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Disturbance rejection in formation keeping control of nonholonomic wheeled robots

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SUMMARY

This paper presents the results of formation keeping control of a group of nonholonomic wheeled robots within the port-Hamiltonian framework and in the presence of matched input disturbances. Two scenarios on the internal damping of the dynamics of the robots are considered: strictly output passive and loss less robots. For strictly output passive robots, the distributed formation keeping controllers drive the robots towards a desired formation, while internal-model-based controllers locally compensate the harmonic input disturbance for each of the robots. Moreover, the effect of constant input disturbances is studied considering internal-model-based controllers. For lossless robots, results on formation keeping control are presented. Simulation results illustrate the effectiveness of the approach. Copyright © 2016 John Wiley & Sons, Ltd.

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KEY WORDS: formation control; passivity; nonholonomic systems; port-Hamiltonian systems; disturbance rejection; internal-model-based approach

1. INTRODUCTION

Coordination of a network of autonomous robots, which allows for the execution of complex tasks, has attracted a great deal of attention in recent years. Formation control is a coordination control problem, which aims at controlling the geometrical shape (and orientation) of the network.

Among different approaches in formation keeping control presented in the literature, an innovative approach using passivity-based techniques appeared in [1], which was further investigated in the literature, for example, [2–6]. In line with the passivity-based approach, the port-Hamiltonian framework, which is an energy-based modeling framework, describes a large class of (nonlinear) systems including passive mechanical and electrical systems [7, 8]. The authors of [9] have shown that the use of energy-based models provides a clear physical interpretation of engineering problems. The recent paper [10] has introduced the concept of *port-Hamiltonian systems on graphs*, which provides tools for the analysis of (complex) networks.

One of the main challenges in the field of formation keeping is to reach and maintain the desired formation despite input disturbances. In this regard, different approaches have been pursued in the literature to deal with disturbances for some classes of dynamical systems. The role of the internal model principle and of the passivity property to deal with input disturbances in coordination of relative degree one and two incrementally passive systems has been discussed in [4].

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The problem of output agreement in networks of nonlinear dynamical systems under time-varying disturbances and dynamic coupling has been studied in [3]. Disturbance rejection (both internal-model-based approach and observer-based designs) in formation control of strictly passive systems with coarse exchanged data has been studied in [6]. A leader–follower coordination has been considered in [11] for single-integrator agents and constant disturbance rejection by proportional-integral controllers with quantized information and time-varying topologies. Moreover, synchronization of the incrementally passive nonlinear systems in the presence of the disturbances has been investigated in [12].

In the port-Hamiltonian framework, disturbance rejection has been mainly studied for a single agent rather than a network. Papers [13] and [14] have analyzed input disturbance suppression for one dynamical system using an adaptive internal-model-based controller.

Another important component in formation control problems is the dynamics of the agents. In particular, there has been a strong interest in formation control of nonholonomic wheeled robots. Each of the robots is subject to a nonholonomic constraint due to the constrained motion of its wheels. Stabilization of the position and heading of this type of robot is challenging, because it does not satisfy the Brockett's necessary condition for stabilization using smooth feedback ([15]). In this regard, both discontinuous and time-varying control laws have been studied to stabilize a single robot (e.g., [16] and [17]). The multiple robot case, which is naturally more challenging, has also been studied (e.g., [18, 19], and [20]). To cope with the restriction of smooth controllers to stabilize both of the position and orientation of the robots, the authors of [18] considered formation keeping control of the front end ('hand position') of the robots. The front end of a wheeled robot lies at a distance $L \neq 0$ along the line that is normal to the wheel axle. This assumption simplifies the problem; however, it is of high practical interests due to numerous applications, that is, controlling the gripper position located on the front end of a wheeled robot. A similar approach in the port-Hamiltonian framework and using passivity-based design techniques has been considered in [21].

Main contribution. This work analyzes the formation control of a network of nonholonomic wheeled robots to achieve a desired formation despite of matched input disturbances. The results are stated within the port-Hamiltonian framework and build upon previous works [6] and [21]. We study the formation control of nonholonomic wheeled robots in the presence of harmonic and constant input disturbances using design tools of passivity-based [1] and internal-model-based [22] approaches. We also consider some relaxation on the assumption of strict passivity condition of the robots' dynamics. To the authors best knowledge, the combination of problems that we considered, that is, disturbance rejection of a network of nonholonomic agents within the port-Hamiltonian framework, has not been considered before. Comparing with the literature on disturbance rejection using internal-model-based approach within the port-Hamiltonian framework (e.g., [13] and [14]), our contribution is to consider a network of agents rather than a single robot as well as considering robots with nonholonomic constraints. A partial and preliminary version of this paper considering only harmonic disturbances and networks with at least one strictly output passive robot has been presented in [23].

The outline of the paper is as follows. Section 1.1 recalls some preliminaries from graph theory and port-Hamiltonian systems. Section 2 continues with the formal problem statement. Section 3 presents the controller design to reach the desired formation in the presence of matched input disturbances. Section 4 presents the stability and convergence analysis of the closed-loop system. Simulation results illustrating the effectiveness of the approach are given in Section 5. Section 6 concludes the paper.

Notation. Given a matrix C of real numbers, we denote by $\mathcal{R}(C)$ and $\mathcal{N}(C)$ the range and the null space, respectively. The symbols $\mathbf{1}$, $\mathbf{0}$ denote vectors or matrices of all 1 and 0 respectively, where the dimension is sometimes explicitly given (e.g., $\mathbf{1}_N$ denotes the N -dimensional column vector of all ones). I_p is the $p \times p$ identity matrix. S^1 denotes the unit circle. Given two matrices A and B , the symbol $A \otimes B$ denotes the Kronecker product. $A = \text{block.diag} \{A_1, \dots, A_N\}$ denotes a diagonal matrix where A_i is its i -th diagonal element. Finally, $\frac{\partial H}{\partial x}(x)$ denotes the column vector of partial derivatives of scalar function H with respect to $x \in \mathbb{R}^n$.

1.1. Preliminaries

We consider a network of N nonholonomic robots evolving in $\mathbb{R}^2$. The way in which the robots exchange information is modeled by a connected undirected graph $G(\mathcal{V}, \mathcal{E})$, where the node-set $\mathcal{V}$ corresponds to N robots and the edge-set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ corresponds to M edges. In order to use the tools from graph theory, we assign an orientation to each edge. The orientation can be imposed in an arbitrary manner, and it does not have any effect on the final results. The incidence matrix B associated to $G(\mathcal{V}, \mathcal{E})$ describes which nodes are coupled by an edge and is defined as

$$b_{ik} = \begin{cases} +1 & \text{if node } i \text{ is the head vertex of edge } k, \\ -1 & \text{if node } i \text{ is the tail vertex of edge } k, \\ 0 & \text{otherwise.} \end{cases}$$

Moreover, we define the relative position z_k between agent i and j as follows

$$z_k = \begin{cases} r_i - r_j & \text{if node } i \text{ is the head vertex of edge } k, \\ r_j - r_i & \text{if node } j \text{ is the tail vertex of edge } k, \end{cases}$$

where $r_i \in \mathbb{R}^2$ is the position of agent i expressed in an inertial reference frame. By definition of B , we can represent the relative position variable z , with $z = (z_1^T \dots z_M^T)^T$, $z \in \mathbb{R}^{2M}$, as

$$z = (B^T \otimes I_2) r,$$

which implies that $z \in \mathcal{R}(B^T \otimes I_2)$.

Now, we provide preliminaries on passivity and the port-Hamiltonian framework. Consider the system

$$\dot{\xi} = f(\xi) + g(\xi)u, \quad (1)$$

with state $\xi \in \mathbb{R}^n$, input $u \in \mathbb{R}^m$, and output $y \in \mathbb{R}^m$. System (1) is assumed to be (strictly) output passive from the input u to the output y if there is a continuously differentiable storage function $S : \mathbb{R}^n \rightarrow \mathbb{R}_+$, which satisfies

$$\frac{\partial S}{\partial \xi} [f(\xi) + g(\xi)u] \leq -\alpha y^T y + y^T u, \quad (2)$$

where $\alpha > 0$. The dynamics of each robot is modeled within the port-Hamiltonian framework. Define the state $x \in \mathbb{R}^n$, input $u \in \mathbb{R}^m$, and output $y \in \mathbb{R}^m$. The product of the input and output $u^T(t)y(t)$ is the power supplied to the system. The general form of a port-Hamiltonian system in the case of linear damping and in the absence of constraints is

$$\begin{aligned} \dot{x} &= [J(x) - R(x)] \frac{\partial H}{\partial x}(x) + g(x)u, \\ y &= g^T(x) \frac{\partial H}{\partial x}, \end{aligned} \quad (3)$$

where $J(x) = -J^T(x)$ and $R(x) = R^T(x) \geq 0$. The Hamiltonian $H(x)$ equals the total energy stored in the system, and its time derivative is given by $\dot{H}(t) \leq u^T(t)y(t)$. Hence, the increase in the stored energy is always less than or equal the power supplied through the power-port (u, y) . Therefore, (if H is bounded from below) (3) is a strictly output passive if $\ker R(x) \subset \ker g^T(x)$ (in particular if $R(x) > 0$). The system is lossless if $R(x) = 0$. See [7] and [8] for a concise overview of the port-Hamiltonian framework.

2. PROBLEM STATEMENT

2.1. Wheeled robot dynamics

Consider a wheeled robot i with heading angle ϕ_i , and let $(x_{A,i}, y_{A,i})$ and $(x_{C,i}, y_{C,i})$ denote the center of the wheel axle and a point at the front end of the robot, respectively (Figure 1). The center

Figure 1. Wheeled robot i .

of mass is assumed to lie at the center of the wheel axle. Furthermore, let $d_{AC,i}$ denote the distance between these points. Each robot is modeled as a rigid body, subject to a nonholonomic constraint. The model presented here is based on a preliminary model in [21], which has been extended in [24]. In this paper, we assume that the center of mass is located at the center of the wheel axle. Let $q_i \in \mathbb{R} \times \mathbb{R} \times S^1$, where S^1 is the unit circle, denote the position, and let $p_i^{rb} \in \mathbb{R}^3$ denote the corresponding momentum of robot i , where $q_i = (x_{A,i}, y_{A,i}, \phi_i)$ and $p_i^{rb} = (p_{x,i}^{rb}, p_{y,i}^{rb}, h_i^{rb})$ (Figure 1). The rigid body momentum is related to the position by $p_i^{rb} = M_i^{rb} \dot{q}_i$, with constant mass-inertia matrix $M_i^{rb} = \text{diag} m_i, m_i, I_{cm,i}$, where m_i denotes the robot's mass and $I_{cm,i}$ denotes the moment of inertia around the center of mass $(x_{A,i}, y_{A,i})$. The rigid body dynamics are given by [8]

$$\begin{pmatrix} \dot{q}_i \\ \dot{p}_i^{rb} \end{pmatrix} = \begin{pmatrix} 0 & I_3 \\ -I_3 & -D_i^{rb} \end{pmatrix} \begin{pmatrix} \frac{\partial H_i^{rb}}{\partial q_i} \\ \frac{\partial H_i^{rb}}{\partial p_i^{rb}} \end{pmatrix} + \begin{pmatrix} 0 \\ S_i(q_i) \end{pmatrix} \bar{u}_i \quad (4)$$

$$\bar{y}_i = S_i^T(q_i) \frac{\partial H_i^{rb}}{\partial p_i^{rb}},$$

with input $\bar{u}_i = (u_{f,i}, u_{\phi,i})^T \in \mathbb{R}^2$ and output $\bar{y}_i = (y_{f,i}, y_{\phi,i})^T \in \mathbb{R}^2$ corresponding to respectively the forward force and the torque acting at point A and the corresponding velocities. The dissipation matrix D_i^{rb} and input matrix $G_i(q_i)$ are given by

$$D_i^{rb} = \begin{pmatrix} d_{f,i} & 0 & 0 \\ 0 & d_{f,i} & 0 \\ 0 & 0 & d_{\phi,i} \end{pmatrix}, \quad S_i(q_i) = \begin{pmatrix} \cos \phi_i & 0 \\ \sin \phi_i & 0 \\ 0 & 1 \end{pmatrix}$$

with friction coefficients $d_{f,i}$ and $d_{\phi,i}$. The Hamiltonian H_i^{rb} is the kinetic energy stored in the rigid body given by $H_i^{rb}(p_i^{rb}) = \frac{1}{2} (p_i^{rb})^T (M_i^{rb})^{-1} p_i^{rb}$. The rigid body dynamics (4) are subject to the *nonholonomic* (kinematic) constraints

$$\sin \phi_i \dot{x}_{A,i} - \cos \phi_i \dot{y}_{A,i} = 0,$$

which may be rewritten in terms of the rigid body dynamics (4) as the algebraic constraints

$$\underbrace{(\sin \phi_i \quad -\cos \phi_i \quad 0)}_{A_i^T(q_i)} \frac{\partial H_i^{rb}}{\partial p_i^{rb}} (p_i^{rb}) = 0. \quad (5)$$

Figure 2. Illustration of force $\bar{u}_i$ acting on the center of wheeled robot i (left) and the front head of the robot u_i (right).

The constraint (5) may be eliminated to obtain the *dynamics on the constrained state space* [8]. To do so, define the coordinate transformation $(q_i, p_i^{rb}) \mapsto (q_i, p_i, p_{s,i})$, where $p_i = (p_{f,i}, h_i) := S_i^T(q_i)p_i^{rb}$ and $p_{s,i} := A_i^T(q_i)p_i^{rb}$. Here, $p_{f,i}$, $p_{s,i}$ refer to, respectively, the *forward* and *sideward* momenta of robot i . In the new coordinates, the $p_{s,i}$ dynamics may be eliminated to obtain the wheeled robot dynamics

$$\begin{pmatrix} \dot{q}_i \\ \dot{p}_i \end{pmatrix} = \begin{pmatrix} 0 & S_i(q_i) \\ -S_i^T(q_i) & -D_i^r \end{pmatrix} \begin{pmatrix} \frac{\partial H_i^r}{\partial q_i} \\ \frac{\partial H_i^r}{\partial p_i} \end{pmatrix} + \begin{pmatrix} 0 \\ I_2 \end{pmatrix} \bar{u}_i, \quad (6)$$

$$\bar{y}_i = \frac{\partial H_i^r}{\partial p_i},$$

with Hamiltonian $H_i^r = \frac{1}{2} p_i^T (M_i^r)^{-1} p_i$, where $M_i^r = \text{diag}(m_i, I_{cm,i})$. The new dissipation matrix D_i^r is defined as $D_i^r = S_i^T(q_i) D_i^{rb} S_i(q_i) = \text{diag}(d_{f,i}, d_{\phi,i})$. For control purposes, we transform the input $\bar{u}_i = (u_{f,i}, u_{\phi,i})^T$ to a new input $u_i = (u_{x,i}, u_{y,i})^T$, where $u_{x,i}$ ($u_{y,i}$) denotes a force along the x -direction (y -direction) acting on the point $(x_{C,i}, y_{C,i})$ (Figure 2). The output $\bar{y}_i$ is transformed accordingly into the new output $y_i = (y_{x,i}, y_{y,i})$. The transformation $(\bar{u}_i, \bar{y}_i) \mapsto (u_i, y_i)$ is defined as $\bar{u}_i = G_i(q_i)u_i$, $y_i = G_i^T(q_i)\bar{y}_i$ [24], where

$$G_i(q_i) = \begin{pmatrix} \cos \phi_i & \sin \phi_i \\ -d_{AC,i} \sin \phi_i & d_{AC,i} \cos \phi_i \end{pmatrix}. \quad (7)$$

The dynamics of robot i with new input u_i and new output y_i follows by substituting $\bar{u}_i = G_i(q_i)u_i$, $y_i = G_i^T(q_i)\bar{y}_i$ into (6).

Furthermore, because each robot is affected by a matched input disturbance d_i , the robot dynamics are given by

$$\begin{pmatrix} \dot{q}_i \\ \dot{p}_i \end{pmatrix} = \begin{pmatrix} 0 & S_i(q_i) \\ -S_i^T(q_i) & -D_i^r \end{pmatrix} \begin{pmatrix} \frac{\partial H_i^r}{\partial q_i} \\ \frac{\partial H_i^r}{\partial p_i} \end{pmatrix} + \begin{pmatrix} 0 \\ G_i(q_i) \end{pmatrix} (u_i + d_i) \quad (8)$$

$$y_i = G_i^T(q_i) \frac{\partial H_i^r}{\partial p_i}.$$

Writing the dynamics of a network of N robots in compact form gives

$$\begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} 0 & S(q) \\ -S^T(q) & -D^r \end{pmatrix} \begin{pmatrix} \frac{\partial H^r}{\partial q} \\ \frac{\partial H^r}{\partial p} \end{pmatrix} + \begin{pmatrix} 0 \\ G(q) \end{pmatrix} (u + d) \quad (9)$$

$$y = G^T(q) \frac{\partial H^r}{\partial p},$$

where

$$\begin{aligned} q &= (q_1^T, \dots, q_N^T)^T, p = (p_1^T, \dots, p_N^T)^T, \\ u &= (u_1^T, \dots, u_N^T)^T, y = (y_1^T, \dots, y_N^T)^T, \\ S(q) &= \text{block.diag} \{S_1(q_1), \dots, S_N(q_N)\}, \\ G(q) &= \text{block.diag} \{G_1(q_1), \dots, G_N(q_N)\}, \\ D^r &= \text{block.diag} \{D_1^r, \dots, D_N^r\}, \end{aligned}$$

and $H^r(p) = \sum_{i=1}^N H_i^r(p_i) = \frac{1}{2} p^T (M^r)^{-1} p$, with $M^r = \text{block.diag} \{M_1^r, \dots, M_N^r\}$.

Remark 1

The dynamics of robot i in (8) is affected by the disturbance d_i , which occurs matched with input u_i . The effect of the disturbance is not modeled in (4) and (6), because the latter are meant to introduce the dynamics of the robot. In fact, considering the addition of disturbance d_i in (8) implies a disturbance $\tilde{d}_i$ (matched with $\tilde{u}_i$) in (6) such that $\tilde{d}_i = G_i(q_i)d_i$.

2.2. Control goals

In this section, we formally define the control goals for the controller design presented in Section 3. We consider the control law u_i as the sum of two control laws u_i^f and u_i^d . Hence, we can rewrite (8) as follows:

$$\begin{aligned} \begin{pmatrix} \dot{q}_i \\ \dot{p}_i \end{pmatrix} &= \begin{pmatrix} 0 & S_i(q_i) \\ -S_i^T(q_i) & -D_i^r \end{pmatrix} \begin{pmatrix} \frac{\partial H_i^r}{\partial q_i} \\ \frac{\partial H_i^r}{\partial p_i} \end{pmatrix} + \begin{pmatrix} 0 \\ G_i(q_i) \end{pmatrix} (u_i^f + u_i^d + d_i) \\ y_i &= G_i^T(q) \frac{\partial H_i^r}{\partial p_i}. \end{aligned} \quad (10)$$

The control law u_i^f achieves the desired formation, based on the results by [24]. The control law u_i^d is designed based on an internal-model-based approach (e.g., [2, 4, 6]) to reject the matched input disturbance d_i in (10).

As mentioned in Section 1.1, the way in which the robots exchange information is modeled with a connected undirected graph. The formation control goal is to make the relative position z_k between robots i and j converge to a desired relative position z_k^* if there exists an edge k in between nodes i and j in the graph. Note that z_k is the relative position between each two robots i and j with respect to their front end position (point C in Figure 1). Hence, assuming that agent i is the head vertex of edge k , we can write

$$\begin{aligned} z_{x,k} &= x_{C,i} - x_{C,j}, \\ z_{y,k} &= y_{C,i} - y_{C,j}. \end{aligned} \quad (11)$$

Defining the error variables $\tilde{z}_k = z_k - z_k^*$ and $\tilde{d}_i = d_i + u_i^d$, we formally state the control goals as

$$\lim_{t \rightarrow \infty} \tilde{d}_i = 0 \text{ for } i = 1, \dots, N, \quad (12)$$

$$\lim_{t \rightarrow \infty} \tilde{z}_k = 0 \text{ for } k = 1, \dots, M. \quad (13)$$

In the next section, we design the control laws u_i^f and u_i^d in (10).

3. CONTROL DESIGN

In Section 3.1, we design the control law u_i^f (in (10)), based on the passivity-based approach [1], to achieve the desired formation. Subsequently, in Section 3.2, the control law u_i^d is designed to reject the matched input disturbances.

3.1. Formation control

The interpretation of the formation control law is as follows. We consider N (strictly output) passive wheeled robots of the form (9), which correspond to the nodes of the graph. The M edges of the graph correspond to virtual couplings, which consist of virtual springs and dampers in parallel. In other words, those robots that are exchanging information are interconnected by a virtual spring and a damper. If all of the springs of the network reach their minimum potential energy, the desired formation is reached.

The state of virtual coupling k is the relative error position $\tilde{z}_{x,k} \in \mathbb{R}$ along x -direction and relative error position $\tilde{z}_{y,k} \in \mathbb{R}$ along y -direction. The velocity inputs are denoted by $v_{x,k}, v_{y,k} \in \mathbb{R}$, while the force outputs are denoted by $\lambda_{x,k}, \lambda_{y,k} \in \mathbb{R}$. Consider $\tilde{z}_k = (\tilde{z}_{x,k}, \tilde{z}_{y,k})$, $v_k = (v_{x,k}, v_{y,k})$, and $\lambda_k = (\lambda_{x,k}, \lambda_{y,k})$, then the dynamics for virtual coupling k are given by

$$\begin{aligned}\dot{\tilde{z}}_k &= v_k \\ \lambda_k &= \frac{\partial H_k^{\tilde{z}}}{\partial \tilde{z}_k} + D_k^v v_k,\end{aligned}\quad (14)$$

with dissipation matrix $D_k^v = \text{diag}(d_{x,k}, d_{y,k}) > 0$, where $d_{x,k}, d_{y,k}$ denote the damping coefficients along the x and y directions, respectively. The Hamiltonian $H_k^{\tilde{z}}$ equals the potential energy stored in the virtual spring, which is given by $H_k^{\tilde{z}} = \frac{1}{2} \tilde{z}_k^T K_k \tilde{z}_k$, with spring constant matrix $K_k = \text{diag}(\kappa_{x,k}, \kappa_{y,k}) > 0$.

Writing the dynamics of M virtual spring-damper systems in compact form gives

$$\begin{aligned}\dot{\tilde{z}} &= v \\ \lambda &= \frac{\partial H^{\tilde{z}}}{\partial \tilde{z}} + D^v v,\end{aligned}\quad (15)$$

with Hamiltonian $H^{\tilde{z}} = \sum_{k=1}^M H_k^{\tilde{z}} = \frac{1}{2} \tilde{z}^T K \tilde{z}$, where

$$\begin{aligned}\tilde{z} &= (\tilde{z}_1^T, \dots, \tilde{z}_M^T)^T, \\ v &= (v_1^T, \dots, v_M^T)^T, \\ \lambda &= (\lambda_1^T, \dots, \lambda_M^T)^T, \\ D^v &= \text{block.diag} \{D_1^v, \dots, D_M^v\}, \\ K &= \text{block.diag} \{K_1, \dots, K_M\}.\end{aligned}$$

Now, let B denote the incidence matrix associated to the undirected connected graph $G(\mathcal{V}, \mathcal{E})$, then the distributed formation keeping control law $u^f = (u_1^f, \dots, u_N^f)$ is ([24])

$$u^f = -(B \otimes I_2) \lambda. \quad (16)$$

3.2. Matched input disturbance rejection: an internal-model-based approach

This section considers that each robot i modeled by (10) is affected by a class of disturbances generated by an autonomous system that we refer to as the exosystem. Given two matrices $\Phi_i^d \in \mathbb{R}^{4 \times 4}$ and $\Gamma_i^d \in \mathbb{R}^{2 \times 4}$, whose properties will be made precise later on, the exosystem of robot i obeys the following equations

$$\begin{aligned}\dot{w}_i^d &= \Phi_i^d w_i^d, \\ d_i &= \Gamma_i^d w_i^d, \quad i = 1, 2, \dots, N,\end{aligned}\quad (17)$$

with exosystem state $w_i^d \in \mathbb{R}^4$ and output $d_i \in \mathbb{R}^2$. In this paper, we consider the case where both the input and the disturbance are decoupled in x and y directions. The exosystem in (17) is assumed to generate $d_i = (d_{x,i}, d_{y,i})^T$ where $d_{x,i}$ and $d_{y,i}$ are either constant or harmonic signals. Let us study constant and harmonic disturbances separately. In fact if both types of disturbances can be

rejected, their combination can also be rejected. The structure in (17) is in accordance with (14) where springs and dampers are considered in x and y directions separately.

Inspired by the theory of output regulation (e.g., [22]), an internal-model-based controller can be adopted to counteract the effect of the disturbance. Let θ_i , $\check{u}_i$, and $\check{d}_i$ denote the state, input, and output of the internal-model-based controller, respectively. Then the internal model dynamics is given by [6]

$$\begin{aligned}\dot{\theta}_i &= \Phi_i^d \theta_i + G_i^d \check{u}_i \\ \check{d}_i &= \Gamma_i^d \theta_i, \quad i = 1, 2, \dots, N,\end{aligned}\quad (18)$$

with internal model controller state $\theta_i \in \mathbb{R}^4$, input $\check{u}_i \in \mathbb{R}^2$, output $\check{d}_i \in \mathbb{R}^2$, and input matrix $G_i^d \in \mathbb{R}^{4 \times 2}$. When $\check{u}_i = 0$ and the system is appropriately initialized, the latter system is able to generate any w_i^d solution to (17).

Define the error variables $\tilde{d}_i = \check{d}_i - d_i$ and $\tilde{\theta}_i = \theta_i - w_i^d$. Set $G_i^d = \Gamma_i^{dT}$ and assume that Φ_i^d is a skew-symmetric matrix, then

$$\begin{aligned}\dot{\tilde{\theta}}_i &= \Phi_i^d \tilde{\theta}_i + \Gamma_i^{dT} \check{u}_i \\ \tilde{d}_i &= \Gamma_i^d \tilde{\theta}_i.\end{aligned}\quad (19)$$

In compact form we write

$$\begin{aligned}\dot{\tilde{\theta}} &= \Phi^d \tilde{\theta} + \Gamma^{dT} \check{u} \\ \tilde{d} &= \Gamma^d \tilde{\theta},\end{aligned}\quad (20)$$

with

$$\begin{aligned}\tilde{\theta} &= (\tilde{\theta}_1^T, \dots, \tilde{\theta}_N^T)^T, \\ \check{u} &= (\check{u}_1^T, \dots, \check{u}_N^T)^T, \\ \tilde{d} &= (\tilde{d}_1^T, \dots, \tilde{d}_N^T)^T, \\ \Phi^d &= \text{block.diag} \{ \Phi_1^d, \dots, \Phi_N^d \}, \\ \Gamma^d &= \text{block.diag} \{ \Gamma_1^d, \dots, \Gamma_N^d \}.\end{aligned}$$

Note that the exosystem (17) itself is not a passive system, because it has no input. However, interconnecting exosystem (17) with the internal model controller (18), the resulting system (19) is lossless with respect to the port variables $\check{u}_i, \tilde{d}_i$ (Figure 3). Furthermore, (20) can easily be

Figure 3. The exosystem (17) together with the disturbance rejecting controller (18) form the lossless subsystem (19) with port variables $\check{u}_i$ and $\tilde{d}_i$.

represented in the port-Hamiltonian framework by defining the Hamiltonian $H^d(\tilde{\theta}) = \frac{1}{2}\tilde{\theta}^T\tilde{\theta}$, such that (20) can be rewritten as

$$\begin{aligned}\dot{\tilde{\theta}} &= \Phi^d \frac{\partial H^d}{\partial \tilde{\theta}} + \Gamma^{dT} \check{u} \\ \tilde{d} &= \Gamma^d \frac{\partial H^d}{\partial \tilde{\theta}}.\end{aligned}\quad (21)$$

Note that in (21), the port-Hamiltonian structure is preserved, because Φ^d is skew-symmetric.

3.3. Closed-loop dynamics

In the previous sections, we designed two passive control systems: one for reaching the desired formation and one for counteracting the matched input disturbances. By interconnecting these systems appropriately, the closed-loop system preserves the passivity properties and the port-Hamiltonian structure. To obtain the closed-loop dynamics, which is the interconnection of the dynamics of the robots with the virtual springs (formation controller) and the disturbance rejection controller, we define the following coupling rules:

$$\begin{cases} u = -(B \otimes I_2)\lambda - \check{d} \\ v = (B^T \otimes I_2)y \\ \check{u} = y, \end{cases}\quad (22)$$

where $-(B \otimes I_2)\lambda = u^f$ (16), $-\check{d} = u^d = -\Gamma^d \theta$ (18). Note that (u, y) , (v, λ) , and $(\check{u}, \check{d})$ are the port-variables for the robots (9), virtual springs (15) and internal-model-based controller (20), respectively. The overall closed-loop dynamics follows from (9), (15), (21), and (22) and are given by

$$\begin{pmatrix} \dot{q} \\ \dot{p} \\ \dot{\tilde{z}} \\ \dot{\tilde{\theta}} \end{pmatrix} = \begin{pmatrix} 0 & S(q) & 0 & 0 \\ -S^T(q) & -\check{D}(q) & -\check{G}(q) & -G(q)\Gamma^d \\ 0 & \check{G}^T(q) & 0 & 0 \\ 0 & \Gamma^{dT}G^T(q) & 0 & \Phi^d \end{pmatrix} \begin{pmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \\ \frac{\partial H}{\partial \tilde{z}} \\ \frac{\partial H}{\partial \tilde{\theta}} \end{pmatrix}, \quad (23)$$

with $\check{G}(q) = G(q)(B \otimes I_2)$ and $\check{D}(q) = D^r + \check{G}(q)D^v\check{G}^T(q)$. Finally, the closed-loop Hamiltonian is given by $H(p, \tilde{z}, \tilde{\theta}) = H^r(p) + H^{\tilde{z}}(\tilde{z}) + H^d(\tilde{\theta})$.

4. RESULTS AND ANALYSIS

In this section, we present the results of the stability and convergence analysis of the closed loop system (23). Based on [24], a network of strictly output passive nonholonomic wheeled robots in the absence of disturbances converges to the desired formation where agents' momenta are zero. Here, we first consider a network of passive agents such that at least one of the agents is strictly output passive and study the closed-loop system considering harmonic and constant disturbances. Next, we study a network of lossless agents. To study the effects of disturbances on a lossless network, we first consider the network without disturbances. Then, we continue with the addition of input disturbances.

4.1. Network of strictly output passive agents

This section studies a network of agents where at least one of the them is strictly output passive.

Proposition 1

Consider (23) and assume that $D^v > 0$, and there is at least one robot i satisfying $D_i^r > 0$. The closed-loop system (23) asymptotically converges to the largest invariant set where $p = \mathbf{0}$.

Proof

Take the closed-loop Hamiltonian H as the candidate Lyapunov function. Calculating the derivative of H , we obtain

$$\begin{aligned}\dot{H} = & -\frac{\partial^T H}{\partial p} \check{D}(q) \frac{\partial H}{\partial p} - \frac{\partial^T H}{\partial p} \check{G}(q) \frac{\partial H}{\partial \tilde{z}} \\ & + \frac{\partial^T H}{\partial \tilde{z}} \check{G}^T(q) \frac{\partial H}{\partial p} + \frac{\partial^T H}{\partial \tilde{\theta}} \Phi^d \frac{\partial H}{\partial \tilde{\theta}} \\ & - \frac{\partial^T H}{\partial p} G(q) \Gamma^d \frac{\partial H}{\partial \tilde{\theta}} + \frac{\partial^T H}{\partial \tilde{\theta}} \Gamma^{dT} G^T(q) \frac{\partial H}{\partial p}.\end{aligned}\quad (24)$$

Due to skew-symmetry of Φ^d , we immediately obtain

$$\dot{H}(x) = -\frac{\partial^T H}{\partial p} \left(D^r + \check{G}(q) D^v \check{G}^T(q) \right) \frac{\partial H}{\partial p}. \quad (25)$$

Because $D^v > 0$ and $D_i^r \geq 0$, we have that $\dot{H}(x) \leq 0$. Therefore, the system is stable and we conclude boundedness of $p, \tilde{z}, \tilde{\theta}$. Also, $\phi_i \in S^1$ is bounded. Hence, we can apply LaSalle's invariance principle. Because there is at least one strictly output passive robot i such that $D_i^r > 0$, by LaSalle's invariance principle, the system converges to the largest invariant set where the following holds:

$$\frac{\partial H}{\partial p_i} = (M_i^r)^{-1} p_i = 0, \quad (26)$$

$$\check{G}^T(q) \frac{\partial H}{\partial p} = (B^T \otimes I_2) G^T(q) (M^r)^{-1} p = \mathbf{0}. \quad (27)$$

Multiplying both sides of (27) by $(B \otimes I_2)$, we have $(L \otimes I_2) G^T(q) (M^r)^{-1} p = \mathbf{0}$, where L is the graph Laplacian matrix. From the properties of an undirected and connected graph, we have that $\mathcal{N}(L) = \alpha \mathbf{1}_N$. Hence, $G^T(q) (M^r)^{-1} p = \alpha (\mathbf{1}_N \otimes I_2)$, and therefore, $p = M^r G^{-T}(q) \alpha (\mathbf{1}_N \otimes I_2)$ (note that $G^T(q)$ depends on q , but has no singularity for any q). Moreover, from (26), we have $p_i = 0$ (because m_i and $I_{CM,i}$ are non-zero and positive). This implies that $\alpha = 0$ and therefore $p = \mathbf{0}$, thereby completing the proof. $\square$

Remark 2

If all the agents of the network are strictly output passive, the results of Proposition 1 hold independently of the coupling damping matrix, that is $D^r > 0$ and $D^v \geq 0$.

Substituting $p = \mathbf{0}$ into (23), Proposition 1 guarantees, in addition to stability, convergence of the closed-loop system to the largest invariant set for the following dynamics

$$\begin{aligned}\dot{q} &= \mathbf{0}, \\ \mathbf{0} &= -(B \otimes I_2) K \tilde{z} - \Gamma^d \tilde{\theta}, \\ \dot{\tilde{z}} &= \mathbf{0}, \\ \dot{\tilde{\theta}} &= \Phi^d \tilde{\theta}.\end{aligned}\quad (28)$$

However, the above guarantees neither reaching the desired formation (i.e., $\tilde{z} = \mathbf{0}$) nor rejecting the input disturbances (i.e., $\tilde{\theta} = \mathbf{0}$). To verify more, we now consider two special types of the disturbances generated by the exosystem (17), namely, harmonic and constant disturbances.

Corollary 1 (Harmonic disturbances)

Assume that the exosystems' matrices (Γ_i^d, Φ_i^d) are of the form

$$\Phi_i^d = \text{block.diag} \left\{ \begin{pmatrix} 0 & \omega_{i1} \\ -\omega_{i1} & 0 \end{pmatrix}, \begin{pmatrix} 0 & \omega_{i2} \\ -\omega_{i2} & 0 \end{pmatrix} \right\}, \quad (29)$$

with $\omega_{i\ell} \neq 0$ for $\ell = 1, 2$, and $\Gamma_i^d = \text{block.diag} \{\Gamma_{i1}^d, \Gamma_{i2}^d\}$, with $\Gamma_{i\ell}^d \neq \mathbf{0}$, for all $\ell = 1, 2$, and the pair $(\Gamma_{i\ell}^d, \Phi_{i\ell}^d)$ is observable. Then the closed loop system (23) converges to the set where $p = \mathbf{0}$, $\tilde{\theta} = \mathbf{0}$, and $\tilde{z} = \mathbf{0}$.

Proof

Corollary 1 is a special case of Proposition 1. Therefore, we can start considering the system (28). First, calculate the time derivative of the second equality. Because $\dot{\tilde{z}} = \mathbf{0}$, we obtain $\Gamma^d \dot{\tilde{\theta}} = \mathbf{0}$. Now, replacing $\dot{\tilde{\theta}}$ from the last equality in (28), we have $\Gamma^d \Phi^d \tilde{\theta} = \mathbf{0}$. Calculate the derivative of the latter, we obtain $\Gamma^d \Phi^d \dot{\tilde{\theta}} = \mathbf{0}$. Because Φ^d is a skew-symmetric matrix with the structure given in (29), $\Phi_{i\ell}^d \Phi_{i\ell}^d$ is equal to γI_2 , where γ is a constant. Hence, $\Gamma^d \Phi^d \dot{\tilde{\theta}} = \mathbf{0}$ implies that $\Gamma^d \tilde{\theta} = \mathbf{0}$. Hence,

$$\begin{aligned}\dot{\tilde{\theta}} &= \Phi^d \tilde{\theta}, \\ \Gamma^d \tilde{\theta} &= \mathbf{0}.\end{aligned}$$

By the condition that the pair $(\Gamma_{i\ell}^d, \Phi_{i\ell}^d)$ is observable, we conclude that $\tilde{\theta} = \mathbf{0}$. Now, consider the second equality in (28). Substituting $\tilde{\theta} = \mathbf{0}$ results in $(B \otimes I_2)K\tilde{z} = \mathbf{0}$. Because $\tilde{z} \in \mathcal{R}(B^T \otimes I_2)$ and K is a diagonal matrix with positive constants on the diagonal, we conclude that $\tilde{z} = \mathbf{0}$, which ends the proof. $\square$

Corollary 1 shows that the internal-model-based controller (20) can counteract harmonic disturbances. The results of Corollary 1 still hold if one of the agents of the network is subject to constant disturbances.

Corollary 2 (Constant disturbances)

If one of the agents of the network (23) is subject to constant disturbances and the other $N - 1$ agents are subject to harmonic disturbances, the network with dynamics in (23) converges to $p = \mathbf{0}$, $\tilde{\theta} = \mathbf{0}$, and $\tilde{z} = \mathbf{0}$. However, if all of the agents are subject to constant disturbances, the network converges to the largest invariant set where $p = \mathbf{0}$, $\dot{\tilde{z}} = \mathbf{0}$ and $(\mathbf{1}^T \otimes I_2)\Gamma^d \tilde{\theta} = \mathbf{0}$.

Proof

First, assume that there is an index $j \in \{1, \dots, N\}$ such that $\omega_{j\ell} = 0$ in (29). Because for $N - 1$ agents, the conditions of Corollary 1 hold, following the proof of Corollary 1, we can show that $\Gamma_i^d \tilde{\theta}_i = \mathbf{0}$ holds for each of the $N - 1$ agents. Also, multiplying the second equality in (28) by $(\mathbf{1}^T \otimes I_2)$ we conclude that

$$(\mathbf{1}^T \otimes I_2) \Gamma^d \tilde{\theta} = \mathbf{0}.$$

Because all components of $\Gamma^d \tilde{\theta}$ except $\Gamma_j^d \tilde{\theta}_j$ are equal to zero, we conclude that $\Gamma_j^d \tilde{\theta}_j$ should also be zero implying that the results of Corollary 1 hold if only one of the agents is subject to non-zero constant disturbances.

Now, consider the case where all of the agents are subject to constant disturbances where $\Phi^d = \mathbf{0}$. Plugging $\Phi^d = \mathbf{0}$ in (28) we obtain

$$\begin{aligned}\dot{q} &= \mathbf{0}, \\ \mathbf{0} &= -(B \otimes I_2)\tilde{z} - \Gamma^d \tilde{\theta}, \\ \dot{\tilde{z}} &= \mathbf{0}, \\ \dot{\tilde{\theta}} &= \mathbf{0}.\end{aligned}\tag{30}$$

In the aforementioned case, we can only conclude that the error position vector $\tilde{z}$ and the error disturbance vector $\tilde{\theta}$ are constant. Also, the sum of disturbance errors in both x and y directions on the invariant set is zero. $\square$

From the aforementioned result, if all of the agents are subject to constant disturbances, we infer that an internal-model-based controller as in (21) neither rejects constant disturbances nor achieves the desired formation. However, the stability of the network is maintained and the robots' velocities converge to zero. To overcome the limitations of the controller (21), the authors of [4] and [6] have proposed an observer-based distributed controller which however does not appear to apply straightforwardly to the present case.

4.2. Network of lossless agents

In this section, we study a network of lossless agents. Proposition 1 requires that at least one of the robots is strictly output passive. If all of the robots are lossless, that is, $\dot{H}_i(t) = u_i^T(t)y_i(t)$, and $D^v > 0$, the formation will converge to the largest invariant set where $(B^T \otimes I_2)G^T(q)\frac{\partial H}{\partial p} = \mathbf{0}$ holds. Because $y = G^T(q)\frac{\partial H}{\partial p}$, the latter implies that the velocity of the front end position of all of the robots converge to the same vector (see the proof of the Proposition 2) and consequently $\dot{z} = \mathbf{0}$. In what follows, first, we present the analysis of a lossless network in the absence of disturbances, and we prove that in this case, the desired formation is achieved. Then, we study the effect of constant disturbances. The analysis of a network of lossless agents in the presence of disturbances is more complicated than a network of the strict output passive agents. Remark 3 refers to this case. For simplicity of the analysis, we assume Assumption 1.

Assumption 1

Assume that $d_{AC,i} = 1$, $m_i = m$ (consequently $I_{cm,i} = m$) and $K = kI_{2M}$.

Proposition 2

Consider system (23) with Assumption 1 and in the absence of disturbances ($\Gamma_i^d = \mathbf{0}$, $\Phi_i^d = \mathbf{0}$). Assume that $D^r = 0$, $D^v > 0$. The closed-loop system (23) asymptotically converges to the largest invariant set where $\tilde{z} = \mathbf{0}$, $(B^T \otimes I_2)y = \mathbf{0}$ and p is a constant vector.

Proof

Consider the system (23) with $D^r = 0$ and $D^v > 0$. Take the closed-loop Hamiltonian $H(q, p, \tilde{z}, \tilde{\theta}) = \frac{1}{2m}p^T p + \frac{k}{2}\tilde{z}^T \tilde{z}$ as the candidate Lyapunov function. Now, following the proof of Proposition 1, the system converges to the largest invariant set where $(B^T \otimes I_2)G^T(q)p = \mathbf{0}$ and the following dynamics holds

$$\begin{aligned}\dot{q} &= \frac{1}{m}S(q)p, \\ \dot{p} &= -k G(q)(B \otimes I_2)\tilde{z}, \\ \dot{\tilde{z}} &= \mathbf{0}.\end{aligned}\tag{31}$$

Recall that $y = G^T(q)p$ and $p = (p_{f,1}, h_1, \dots, p_{f,N}, h_N)^T$. Because $(B^T \otimes I_2)y = \mathbf{0}$, then all agents' velocities in x -direction are equal to v_x and in y -direction to v_y . From (7), for each agent i we can write

$$\begin{aligned}v_x &= \cos(\phi_i)p_{f,i} - \sin(\phi_i)h_i \\ v_y &= \sin(\phi_i)p_{f,i} + \cos(\phi_i)h_i.\end{aligned}\tag{32}$$

Moreover, considering $y = G^T(q)p$, we calculate $\dot{y}$ as follows

$$\dot{y} = \frac{d(G^T)}{dt}p + G^T \dot{p}.\tag{33}$$

From $(B^T \otimes I_2)y = \mathbf{0}$, the aforementioned simplifies to

$$\begin{pmatrix} \dot{v}_x \\ \dot{v}_y \\ \vdots \\ \dot{v}_x \\ \dot{v}_y \end{pmatrix} = \begin{pmatrix} -v_y h_1 \\ v_x h_1 \\ \vdots \\ -v_y h_N \\ v_x h_N \end{pmatrix} + G^T \dot{p}.\tag{34}$$

Because $(B^T \otimes I_2)y = \mathbf{0}$, we conclude that $(B^T \otimes I_2)\dot{y} = \mathbf{0}$. Multiplying the equality (34) by $(B^T \otimes I_2)$ results in

$$(B^T \otimes I_2) \begin{pmatrix} -v_y h_1 \\ v_x h_1 \\ \vdots \\ -v_y h_N \\ v_x h_N \end{pmatrix} = -(B^T \otimes I_2)G^T \dot{p}. \quad (35)$$

Now, we prove by contradiction that on the invariant set (31) $\dot{p} = \mathbf{0}$ and $\tilde{z} = \mathbf{0}$. Assume that $\tilde{z} \neq \mathbf{0}$. Multiplying the second equality of (31) by $(B^T \otimes I_2)$, we obtain

$$(B^T \otimes I_2)G^{-1}(q)\dot{p} = -k(B^T \otimes I_2)(B \otimes I_2)\tilde{z}.$$

From $d_{AC,i} = 1$, we have $G^{-1}(q) = G^T(q)$. Considering (35), the aforementioned equality results in

$$(B^T \otimes I_2) \begin{pmatrix} -v_y h_1 \\ v_x h_1 \\ \vdots \\ -v_y h_N \\ v_x h_N \end{pmatrix} = k(B^T \otimes I_2)(B \otimes I_2)\tilde{z}. \quad (36)$$

By assuming $\tilde{z} \neq \mathbf{0}$, the term $k(B^T \otimes I_2)(B \otimes I_2)\tilde{z}$ is also a non-zero constant vector. Otherwise $(B \otimes I_2)\tilde{z}$ should belong to the null-space of $(B^T \otimes I_2)$. Hence, $(B \otimes I_2)\tilde{z} \in \text{Span}(1 \otimes I_2)$ should hold because the graph topology is connected. However, the latter holds only if $\tilde{z} = \mathbf{0}$, because otherwise $(1^T \otimes I_2)(B \otimes I_2)\tilde{z}$ will not be zero, that contradicts the graph's properties. As a result, the right-hand side of (36) is a non-zero constant vector. Hence,

$$\begin{pmatrix} -v_y h_1 \\ v_x h_1 \\ \vdots \\ -v_y h_N \\ v_x h_N \end{pmatrix} \notin \ker(B^T \otimes I_2). \quad (37)$$

In other words, there exists at least one agent i such that its h_i is different from other agents.

Now, denote the vector $k(B \otimes I_2)\tilde{z}$ as follows

$$k(B \otimes I_2)\tilde{z} = \begin{pmatrix} b_1 \\ \bar{b}_1 \\ \vdots \\ b_n \\ \bar{b}_n \end{pmatrix}.$$

From (34), we can write

$$\begin{aligned} \dot{v}_x &= -v_y h_i - b_i \\ \dot{v}_y &= v_x h_i - \bar{b}_i. \end{aligned} \quad (38)$$

Based on (38), writing $\dot{v}_x$ and $\dot{v}_y$ for two agents i and j , we obtain

$$\begin{aligned} \dot{v}_x &= -v_y h_i - b_i = -v_y h_j - b_j \\ \dot{v}_y &= v_x h_i - \bar{b}_i = v_x h_j - \bar{b}_j. \end{aligned} \quad (39)$$

From (39), we calculate $v_x = \frac{\bar{b}_i - \bar{b}_j}{h_i - h_j}$ and $v_y = \frac{b_j - b_i}{h_i - h_j}$. We assume that $\bar{b}_i - \bar{b}_j \neq 0$ and $b_j - b_i \neq 0$ (otherwise, $y = \mathbf{0}$ and consequently $p = \mathbf{0}$ and $\tilde{z} = \mathbf{0}$, which contradicts the assumption $\tilde{z} \neq \mathbf{0}$). Because (39) holds for all agents, we conclude that $v_y = kv_x$ where $k = \frac{b_j - b_i}{\bar{b}_i - \bar{b}_j}$ is a non-zero constant. Now, consider (38). Replacing v_y with kv_x , we calculate

$$h_i = \frac{d_i}{v_x}, \quad (40)$$

where $d_i = \frac{\bar{b}_i - kb_i}{k^2 + 1}$. Now, assume that $v_x \neq \mathbf{0}$ (otherwise if $v_x = \mathbf{0}$, from (34), we conclude that $p = \mathbf{0}$ and $\tilde{z} = \mathbf{0}$, which contradicts the assumption on $\tilde{z} \neq \mathbf{0}$). From (38) and considering $v_y = kv_x$ and $h_i = \frac{d_i}{v_x}$, we obtain

$$\begin{aligned} \dot{v}_x &= -kd_i - b_i \\ \dot{v}_y &= d_i - \bar{b}_i. \end{aligned} \quad (41)$$

From the aforementioned equalities, both of the $\dot{v}_x$ and $\dot{v}_y$ are constants. Hence, v_x is either an unbounded function or a non-zero constant. Recall that $h_i = \frac{d_i}{v_x}$. If $d_i = 0$, $h_i = 0$ and if $d_i \neq 0$ then h_i should converge to zero because v_x increases by time. But from (32), this implies that $p_{f,i}$ is increasing by time, which contradicts the boundedness of p . Hence, v_x should be constant. As a result, h_i is either zero and consequently $d_i = 0$ or h_i is a non-zero constant and $\phi_i = \int h_i = \alpha_i t$. Consider the latter case and notice that in this case, ϕ_i takes values in $[0, 2\pi)$. Recall from (32) that

$$v_y \cos(\phi_i) - v_x \sin(\phi_i) = h_i. \quad (42)$$

Substituting $v_y = kv_x$ and $h_i = \frac{d_i}{v_x}$ in (40), we obtain

$$h_i^2 = kd_i \cos(\phi_i) - d_i \sin(\phi_i).$$

Now, we argue that the aforementioned equality holds only if $d_i = 0$. Consider that case where $\cos(\phi_i) = 0, \sin(\phi_i) = 1$, then d_i should be non-positive. On the other hand, when $\cos(\phi_i) = 0, \sin(\phi_i) = -1$, d_i should be non-negative. Hence, $d_i = 0$. Now, from (40), we conclude that $h_i = 0, \forall i$, which contradicts (37). Hence, we conclude that the assumption $\tilde{z} \neq \mathbf{0}$ is not correct. The latter implies that $\tilde{z} = \mathbf{0}$. Also, from (31), $\tilde{z} = \mathbf{0}$ implies $\dot{p} = 0$, which ends the proof. $\square$

Now, we consider the lossless network in the presence of constant disturbances. Consider system (23) and assume that $D^r = 0, D^v > 0, \Phi_i^d = \mathbf{0}$ and $\Gamma^d \Gamma^{dT} = I_{2N}$. Take $H(q, p, \tilde{z}, \tilde{\theta}) = \frac{1}{2} p^T (M^r)^{-1} p + \frac{1}{2} \tilde{z}^T K \tilde{z} + \frac{1}{2} \tilde{\theta}^T \tilde{\theta}$ as the candidate Lyapunov function of the system. Following the proof of Proposition 2, the closed-loop system (23) asymptotically converges to the largest invariant set where $(B^T \otimes I_2)y = 0$ with the following dynamics

$$\begin{aligned} \dot{q} &= \frac{1}{m} S(q) p, \\ \dot{p} &= -G(q)(B \otimes I_2)\tilde{z} - G(q)\Gamma^d \tilde{\theta}, \\ \dot{\tilde{z}} &= \mathbf{0}, \\ \dot{\tilde{\theta}} &= \Gamma^{dT} y. \end{aligned} \quad (43)$$

From $(B^T \otimes I_2)y = 0$, we conclude that all agents' velocities in x -direction are equal to v_x and in y -direction equal to v_y . Now, considering the last equality in (43), v_x and v_y should not contain constant values otherwise $\theta(t)$ will increase by time, which contradicts boundedness of the system. Therefore, v_x and v_y are either equal to zero or a periodic bounded function oscillating around zero (no constant component).

Remark 3

The set $\{y = \mathbf{0}, p = \mathbf{0}, \dot{\tilde{z}} = \mathbf{0}, (1^T \otimes I_2)\Gamma^d \tilde{\theta} = \mathbf{0}\}$, which is a subset of (43) is analogous to the results of Corollary (2). The simulation results of a network of lossless agents in the presence of constant disturbances is also similar to the one related to Corollary (2) (Figure 5).

5. SIMULATION RESULTS

We run simulations for a network of $N = 5$ wheeled robots, where the robots exchange information according to the following incidence matrix:

$$B = \begin{pmatrix} -1 & 0 & 0 & 0 \\ +1 & -1 & 0 & 0 \\ 0 & +1 & -1 & 0 \\ 0 & 0 & +1 & -1 \\ 0 & 0 & 0 & +1 \end{pmatrix}.$$

The desired formation is a line formation defined by the following inter-robot position vectors: $z_k^* = [1 \ 0]^T$ for $k = 1, \dots, 4$. Note that the number of edges of the graph is four. The initial position of the robots is set to $x_B(0) = (0, 0.1, 0.2, 0.3, 0.4, 0.5)$, $y_B(0) = (0, 0, 0, 0, 0)$, $\phi(0) = (1.5124, 4.2482, 1.8162, 4.2211, 4.3677)$.

In this example, the robot's parameters are based on the e-puck wheeled robot developed by the authors of [25] and are taken as $m_i = 0.167$ kg, $d_{f,i} = 2$ kg/s, $d_{\phi,i} = 0.2$ kg m²/s, $d_{AC,i} = 0.06$ m, $I_{CM,i} = 9.69 \cdot 10^{-5}$ kg m² for $i = 1, \dots, 5$. The virtual coupling parameters are chosen as $\kappa_{x,k} = \kappa_{y,k} = 2$, $d_{x,k} = d_{y,k} = 1$ for $k = 1, \dots, 4$.

Here, we present simulation results to illustrate the effectiveness of the approach, by considering harmonic disturbances (Corollary 1) and constant disturbances (Corollary 2). For the harmonic disturbances, we set

$$\Phi_i^d = I_2 \otimes \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \Gamma_i^d = I_2 \otimes (0 \ 2),$$

$$w_i^d(0) = (1, 1, 1, 1)^T, \quad \theta_i^d(0) = (0, 0, 0, 0)^T,$$

Figure 4. Time evolution of the relative distance $\tilde{z}_x$, velocity v_x , and internal-model-based controller state $\tilde{\theta}_x$ in the presence of harmonic disturbances.

for $i = 1, \dots, 5$, while for the constant disturbances, $\Phi_i^d = 0$. We set the robot parameters such that all robots are lossless except for robot 1, which is strictly output passive (i.e., $d_{f,1} = 2$, $d_{\phi,1} = 0.2$ and $d_{f,i} = d_{\phi,i} = 0$ for $i = 2, \dots, 5$). For reasons of readability, we only present the plots along the x -direction here. The plots along the y -direction show a similar trend.

Figure 4 shows the time evolution of the relative distance $\tilde{z}_x$, velocity v_x , and internal-model-based controller state $\tilde{\theta}_x$ in the presence of harmonic disturbances, while Figure 5 shows the same variables for constant disturbances. For the harmonic disturbances, all three variables converge to

Figure 5. Time evolution of the relative distance $\tilde{z}_x$, velocity v_x , and internal-model-based controller state $\tilde{\theta}_x$ in the presence of constant disturbances.

Figure 6. Trajectories of the front ends of the robots (x_C, y_C) in the presence of harmonic disturbances.

Figure 7. Time evolution of the relative distance $\tilde{z}_x$, velocity v_x , and internal-model-based controller state $\tilde{\theta}_x$ in the presence of harmonic disturbances when all robots are strictly output passive and $D^v = 0$.

Figure 8. Time evolution of the relative distance $\tilde{z}_x$ and velocity v_x in the absence of disturbances when all robots are lossless and $D^v > 0$.

zero in accordance with Corollary 1 (Figure 4). For the constant disturbance case, as shown in Figure 5, v_x does converge to zero, while $\tilde{z}_x$ and $\tilde{\theta}_x$ converge to a constant different from zero (Corollary 2). Moreover, Figure 6 shows the trajectories of the five robots in the presence of harmonic disturbances with $w_i^d(0) = (0.1, 0.1, 0.1, 0.1)$. As shown, the front ends converge to a formation where $z_{k,x}^* = 1$ and $z_{k,y}^* = 0$ (the desired formation).

To illustrate Remark 2, we simulate a network where all five robots are strictly output passive. The virtual damping coefficients are set to zero, $d_{x,k} = d_{y,k} = 0$ for $k = 1, \dots, 4$. Here, the robots are subject to the same harmonic disturbances as in Figure 4. For this setting all variables converge to zero as stated in Remark 2 (Figure 7). Finally, the simulation result corresponding to Proposition 2 is shown in Figure 8. All agents are lossless $D^r = 0$, and there is no disturbances. As shown, the formation is achieved, while the velocities of the agents converge to a sinusoidal function. In this case, the addition of constant disturbances will make the agents' velocities converge to zero. The simulation results will be similar to the one shown in Figure 5.

6. CONCLUSIONS

This paper considered formation keeping control with matched input disturbance rejection for a network of wheeled robots. The model of the network and the control design were presented within the port-Hamiltonian framework. We used tools from the passivity and the internal-model-based approaches to design the controllers. The tasks of reaching the desired formation and rejecting the input disturbances were successfully achieved for a class of (strictly) output passive robots affected by harmonic disturbances. Moreover, the study of a network of lossless wheeled robots in the absence of disturbances resulted in achieving the desired formation. However, the effects of disturbances on a lossless network requires more investigations and serves as a future extension. Future avenues of research includes extending the results to counteract constant disturbances. In addition, tracking a desired reference velocity while rejecting input disturbances is another topic that deserves future investigations.

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