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Two phenomenological models to predict the single peak flow stress curves up to the peak during hot deformation

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ABSTRACT

Based on both linear and nonlinear estimations of work hardening rate versus strain curves, two mathematical models have been developed to predict the stress-strain curves under hot working conditions up to the peak stress. The models were validated for a mechanically alloyed $\text{Al6063}/0.75\text{Al}_2\text{O}_3/0.75\text{Y}_2\text{O}_3$ nanocomposite compressed under different hot forming conditions. The predicted results from both models are found to be in accord with the experimental flow stress curves up to the peak. For the present system, the linear model is found to more accurately predict the flow stress (with an average error of 0.81%) relative to the nonlinear model (with an average error of 2.06%).

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1. Introduction

The thermomechanical behavior of metals and metallic alloys is complex, and is usually described in terms of various microstructural dynamic restoration processes including recovery (DRV) and recrystallization (DRX). In materials with high stacking fault energy (SFE), DRV is very rapid, and is usually the only form of dynamic restoration. The flow stress of these materials during thermomechanical processes increases up to a constant saturation stress. On the other hand, for those materials with relatively low to medium SFE, recovery processes are slow; therefore, DRX processes may take place when the strain reaches a critical value (ϵ_c). Consequently, the softening mechanisms prevail over the hardening mechanisms, resulting in a single peak in the flow stress curve that is a characteristic of DRX. The flow stress drops until the equilibrium between the softening and hardening mechanisms is reached, where the flow stress has a steady state value (σ_{ss}) (Solhjoo and Ebrahimi, 2007; Solhjoo, 2010, 2014a,b). Fig. 1 shows a schematic hot deformation flow stress curve with a single peak.

The knowledge of the deformation behavior of metals and alloys at hot deformation conditions plays a key role in designing hot metal forming processes such as hot rolling, forging, and extrusion (Solhjoo and Ebrahimi, 2010). Generally, hot deformation behaviors are described by a constitutive equation, which connects flow stress and process variables like strain, strain rate, and temperature. Many constitutive models are available in the literature to describe the flow behavior of metals and alloys, which can be

categorized into three groups: physical models, phenomenological models, and artificial neural networks (ANNs). Physics-based models describe the flow behavior as a summation of athermal, thermal, and viscous drag components (Follansbee and Kocks, 1988; Nemat-Nasser et al., 2001; Balokhonov et al., 2009). Although these models can be very accurate, they are not always preferred as they often require a large number of material constants that should be determined from precisely controlled experiments (Gray et al., 1999). On the other hand, phenomenological models and ANNs are based on empirical results. ANNs require the application of a statistical learning process beforehand, through which a pattern is constructed to connect the deformation variables (inputs) to the flow stress values (outputs). Due to the nature of ANNs, their applicability is very limited to the preparation range of data. While the usage of physics-based models and ANNs is limited, phenomenological models are used in practical applications such as finite element analysis (Paul, 2012). These models describe flow stress behavior through an equation with a limited number of variables that can be easily calibrated with experimental results. Generally, phenomenological models describe the flow stress values either before (Solhjoo, 2010, 2014a, b, 2009), or after (Solhjoo, 2012a) the peak stress, contrary to physics-based models and ANNs.

In this investigation, two new phenomenological models are proposed based on the estimation of the work hardening rate as a linear and nonlinear function of strain rate, respectively. The models describe the flow stress behavior of metals and alloys up to the peak stress under hot deformation processes; therefore, they are useful only for the materials with medium to low SFE that show a single peak in their stress-strain curves. Validation analyses were performed to determine the flow stress values

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Fig. 1. Schematic representation of a single peak flow stress curve at high temperature. C and P correspond to the critical point for the onset of DRX and the peak point, respectively.

of Al6063/0.75Al₂O₃/0.75Y₂O₃ nanocomposite. The calculated flow stresses match the experimentally measured ones with a high degree of accuracy.

2. Modeling of flow stress curves

The slope of the flow stress curve determined at a constant strain rate and temperature corresponds to the work hardening rate, i.e. $\theta = \left(\frac{d\sigma}{d\varepsilon}\right)_{\dot{\varepsilon}, T}$, where $\dot{\varepsilon}$ and T are strain rate and temperature, respectively. It is known that the linear fit of the $\theta - \varepsilon$ (Solhjoo, 2010), or $\theta - \sigma$ (Solhjoo, 2009) curves can be used to develop a constitutive equation to estimate the flow stress up to the peak, while the results of the linear fit of the $\theta - \varepsilon$ curve show smaller errors compared to the linear fit of the $\theta - \sigma$ curve (Solhjoo, 2012a). In this investigation, both the linear and nonlinear estimations of the work hardening rate versus strain are taken into account in order to build up two formulations for the flow stress up to the peak.

2.1. Work hardening rate as a linear function of strain

This linear fit of the $\theta - \varepsilon$ curve is given by Solhjoo (2010) as

$$\left(\frac{d\sigma}{d\varepsilon}\right)_{\dot{\varepsilon}, T} = A\varepsilon + C \quad (1)$$

where A and C are constants. Using the root of this equation ($\varepsilon = \varepsilon_p$, $\theta = 0$), the value of C is obtained to be $-A\varepsilon_p$. In previous work by one of the authors (Solhjoo, 2010), Eq. (1) was solved for certain limits of integration to predict the critical strain for the initiation of dynamic recrystallization; however, those integration limits resulted in a model that could not account for an initial value of stress (σ_0) without additional modifications (Solhjoo, 2012a,b). On the other hand, by the indefinite integration of Eq. (1), another solution could be found:

$$\sigma = \sigma_p - \frac{A}{2}(\varepsilon_p - \varepsilon)^D, \quad (2)$$

where σ_p and ε_p are the peak stress and the peak strain, respectively. Analytically, the strain hardening exponent D is determined to be 2 (see “Appendix A”); however, because this is a phenomenological model, it should have some flexibility to adjust itself to the experimental data. Regarding this matter, the strain hardening exponent D is considered as a variable. Using the coordinates of the origin ($\varepsilon = 0$, $\sigma = \sigma_0$) as a solution to Eq. (2), the coefficient A is found to be $2\frac{\sigma_p - \sigma_0}{\varepsilon_p^D}$. In this manner, the initial stress becomes part of the model with no need for further modification. By replacing

Fig. 2. The schematic normalized flow stress curves calculated by Eq. (3) as a function of the strain hardening exponent D and the initial normalized stress σ_0/σ_p .

Fig. 3. The schematic normalized flow stress curves calculated by Eq. (5) as a function of G .

the value of A in Eq. (2), this can be rewritten as follows:

$$\sigma = \sigma_p - (\sigma_p - \sigma_0) \left(1 - \frac{\varepsilon}{\varepsilon_p}\right)^D. \quad (3)$$

Fig. 2 plots the stress-strain dependency predicted by Eq. (3) normalized by the peak values. As it is shown, this equation can be adjusted easily by controlling the values of D and σ_0 .

2.2. Work hardening rate as a nonlinear function of strain

A nonlinear fit of the $\theta - \varepsilon$ curve up to the peak stress can be expressed by the following equation:

$$\left(\frac{d\sigma}{d\varepsilon}\right)_{\dot{\varepsilon}, T} = \frac{G}{\varepsilon} - F\varepsilon, \quad (4)$$

where G and F are constants. Using a known solution of the $\theta - \varepsilon$ curve, i.e. $\varepsilon = \varepsilon_p$ and $\theta = 0$, the value of F is found to be equal to $\frac{G}{\varepsilon_p^2}$. Solution of the differential Eq. (4) would yield

$$\sigma = \sigma_p + G \left[\ln\left(\frac{\varepsilon}{\varepsilon_p}\right) - \frac{1}{2} \left(\left(\frac{\varepsilon}{\varepsilon_p}\right)^2 - 1 \right) \right]. \quad (5)$$

The normalized flow stress curves calculated by Eq. (5) are plotted in Fig. 3. As can be noted, this model has no solution for $\varepsilon = 0$. In other words, it is unable to capture the initial value of stress, and the model is expected to be applicable only for the ranges of the deformation conditions over which the constant G is calculated.

3. Numerical results

Experimental flow stress curves measured from a series of uniaxial hot compression tests on a mechanically alloyed

Fig. 4. The plots used for evaluation of the exponent D of Eq. (3). The value of D was chosen as the mean value of the slopes of the plots, which was found to be 2.036.

Al6063/0.75Al₂O₃/0.75Y₂O₃ nanocomposite published in the literature (Ahmed and Senthilkumar, 2012) were used for the mathematical analyses in this investigation. The considered deformations were performed in the temperature range of 300 and 500 °C, and at three strain rates of 0.01, 0.1 and 1 s⁻¹. Ahmed and Senthilkumar showed that the flow stress of the alloy is a function of deformation conditions: it decreases with increasing temperature, and increases with increasing strain rate (Ahmed and Senthilkumar, 2012). Also, the initiation of the dynamic recrystallization during the hot deformation results in a single peak in the stress-strain curves.

3.1. Linear approximation

In order to estimate the flow stress using the linear fit, the stress must be calculated from Eq. (3). As Fig. 4 shows, the plots of $\ln(\sigma_p - \sigma)$ versus $\ln(1 - \frac{\epsilon}{\epsilon_p})$ were used to determine the value of D for all sets of temperatures and strain rates by fitting the best line for each set; the average value was assigned to the work hardening exponent D , and was found to be 2.036.

3.2. Nonlinear approximation

The stress must be determined from Eq. (5) for a nonlinear approximation of the flow curves. Linear plots of $(\sigma_p - \sigma)$ versus $\ln(\frac{\epsilon}{\epsilon_p}) - \frac{1}{2}((\frac{\epsilon}{\epsilon_p})^2 - 1)$ were used to determine the parameter G ; this parameter is found to be a function of hot deformation conditions. The following formula (Solhjoo, 2012a) was used to describe the values of G :

$$G = \bar{G} \dot{\epsilon}^E \exp(\phi T) \quad (6)$$

where $\bar{G}$, E and ϕ are constants, and T is the deformation absolute temperature. As was explained in a previous work (Solhjoo, 2012a), the constants E and ϕ should be calculated from $\ln \dot{\epsilon}$ versus $\ln G$ (Fig. 5) and T versus $\ln G$ (Fig. 6) plots, respectively. Using the calculated values of E and ϕ , $\bar{G}(\dot{\epsilon}, T)$ was calculated for each set of deformation conditions, and their average was used as $\bar{G}$. In this research, G can be calculated with the following equation:

$$G = 17247.8 \dot{\epsilon}^{0.131} \exp(-0.0076 T). \quad (7)$$

4. Discussion

In order to utilize either of these models, the peak values of stress and strain are needed. In the present study, the available experimental measurements were used for the peak values.

Fig. 5. Dependence of the parameter G on strain rate. The mean value of the slopes, i.e. E of Eq. (6), was determined to be 0.131.

Fig. 6. Dependence of the parameter G on temperature. The mean value of the slopes, i.e. ϕ of Eq. (6), was determined to be -0.0076.

4.1. Linear approximation

The numerical value of D was calculated to be 2.036, which is very close to its reported analytical value, i.e. 2 (see “Appendix A”). Although the calculated value of D is in agreement with its analytical solution for the studied material, a large number of experiments on different materials and at different deformation conditions is needed to draw any conclusions about the value of D and whether it, indeed, has a universal value of 2.

In order to work with this model, the value of the initial stress is also needed; however, its value might be unavailable, e.g. due to the resolution of the testing machine, which would be a problem in utilizing Eq. (3). A solution for this problem is to collect the x-intercept of $\ln(\sigma_p - \sigma)$ versus $\ln(1 - \frac{\epsilon}{\epsilon_p})$ plots (see Fig. 4), which equals $\ln(\sigma_p - \sigma_0)$, and use that for the estimation of σ_0 value. Also, this problem can be solved through another approach: using a known experimental data point of the stress-strain curve as a solution of Eq. (3). Let's call the stress and strain of a known point σ_N and ϵ_N , respectively. Eq. (3) would be rewritten for this point as

$$\sigma_N = \sigma_p - (\sigma_p - \sigma_0) \left(1 - \frac{\epsilon_N}{\epsilon_p}\right)^D, \quad (8)$$

and the value of σ_0 can be found as

$$\sigma_0 = \sigma_p - (\sigma_p - \sigma_N) \left(1 - \frac{\epsilon_N}{\epsilon_p}\right)^{-D}. \quad (9)$$

Inserting σ_0 of the form of Eq. (9) into Eq. (3), one gets

$$\sigma = \sigma_p - (\sigma_p - \sigma_N) \left(\frac{\epsilon_p - \epsilon}{\epsilon_p - \epsilon_N}\right)^D. \quad (10)$$

Both solutions, i.e. estimating the initial stress value from the x-intercept of the plots of $\ln(\sigma_p - \sigma)$ versus $\ln(1 - \frac{\epsilon}{\epsilon_p})$, and using a known value (Eq. (10)), were compared for the system with the deformation conditions of $\dot{\epsilon} = 1 \text{ s}^{-1}$ and $T = 300 \text{ °C}$, as shown in

Fig. 7. Comparison between experimental and predicted flow stress of a strain rate of 1 s^{-1} at a temperature of 300°C . (Notice the stress and the strain ranges.) The needed value of the initial stress in Eq. (3) was estimated based on the x-intercept of the corresponding $\ln(\sigma_p - \sigma)$ versus $\ln(1 - \frac{\varepsilon}{\varepsilon_p})$ linear plot. Moreover, in order to work with Eq. (10), the experimental data point at the lowest strain was used as the known point.

Fig. 8. Comparison between experimental and predicted flow stress by the linear approximation model. The data point at the lowest available strain at each set of deformation condition was used as the corresponding known point.

Fig. 7. Eq. (3) was used by extracting the value of $\ln(\sigma_p - \sigma_0)$, which resulted in $\sigma_0 = 5.642 \text{ MPa}$. Also, Eq. (10) was utilized with the coordinates of the known point: $\varepsilon_N = 0.05$ and $\sigma_N = 153 \text{ MPa}$. As can be seen, both solutions resulted in good agreement with the experimental data.

It should be noted that predicted values are only reliable for the validated domain, and there is a possibility that the solutions outside the fitted range result in some unexpected and unrealistic values (see Fig. 1-B of (Solhjoo, 2014c)). This problem can be overcome if the values of the initial stress are known.

Using the available experimental data at the lowest strain for each deformation condition, the flow stress values were estimated via Eq. (10), and the values were compared with the experimental ones. The results are shown in Fig. 8, which shows the data of the best fitted line with a coefficient of determination of 0.9998, a slope of 0.9919, and a constant term of 0.0472. All these parameters confirm the high accuracy of Eq. (10) in estimating the flow stress values.

4.2. Nonlinear approximation

In order to estimate the flow stress values using the nonlinear approximation method, Eq. (5) should be used, where the value of G should be calculated from Eq. (7) for each deformation condition. Fig. 9 shows the comparison between the results of this method and the experimental values. In order to have a better view of the

Fig. 9. Comparison between experimental and predicted flow stress by the nonlinear approximation model. The needed values of G were estimated from Eq. (7) for each set of deformation conditions.

Fig. 10. The plot used for the evaluation of the exponent S of Eq. (12). The slope of the plot determines the value of S , which was found to be 1.23.

accuracy of the model, using a simple linear regression model, a line was fitted to the data in Fig. 9. The fitted line had a coefficient of determination of 0.9985, a slope of 0.987, and a constant term of -0.1616 . The low value of the constant term (≈ 0), the slope of the fitted line (≈ 1), and the value of coefficient of determination (≈ 1), show a high accordance between the experimental and calculated flow stress.

4.3. Comparison between the models

The previous model based on a linear estimation of the work hardening rate as a function of strain was presented as (Solhjoo, 2010, 2012a,b)

$$\sigma = \sigma_p \left(\left(\frac{\varepsilon}{\varepsilon_p} + N \right) \left(2 - \frac{\varepsilon}{\varepsilon_p} \right) \right)^S, \quad (12)$$

with

$$N = \frac{1}{2} \frac{\varepsilon_p - \varepsilon}{\varepsilon_p} \left(\frac{\sigma_0}{\sigma_p} \right)^{1/S}, \quad (13)$$

where S is a constant, which could be found from the slope of the plot of $\ln(\sigma/\sigma_p)$ versus $\ln((\varepsilon/\varepsilon_p)(2 - \varepsilon/\varepsilon_p))$. Fig. 10 shows that the value of S was found to be 1.23, and its value was insensitive to temperature.

In order to work with this model, the value of the initial stress has to be known, otherwise, the value of the parameter N cannot be found. Considering the missing data of the initial stress in the current study, one way is to assume that $\sigma_0 = 0$, which results in $N = 0$; however, it is possible to redefine the parameter N as a

Fig. 11. Comparison between the experimental and predicted flow stress curves at different temperatures, and for strain rates of (a) 1 s^{-1} , (b) 0.1 s^{-1} , and (c) 0.01 s^{-1} . The results show that the models are not reliable for strain values outside the fitted range.

function of the known point of $(\varepsilon_N, \sigma_N)$ by the following formula:

$$N = \frac{\varepsilon_p - \varepsilon}{\varepsilon_p - \varepsilon_N} \left(\frac{\varepsilon_N}{\varepsilon_p} - \frac{\varepsilon_p}{2\varepsilon_p - \varepsilon_N} \left(\frac{\sigma_N}{\sigma_p} \right)^{1/5} \right). \quad (14)$$

The details of the redefinition of N can be found in Appendix B. The experimental and estimated flow stress curves for different deformation conditions are shown in Fig. 11 and discussed below.

Fig. 11 shows that the newly presented models can predict the flow stress values up to the peak with a high accuracy, as was concluded from Figs. 8 and 9. Moreover, Fig. 11 shows that the models deviate from each other significantly for lower strains. In other words, the models are not reliable for the ranges of strain where experimental data is not available, e.g. lower than 0.05 at the current study.

In order to compare the accuracy of the proposed models of the current study with the previous one (Eq. 12), the errors of the predicted values were calculated using the mean absolute percentage error (MAPE)

Table 1

The absolute percentage error of the predicted flow stress values.

	Minimum	Maximum	MAPE (Eq. (15))
Linear	0.00	2.86	0.81
Nonlinear	0.00	8.90	2.06
Eq. (12) ($\sigma_0 = 0$)	0.00	4.31	1.19
Eq. (12) (N from Eq. (14))	0.00	7.39	2.00

$$MAPE = \frac{100\%}{n} \sum_n \left| \frac{\sigma_{\text{exp}} - \sigma_{\text{pre}}}{\sigma_{\text{exp}}} \right|, \quad (15)$$

where σ_{exp} and σ_{pre} are the experimental and the predicted values of the flow stress, respectively, and n is the total number of data points used for the comparison. In this study, more than 200 data points were collected for the flow stress values extracted for strains ≥ 0.05 , with an interval of 0.005, up to the peak strain. The calculated errors are listed in Table 1. The linear model is found to be more accurate (with an MAPE of 0.81%) compared to the nonlinear model (with an MAPE of 2.06%). This suggests that the linear estimation of the work hardening rate as a function of strain shows better results than a nonlinear estimation in the form of Eq. (5). Moreover, the previous model (Solhjoo, 2010, 2012a,b), either with the assumption that $\sigma_0 = 0$ (with an MAPE of 1.19%) or the calculation of N from Eq. 14 (with an MAPE of 2.00%), shows a lower accuracy compared to the newly presented linear model. Furthermore, if the experimental data for the initial stress are missing, the previous model results would be made more accurate by assuming that $\sigma_0 = 0$, as opposed to estimating the values of N from Eq. (14).

Considering the applicability of the compared models, and in order to work with Eq. (12), one must either assume that $\sigma_0 = 0$, which results in $N = 0$, or estimate the value of N from Eq. (14) which lowers the accuracy of the model. On the other hand, the unavailability of data for the initial stress does not restrict the applicability of the models proposed in the current work.

5. Conclusions

In this paper, two new phenomenological models have been developed and discussed, which can be used to predict the hot deformation flow stress up to the peak of the stress-strain curve. The models are derived from defining the work hardening rate as a function of strain, with linear and nonlinear approximations, respectively. The linear approximation formula can capture the initial stress value. In the case where the initial stress value is unavailable, the formulation is still applicable by means of different analytical solutions. On the other hand, the model based on the nonlinear approximation has a material constant G , which is found to be a function of temperature and strain rate, and has no solution to the initial stress value. Moreover, the proposed models in the current study are more accurate than the older one (Solhjoo, 2010) and its modified form (Solhjoo, 2012a,b), regarding, both, the derivation method (see Section 2.1) and the predicted values of flow stress (see Table 1). Furthermore, the applicability of the previous model would be limited if the data of the initial stress were missing; this limitation was treated in this work by redefining the parameter N .

The proposed models may be used to develop numerical solutions, e.g. using the finite element method, to describe metallic material behavior in processes where the hot deformation conditions should be considered. Both models have been validated for the case of an aluminum alloy and can predict the flow stress values with high accuracy, with the linear approximation showing smaller errors.

Appendix A

Starting with the integration of Eq. (1), the solution would be:

$$\sigma = A\varepsilon\left(\frac{1}{2}\varepsilon - \varepsilon_p\right) + \text{Const.} \quad (\text{A1})$$

In order to determine the value of *Const*, one may use the value of stress and strain corresponding to the peak. Therefore, we have:

$$\sigma_p = A\varepsilon_p\left(\frac{1}{2}\varepsilon_p - \varepsilon_p\right) + \text{Const}, \quad (\text{A2})$$

which results in:

$$\text{Const} = \sigma_p + \frac{1}{2}A\varepsilon_p^2. \quad (\text{A3})$$

Using this value, Eq. (A.1) changes into

$$\sigma = \sigma_p + \frac{1}{2}A(\varepsilon_p - \varepsilon)^2. \quad (\text{A4})$$

Appendix B

The parameter *N* in Eq. (12) is introduced for implementing the initial stress into the original model (Solhjoo, 2010, 2012a, b). This parameter is defined as a linear function between two points of ($\varepsilon = 0$, $\sigma = \sigma_0$) and ($\varepsilon = \varepsilon_p$, $\sigma = \sigma_p$), where $N_{\varepsilon=0} = \frac{1}{2}\left(\frac{\sigma_0}{\sigma_p}\right)^{1/S}$ and $N_{\varepsilon=\varepsilon_p} = 0$, respectively. Following the same procedure, it is possible to calculate the value of *N* at a known point of (ε_N , σ_N) from Eq. (12), which results in $N_{\varepsilon=\varepsilon_N} = \frac{\varepsilon_N}{\varepsilon_p} - \frac{\varepsilon_p}{2\varepsilon_p - \varepsilon_N}\left(\frac{\sigma_N}{\sigma_p}\right)^{1/S}$. Assuming that the parameter *N* would change linearly with strain (Solhjoo, 2012a), its function can be found to be $N = N_{\varepsilon=\varepsilon_N}\left(\frac{\varepsilon_p - \varepsilon}{\varepsilon_p - \varepsilon_N}\right)$. Inserting the value of $N_{\varepsilon=\varepsilon_N}$ in this formula results in:

$$N = \frac{\varepsilon_p - \varepsilon}{\varepsilon_p - \varepsilon_N}\left(\frac{\varepsilon_N}{\varepsilon_p} - \frac{\varepsilon_p}{2\varepsilon_p - \varepsilon_N}\left(\frac{\sigma_N}{\sigma_p}\right)^{1/S}\right). \quad (\text{B1})$$

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