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Formation Control and Velocity Tracking for a Group of Nonholonomic Wheeled Robots

Ewoud Vos, Arjan J. van der Schaft, *Fellow, IEEE*, and Jacquelin M. A. Scherpen, *Senior Member, IEEE*

Abstract—This technical note presents an integrated approach for formation control and velocity tracking of a group of nonholonomic wheeled robots. The solution is defined within the port-Hamiltonian framework, providing a clear interpretation of the results. The controller consists of a local nonlinear heading and velocity tracking controller combined with a distributed formation controller. The formation controller achieves formations by assigning virtual couplings in between the robots. Experimental results are provided to illustrate the effectiveness of the approach.

Index Terms—Decentralized control, robot control, velocity control.

I. INTRODUCTION

This technical note considers the problem of formation control for a group of nonholonomic wheeled robots, while tracking a prescribed reference velocity. Formation control fits into the broader category of group coordination problems, where the main challenge is to achieve prescribed group behaviors with local feedback rules, rather than centralized controllers [1]. Specifically formation control aims to achieve a prescribed geometrical shape, by controlling the inter-robot distances or positions. Velocity tracking enables the group of robots to move as a formation along a surface. Applications may be found in sweeping [4], [10] and deployment for coverage [5], [13].

The motivating application for the present technical note is the Energy-efficient design and control of mobile Robotic Sensor networks (ROSE) project which is devoted to control of a group of mobile sensor-integrated robotic devices for data acquisition on dikes. The motivation for the current technical note stems from the project goal to make a group of robotic sensors move in formation along the dike surface to obtain knowledge on the dike interior.

Due to its practical interest, many works have focused on the control of the wheeled robot (see e.g. [8], [11] and references therein). Compared to many other types of mobile robots, the wheeled robot possesses a nonholonomic constraint on the sideways movement of the wheel axle. Due to this constraint, the wheeled robot does not satisfy the well-known Brockett's condition for smooth feedback stabilization [3] as was shown in [18]. Efforts to circumvent Brockett's condition include switching control for posture stabilization by decoupling the position and heading of the robot [11] and path tracking and formation control using linearization techniques with gain scheduling [8]. In this technical note, we are concerned with controlling the heading of each robot and the relative position between the front ends of the robots, which is shown in the technical note not to be hindered by Brockett's condition.

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Our solution is defined in the port-Hamiltonian framework [16], which is an energy-based modeling framework (another well known example is the Euler-Lagrange framework). Adopting an energy-based approach has several advantages over other modeling frameworks, two important ones being: energy concepts are familiar to practitioners and may therefore serve as a *lingua franca* and the energy function of the system determines both the static and the transient behavior [14]. A port-Hamiltonian system consists of energy-storing and energy-dissipating components, which are interconnected in a power preserving manner via so-called power-ports [16]. The interconnection topology of the system is modelled by a *Dirac* structure. The Hamiltonian equals the total energy stored in the system and may be used as a Lyapunov function if it is lower bounded.

The starting point for this work is the passivity-based design tool from [1], [2]. In [1] and [2], each agent is rendered passive with respect to a reference velocity, while the inter-robot position converges to a prescribed compact set. Furthermore we build upon the recently introduced theory of *port-Hamiltonian systems on graphs* [17], which provides powerful tools for modeling and analyzing complex physical network dynamics as port-Hamiltonian systems on open graphs.

The main contribution of this technical note is the design of a controller that is a combination of a local heading and velocity tracking controller, and a distributed formation controller. While stability is easily proven for the separate controllers, the combination poses a significant challenge due to the conflicting interests of the heading and the formation controller. Using a nonlinear heading controller design, we are able to prove stability for the joint controller scheme under a mild condition on the initial heading of the robots. Furthermore we derive a dynamical model for the nonholonomic wheeled robot in the port-Hamiltonian framework, which has not been considered before.

The formation controller assigns virtual springs and dampers between the front ends of the robots, which aligns with the passivity-based approach in [1] and [2]. Relative to [1], [2] we do not require full actuation of the agents, which is of high practical interest. Furthermore, by designing the velocity tracking controller ourselves, we relax the assumption on the passivity of the robots with respect to the reference velocity [1]. Using tools from port-Hamiltonian systems theory [16], [17] has not been considered before in this setting. Preliminary work considering only the dynamical modeling and formation control was presented at [18].

The outline of the technical note is as follows. First, we provide the preliminaries for the remainder in Section II. In Section III, we derive the dynamical model of the wheeled robot. Then we continue with the formal problem formulation in Section IV, and present the controller design and closed-loop analysis. The effectiveness of the design is illustrated by experimental results in Section V. The technical note is concluded in Section VI with concluding remarks and directions for future research.

II. PRELIMINARIES

The port-Hamiltonian framework is an energy-based framework which describes a large class of (nonlinear) systems including passive mechanical and electrical systems [16]. Control is achieved via

Fig. 1. Wheeled robot i with center of the axle $(x_{A,i}, y_{A,i})$ and front end $(x_{C,i}, y_{C,i})$ in an inertial coordinate frame.

so-called power-ports, which consist of a pair of port-variables (inputs and outputs) whose product equals the power supplied to the system. Let $x \in \mathbb{R}^n$ denote the state of the system, $u \in \mathbb{R}^m$ the input, and $y \in \mathbb{R}^m$ the output, then the dynamics of a port-Hamiltonian system are given by

$$\begin{aligned} \dot{x} &= (J(x) - R(x)) \frac{\partial H}{\partial x}(x) + g(x)u \\ y &= g^T(x) \frac{\partial H}{\partial x}(x) \end{aligned} \quad (1)$$

with skew-symmetric structure matrix $J(x) = -J^T(x) \in \mathbb{R}^{n \times n}$ and positive semi-definite dissipation matrix $R(x) = R^T(x) \geq 0 \in \mathbb{R}^{n \times n}$. The Hamiltonian $H(x)$ equals the total energy stored in the system, and its time derivative satisfies $\dot{H} \leq u^T(t)y(t)$. Hence the increase in stored energy is always equal to or smaller than the power supplied through the power-port (u, y) . Therefore, (1) is a passive system, if $H(x)$ is bounded from below [15]. This property provides an excellent starting point for the stability analysis of many systems.

The interaction amongst the robots in the formation is described by an undirected, connected, acyclic graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ denotes the set of N nodes and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ denotes the set of M edges. The nodes of the graph correspond to the robots, while the edges correspond to virtual couplings. Robot i and j are interconnected by a virtual coupling if there is an edge $(i, j) \in \mathcal{E}$. Each edge has an orientation by arbitrarily assigning a positive sign to one end and a negative sign to the other end. The incidence matrix B associated to $\mathcal{G}(\mathcal{V}, \mathcal{E})$ describes which nodes are coupled by an edge, and is defined as

$$b_{ik} = \begin{cases} +1 & \text{if node } i \text{ is at the positive side of edge } k \\ -1 & \text{if node } i \text{ is at the negative side of edge } k \\ 0 & \text{otherwise.} \end{cases}$$

For more details on graphs see, e.g., [9].

III. DYNAMICAL MODEL OF THE WHEELED ROBOT

Consider wheeled robot i depicted in Fig. 1 with heading ϕ_i . The center of mass is assumed to be at the center of the wheel axle A . Let $d_{AC,i}$ denote the distance between the center point on the wheel axle A and the point on the front end of the robot C . Note that point C is used to define the relative position between robots in the formation controller.

In the remainder, we consider a network of N wheeled robots, but first we derive the dynamics of a single robot i . The dynamics of the wheeled robot are modeled as a rigid body with a nonholonomic constraint on its axle. Let $q_i, p_i^{rb} \in \mathbb{R}^3$ denote, respectively, the position and momentum of robot i , where $q_i = (x_{A,i}, y_{A,i}, \phi_i)^T$ and $p_i^{rb} =$

$M_i^{rb} \dot{q}_i = (p_{Ax,i}, p_{Ay,i}, h_i^{rb})^T$.¹ The mass-inertia matrix M_i^{rb} is given by $M_i^{rb} = \text{diag}(m_i I_2, I_{cm,i})$ with robot mass m_i , moment of inertia $I_{cm,i}$, and identity matrix I_2 . The inputs for robot i are a forward force $u_{f,i} \in \mathbb{R}$ and a torque $u_{\phi,i} \in \mathbb{R}$ acting at the axis of rotation (point A in Fig. 1), to be delivered by the robot's actuators. The outputs are the corresponding forward velocity $y_{f,i} \in \mathbb{R}$ and angular velocity $y_{\phi,i} \in \mathbb{R}$. Let $u_i = (u_{f,i}, u_{\phi,i})^T$ and $y_i = (y_{f,i}, y_{\phi,i})^T$, then the rigid body dynamics of robot i are given by [16]

$$\begin{aligned} \begin{pmatrix} \dot{q}_i \\ \dot{p}_i^{rb} \end{pmatrix} &= \begin{pmatrix} 0 & I_3 \\ -I_3 & -D_i^{rb} \end{pmatrix} \begin{pmatrix} \frac{\partial H_i^{rb}}{\partial q_i} \\ \frac{\partial H_i^{rb}}{\partial p_i^{rb}} \end{pmatrix} + \begin{pmatrix} 0 \\ F(q_i) \end{pmatrix} u_i \\ y_i &= F^T(q_i) \frac{\partial H_i^{rb}}{\partial p_i^{rb}} \end{aligned} \quad (2)$$

where

$$D_i^{rb} = \begin{pmatrix} d_{f,i} & 0 & 0 \\ 0 & d_{\phi,i} & 0 \\ 0 & 0 & d_{\phi,i} \end{pmatrix}, \quad F(q_i) = \begin{pmatrix} \cos \phi_i & 0 \\ \sin \phi_i & 0 \\ 0 & 1 \end{pmatrix}$$

with friction coefficients $d_{f,i}, d_{\phi,i}$. The Hamiltonian $H_i^{rb} : \mathbb{R}^3 \mapsto \mathbb{R}$ equals the kinetic energy stored in the robot mass and is given by

$$H_i^{rb}(p_i) = \frac{1}{2} p_i^{rbT} (M_i^{rb})^{-1} p_i^{rb}.$$

In (2) and in the remainder of this technical note, the arguments of the partial derivatives are left out for simplicity.

Robot i is subject to a nonholonomic constraint on its wheel axle, which is given by

$$\sin \phi_i \dot{x}_{A,i} - \cos \phi_i \dot{y}_{A,i} = 0 \quad (3)$$

which can be rewritten in terms of (2) as

$$\underbrace{(\sin \phi_i \quad -\cos \phi_i \quad 0)}_{A_i^T(q_i)} \frac{\partial H_i^{rb}}{\partial p_i^{rb}}(p_i^{rb}) = 0. \quad (4)$$

Now, we solve for constraint (4) using a coordinate transformation $(q_i, p_i^{rb}) \mapsto (q_i, p_i, p_s^i)$ given by

$$\begin{aligned} p_i &:= S_i^T(q_i) p_i^{rb} \\ p_{s,i} &:= A_i^T(q_i) p_i^{rb} \end{aligned} \quad p_i = (p_{f,i}, h_i)^T \in \mathbb{R}^2, \quad p_{s,i} \in \mathbb{R} \quad (5)$$

where

$$S_i(q_i) = \begin{pmatrix} \cos \phi_i & 0 \\ \sin \phi_i & 0 \\ 0 & 1 \end{pmatrix}.$$

This coordinate transformation corresponds to splitting the rigid body momentum p_i^{rb} into (constrained) *sideway* momentum $p_{s,i}$, and *forward* and angular momentum $p_{f,i}, h_i$.

In the new coordinates, the $p_{s,i}$ dynamics are eliminated to obtain the dynamics on the constrained state space [16], [18]

$$\begin{aligned} \begin{pmatrix} \dot{q}_i \\ \dot{p}_i \end{pmatrix} &= \begin{pmatrix} 0 & S_i(q_i) \\ -S_i^T(q_i) & D_i^r \end{pmatrix} \begin{pmatrix} \frac{\partial H_i^r}{\partial q_i} \\ \frac{\partial H_i^r}{\partial p_i} \end{pmatrix} + \begin{pmatrix} 0 \\ I_2 \end{pmatrix} u_i \\ y_i &= \frac{\partial H_i^r}{\partial p_i} \end{aligned} \quad (6)$$

with $D_i^r = S_i^T(q_i) D_i^{rb} S_i(q_i)$ and $H_i^r = (1/2) p_i^T (M_i^r)^{-1} p_i$, where $M_i^r = \text{diag}(m_i, I_{cm,i})$.

¹The superscript $\cdot^{rb}$ refers to the *rigid body* dynamics.

Fig. 2. Robot heading ϕ_i is defined with respect to the x -axis, while the desired heading $\phi_i^* = \pi/2$ is along the y -axis.

Remark 1 (Brockett's Condition): Brockett's condition for a stabilizing continuous feedback controller [3] is not satisfied for (6) (see [15, Proposition 4.2.14] for more details). However, for our design this raises no problems since we focus on controlling the *relative* position of the robot front ends.

We now continue with the problem formulation, controller design, and closed-loop analysis.

IV. VELOCITY TRACKING CONTROL AND FORMATION CONTROL USING VIRTUAL COUPLINGS

The goal of this technical note is to make N wheeled robots of the form (6) track a reference velocity along a prescribed heading angle, while converging to a desired formation shape. The velocity tracking controller is a local controller based on generalized canonical transformations [7] which guarantees that each wheeled robot tracks a predefined *forward* velocity v_i^* . To ensure the movement of the robots is along the desired heading ϕ_i^* , we propose an additional nonlinear heading controller. In this work, the desired heading is along the y -axis, such that $\phi_i^* = \pi/2$ (see Fig. 2).

To achieve the desired formation shape, we assign virtual couplings between the robots. Each virtual coupling consists of a virtual spring to steer the robots to the desired formation and a virtual damper in parallel to shape the transient response. The interconnection topology (i.e., which robots are interconnected by a virtual coupling) is modeled by an undirected graph which is connected and acyclic. Consider two robots i and j , which are interconnected using virtual coupling k , where robot j is at the positive end of the corresponding edge. The inter-robot position z_k is defined as $z_k = (x_{C,j}, y_{C,j})^T - (x_{C,i}, y_{C,i})^T$.

Then, the control goals are more formally stated as

$$\begin{cases} \phi_i \rightarrow \phi_i^* \\ p_i \rightarrow p_i^* \\ z_j \rightarrow z_j^* \end{cases} \quad \text{as } t \rightarrow \infty \quad (7)$$

for all $i = 1, \dots, n$ robots and $j = 1, \dots, m$ virtual couplings. Here, p_i^* denotes the desired momentum corresponding to the reference velocity v_i^* (to be defined later on).

A. Control Design

The controller consists of three parts: a nonlinear heading controller, a velocity tracking controller based on generalized canonical transformations, and a distributed formation controller. Correspondingly the control input for the robots u is the sum of three terms $u = u_1 + u_2 + u_3$. Below we first design the three controllers, followed by the analysis of the closed-loop system.

1) *Heading Control:* Each robot is required to track a forward reference velocity along $\phi_i^* = \pi/2$ (see Fig. 2). Under the following mild condition on the initial heading, convergence to the desired heading is achieved.

Assumption 1: The initial heading $\phi_i(0)$ of robot i is contained in the interval $\phi_i(0) \in (0, \pi)$.

Remark 2: We use the interval $(0, \pi)$ for simplicity of notation. This interval may be 'stretched' to, e.g., the interval $(-\pi/2, 3\pi/2)$ resulting however in a much more complicated expression.

Denote the error variable $\hat{\phi}_i = \phi_i - \phi_i^*$, then the heading control dynamics are given by

$$\begin{aligned} \dot{\hat{\phi}}_i &= u_i^h \\ y_i^h &= \frac{\partial H_i^h}{\partial \hat{\phi}_i} \end{aligned} \quad (8)$$

where the Hamiltonian is defined as

$$H_i^h(\hat{\phi}_i) := -k_i^\phi \ln |\cos \hat{\phi}_i|$$

with $k_i^\phi > 0$ the controller gain. This nonlinear Hamiltonian guarantees under Assumption 1 that $\phi_i(t) \in (0, \pi)$ for all t (see proof of Theorem 4). On the interval $\hat{\phi}_i \in (-\pi/2, \pi/2)$, it is easy to verify that $H_i^h(\hat{\phi}_i) \geq 0$ since $0 < |\cos \hat{\phi}_i| \leq 1$ and hence $\ln |\cos \hat{\phi}_i| \leq 0$. Note that the partial derivative of H_i^h with respect to $\hat{\phi}_i$ is simply $(\partial H_i^h / \partial \hat{\phi}_i) = k_i^\phi \tan \hat{\phi}_i$.

The coupling law for robot (6) and heading controller (8) is given by

$$\begin{cases} u_{1i} = -L_i y_i^h \\ u_i^h = L_i^T y_i \end{cases} \quad (9)$$

where $L_i^T = (0 \ 1)$.

2) *Velocity Control:* We design a velocity tracking controller based on generalized canonical coordinate transformations [7]. The main idea is to derive the error dynamics with respect to (w.r.t.) the reference velocity and then stabilize these dynamics. In this technical note, we consider a constant velocity without loss of generality w.r.t. time varying velocities. Time-varying velocities complicate the calculations, which are omitted here due to space limitations. Let $v_i^* \in \mathbb{R}^2$ denote the desired velocity and define the error variables

$$\begin{pmatrix} \bar{q}_i \\ \bar{p}_i \end{pmatrix} = \begin{pmatrix} \hat{q}_i - \hat{S}_i(\hat{q}_i) v_i^* t \\ p_i - M_i^T v_i^* \end{pmatrix} =: \Phi_i(q_i, p_i, t) \quad (10)$$

where $v_i^* = (v_{f,i}^*, 0)^T$ with forward reference velocity $v_{f,i}^* \in \mathbb{R}$. Note that the heading $\hat{\phi}_i$ and angular momentum h_i are not affected by (10) since $h_i^* = 0$ (i.e., $\hat{\phi}_i = \phi_i$, $h_i = h_i$). The corresponding error Hamiltonian is defined as

$$\begin{aligned} \bar{H}_i^r(\bar{q}_i, \bar{p}_i) &:= -k_i^\phi \ln |\cos \bar{\phi}_i| + \frac{1}{2} \bar{p}_i^T (M_i^r)^{-1} \bar{p}_i \\ &= \underbrace{-k_i^\phi \ln |\cos \hat{\phi}_i| + \frac{1}{2} p_i^T (M_i^r)^{-1} p_i}_{\bar{H}_i^r(\hat{\phi}_i, p_i)} \\ &\quad \underbrace{-p_i^T v_i^* + \frac{1}{2} v_i^{*T} M_i^r v_i^*}_{U_i(p_i)} \end{aligned} \quad (11)$$

where $U_i(p_i)$ is a fictitious potential. The resulting control input for (6) is given by

$$u_{2i} = -\beta - D_i^t (y_i + \alpha_i) \quad (12)$$

with $\beta_i = D_i^r v_i^*$, $\alpha_i = (\partial U_i / \partial p_i) = -(v_{f,i}^*, 0)^T$, and $D_i^t \geq \epsilon I_2 > 0 \in \mathbb{R}^{2 \times 2}$.

Remark 3 (Generalized Canonical Transformation): The quadruple $(\Phi_i, U_i, \alpha_i, \beta_i)$ is a generalized canonical transformation if and only if $\Phi_i, U_i, \alpha_i, \beta_i$ satisfy the partial differential equation

$$\frac{\partial \Phi}{\partial(x, t)} \left((J - R) \frac{\partial U}{\partial x} + (K - S) \frac{\partial(H+U)}{\partial x} + g\beta \right) = 0 \quad (13)$$

for a skew-symmetric matrix $K(x, t) \in \mathbb{R}^{n \times n}$ and a symmetric matrix $S(x, t) \in \mathbb{R}^{n \times n}$ satisfying $R + S \geq 0$ [7]. Setting $K = 0$, $S = 0$, and substituting system matrices J, R from (6) it is easily verified that (13) holds for our choice of $\Phi_i, U_i, \alpha_i, \beta_i$.

3) *Formation Control*: The third controller is a distributed formation controller, where robots exchange local information. The controller assigns virtual couplings between the front ends of the wheeled robots (point C in Fig. 1) [18]. Each virtual coupling consists of a virtual spring and a virtual damper in parallel. The virtual springs determine the formation shape, while the virtual dampers shape the transient response. Virtual couplings provide a clear interpretation of the results and the control law, which is illustrated by experimental results in Section V.

Consider virtual coupling j with spring elongation $z_j \in \mathbb{R}^2$ and nominal spring length $z_j^* \in \mathbb{R}^2$, where each virtual spring consists of a separate spring along the x and y direction. Since the virtual springs are assigned between the robots in the network, spring elongation z_j equals the relative inter-robot position. The inputs $w_j \in \mathbb{R}^2$ for each virtual coupling are the velocities along x and y , while the outputs $\tau_j \in \mathbb{R}^2$ are the corresponding forces. Let $\bar{z}_j = z_j - z_j^*$, then the virtual coupling dynamics [16] are given by

$$\begin{aligned} \dot{\bar{z}}_j &= w_j \\ \tau_j &= \frac{\partial H^v}{\partial \bar{z}_j} + D_j^v w_j \end{aligned} \quad (14)$$

with virtual damping coefficients D_j^v . The Hamiltonian equals the potential energy stored in the spring and is given by

$$H_j^v = \frac{1}{2} \bar{z}_j^T K_j^v \bar{z}_j$$

with spring constants $K_j^v = \text{diag}(k_{xj}^v, k_{yj}^v)$.

The interconnection topology amongst robots and virtual couplings is described by a undirected, connected, acyclic graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$. The node-set $\mathcal{V}$ corresponds to N wheeled robots, while the edge-set $\mathcal{E}$ corresponds to M virtual couplings. Let B denote the incidence matrix associated to $\mathcal{G}(\mathcal{V}, \mathcal{E})$ then the interconnection constraint of the formation controller may be written as

$$\begin{cases} u_3 = -G(q)(B \otimes I_2)\tau \\ w = (B^T \otimes I_2)G^T(q)y \end{cases} \quad (15)$$

with $u_3 = (u_{31}, \dots, u_{3N})^T$, $y = (y_1, \dots, y_N)^T$, $w = (w_1, \dots, w_M)^T$, $\tau = (\tau_1, \dots, \tau_M)^T$. Matrix $G(q)$ assigns the virtual couplings to the front ends of robots (see [18] for more details) and is defined as $G(q) = \text{block.diag}(G_1(q_1), \dots, G_N(q_N))$, where

$$G_i(q_i) = \begin{pmatrix} \cos \phi_i & \sin \phi_i \\ -d_{AC,i} \sin \phi_i & d_{AC,i} \cos \phi_i \end{pmatrix}.$$

We now continue with the main result and analysis of the closed-loop system resulting from (6), (8), (9), (12), (14), (15).

B. Analysis

Let $u_1 = (u_{11}, \dots, u_{1N})^T$, $u_2 = (u_{21}, \dots, u_{2N})^T$. Then combining the three control inputs (9), (12), (15) from the three controllers we are now ready to state our main result.

Theorem 4: Using $u = u_1 + u_2 + u_3$, the closed-loop system converges to $\phi = \phi^*$, $p = M^r v^*$, $z = z^*$, thereby achieving the control goals (7).

Proof: To analyze the closed-loop dynamics of the network, first let $\bar{q} = (\bar{q}_1, \dots, \bar{q}_N)^T$, $\bar{p} = (\bar{p}_1, \dots, \bar{p}_N)^T$, $D^r = \text{block.diag}(D_1^r, \dots,$

$D_N^r)$, $D^t = \text{block.diag}(D_1^t, \dots, D_N^t)$, and $\bar{z} = (\bar{z}_1, \dots, \bar{z}_M)^T$, $D^v = \text{block.diag}(D_1^v, \dots, D_M^v)$, $K^v = \text{block.diag}(K_1^v, \dots, K_M^v)$.

From (6), (8), (9), (12), (14), (15) we obtain the closed-loop dynamics

$$\begin{pmatrix} \dot{\bar{q}} \\ \dot{\bar{p}} \\ \dot{\bar{z}} \end{pmatrix} = \begin{pmatrix} 0 & \bar{S}(\bar{q}) & 0 \\ -\bar{S}^T(\bar{q}) & -\bar{D}(\bar{q}) & -\bar{G}(\bar{q}) \\ 0 & \bar{G}^T(\bar{q}) & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial \bar{H}}{\partial \bar{q}} \\ \frac{\partial \bar{H}}{\partial \bar{p}} \\ \frac{\partial \bar{H}}{\partial \bar{z}} \end{pmatrix} \quad (16)$$

where $\bar{G}(\bar{q}) = G(\bar{q})(B \otimes I_2)$, $\bar{D}(\bar{q}) = (D^r + D^t + G(\bar{q})(B \otimes I_2)D^v(B^T \otimes I_2)G^T(\bar{q}))$, and $\bar{S}(\bar{q}) = \text{block.diag}(\bar{S}_1(\bar{q}_1), \dots, \bar{S}_N(\bar{q}_N))$ where $\bar{S}_i(\bar{q}_i)$ follows from (5) and (6) as:

$$\bar{S}_i(\bar{q}_i) = \begin{pmatrix} \cos(\bar{\phi}_i + \phi_i^*) & 0 \\ \sin(\bar{\phi}_i + \phi_i^*) & 0 \\ 0 & 1 \end{pmatrix}.$$

Note that in (16) the controller state $\bar{\phi}$ is contained in $\bar{q}$. The closed-loop Hamiltonian $\bar{H}$ is given by

$$\bar{H}(\bar{\phi}, \bar{p}, \bar{z}) = -\sum_{i=1}^N k_i^\phi \ln |\cos \bar{\phi}_i| + \frac{1}{2} \bar{p}^T M^{-1} \bar{p} + \frac{1}{2} \bar{z}^T K^v \bar{z}.$$

Take the closed-loop Hamiltonian $\bar{H}(\bar{q}, \bar{p}, \bar{z})$ as a candidate Lyapunov function. It is easily verified that $\bar{H}(\bar{q}, \bar{p}, \bar{z}) \geq 0$ and that the sets $\{(\bar{q}, \bar{p}, \bar{z}) \mid \bar{H}(\bar{q}, \bar{p}, \bar{z}) \leq a\}$ are compact for every $a > 0$. Furthermore, the time derivative along solutions is given as

$$\dot{\bar{H}}(\bar{q}, \bar{p}, \bar{z}) = -\frac{\partial^T \bar{H}}{\partial \bar{p}} \bar{D}(\bar{q}) \frac{\partial \bar{H}}{\partial \bar{p}}.$$

Invoking LaSalle's invariance principle gives that the system converges to the largest invariant set where $\dot{\bar{H}} = 0$. On this set $(\partial \bar{H} / \partial \bar{p}) = 0$, which implies that $\bar{p} = 0$ and therefore that $\dot{\bar{p}} = 0$. Substituting $\bar{p} = 0$, $\dot{\bar{p}} = 0$ into (16) yields

$$-\bar{S}^T(\bar{q}) \frac{\partial \bar{H}}{\partial \bar{q}} - G(\bar{q})(B \otimes I_2) \frac{\partial \bar{H}}{\partial \bar{z}} = 0. \quad (17)$$

Rearranging the terms, (17) can be rewritten as

$$\cos(\bar{\phi} + \phi^*) B K^x \bar{z}_x + \sin(\bar{\phi} + \phi^*) B K^y \bar{z}_y = 0 \quad (18)$$

$$\begin{aligned} K^\phi \tan \bar{\phi} - D_{AC} \sin(\bar{\phi} + \phi^*) B K^x \bar{z}_x \\ + D_{AC} \cos(\bar{\phi} + \phi^*) B K^y \bar{z}_y = 0 \end{aligned} \quad (19)$$

with $K^x = \text{diag}(k_1^x, \dots, k_N^x)$, $K^y = \text{diag}(k_1^y, \dots, k_N^y)$, $K^\phi = \text{diag}(k_1^\phi, \dots, k_N^\phi)$, $D_{AC} = \text{diag}(d_{AC,1}, \dots, d_{AC,N})$, and $\cos(\bar{\phi} + \phi^*)$, $\sin(\bar{\phi} + \phi^*)$, $\tan \bar{\phi}$ are diagonal matrices. Now multiply (18) by $\sin(\bar{\phi} + \phi^*)$, multiply (19) by $\cos(\bar{\phi} + \phi^*) D_{AC}^{-1}$ and sum the result to obtain

$$\cos(\bar{\phi} + \phi^*) D_{AC}^{-1} K^\phi \tan \bar{\phi} + B K^y \bar{z}_y = 0. \quad (20)$$

Since the graph is connected, it follows that $\ker B^T = \alpha \mathbb{1}_N$ with α some arbitrary constant. Multiplying (20) by $\mathbb{1}_N^T$ from the left results in

$$\mathbb{1}_N^T \cos(\bar{\phi} + \phi^*) D_{AC}^{-1} K^\phi \tan \bar{\phi} = 0. \quad (21)$$

Then, since $\cos(\bar{\phi} + \phi^*)$, D_{AC} , and K^ϕ are diagonal matrices, (21) can be rewritten element wise as

$$\sum_{i=1}^N \frac{k_i^\phi}{d_{AC,i}} \cos(\bar{\phi}_i + \phi_i^*) \tan \bar{\phi}_i = 0. \quad (22)$$

To prove that (22) implies that $\bar{\phi}_i = 0$, we first show that under Assumption 1 it follows that $\bar{\phi}_i(t) \in (-(\pi/2), (\pi/2))$ for all t . From Assumption 1 and $\phi_i^* = (\pi/2)$ it follows that $\bar{\phi}_i(0) = \phi_i(0) - \phi_i^* \in (-(\pi/2), (\pi/2))$. Then, since $\bar{H}(t)$ is bounded from above ($\bar{H}(t) \leq \bar{H}(0) \leq M < \infty$ for some $M > 0 \in \mathbb{R}$ for all t) and $\bar{H}_i \rightarrow \infty$ as $\bar{\phi}_i \rightarrow \pm(\pi/2)$, it follows that $\bar{\phi}_i(t) \in (-(\pi/2), (\pi/2))$ for all t .

Now we continue to prove that (22) implies $\bar{\phi}_i = 0$ by contradiction. Assume that $\bar{\phi}_i \neq 0$, then either $\bar{\phi}_i \in (-(\pi/2), 0)$ or $\bar{\phi}_i \in (0, (\pi/2))$. For the case that $\bar{\phi}_i \in (-(\pi/2), 0)$, it immediately follows that $\cos(\bar{\phi}_i + \phi_i^*) > 0$ while $\tan \bar{\phi}_i < 0$. On the other hand for $\bar{\phi}_i \in (0, (\pi/2))$, it follows that $\cos(\bar{\phi}_i + \phi_i^*) < 0$ and $\tan \bar{\phi}_i > 0$. Hence, if $\bar{\phi}_i \neq 0$ always $\cos(\bar{\phi}_i + \phi_i^*) \tan \bar{\phi}_i < 0$ for all $i = 1, \dots, N$. Substituting into (22) gives

$$\sum_{i=1}^N \frac{k_i^\phi}{d_{AC,i}} \cos(\bar{\phi}_i + \phi_i^*) \tan \bar{\phi}_i < 0$$

which contradicts (22) and hence one may conclude that $\bar{\phi} = 0$. Substituting $\bar{\phi} = 0$, $\phi^* = (\pi/2)$ into (18), (19) yields

$$\begin{aligned} BK^y \bar{z}_y &= 0 \\ D_{AC} BK^x \bar{z}_x &= 0. \end{aligned}$$

The graph is assumed to be acyclic, implying that $\ker B = 0$. Since D_{AC} is a positive definite diagonal matrix, it follows that $\bar{z}_x = 0$, $\bar{z}_y = 0$, thus completing the proof. ■

Remark 5 (Time-Varying Velocities): Instead of a constant forward velocity v_i^* , we can also incorporate a time-varying forward velocity $v_i^*(t)$. Then transformation (10) needs to be redefined accordingly and the state feedback term $\beta_i(p_i)$ requires an additional term $M_i^T \dot{v}_i^*(t)$, corresponding to the time derivative of the reference velocity $v_i^*(t)$; because of length constraints and burdensome notation we leave the details to the reader.

Remark 6 (Cyclic Graphs): In this technical note, the graph topology is a design freedom and considering only acyclic graphs is not restrictive. However, cyclic graphs may provide robustness against e.g., communication failures. Note that cyclic graphs do impose an additional constraint of the form $K_y \bar{z}_y \in \ker B$, $K_y \bar{z}_y \in \ker B$ (see [17]).

To illustrate the effectiveness of Theorem 4, we present experimental results in the following section.

V. EXPERIMENTAL RESULTS

We performed an experiment using the e-puck robot [12], where the experimental setup consists of 2.6×2.0 m table with an overhead camera for localization. A vision algorithm runs in parallel with MATLAB and provides the ID, global position, and heading of each e-puck. MATLAB calculates the corresponding control input and sends it to the e-pucks via a Bluetooth connection.

For the experiments, we consider a group of $N = 3$ e-pucks. For the heading control and velocity tracking control, we set respectively $\phi_i^* = (\pi/2)$ rad, $k_i^\phi = 0.1$ kg/s² and $v_{f,i}^* = 0.05$ m/s, $D_i^t = 0$ for $i = 1, 2, 3$. Finally, for the formation control, we set $z_{x,j}^* = 0.4$ m, $z_{y,i}^* = 0$ m, $k_j^x = 2$ kg/s², $k_j^y = 2$ kg/s², $d_{xj}^v = d_{yj}^v = 0$ kg/s for $j = 1, 2$. Note that the main result holds for all desired relative positions. Equal desired relative positions are chosen here for clarity of the illustrations.

The experiment is run for $t = 21.4$ s. The results are shown in Fig. 3, which shows the time evolution of the heading ϕ , forward velocity v_f , and relative position z_x . The dotted lines correspond the reference values ϕ^* , v_f^* , z_x^* respectively. The angular velocity v_ϕ and relative position z_y show a similar trend and are omitted here. Fig. 3 shows that for the experiment all variables converge to their reference value in accordance with Theorem 4.

Fig. 3. Time evolution of the heading ϕ , forward velocity v_f , and relative position z_x . The dotted line shows the reference value for each variable.

VI. CONCLUSION

In this technical note, we consider the problem of formation control of a group of wheeled robots, while tracking a constant reference velocity along a prescribed heading. Our solution is a combination of a local nonlinear heading controller, a local velocity tracking controller, and a distributed formation controller. Under a mild condition on the initial heading of the robots (which may be even more relaxed, see Remark 2), we have proven that our approach achieves the control objectives. Experimental results are presented to illustrate the effectiveness of the approach.

In the current setup, each robot has a local controller for tracking the reference velocity: i.e., all robots know the reference velocity. Future research aims at relaxing this condition, by either assigning only a subset of the robots (i.e., leaders) to know the reference velocity or to add a dynamic extension to each robot to estimate its velocity [6]. Other avenues for research include the generalization of the graph topologies considered.

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