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Brief paper

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ABSTRACT

In this note we identify a class of underactuated mechanical systems whose desired constant equilibrium position can be globally stabilised with the ubiquitous PID controller. The class is characterised via some easily verifiable conditions on the systems inertia matrix and potential energy function, which are satisfied by many benchmark examples. The design proceeds in two main steps, first, the definition of two new passive outputs whose weighted sum defines the signal around which the PID is added. Second, the observation that it is possible to construct a Lyapunov function for the desired equilibrium via a suitable choice of the aforementioned weights and the PID gains.

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1. Introduction

A major breakthrough in robotics was the proof by Takegaki and Arimoto (Takegaki & Arimoto, 1981) that, in spite of its highly complicated nonlinear dynamics, a simple PD law provides a global solution to the point-to-point positioning task for fully actuated robot manipulators. As shown in Ortega and Spong (1989) the key property underlying the success of such a simple scheme is the passivity of the system dynamics. Indeed, as is now well-known, mechanical systems define passive maps from the external forces to the generalised coordinate velocities, see Kelly and Ortega (1988) for an early reference. Invoking this property the derivative term of the aforementioned PD is assimilated to a constant feedback around the passive output, while the proportional one adds a quadratic term to the systems potential energy to assign a minimum at the desired equilibrium, making the total energy function a *bona fide* Lyapunov function. It should be mentioned that this “energy-shaping plus damping injection” construction proposed 34 years ago is still the basis of most developments in

Passivity-Based Control (PBC)—a term coined in Ortega and Spong (1989) to describe a controller design procedure where the control objective is achieved via passivation.

While fully actuated mechanical systems admit an arbitrary shaping of the potential energy by means of feedback, and therefore stabilisation to any desired equilibrium via PD control, this is in general not possible for *underactuated* systems,¹ that is, for systems where the number of degrees of freedom is strictly larger than the number of control variables. In certain cases this problem can be overcome by also modifying the kinetic energy of the system—as done, for instance in interconnection and damping assignment (IDA) PBC (Ortega, Donaire, & Romero, 2017), the controlled Lagrangians (Bloch, Chang, Leonard, & Marsden, 2001) or the canonical transformation (Fujimoto & Sugie, 2001) methods. There are two major drawbacks to this total energy shaping controllers, first, that they require the use of complicated full-state feedback controllers, which makes them more fragile and difficult to tune. Second, that the derivation of the control law relies on the solution of a partial differential equation (PDE), a difficult task that hampers its wider application. See Hatanaka, Chopra, Fujita, and Spong (2015) and Ortega et al. (2017) for recent reviews of the literature of PBC of mechanical systems.

A key feature of total energy shaping methods is that the mechanical structure of the system is preserved in closed-loop, a

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¹ An interesting survey on the modelling, classification and control of underactuated systems is presented in Liu and Yu (2013).

condition that gives rise to the aforementioned PDE, which characterises the assignable energy functions. In Donaire, Mehra et al. (2016) it was recently proposed to *relax* this constraint, and concentrate our attention on the energy shaping objective only. That is, we look for a control law – which is of the PID structure – that stabilises the desired equilibrium assigning to the closed loop a Lyapunov function of the same form as the energy function of the open-loop system but with new, desired inertia matrix and potential energy function. However, we *do not require* that the closed-loop system is a mechanical system with this Lyapunov function qualifying as its energy function. In this way, the need to solve the PDE is avoided.

The PID-PBC design of Donaire, Mehra et al. (2016), which requires a non-robust (collocated) partial feedback linearisation stage, *à la* (Spong, 1994), has been improved in Romero, Donaire, and Ortega (2016a) where the feedback linearisation step is obviated. The main objective of this paper is to extend the results of Donaire, Mehra et al. (2016) and Romero et al. (2016a) in the following directions.

- D1 Impose a strictly weaker condition on the inertia matrix to enlarge the class of systems for which the PID-PBC ensures Lyapunov stability.
- D2 Identify a class of systems for which the (non robust) step of cancellation of a term of the potential energy required in Donaire, Mehra et al. (2016) and Romero et al. (2016a) is avoided.
- D3 Relax the conditions on the PID-PBC tuning gains imposed in Donaire, Mehra et al. (2016) and Romero et al. (2016a) to construct a Lyapunov function, carrying out instead a LaSalle's analysis.
- D4 Give more flexibility to the PID-PBC design replacing the *linear* integral term by an arbitrary function.

Two additional contributions of the paper are, first, the role of the initial conditions of the controller state on the equilibrium assignment task has been clarified. Namely, it is shown that the if the controller is implemented as a static state-feedback the is no need to make the controller dependent on the plants initial conditions, see Remark 6. Second, that we provide coordinate-free characterisations of some of the conditions that identify the class of systems considered in the paper.

The remainder of the paper is structured as follows. In Section 2 the new passive outputs are identified. In Section 3 we present the proposed PID-PBC and establish the conditions for well-posedness of the control law. In Section 4 we show that the PID-PBC can assign the desired equilibrium and shape the energy function to make it a Lyapunov function and present some extensions in Section 5. We wrap-up the paper with concluding remarks and future research in Section 6.

Notation. I_n is the $n \times n$ identity matrix and $0_{n \times s}$ is an $n \times s$ matrix of zeros, 0_n is an n -dimensional column vector of zeros, $e_i \in \mathbb{R}^n$, $i \in \bar{n} := \{1, \dots, n\}$, are the n -dimensional Euclidean basis vectors. For $x \in \mathbb{R}^n$, $S \in \mathbb{R}^{n \times n}$, $S = S^\top > 0$, we denote the Euclidean norm $\|x\|^2 := x^\top x$, and the weighted-norm $\|x\|_S^2 := x^\top S x$. All mappings are assumed smooth. Given a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ we define the differential operator $\nabla_{x_i} f := \left(\frac{\partial f}{\partial x_i} \right)^\top$ and for a mapping $F : \mathbb{R}^n \rightarrow \mathbb{R}^m$ we define the ij th element of its $m \times n$ Jacobian matrix as $(\nabla F)_{ij} := \frac{\partial F_i}{\partial x_j}$. When clear from the context the subindex in ∇ is omitted.

Caveat Emptor This is an abridged version of the full paper that is available from Romero, Donaire, and Ortega (2016b). In particular, the proofs of the propositions are considerably shortened and the illustrative examples are omitted.

2. New passive outputs

Following Donaire, Mehra et al. (2016) and Romero et al. (2016a) the first step for the characterisation of a class of mechanical systems whose position can be regulated with classical PIDs is the identification of two new passive outputs, which is carried out in the section.

2.1. Identification of the class

We consider general underactuated mechanical systems whose dynamics is described by the well-known Euler–Lagrange equations of motion

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + \nabla V(q) = G(q)\tau, \quad (1)$$

where $q \in \mathbb{R}^n$ are the configuration variables, $\tau \in \mathbb{R}^m$, with $m < n$, are the control signals, $M : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$, is the positive definite generalised inertia matrix, $C(q, \dot{q})\dot{q}$ are the Coriolis and centrifugal forces, with $C : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$ defined via the Christoffel symbols of the first kind (Kelly, Santibáñez, & Loria, 2005; Ortega & Spong, 1989), $V : \mathbb{R}^n \rightarrow \mathbb{R}$ is the systems potential energy and $G : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times m}$ is the input matrix, which is assumed to verify the following assumption.

- A1.** The distribution spanned by the columns of the input matrix is *involutive* (Spivak, 1999). Equivalently, there exists a coordinate and input change of coordinates so that it takes the form ²

$$G = \begin{bmatrix} 0_{s \times m} \\ I_m \end{bmatrix}, \quad (2)$$

where $s := n - m$.

To simplify the definition of the class, and consistent with Assumption A1, we partition the generalised coordinates into its unactuated and actuated components as $q = \text{col}(q_u, q_a)$, with $q_u \in \mathbb{R}^s$ and $q_a \in \mathbb{R}^m$. Similarly, the inertia matrix is partitioned as

$$M(q) = \begin{bmatrix} m_{uu}(q) & m_{au}^\top(q) \\ m_{au}(q) & m_{aa}(q) \end{bmatrix},$$

where $m_{uu} : \mathbb{R}^n \rightarrow \mathbb{R}^{s \times s}$, $m_{au} : \mathbb{R}^n \rightarrow \mathbb{R}^{m \times s}$ and $m_{aa} : \mathbb{R}^n \rightarrow \mathbb{R}^{m \times m}$. Using this notation the class is identified imposing the following assumptions.

- A2.** The inertia matrix depends *only* on the unactuated variables q_u , i.e., $M(q) = M(q_u)$.
- A3.** The sub-block matrix m_{aa} of the inertia matrix is *constant*.
- A4.** The potential energy can be written as

$$V(q) = V_a(q_a) + V_u(q_u),$$

and $V_u(q_u)$ is bounded from below.

Remark 1. The lower boundedness assumption on $V_u(q_u)$ can be relaxed and is only introduced to simplify the notation avoiding the need to talk about cyclo-passive (instead of passive) outputs—see Footnote 2 of Romero et al. (2016a).

² To avoid cluttering in the sequel it is assumed that $G(q)$ in (1) is already of the form (2).

2.2. Passive outputs and storage functions

The proposition below, whose proof is given in [Appendix A](#), identifies two new passive outputs used in the PID controller design.

Proposition 1. Consider the underactuated mechanical system (1) verifying Assumptions **A1**–**A4**. Define the new input

$$u = \tau - \nabla V_a(q_a) \quad (3)$$

and the outputs

$$y_u := -m_{aa}^{-1} m_{au}(q_u) \dot{q}_u, \quad y_a := m_{aa}^{-1} m_{au}(q_u) \dot{q}_u + \dot{q}_a. \quad (4)$$

The operators $u \mapsto y_u$ and $u \mapsto y_a$ are passive with storage functions

$$H_u(q_u, \dot{q}_u) = \frac{1}{2} \dot{q}_u^\top m_{uu}^s(q_u) \dot{q}_u + V_u(q_u) \quad (5)$$

$$H_a(q_u, \dot{q}) = \frac{1}{2} \dot{q}^\top M_a(q_u) \dot{q}, \quad (6)$$

where

$$m_{uu}^s(q_u) := m_{uu}(q_u) - m_{au}^\top(q_u) m_{aa}^{-1} m_{au}(q_u) \\ M_a(q_u) := \begin{bmatrix} m_{au}^\top(q_u) m_{aa}^{-1} m_{au}(q_u) & m_{au}^\top(q_u) \\ m_{au}(q_u) & m_{aa} \end{bmatrix}. \quad (7)$$

More precisely,

$$\dot{H}_a = u^\top y_a, \quad \dot{H}_u = u^\top y_u. \quad (8)$$

Remark 2. Assumption **A1** means that the input force co-distribution is integrable ([Bullo & Lewis, 2004](#)). Assumption **A2** implies that the shape coordinates coincide with the un-actuated coordinates and are referred as “Class II” systems in [Olfati-Saber \(2001\)](#). Assumptions **A3** and **A4** are technical conditions. In particular, the latter condition is required because, as clear from (3), the first step in our design is to apply a preliminary feedback that cancels the term $V_a(q_a)$. See Section 5.1 for a removal of this condition for affine functions $V_a(q_a)$. The class of systems verifying all these assumptions contains many benchmark examples, including the spider crane, the 4-DOF overhead crane and the spherical pendulum on a puck.

Remark 3. In [Olfati-Saber \(2001\)](#), see also [Olfati-Saber \(2002\)](#), similar assumptions are imposed to identify mechanical systems that are transformable, via a change of coordinates, to the classical feedback and feedforward forms. The motivation is to invoke the well-known backstepping or forwarding techniques of nonlinear control to design stabilising controllers. It should be noticed that, although these two techniques are claimed to be systematic, similarly to total energy shaping techniques ([Ortega et al., 2017](#)), they still require the solution of PDEs, see Subsections 4.2 and 4.3 of ([Astolfi et al., 2007](#)).

Remark 4. From (4), (7) it is clear that $y_u + y_a = \dot{q}_a$ and $H_u(q_u, \dot{q}) + H_a(q_u, \dot{q}) = H(q_u, \dot{q})$, with

$$H(q_u, \dot{q}) = \frac{1}{2} \dot{q}^\top M(q_u) \dot{q} + V_u(q_u),$$

that is, the total co-energy of the system in closed-loop with (3). In other words, the new passive outputs are obtained splitting the (1, 1) block of the kinetic energy function into two components with one containing the Schur complement of the (2, 2) block. The same passive outputs were derived in [Donaire, Mehra et al. \(2016\)](#) for a system in Spong’s normal form, while in [Romero et al. \(2016a\)](#) related outputs were obtained proceeding from the Hamiltonian description of the system—see [Romero et al. \(2016b\)](#) for further details.

3. Proposed PID-PBC

Define the new output

$$y_d := k_a y_a + k_u y_u, \quad (9)$$

where $k_a, k_u \in \mathbb{R}$, $k_a \neq k_u$ are nonzero constants. From [Proposition 1](#) we have that

$$k_a \dot{H}_a + k_u \dot{H}_u = y_d^\top u. \quad (10)$$

To extend the results of [Donaire, Mehra et al. \(2016\)](#) and [Romero et al. \(2016a\)](#) we introduce a full rank matrix $T : \mathbb{R}^n \rightarrow \mathbb{R}^{m \times m}$ and propose input and output change of coordinates

$$\bar{y}_d := T(q) y_d, \quad \bar{u} := T^{-\top}(q) u, \quad (11)$$

which clearly satisfy

$$\bar{y}_d^\top \bar{u} = y_d^\top u. \quad (12)$$

We propose the PID-PBC

$$k_e \bar{u} = -(K_p \bar{y}_d + K_I x_C + K_D \dot{\bar{y}}_d) \\ \dot{x}_C = \bar{y}_d, \quad (13)$$

where $k_e \in \mathbb{R}$, $K_p, K_I, K_D \in \mathbb{R}^{m \times m}$, $K_p, K_I > 0$ and $K_D \geq 0$. We remark the presence of the sign-indefinite gains k_e, k_a, k_u in (13) and (9) whose motivation is to give more flexibility to achieve Lyapunov stabilisation and is discussed at the end of Section 4.2.

Before proceeding to analyse the stability of the closed loop it is necessary to ensure that the control law (13) can be computed without differentiation nor singularities that may arise due to the presence of the derivative term $\dot{\bar{y}}_d$. For, after some lengthy but straightforward calculations to compute $\dot{\bar{y}}_d$, we can prove that (13) is equivalent to

$$K(q_u) \bar{u} = -K_p \bar{y}_d - K_I x_C - S(q, \dot{q}), \quad (14)$$

where the mapping $K : \mathbb{R}^s \rightarrow \mathbb{R}^{m \times m}$ is given by

$$K(q_u) := k_e I_m + k_a K_D T m_{aa}^{-1} T^\top \\ + k_u K_D T [m_{aa}^{-1} m_{au}(m_{uu}^s)^{-1} m_{au}^\top m_{aa}^{-1}] T^\top.$$

and $S : \mathbb{R}^n \times \mathbb{R}^s \mapsto \mathbb{R}^m$ is the globally defined mapping

$$S(q, \dot{q}_u) := K_D T \left\{ k_u m_{aa}^{-1} m_{au}(m_{uu}^s)^{-1} \right. \\ \left. - [k_a m_{aa}^{-1} + k_u m_{aa}^{-1} m_{au}(m_{uu}^s)^{-1} m_{au}^\top m_{aa}^{-1}] \nabla(m_{au} \dot{q}_u) \dot{q}_u \right. \\ \left. (k_a - k_u) m_{aa}^{-1} \dot{m}_{au} \dot{q}_u \right\},$$

with β defined in (39). Notice that $S(q, \dot{q}) = 0$ if there is no derivative action in the control.

To ensure that the control law (14) – and consequently (13) – are well-defined we impose the full rank assumption.

A5. The controller gains k_e, k_a, k_u and K_D are such that

$$\det[K(q_u)] \neq 0.$$

4. Lyapunov stabilisation

As is well known, PID controllers define input strictly passive mappings. Thus, the passivity theorem allows to immediately conclude output strict passivity – hence, $\mathcal{L}_2$ -stability – of the closed-loop system. $\mathcal{L}_2$ -stability is a rather weak property. For instance, boundedness of trajectories is not guaranteed and the system can be destabilised by a constant external disturbance. Hence, we are interested in establishing a stronger property, e.g., Lyapunov stability of a desired equilibrium. In this section we prove that, under some additional conditions on the inertia matrix, it is possible to ensure Lyapunov stability of a desired equilibrium position via PID-PBC.

4.1. Equilibrium assignment

A first step for Lyapunov stabilisation of a desired constant state is, obviously, to ensure that it is an *equilibrium* of the closed loop. Since the system (1) is underactuated it is not possible to choose an arbitrary desired equilibrium, instead, it must be chosen as a member of the assignable equilibrium set. An additional difficulty stems from the fact that the signal y_d is equal to zero for *all* constant values of q . Hence, the PID control (13), (18) does not allow us to impose an assignable equilibrium to the closed loop. To overcome this obstacle we add a condition to the system inertia matrix to be able to express the integral term of the PID control, i.e., the signal x_c , as a function of q .

Towards this end, introduce the matrix $L : \mathbb{R}^s \rightarrow \mathbb{R}^{n \times s}$

$$L(q_u) := \begin{bmatrix} I_s \\ \frac{k_u - k_a}{k_a} m_{aa}^{-1} m_{au}(q_u) \end{bmatrix}$$

and define the PDE

$$\nabla \gamma(q) L(q_u) = 0, \quad (15)$$

with unknown the mapping $\gamma : \mathbb{R}^n \rightarrow \mathbb{R}^m$.

A6. The columns of $L(q_u)$ span an involutive distribution³ and the PDE (15) admits a solution such that

$$\det[\nabla_{q_a} \gamma(q)] \neq 0. \quad (16)$$

Lemma 1. In (11) select

$$T(q) := \frac{1}{k_a} \nabla_{q_a} \gamma(q). \quad (17)$$

Then, $\bar{y}_d = \dot{\gamma}$.

Proof. Replacing (4) in (9), we can write y_d in the form

$$y_d = k_a \dot{q}_a + (k_a - k_u) m_{aa}^{-1} m_{au}(q_u) \dot{q}_u. \quad (18)$$

Replacing (18) in (11), with (17), we get

$$\bar{y}_d = \nabla_{q_a} \gamma(q) \dot{q}_a + \frac{k_a - k_u}{k_a} \nabla_{q_a} \gamma(q) m_{aa}^{-1} m_{au}(q_u) \dot{q}_u.$$

On the other hand,

$$\dot{\gamma} = \nabla_{q_a} \gamma(q) \dot{q}_a + \nabla_{q_u} \gamma(q) \dot{q}_u.$$

The proof is completed by noting that (15) ensures $\bar{y}_d = \dot{\gamma}$.

A corollary of Lemma 1, that follows from (13), is that

$$x_c = \gamma(q) + \kappa, \quad (19)$$

with $\kappa \in \mathbb{R}^m$ a free integration constant. Hence the control (13) can be written as

$$k_e \bar{u} = -[K_P \bar{y}_d + K_I(\gamma(q) + \kappa) + K_D \dot{\gamma}_d] \quad (20)$$

In this way we have achieved the desired objective of adding a term dependent on q in the control signal.

We have the following simple proposition.

Proposition 2. Consider the underactuated mechanical system (1) satisfying Assumptions A1–A4 and A6, together with (3) and the PID-PBC (20), with (11), (17) and (18), verifying the well-posedness Assumption A5. Fix $q_u^* \in \mathbb{R}^s$ such that

$$\nabla V_u(q_u^*) = 0 \quad (21)$$

³ From Frobenius theorem (Spivak, 1999) this is a necessary and sufficient condition for solvability of the PDE (15).

and

$$\kappa = -\gamma(q^*), \quad (22)$$

where $q_a^* \in \mathbb{R}^m$ is arbitrary. Then, $(q, \dot{q}) = (q^*, 0)$ is an equilibrium point of the closed-loop system.

Proof. First, recall the system dynamics is given by (37) and (38) and notice from (4) that $D_{mu}(q_u, 0) = 0$. The assignable equilibrium set is then given as

$$\mathcal{E} := \{(q, \dot{q}) \in \mathbb{R}^n \times \mathbb{R}^n \mid \dot{q} = 0 \text{ and } \nabla V_u(q_u) = 0\},$$

with $u|_{(q^*, 0)} = 0$. Now, (21) ensures $(q^*, 0) \in \mathcal{E}$ for any $q_a^* \in \mathbb{R}^m$. Evaluating the controller signals (18), (20) at $\dot{q} = 0$ yields

$$u|_{\dot{q}=0} = -\frac{K_I}{k_e}(\gamma(q) + \kappa) \quad (23)$$

that, with the choice (22) is zero at $q = q^*$, completes the proof.

Remark 5. In Donaire, Mehra et al. (2016) and Romero et al. (2016a) it was assumed that the rows of $m_{au}(q_u)$ are *gradient mappings*, that is,

$$\nabla(m_{au}^\top(q_u) e_i) = [\nabla(m_{au}^\top(q_u) e_i)]^\top, \quad \forall i \in \bar{m}. \quad (24)$$

Under this condition, a *particular* solution of the PDE (15) that verifies (16) is

$$\gamma(q) = k_a q_a + (k_a - k_u) V_N(q_u),$$

where $V_N : \mathbb{R}^s \rightarrow \mathbb{R}^m$ is given by

$$V_N(q_u) = m_{aa}^{-1} \int_0^1 m_{au}(sq_u) q_u ds + V_N(0).$$

It is shown in Romero et al. (2016b) that the condition (24) is *equivalent* to having all Lie brackets of the columns of the matrix $L(q_u)$ equal to zero. This condition is *strictly stronger* than the involutivity Assumption A6—enlarging, in this way, the realm of applicability of the PID-PBC.

Remark 6. In our previous works (Donaire, Mehra et al., 2016; Romero et al., 2016a) it was proposed to implement the PID-PBC with the dynamic extension $\dot{x}_c$, that is, in the form (13). However, it was not clarified that, in this case, it is necessary to select the initial conditions of the controller state as a function of the systems initial conditions to ensure that $x_c(t) \equiv \gamma(t)$, $\forall t \geq 0$. On the other hand, if we implement it as a static state-feedback – as it is proposed in (20) – this restriction is avoided, as we only need to satisfy that $\bar{y}_d = \dot{x}_c \equiv \dot{\gamma}$.

4.2. Main stability result

To streamline the presentation of our main result we introduce the matrix $M_d : \mathbb{R}^q \rightarrow \mathbb{R}^{n \times n}$

$$M_d(q) := \begin{bmatrix} A(q) & B(q) \\ B^\top(q) & k_e k_a m_{aa} + k_a^2 \bar{K}_D(q) \end{bmatrix}, \quad (25)$$

where

$$\begin{aligned} A(q) &:= k_e k_u m_{uu}^s(q_u) + k_e k_a m_{au}^\top(q_u) m_{aa}^{-1} m_{au}(q_u) + \\ &\quad + (k_a - k_u)^2 m_{au}^\top(q_u) m_{aa}^{-1} \bar{K}_D(q) m_{aa}^{-1} m_{au}(q_u) \\ B(q) &:= k_e k_a m_{au}^\top(q_u) + k_a (k_a - k_u) m_{au}^\top(q_u) m_{aa}^{-1} \bar{K}_D(q) \\ \bar{K}_D(q) &:= \frac{1}{k_a^2} \nabla_{q_a} \gamma(q) K_D \nabla_{q_a}^\top \gamma(q), \end{aligned}$$

and the function $V_d : \mathbb{R}^n \rightarrow \mathbb{R}$

$$V_d(q) := k_e k_u V_u(q_u) + \frac{1}{2} \|\gamma(q) - \gamma(q^*)\|_{K_I}^2, \quad (26)$$

and introduce the assumption.

A7. The controller gains k_e, k_a, k_u, K_D and K_I are such that the matrix $M_d(q)$ defined in (25) is *positive definite* and the function $V_d(q)$ defined in (26) has an *isolated minimum* at $q^* \in \mathbb{R}^n$.

Proposition 3. Consider the underactuated mechanical system (1) satisfying Assumptions **A1–A4** and **A6**, in closed-loop with (3) and the PID-PBC (20), with (11), (17) and (18) verifying Assumptions **A5** and **A7**.

(i) The closed-loop system has a stable equilibrium at the desired point $(q, \dot{q}) = (q^*, 0)$ with Lyapunov function

$$H_d(q, \dot{q}) := \frac{1}{2} \dot{q}^T M_d(q_u) \dot{q} + V_d(q), \quad (27)$$

where $M_d(q_u)$ and $V_d(q_u)$ are defined in (25) and (26), respectively.

(ii) The equilibrium is asymptotically stable if the signal y_d (18) is a detectable output for the closed-loop system.

(iii) The stability properties are global if $V_d(q)$ is radially unbounded.

Proof. Some simple calculations show that the function $H_d(q, \dot{q})$ can be written as follows:

$$H_d(q, \dot{q}) = k_e[k_a H_a(q_u, \dot{q}) + k_u H_u(q_u, \dot{q}_u)] + \frac{1}{2} \|\bar{y}_d\|_{K_D}^2 + \frac{1}{2} \|\gamma(q) - \gamma(q^*)\|_{K_I}^2, \quad (28)$$

that, in view of Assumption **A7**, is positive definite. Computing its time derivative we get

$$\dot{H}_d = k_e k_a \dot{H}_a + k_e k_u \dot{H}_u + \bar{y}_d^T K_D \dot{\bar{y}}_d + \dot{\gamma}(q)^T K_I (\gamma(q) - \gamma(q^*))$$

and using (10), (12) and (20) with (22) yields

$$\dot{H}_d = -\|\bar{y}_d\|_{K_P}^2 \leq 0,$$

from which we conclude Lyapunov stability of $(q^*, 0)$. To prove (ii) notice that $\bar{y}_d = 0$ iff $y_d = 0$ therefore the standard detectability assumption can be imposed to y_d . Finally, (iii) invokes standard Lyapunov arguments (Khalil, 2002).

Remark 7. We elaborate on the remark made about the coefficient k_e in Section 3. For, we attract the readers attention to the presence of the term $k_e k_u$ in the first right hand term of (26). In most pendular systems q_u represents the pendulum angle and the potential energy function $V_u(q_u)$ has a *maximum* at its upward position, which is an unstable equilibrium. If the control objective is to swing up the pendulum and stabilise this equilibrium this maximum can be transformed into a minimum choosing the controller gains such that $k_e k_u < 0$. See the example in Subsection 6.2 of Romero et al. (2016b).

5. Extensions

In this section we present the following three extensions to Proposition 3.

- The proof that the cancellation of the potential energy term $V_a(q_a)$ of the preliminary feedback (3) can be obviated for a certain class of potential energy functions.
- The relaxation – in two directions – of the conditions imposed on $H_d(q, \dot{q})$ by Assumption **A7**.
 - Removing the requirement of positivity and carrying-out a LaSalle's analysis.
 - Replacing the quadratic integral term by a general function.

5.1. Removing the cancellation of $V_a(q_a)$

As clear from the derivations above the key step for the design of the PID-PBC is to prove that the mappings $\tau \mapsto y_u$ and $\tau \mapsto y_a$ are *passive*—avoiding the cancellation of the term $V_a(q_a)$. This fact is stated in the proposition below whose proof is given in Appendix B and requires the following assumption.

A8. The function $V_a(q_a)$ is of the form

$$V_a(q_a) = s_a^T q_a + c_0, \quad (29)$$

with $s_a \in \mathbb{R}^m$ and $c_0 \in \mathbb{R}$.

Proposition 4. Consider the underactuated mechanical system (1) satisfying Assumptions **A1–A4**, **A8** and (24). The operators $\tau \mapsto y_u$ and $\tau \mapsto y_a$ are passive with storage functions

$$\tilde{H}_u(q_u, \dot{q}_u) := \frac{1}{2} \dot{q}_u^T m_{uu}^s(q_u) \dot{q}_u + V_u(q_u) - V_0(q_u) \quad (30)$$

$$\tilde{H}_a(q, \dot{q}) := \frac{1}{2} \dot{q}^T M_a(q_u) \dot{q} + V_a(q_a) + V_0(q_u), \quad (31)$$

where

$$V_0(q_u(t)) = s_a^T \int_0^t y_u(s) ds + c_0, \quad (32)$$

where $c_0 \in \mathbb{R}$.

Remark 8. In contrast to Proposition 3, the proposition above does not include the nonlinearity cancellation due to the feedback (3). On the other hand, we impose Assumption **A8** and the integrability condition (24), that ensures $V_0(q_u)$ is well-defined.

5.2. Convergence analysis via LaSalle's principle

In this subsection we remove Assumption **A7** and we perform a LaSalle-based convergence analysis using the following – rather weak – assumption.

A9. The system is *strongly inertially coupled* (Spong, 1994), that is,

$$\text{rank}[m_{au}(q_u)] = s.$$

Proposition 5. Consider the underactuated mechanical system (1) satisfying Assumptions **A1–A4**, **A6** and **A9**, in closed-loop with (3) and the PID-PBC (20), with (11), (17) and (18) verifying Assumption **A5**. All bounded trajectories of the closed loop system verify

$$\lim_{t \rightarrow \infty} \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} q^* \\ 0 \end{bmatrix}.$$

Remark 9. Assumption **A7**, which imposes constraints on the PID-PBC gains to ensure positive definiteness of the function $H_d(q, \dot{q})$ – and, consequently, Lyapunov stability of the desired equilibrium – is conspicuous by its absence. The prize that is paid for this relaxation is that there is no guarantee that trajectories remain bounded. However, there are cases where boundedness of trajectories can be established invoking other (non Lyapunov-based) considerations.

Remark 10. We impose an additional condition on the coupling between the actuated and unactuated dynamics captured in Assumption **A9**. As discussed in Spong (1994) this is, essentially, a controllability condition and it requires that $s \leq m$. In Exercise E10.17 of Bullo and Lewis (2004) it is shown that this assumption is not coordinate-invariant nor is it related to stabilisability of the system. In the sense that there are strongly inertially coupled systems that cannot be stabilised using any kind of feedback.

5.3. Enlarging the class of integral actions

An additional degree of freedom to verify Assumption **A7** is obtained replacing the integral term in the controller (20) by

$$k_e \bar{u} = -K_P \bar{y}_d - \nabla_{\gamma} \Phi(\gamma) - K_D \dot{\bar{y}}_d,$$

where $\Phi : \mathbb{R}^m \rightarrow \mathbb{R}_+$ is an arbitrary function verifying $\Phi(\gamma(q^*)) = 0$ and using the function

$$\bar{V}_d(q) = k_e k_u V_u(q_u) + \Phi(\gamma(q)),$$

instead of (26) in the Lyapunov function (27). The details may be found in Romero et al. (2016b) and are omitted for brevity.

Remark 11. The three extensions above can be combined yielding a robust, higher performance controller that does not cancel the term $V_a(q_a)$, relaxes Assumption **A7** and allows for nonlinear integral action—e.g., using a saturation function.

6. Conclusions and future research

We have identified in this paper a class of underactuated mechanical systems whose constant position can be stabilised with a PID controller. Some of the assumptions that identify the class being coordinate-free involutivity conditions. It should be underscored that in view of the freedom in the choice of the signs of the constants entering into the control design, i.e., k_e , k_a and k_u , the proposed PID-PBC is far from being standard.

Given the popularity and simplicity of this controller, and the fact that the class contains some common benchmark examples, the result is of practical interest. Moreover, the performance of PID-PBC sometimes parallels the one of total energy shaping controllers like IDA PBC. For instance, it has been proved that in some benchmark examples (like the cart–pendulum system and the inertia wheel), the desired equilibrium has the same estimated domain of attraction for both designs (Donaire, Mehra et al., 2016; Romero et al., 2016a). See also the example of Romero et al. (2016b) where it is shown, via simulations, that the transient performance of the PID-PBC is far superior to the one of the total energy shaping controller reported in Bloch et al. (2001).

Besides the usual Lyapunov stability analysis, that imposes some constraints on the controller tuning gains to shape the energy function, a LaSalle-based study of attractivity has been carried removing the conditions on these gains. Two additional contributions of the paper are the proof that it is possible to obviate the cancellation of the actuated part of the potential energy – provided this is described by an affine function – and allowing for nonlinear integral action to improve its performance.

Current research is under way along the following directions.

- To replace Assumption **A8**, which is rather restrictive, by some integrability-like condition.
- Investigate alternatives to the classical LaSalle analysis used in the proof of Proposition 5, e.g., invoking Matrosov-like theorems of Scariotti, Praly, and Astolfi (2016). Also, to identify classes of systems for which it is possible to prove boundedness of trajectories without Lyapunov stability.
- Explore the effect on the inertia matrix and the potential energy of the change of coordinates – required in Assumption **A1** – to verify the key Assumptions **A2–A4**. Also, following the port-Hamiltonian approach to the problem adopted in Romero et al. (2016a), investigate other choices of factorisations of the inertia matrix to relax Assumptions **A2, A3** and **A6**.
- Further explore the relationship between total energy shaping PBC and the proposed PID-PBC as done in Donaire, Ortega, and Romero (2016), where it has been shown that the

PID-PBC of Donaire, Mehra et al. (2016) can be recast as a classical IDA-PBC with generalised dissipative forces.

Appendix A. Proof of Proposition 1

To establish the proof we, first, invoke **A1–A4** to rewrite the system (1) in a more suitable form. Towards this end, we define the mapping

$$\alpha(q_u, \dot{q}) := \nabla_{q_u} [m_{uu} \dot{q}_u + m_{au}^T(q_u) \dot{q}_a]. \quad (33)$$

Therefore, under Assumptions **A2** and **A3**,

$$\nabla_q(M\dot{q}) = \begin{bmatrix} \alpha(q_u, \dot{q}) & 0_{s \times m} \\ \nabla_{q_u} [m_{au}(q_u) \dot{q}_u] & 0_{m \times m} \end{bmatrix}. \quad (34)$$

Now, we recall the following well-known identity given in Eq. 3.19, page 72 of Kelly et al. (2005)

$$C(q, \dot{q})\dot{q} = \left[\nabla_q(M(q)\dot{q}) - \frac{1}{2} \nabla_q^T(M(q)\dot{q}) \right] \dot{q}. \quad (35)$$

Hence, replacing (34) in (35) we get⁴

$$\begin{aligned} C\dot{q} &= \begin{bmatrix} \alpha - \frac{1}{2} \alpha^T & -\frac{1}{2} \nabla_{q_u}^T(m_{au} \dot{q}_u) \\ \nabla_{q_u}(m_{au} \dot{q}_u) & 0_{m \times m} \end{bmatrix} \dot{q} \\ &= \begin{bmatrix} \alpha \dot{q}_u - \frac{1}{2} [\alpha^T \dot{q}_u + \nabla_{q_u}^T(m_{au} \dot{q}_u) \dot{q}_a] \\ \nabla_{q_u}(m_{au} \dot{q}_u) \dot{q}_u \end{bmatrix} \\ &= \begin{bmatrix} C_{mu}(q_u, \dot{q}_u) \dot{q}_u + D_{mu}(q_u, \dot{q}) \\ \nabla_{q_u}(m_{au} \dot{q}_u) \dot{q}_u \end{bmatrix} \end{aligned}$$

with

$$\begin{aligned} C_{mu}(q_u, \dot{q}_u) &:= \nabla_{q_u}(m_{uu} \dot{q}_u) - \frac{1}{2} \nabla_{q_u}^T(m_{uu} \dot{q}_u) \\ D_{mu}(q_u, \dot{q}) &:= \nabla_{q_u}(m_{au}^T \dot{q}_a) \dot{q}_u - \nabla_{q_u}^T(m_{au}^T \dot{q}_a) \dot{q}_u, \end{aligned}$$

where we used the definition (33) and the fact that

$$\nabla_{q_u}^T(m_{au}^T \dot{q}_a) \dot{q}_u = \nabla_{q_u}^T(m_{au} \dot{q}_u) \dot{q}_a. \quad (36)$$

Furthermore, invoking Assumption **A4**, the system (1) with the inner-loop control (3) can be written as

$$m_{uu} \ddot{q}_u = -m_{au}^T \ddot{q}_a - \beta(q_u, \dot{q}) \quad (37)$$

$$\ddot{q}_a = -m_{aa}^{-1} [m_{au} \ddot{q}_u + \nabla_{q_u}(m_{au} \dot{q}_u) \dot{q}_u - u]. \quad (38)$$

with

$$\beta(q_u, \dot{q}) := C_{mu} \dot{q}_u + D_{mu} + \nabla V_u. \quad (39)$$

Moreover, replacing (38) in (37), we have

$$m_{uu}^s \ddot{q}_u = m_{au}^T m_{aa}^{-1} [\nabla_{q_u}(m_{au} \dot{q}_u) \dot{q}_u - u] - \beta \quad (40)$$

We proceed now to prove (8). The time derivative of the storage function (5) along the solution of (40) yields

$$\begin{aligned} \dot{H}_u &= \dot{q}_u^T m_{au}^T m_{aa}^{-1} [\nabla_{q_u}(m_{au} \dot{q}_u) \dot{q}_u - u] \\ &\quad - \dot{q}_u^T (C_{mu} \dot{q}_u + D_{mu}) + \frac{1}{2} \dot{q}_u^T \dot{m}_{uu}^s \dot{q}_u := u^T y_u \end{aligned}$$

⁴ In the sequel the arguments of some mappings are omitted to simplify the notation.

where we used (4), Assumption A3, (36), the definition of m_{uu}^s given in (7) and the following identities:

$$\begin{aligned} \dot{q}_u^\top D_{mu} &= 0 \\ \dot{q}_u^\top [\dot{m}_{uu} - 2C_{mu}] \dot{q}_u &= 0 \\ \dot{q}_u^\top \left[\frac{d}{dt} (m_{au}^\top m_{aa}^{-1} m_{au}) - 2m_{au}^\top m_{aa}^{-1} \nabla_{q_u} (m_{au} \dot{q}_u) \right] \dot{q}_u &= 0. \end{aligned}$$

To prove (8) we, first, notice that the storage function (6) can be rewritten as follows:

$$\begin{aligned} H_a &= \frac{1}{2} \dot{q}^\top M_a \dot{q} + H_u - H_u \\ &= \frac{1}{2} \dot{q}^\top M \dot{q} - H_u + V_u(q_u). \end{aligned} \quad (41)$$

Hence, computing the time derivative of (41) along the solutions of (1) with the inner-loop control (3) yields

$$\begin{aligned} \dot{H}_a &= \frac{1}{2} \dot{q}^\top \dot{M} \dot{q} + \dot{q}^\top M \ddot{q} - \dot{H}_u + \nabla V_u^\top \dot{q}_u \\ &= \dot{q}_a^\top u - y_u^\top u \\ &= u^\top y_a \end{aligned} \quad (42)$$

where we used (35) and (41). $\square\square\square$

Appendix B. Proof of Proposition 4

The proof follows closely the proof of Proposition 1, with the difference of the inclusion of $V_a(q_a)$ via Assumption A8. The time derivative of the storage function (30) along the solutions of (40) is given by

$$\dot{H}_u = \dot{H}_u - \dot{V}_0 = u^\top y_u - s_a^\top y_u = \tau^\top y_u$$

where we used (41), (32) and Assumption A8, (3) to obtain the second and the third equality, respectively.

We now proceed to prove passivity of the operator $\tau \mapsto y_a$. Towards this end, we compute its time derivative along the solutions of (1) yielding

$$\dot{H}_a = \dot{H}_a + \dot{V}_a + \dot{V}_0 = u^\top y_a + s_a^\top y_u + s_a^\top \dot{q}_a = \tau^\top y_a$$

where we used Assumption A8, (32) and (42) to obtain the second equality, and (3), (4) to obtain the third one. $\square\square\square$

Appendix C. Proof of Proposition 5

It is shown in Section 4.2 that, independently of Assumption A7, the function $H_d(q, \dot{q})$ given in (27) verifies

$$\dot{H}_d = -\|y_d\|_{K_p}^2,$$

where y_d is defined in (18). Invoking LaSalle's invariance principle (Khalil, 2002), we can conclude that all bounded trajectories converge to the maximum invariant set contained in

$$\mathcal{X} := \{(q, \dot{q}) \in \mathbb{R}^n \times \mathbb{R}^n \mid k_a \dot{q}_a + (k_a - k_u) m_{aa}^{-1} m_{au}(q_u) \dot{q}_u \equiv 0\}.$$

Now, from the PID controller (13) it is clear that $y_d \equiv 0$ implies $x_c = c_0$ and, consequently, $u = c_0$ where here, and throughout the rest of the proof, c_0 denotes a (generic) constant vector (of suitable dimensions).

We now compute the time derivative of y_d , which results as follows:

$$\begin{aligned} \dot{y}_d &= (k_a - k_u) m_{aa}^{-1} \nabla_{q_u} (m_{au} \dot{q}_u) \dot{q}_u + \\ &\quad + (k_a - k_u) m_{aa}^{-1} m_{au} \ddot{q}_u + k_a \ddot{q}_a \\ &= (k_a - k_u) m_{aa}^{-1} u + k_u \ddot{q}_a, \end{aligned}$$

where we have used (38) to get the second identity. From the equation above $\dot{y}_d = 0$ and $u = c_0$ we conclude that, in $\mathcal{X}$, $\dot{q}_a = c_0$ also. Since we are looking only at bounded trajectories this implies that $\dot{q}_a = \dot{q}_a = 0$ and $q_a = c_0$. Setting $\dot{q}_a = 0$ and $y_d = 0$ in (18) yields $m_{aa}^{-1} m_{au}(q_u) \dot{q}_u = 0$ that, invoking the full rank Assumption A9, allows us to conclude that $\dot{q}_u = 0$, which implies that $q_u = c_0$. This proves that the only bounded trajectories living in the residual set $\mathcal{X}$ are of the form $(q(t), \dot{q}(t)) = (\bar{q}, 0)$, with $\bar{q}$ a constant vector.

It only remains to prove that the only point $(\bar{q}, 0)$, which is invariant to the dynamics, is with $\bar{q} = q^*$. This follows from injectivity of $\nabla V_u(q_u)$ that, setting it equal to zero, ensures $\bar{q}_u = q_u^*$ and the following chain of implications:

$$\begin{aligned} (q(t), \dot{q}(t)) = (\bar{q}, 0) &\Rightarrow u = 0 \\ &\Rightarrow \bar{q}_a = q_a^* \\ &\Rightarrow \bar{q} = q^*, \end{aligned}$$

where we have used (38) in the first implication and (23) together with $\bar{q}_u = q_u^*$ in the second one. $\square\square\square$

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