” the obstacle avoidance action to the other agents, we introduce a dynamic obstacle avoidance controller whose state variable ζ_i is communicated to its neighbors. More precisely, the local dynamic obstacle avoidance controller is described by

$$\left. \begin{aligned} \dot{\zeta}_i &= c_\zeta \sum_{j \in \mathcal{N}_i} w_{ij} (\zeta_j - \zeta_i) + u_{\alpha_i} \\ u_i^o &= \zeta_i, \end{aligned} \right\} \quad (12)$$

where $c_\zeta \in \mathbb{R}_{>0}$ is the diffusion gain of obstacle avoidance control law, $w_{ij} \in \mathbb{R}_{>0}$, $i, j \in \mathcal{V}$, is the weight of the edge $(i, j) \in \mathcal{E}$ and u_{α_i} is given by

$$u_{\alpha_i} = \alpha_i(p_i) = \sum_{k=1}^K \alpha_i^k(p_i).$$

In a compact form, the distributed dynamic obstacle avoidance control law is given by

$$\begin{aligned} \dot{\zeta} &= -c_\zeta \tilde{L} \zeta + U_\alpha \\ U^o &= \zeta, \end{aligned}$$

where U^o is the stacked vector of u_i^o , $i \in \mathcal{V}$, and respectively, U_α is the stacked vector of u_{α_i} . The state variable ζ can be seen as an aggregation of the obstacle behavior of the individual agent.

When we pre-multiply the first equation in (12) by $\frac{1}{N} \tilde{\mathbf{1}}_N^\top$, we get

$$\frac{1}{N} \tilde{\mathbf{1}}_N^\top \dot{\zeta} = \frac{1}{N} \tilde{\mathbf{1}}_N^\top (-c_\zeta \tilde{L} \zeta + U_\alpha) \Rightarrow \dot{\zeta}_{\text{avg}} = \frac{1}{N} \sum_{i=1}^N u_{\alpha_i},$$

where $\zeta_{\text{avg}} := \frac{1}{N} \tilde{\mathbf{1}}^\top \zeta$. Thus whenever the agents are already free from obstacles after some finite time $\tilde{t} > 0$ then $U_\alpha(t) = 0$ for all $t > \tilde{t}$. In this case, the distributed obstacle avoidance control law becomes a consensus system which implies that ζ_i will converge to a common value given by $\frac{1}{N} \sum_j \zeta_j(\tilde{t})$ due to the average consensus protocol. Due to the integral action in the group stabilization controller, such constant bias from the asymptote of $\zeta_i(t)$ will be compensated and the integral controller ensures that the formation centroid converges to the origin.

D. Stability analysis of the unified control law

In this subsection, we will prove that the unified control law (2), where the individual control laws u_i^f, u_i^g, u_i^o are as in (3), (6) and (12), is able to achieve the control objectives of reaching and maintaining desired formation, of coordinated group stabilization and of performing obstacle avoidance.

Theorem 3.1: Consider a group of N -agents modeled as single integrators as in (1) and the multitasking control problem as given before with obstacles $\mathcal{O}_k$, $k = 1, \dots, K$ and with the target formation shape encoded in the desired relative position P^* . Then the unified control law (2) with u_i^f, u_i^g, u_i^o given by (3), (6) and (12), respectively, solve simultaneously the obstacle avoidance, formation and group stabilization control problems globally. More precisely, for all initial conditions $p_i(0) \in \mathbb{R}^2 \setminus \bigcup_k \mathcal{O}_k$, $i \in \mathcal{V}$, $k = 1, \dots, K$,

- (O1) $\lim_{t \rightarrow \infty} \tilde{B}^\top (P^* - P(t)) = \mathbf{0}_{2M}$ (i.e., the desired formation is asymptotically reached);
- (O2) $p_{\text{cen}}(t) = \frac{1}{N} \tilde{\mathbf{1}}_N^\top P(t) \rightarrow \mathbf{0}_2$ as $t \rightarrow \infty$ (i.e., the group centroid converges to the origin);
- (O3) $d_k(p_i(t)) > 0$ for all $i \in \mathcal{V}$, $k = 1, \dots, K$ and $t \in \mathbb{R}_{\geq 0}$ (i.e., every agent avoids all obstacles at all time).

Before giving the proof of Theorem 3.1, we first present an auxiliary result.

Lemma 3.1: Assume the group of agents successfully carried out the obstacle avoidance task. Then the unique equilibrium point of the closed loop system is $\hat{P} =$

$(I_{2N} - \tilde{J})P^*$ satisfying $\tilde{B}^\top(P^* - \hat{P}) = \mathbf{0}_{2M}$ and $p_{\text{cen}} = \frac{1}{N}\mathbb{1}_N^\top \hat{P} = \mathbf{0}_2$.

The proof of the lemma will be provided in the full version of the paper.

Now, we are ready to prove Theorem 3.1.

PROOF (THEOREM 3.1). Consider the Lyapunov-like function

$$V(P, \dot{P}) = \frac{1}{2}\dot{P}^\top \dot{P} + \frac{1}{2}c_f c_\zeta (P^* - P)^\top \tilde{L}^2 (P^* - P) + \frac{1}{2}c_1 N p_{\text{cen}}^\top p_{\text{cen}} + \sum_{i=1}^N \sum_{k=1}^K W_k(p_i) \quad (13)$$

where $W_k(p_i)$ is the logarithmic barrier function with nonzero value when $d_k(p_i) \leq R^{\text{safe}}$. The time derivative of V is

$$\begin{aligned} \dot{V} &= \dot{P}^\top \ddot{P} - c_f c_\zeta (P^* - P)^\top \tilde{L}^2 \dot{P} \\ &\quad + c_1 N p_{\text{cen}}^\top \dot{p}_{\text{cen}} + \sum_{i=1}^N \sum_{k=1}^K \frac{d}{dt} W_k(p_i) \\ &= \dot{P}^\top \left[- \left[(c_f + c_\zeta) \tilde{L} + c_p \tilde{J} \right] \dot{P} - \left[c_f c_\zeta \tilde{L}^2 + c_1 \tilde{J} \right] P \right. \\ &\quad \left. + c_f c_\zeta \tilde{L}^2 P^* + U_\alpha \right] - \dot{P}^\top c_f c_\zeta \tilde{L}^2 (P^* - P) \\ &\quad + c_1 N \left(\frac{1}{N} \mathbb{1}_N^\top P \right)^\top \left(\frac{1}{N} \mathbb{1}_N^\top \dot{P} \right) + \sum_{i=1}^N \sum_{k=1}^K \frac{\partial W_k}{\partial p_i} \dot{p}_i \\ &= \dot{P}^\top \left[- \left[(c_f + c_\zeta) \tilde{L} + c_p \tilde{J} \right] \dot{P} - c_1 \tilde{J} P + U_\alpha \right] \\ &\quad + c_1 P^\top \frac{1}{N} \mathbb{1}_N \mathbb{1}_N^\top \dot{P} - \sum_{i=1}^N u_{\alpha_i} \dot{p}_i \\ &= -\dot{P}^\top \left[(c_f + c_\zeta) \tilde{L} + c_p \tilde{J} \right] \dot{P} \\ &< -\ell \|\dot{P}\|^2, \end{aligned} \quad (14)$$

where $\ell > 0$. The last inequality is due to the fact that $(c_f + c_\zeta) \tilde{L} + c_p \tilde{J}$ is positive definite by Lemma 2.1. Since the function V is proper and V is non-increasing, it follows immediately that **(O3)** is achieved. Moreover the application of La-Salle invariance principle implies that the state trajectories $(P(t), \dot{P}(t))$ converges to the largest invariant set where $\dot{P} = 0$. By Lemma 3.1, such invariant set is given by $\tilde{B}^\top(P^* - P) = \mathbf{0}_{2M}$ and $p_{\text{cen}} = \frac{1}{N}\mathbb{1}_N^\top P = \mathbf{0}_2$. Thus this implies that both **(O1)** and **(O2)** hold. This proves the claim. $\square$

IV. NUMERICAL SIMULATION

A. Simulation setup

To demonstrate the applicability of the proposed control law, we perform numerical simulations with a network of 3 agents.

We consider two circular obstacles in the plane with center at $p_{\text{obs}_1} = (4, 7)$ and $p_{\text{obs}_2} = (1, 4)$ (shown in red in the figures). The radii of the obstacles is set equally to $R_{\text{obs}} = 0.75$. For the obstacle avoidance control law, we take $R^{\text{safe}} = 0.75$. In the figures, the blue annulus shows the area within R^{safe} from the obstacles where the obstacle avoidance control law is activated.

We set the gains of the control law to be $c_f = 10$, $c_p = 1$, $c_1 = 0.75$, $c_\alpha = 10$, $c_\zeta = 10$ and the initial positions of the agents to be $p_1 = [5, 10]^\top$, $p_2 = [5, 13]^\top$, and $p_3 = [3, 10]^\top$. The desired relative positions for the formation are set to be $p_{12}^* = [0, 3]^\top$, $p_{13}^* = [-2, 0]^\top$, and $p_{23}^* = [-2, -3]^\top$.

B. Simulation results

The proposed control law (2) is compared against the control mechanism where the obstacle avoidance control law is only locally applied to the agent which is within a distance R^{safe} from an obstacle. No diffusion of this control action takes place to the neighbors and as such the other agents will solely be driven by the formation control law (3) and the group stabilization control law (6) when they are individually not in close vicinity to any obstacles. From the figure, we can observe that whenever an agent is within a distance R^{safe} from the boundary of the obstacle (the blue region) then the obstacle avoidance behavior of the particular agent is ‘activated’. In the top right figure, as a direct consequence of the diffusive action in the proposed control law, the neighboring agents undergo similar trajectories as that of the manoeuvring agent. However, this is not the case in the bottom left figure. Because of this, we can observe larger distortion caused by the obstacle manoeuvre to the formation (compare the top right and the bottom right figure) for the “diffusiveless” case. Besides, in the top right figure, the length of the edge between agents 2 and 3 is not affected by the obstacle manoeuvre due to the fact that we started with the prescribed configuration, set the initial condition for ζ_i , $i = 2, 3$ for both agents to be the same ($\mathbf{0}_2$) and both agents are connected to agent 1. In the bottom right figure, during the time interval $[0.14, 0.6]$ s, the edge between agent 2 and 3 is not affected by the obstacle manoeuvre of agent 1. This also occurs at the time interval $[1.0, 1.4]$ s, but now for the edge between 1 and 3 while agent 2 is avoiding obstacle $\mathcal{O}_2$. Finally, one can observe that the group stabilization control law is able to steer the whole group towards the origin.

V. CONCLUSIONS

In this paper, we propose a new distributed control design law for achieving formation while avoiding obstacles for a group of agents. The local obstacle avoidance is achieved by a potential-based control law. By employing a dynamical controller, the local obstacle behavior of a particular agent is diffused through the network. In combination with a coordinated group controller, we are able to steer the formation centroid to the origin while

Fig. 1: Simulation results of simultaneous obstacle avoidance, formation and group stabilization control tasks problem where we compare the performance of our diffusive obstacle avoidance distributed control approach with that of standard gradient-based local obstacle avoidance control method. Left column: The plot of three agents' position trajectories transitioning from a given initial position to the desired formation and final desired centroid position; Right column: The plot of distances between the agents; Top row: The simulation result using our proposed diffusive obstacle avoidance distributed control law; Bottom row: The simulation result using local obstacle avoidance control method and hence no "diffusion" to the neighbors.

maintaining formation shape during and after the obstacle avoidance manoeuvre. This is shown through an example in which we compare the proposed control law with the case when the local obstacle avoidance behavior is not diffused.

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