A threshold uncertainty investment model for the Netherlands

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This paper presents a threshold uncertainty investment model for Dutch firms. The proposed uncertainty measure is constructed as an empirical proxy for the standard real options multiple. The uncertainty measure serves as the threshold variable in estimating a piecewise linear accelerator investment model using Hansen’s panel data threshold estimation procedure. It is found that in the regime of low uncertainty in which the empirical proxy for the real options multiple is below the estimated threshold, the estimated accelerator effect on investment is higher than that in the regime of high uncertainty. The result indicates that firms delay investment due to positive values of waiting.

I. Introduction

Recent literature pays a lot of attention to the theoretical relation between investment and uncertainty. The traditional criterion for making investment decisions is the Net Present Value (NPV) rule. If the expected NPV of a project is positive, it should be undertaken. The NPV-decision is in principle a static decision, since there is no timing flexibility. Correspondingly, the optimal neoclassical investment rule is simply the equality between the marginal revenue of capital and the marginal cost of capital ($MR = MC$). In this line of thinking, there is no role for uncertainty. The recently popularized real options theory of investment assumes that a firm holds call options on the sequence of net returns the investment project is expected to generate (Dixit and Pindyck, 1994). According to the real options theory of investment the option value of the investment opportunity is a component of the marginal revenue of capital, without affecting the general rule ($MR = MC$). The intuition is that once a firm decides to invest rightaway, the opportunity of obtaining more information about uncertain variables is lost. It is equivalent to the situation that the firm gives up the possibility to improve the outcome if necessary. The general empirical conclusion from this class of models is that one is likely to observe a negative relation between investment and uncertainty.

Despite its theoretical attractiveness, empirical tests of the real options theory of investment are scant in the literature. One difficulty concerns the construction of the empirical proxy for the real options effect. Caballero and Pindyck (1996) and Pindyck and Solimano (1993) calculate the investment threshold by using extreme values of the marginal profitability of capital. They regress the computed threshold on the drift and the standard deviation of the marginal profitability of capital (as a measure of uncertainty) to test the effect of uncertainty on investment. Pattillo (1998) tests the threshold effect on investment for manufacturing firms in Ghana using survey data. She estimates the investment threshold in a reduced-form equation in which the investment trigger is a function of both demand uncertainty and the cost of capital variables. Although the ideas of these empirical papers are based on the real options theory of investment, i.e. uncertainty directly increases the investment threshold and through the threshold it
depresses investment, the investment threshold is not derived from the structure of the real options model of investment.

In this paper, we propose a simple empirical way to approximate the effects of real options in a simple piecewise linear accelerator model. More specifically, we construct an uncertainty measure as an empirical proxy for the standard real options multiple. We employ the constructed uncertainty measure as the threshold variable to test the threshold effect of uncertainty on investment using the threshold panel data estimation technique of Hansen (1999). Up to a specific threshold the investment decision is linear homogeneous in its value driver, in our case, the accelerator. If the threshold is hit, we should observe a change of the nature of the equation. Using a balanced panel of 55 listed Dutch manufacturing firms during the period 1985–1997, we find threshold effects of uncertainty on firm investment. When our uncertainty measure exceeds the estimated threshold value, the accelerator effect on investment decreases. Since our uncertainty measure captures the real options effect, the results indicate the investment waiting behaviour.

The remainder of the paper is organized as follows. In Section II we explain the construction of our uncertainty measure, an empirical proxy for the standard real options multiple. Section III sets up the empirical investment model and sketches Hansen’s (1999) threshold estimation procedure. The estimation results are discussed in Section IV and Section V concludes.

II. Uncertainty Measure

In this section, we discuss our approach to construct an empirical proxy for the standard real options multiple as the measure of uncertainty. The complete derivation of the standard real options multiple is presented in Appendix A. A risk-averse firm considers whether to start a new investment project in the current year. Suppose that the value of the firm is the expected discounted value of future profits that is generated by the investment project. Assume that profits ($\pi$) are stochastic due to uncertain operating conditions and it follows a geometric Brownian motion. According to the real options model the threshold value of profits $\pi^*$ is given by

$$\pi^* = \frac{\kappa}{\kappa - 1} (\rho - \mu_\pi) IC_t$$

where $\kappa$ is the positive root of the quadratic equation that defines the value of the investment project.

$$\kappa = \frac{1}{2} - \frac{\mu_\pi}{\sigma^2_\pi} + \left[ \left( \frac{\mu_\pi}{\sigma^2_\pi} - \frac{1}{2} \right)^2 + \frac{2 \rho \sigma^2_\pi}{\sigma^4_\pi} \right]^{1/2}$$

$\mu_\pi$, $\sigma^2_\pi$ are the drift and the variance of the profit process, $\rho$ is the risk-adjusted discount rate of the firm, and $IC$ denotes the investment cost. The term $\kappa/(\kappa - 1)$ is the so-called ‘option value multiple’. According to the neoclassical investment theory the project is undertaken if the expected discounted return from investment is not smaller than the investment cost: the NPV rule indicates to invest if $\pi^* \geq (\rho - \mu_\pi) IC_t$. The real options theory of investment suggests to invest if the discounted expected return from investment is larger than the investment outlay, or to invest if $\pi^* \geq \kappa/(\kappa - 1) (\rho - \mu_\pi) IC_t$.

In this case a higher expected return is required to compensate the possible loss due to uncertainty, since $\kappa/(\kappa - 1) > 1$. As a consequence, the possible delay of investment makes fewer projects accepted as compared to the acceptable projects based on the NPV rule.

The definition of $\kappa$ shows that the key parameter that represents the real options effect depends on the distribution of profits and the risk-adjusted discount rate of the firm. Therefore, the problem of constructing an empirical proxy for the real options multiple $\kappa/(\kappa - 1)$ is in fact reduced to finding a way to model the expected distribution of the stochastic profit process.

To construct the empirical proxy for the real option parameter $\kappa$ we need the drift and the variance of the profit process, and the risk-adjusted discount rate of the firm (see Equation 2). The drift and the variance of profits should be forward-looking. When making irreversible investment decisions, the firm assigns subjective probabilities to the future development of profits, on the basis of which the firm decides whether to postpone investment. If the objective distribution of profits exists, historical data can be employed to model the firm’s expectations on the future development of profits. Using historical data to construct the proxy for the expected movement of stochastic variables is now a standard approach in the empirical investment literature (Lensink et al., 2001, Chapter 6). We take into account all historical information on the movement of profits available to the firm. Firms are assumed to update their expectations on the future development of profits every year using the whole history of the
profit process. More specifically, we calculate the drift of profits for the current year by taking the average of the growth rate of profits over the whole sample period in the past. For example, in 1988 the drift is approximated by the average of the growth rates using the 1986, 1987 and 1988 information. For the year 1989 the drift is computed on the basis of average growth rates of profits in the years 1986 to 1989, etc. Similarly, we calculate the variance of the profit process for the current year by taking the variance of the growth rate of profits over the whole past sample period.

Turning to the risk-adjusted discount rate, in the standard real options model of investment it is often assumed that capital markets are sufficiently complete, i.e. the stochastic fluctuations in the value of the underlying asset are spanned by the fluctuations in the value of tradeable assets in financial markets. Thus, stochastic changes in the value of the firm are captured by existing financial assets. Therefore, it is not necessary to compute the risk-adjusted discount rate itself. Instead the risk-adjusted expected rate of return on the underlying asset is often modeled by using the capital asset pricing model (CAPM) (Dixit and Pindyck, 1994, Chapters 5 and 6). Applying the CAPM model to construct the risk-adjusted expected rate of return for the underlying asset requires the data on the market price of risk and the data on the coefficient of correlation between returns on the underlying asset and the whole market portfolio. The data on the market price of risk of the underlying assets is not available at the firm level. Moreover, we believe that it is more realistic to assume that some financial markets are imperfect. For example, the irreversibility property of fixed investment implies that once invested the investor loses some value of the investment project due to sunk costs and inefficient capital markets (Pindyck, 1991). Given that the degree of irreversibility is unobservable, it is difficult to find a perfect substitute for the underlying investment opportunity from the existing assets. Therefore, we take the alternative to proxy the risk-adjusted discount rate of the firm. As pointed out by Dixit and Pindyck (1994), without the spanning assumption (where the dynamic programming approach applies) all investment is financed by equity. If the firm is assumed to be risk-neutral, the discount rate is just the risk-free interest rate. When risk-aversion is assumed, the convenience yield equals the dividend ratio paid by the underlying asset (for example, the investment project). Therefore, the risk-adjusted discount rate of the firm is the sum of the average growth rate of the asset and the dividend ratio paid by the underlying asset. Using the symbols of our model: $\rho = \mu_x + \omega$, where $\omega$ is the payout ratio of the firm. The payout ratio is constructed as the ratio of dividend per share to net profit per share. Therefore, in the real options investment model without the spanning assumption, a risk-averse firm discounts the investment opportunity partially due to the fact that the asset grows itself and partially because of the convenience yield by simply holding the dividend-paying asset.

Based on the above information the key parameter of the real options model of investment $\kappa$ and the real options multiple $\kappa/(\kappa - 1)$ are constructed for each firm each year. The original data covers a balanced panel of 55 Dutch firms during the period 1985–1997. These firms are all listed on the Amsterdam Stock Exchanges (AEX). The quoted firms are relatively big in size. Historically, a majority of the Dutch listed firms is internationally oriented. Many of the sample firms can be considered to be monopolistic competitors on a world scale. The domestic firms in the sample are also relatively large compared to the average firm size in the industry. These sample characteristics enable us to apply data to the standard real options model of investment, in which imperfect competition is one of important assumptions of the model. The mean value of the constructed proxy for the real options multiple is 2.728 with a standard deviation of 4.431 (see Table 1). We observe that the empirical proxy for the real options multiple varies a lot across firms, which indicates that the real options multiple is firm-specific. Therefore, the impact of the real options on investment depends on firm-specific characteristics.

III. A Piecewise Linear Investment Model

We start with the standard accelerator investment model in which the ratio of net investment to the beginning-of-period capital stock ($NI/K$) is explained by the growth rate of sales ($S$). Dummy variables capturing fixed effects of the firm and time effects are included. We omit the fixed effects and time effects from the equations and do not report the relevant estimation results to save space.

In the previous section we explained the construction of our uncertainty measure, which captures the real options effect. To test the threshold effect of uncertainty, we introduce piecewise linearity into the standard investment model by treating the constructed uncertainty measure as the threshold variable (or regime switching variable) that affects the impact of the accelerator. We estimate threshold
regression models for $i = 1, \ldots, n$ firms and $t = 1, \ldots, T$ observations of the following form:

$$
\left( \frac{NI}{K} \right)_{it} = \alpha z_{it-j} + \beta_1 S_{it} I(M_{it} \leq \theta) + \beta_2 S_{it} I(M_{it} > \theta) + \epsilon_{it} \quad j = 0, 1 \tag{3}
$$

where $z$ is the vector of regime-independent (or control) variables, such as cash flow; $S$ is the regime dependent variable taken to be the growth rate of sales; $\alpha$, $\beta_1$, and $\beta_2$ are (vectors of) parameters; and the error term $\epsilon_{it}$ is iid with mean zero and finite variance.\(^1\) The empirical proxy for the real options multiple as defined in the previous section

$$
M_{it} = \left( \frac{\kappa}{\kappa - 1} \right)_{it}
$$

is the threshold variable and $\theta$ is the threshold value to be estimated. $I$ is the indicator function, which has the value one if the argument is true and zero otherwise. As explained above, the real options multiple captures the information on uncertainty the firm faces and its risk attitude. If the risk attitude of the firm does not change very often, the real options multiple varies mainly with uncertainty. The threshold variable defines two regimes: a low uncertainty regime with $M_{it} \leq \theta$ and a high uncertainty regime with $M_{it} > \theta$. Based on the predictions of the standard real options theory of investment, we expect that when uncertainty is below the threshold, the impact of the real options on investment is low and hence the firm is probably investing more. But if uncertainty exceeds the threshold, the real options effect leads a firm to delay investment. This suggests that the estimated coefficients for $\beta_1$ and $\beta_2$ are expected to differ. In other words, sales growth may have different effects on firm investment depending on the magnitude of the real options effect.

The empirical model is estimated by conditional least squares. For that purpose the observations are sorted on the threshold variable and the sums of squared residuals are computed for all values of the threshold variable. The optimal value of the threshold variable is the value that minimizes the sum of squared residuals. The optimal parameter estimates are the estimated $\alpha$s and $\beta$s that belong to this optimal threshold value. An important question is whether the threshold regression model of Equation 3 is statistically significant to its linear counterpart, which has the null hypothesis $H_0$: $\beta_1 = \beta_2$. Hence the threshold parameter is not defined under the null hypothesis, which makes the testing problem complex. However, Hansen (1996) shows that asymptotically valid $p$-values can be constructed by bootstrapping.

Valid confidence intervals for the threshold parameter can be based on the likelihood ratio (or $F$) statistic $LR(\theta) = (S(\hat{\theta}) - S(\hat{\theta})) / \hat{\sigma}^2$, which tests the null hypothesis $H_0$: $\theta = 0$. Here $S(\hat{\theta})$ is the sum of squared errors of the estimated threshold regression when the threshold parameter equals $\theta$, $S(\hat{\theta})$ is the sum of squared residuals belonging to the optimal threshold parameter $\hat{\theta}$, and $\hat{\sigma}^2$ is the residual variance belonging to the optimal threshold parameter $\hat{\theta}$. The likelihood ratio statistic is equal to zero at $\theta = \hat{\theta}$. Confidence intervals for the threshold parameter can be constructed by inverting the distribution function of the likelihood ratio statistic. A graphical method to find the confidence interval of the threshold parameter is to plot the likelihood ratio statistic $LR(\theta)$ against all values of $\theta$ and to check for which values of $\theta$ crosses the horizon line that shows the confidence

\(^1\) As noted by Hansen (1999) this assumption excludes lagged dependent variables from the model.
level of the test. Confidence intervals of the other parameters in the threshold regression, the $\alpha$s and $\beta$s, can be approximated by the conventional normal approximation as if the threshold estimate $\hat{\theta}$ were the true value.

**IV. Results**

We use a balanced panel of 55 Dutch listed firms over the period 1985–1997. After computing the changes in the capital stock and the annual growth rate of sales, the first year observation is lost for each firm. Due to the construction of the empirical proxy for the real options multiple, the time span is further reduced to 1988–1997 in the threshold estimations. Information on the source of the data is available in Appendix B. Table 1 lists some descriptive statistics of the variables that enter the investment equations.

We start by estimating the linear models (the standard accelerator model and its variants). The results are shown in Table 2. Column 1 of Table 2 corresponds to the standard accelerator model of investment. In column 2, we include lagged cash flow. We further add average $Q$ as an additional control variable in the investment equation in column 3. Both cash flow and average $Q$ are standard explanatory variables in the investment equation. To eliminate the multicolinearity problem we use cash flow lagged one year. Since average $Q$ is already a forward looking variable we use the contemporary value of average $Q$ in the estimations.

In all three linear models the estimated coefficient for sales growth is significantly positive as expected. The magnitude of the investment accelerator does not differ across estimations in the linear models. The standardized coefficients of the estimated accelerator effect are shown by the bold figures under the brackets. The standardized coefficient of sales growth is calculated by multiplying the relevant regression coefficient by the standard deviation of sales growth and then dividing it by the standard deviation of the dependent variable in the estimations. On average the linear effect (standardized) of sales growth on the investment rate is 0.14, which implies that a one standard deviation increase in sales growth induces 0.14 standard deviation increase in the investment rate. The estimated coefficient for the lagged cash flow is significant with the positive sign, consistent with the literature. However, the contemporary average $Q$ is not significant.

Next we treat the investment models as piecewise linear by introducing the threshold effect of uncertainty. The empirical proxy for the real options multiple constructed in Section II is used as the uncertainty measure and serves as the threshold variable in the Hansen (1999) panel data threshold estimation procedure. Table 3 shows the results of estimating the threshold model of Equation 3. Column 1 presents the threshold estimation results when there is no other control variable except for fixed effects and time effects dummies. In column 2 of Table 3, we control for lagged cash flow, while in column 3 of Table 3, both the lagged cash flow and

<table>
<thead>
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<th>Table 2. Estimation results: linear models</th>
</tr>
</thead>
</table>

\[
NI_{it} = \alpha_1 S_{it} + \alpha_2 CF_{i,t-1} + \alpha_3 Q_{it} + e_{it}
\]

<table>
<thead>
<tr>
<th>Regressor</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{it}$</td>
<td>0.197</td>
<td>0.199</td>
<td>0.199</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.021)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>$CF_{i,t-1}$</td>
<td></td>
<td>0.139</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.141)</td>
<td></td>
</tr>
<tr>
<td>$Q_{it}$</td>
<td>0.139</td>
<td>0.230</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>(0.141)</td>
<td>(0.080)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>SSR</td>
<td>9.967</td>
<td>9.666</td>
<td>9.658</td>
</tr>
<tr>
<td>No. of obs.</td>
<td>495</td>
<td>495</td>
<td>495</td>
</tr>
</tbody>
</table>

**Notes:**

1. Data source: *Jaarboek van Nederlandse Ondernemingen*.
2. Bold figures below $t$-statistics are the standardised coefficients.
3. Explanation of variables:
   - $NI$: ratio of net investment to the beginning-of-period capital stock.
   - $S$: growth rate of sales.
   - $M$: the constructed empirical real options multiple.
   - $Q$: average $Q$.
   - $CF$: cash flow scaled by the beginning-of-period capital stock.
the contemporary $Q$ are controlled for. Some observations deserve attention. First, we do find evidence of threshold effects on investment at the 10% significant level. The $p$-value of the test for the threshold effect, which is based on 300 bootstrap replications, indicates the existence of one threshold value: the null hypothesis of linearity can be rejected (at the 10% significance level) in all regressions. For all specifications we also investigate the possibility of a second threshold value. Tests for a second threshold value, however, are negative. Second, the estimated coefficient of sales growth varies between the two regimes. When uncertainty is lower than the estimated threshold, the positive effect of sales on net investment is larger; while the positive effect of sales on investment is much lower if uncertainty exceeds the threshold. More specifically, for low levels of uncertainty (i.e. $M_{it} \leq 1.72$) the estimated accelerator effect is on average 2.81 (unstandardized) and 1.21 (standardized) times higher than that for high levels of uncertainty ($M_{it} > 1.72$). The estimated threshold value of uncertainty is equal to 1.72. This result is important because it provides the evidence supporting the notion that the firm probably delays new investment due to the positive value of waiting. It suggests that the investment–sales relation-ship may be piecewise linear if the threshold effect of uncertainty is taken into account.\(^2\) Third, we also notice an important difference in the estimated accelerator effect between the linear and piecewise linear models. Comparing the estimated coefficient for sales growth, we observe that the standardized linear accelerator effect on investment is, in general, lower than that in the piecewise linear models. As shown in Table 3, when uncertainty is below the estimated threshold, one standard deviation increase in the sales growth is associated with 0.43 standard deviation increase in the investment rate, which is much higher than the linear effect (0.14). When uncertainty is above the estimated threshold, one standard deviation increase in the sales growth is associated with 0.35 standard deviation increase

\[^2\text{In a non-reported estimation, we treat } Q \text{ as the regime-dependent variable in the estimations. We notice that when the uncertainty measure is smaller than the threshold value, the impact of } Q \text{ is insignificant, while when the uncertainty proxy is higher than the estimated threshold, } Q \text{ has a significantly negative effect on investment, which may indicate the investment waiting behaviour. However, although the } p\text{-values of the investment models indicate that there is a threshold effect, the results are less attractive. We find a negative impact of } Q \text{ on investment for high levels of uncertainty } (M_{it} > 3.54). \text{ For low levels of uncertainty } (M_{it} \leq 3.54) Q \text{ exerts no influence on investment.}\]

### Table 3. Threshold estimation results

\[
NI_{it} = \beta_1 x_{it} I(M_{it} \leq \theta) + \beta_2 x_{it} I(M_{it} > \theta) + \alpha_1 CF_{i,t-1} + \alpha_2 Q_{it} + \epsilon_{it}
\]

<table>
<thead>
<tr>
<th>Regressor</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{it}(M_{it} \leq \theta)$</td>
<td>0.489</td>
<td>0.474</td>
<td>0.476</td>
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<tr>
<td></td>
<td>(0.081)</td>
<td>(0.079)</td>
<td>(0.078)</td>
</tr>
<tr>
<td>$S_{it}(M_{it} &gt; \theta)$</td>
<td>0.168</td>
<td>0.172</td>
<td>0.172</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>$CT_{i,t-1}$</td>
<td>0.350</td>
<td>0.358</td>
<td>0.358</td>
</tr>
<tr>
<td></td>
<td>(0.080)</td>
<td>(0.078)</td>
<td>(0.078)</td>
</tr>
<tr>
<td>$Q_{it}$</td>
<td>0.012</td>
<td>0.012</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.016)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>1.720</td>
<td>1.720</td>
<td>1.720</td>
</tr>
<tr>
<td>95% interval</td>
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<td>(1.528, 1.752)</td>
<td>(1.528, 1.752)</td>
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<td>LR-statistic</td>
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<tr>
<td>$p$-value</td>
<td>0.073</td>
<td>0.080</td>
<td>0.067</td>
</tr>
</tbody>
</table>

**Notes:**
1. Data source: *Jaarboek van Nederlandse Ondernemingen*.
2. Bold figures below $t$-statistics are the standardised coefficients.
3. Explanation of variables:
   - $NI$: ratio of net investment to the beginning-of-period capital stock.
   - $S$: growth rate of sales.
   - $M$: the constructed empirical real options multiple.
   - $Q$: average $Q$.
   - $CF$: cash flow scaled by the beginning-of-period capital stock.
in the investment rate, which is also higher than the linear effect. Therefore, by treating the accelerator model as a linear model, the impact of sales on investment is reduced. If the threshold effect of uncertainty is included in the accelerator investment model, our estimations suggest that sales have, in fact, a much more important impact on investment.

To summarize, Table 3 provides evidence of the threshold effect of uncertainty on the investment of Dutch firms. Sales growth is a standard explanatory variable in the investment equation. In the literature the impact of sales growth is often tested based on the assumption that the relationship between investment and sales is linear. Our results indicate that the impact of sales on firm investment may be piecewise linear. We introduce piecewise-linearity by allowing the threshold value of uncertainty to affect the investment accelerator. As we find, when the constructed uncertainty measure is high (beyond the estimated threshold), investment is discouraged compared to the regime in which uncertainty is low. This result supports the existence of the real options effect on investment. It suggests that the firm is concerned with the options value of investment, if the options value is high enough, the firm delays new investment.

V. Conclusions

We document the threshold effect of uncertainty on investment using a panel of Dutch listed firms over the period 1985–1997. An empirical proxy for the standard real options multiple is used as the uncertainty measure and serves as the threshold variable in an empirical accelerator-type of investment model using the panel data threshold estimation procedure of Hansen (1999). We find clear evidence showing that when uncertainty is below the estimated threshold, the investment accelerator effect is larger; while when uncertainty is above the estimated threshold, the accelerator effect decreases sharply. Since our uncertainty measure captures the real options effects, these results indicate the investment waiting behaviour of the sample firms.

One caveat is that the real options model is ideally applied to analysing the investment behaviour at the project level. In this paper we find evidence of the real options effect on investment using firm-level data. We believe that the real options effect would be stronger if project-level investment data is applied.

Acknowledgements

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References


Appendix A: Derivation of the Real Options Multiple

An empirical proxy for the real options multiple is used as the measure of uncertainty in the paper. Below we present the derivation of the real options multiple based on the standard real options model of investment (Dixit and Pindyck, 1994, Chapter 5). A risk-averse firm considers whether to start a new capital investment project in the current year. Suppose that the value of the firm is the expected value of the profit stream that is generated by the new capital. The profit process is stochastic due to uncertain operating conditions, which is assumed to follow a geometric Brownian motion:

\[
\frac{d\pi}{\pi} = \mu_{\tau} dt + \sigma_{\tau} dz \tag{A1}
\]

where \(\pi\) is the profit generated by the investment; \(dz\) is the incremental of a standard Wiener process, with \(E[dz] = 0\), and \(E[(dz)^2] = dt\); \(\mu_{\tau}\) is the drift and \(\sigma_{\tau}\) is the volatility of profit.
is the standard deviation of profit. The value of the firm fluctuates with the changes in profits. Suppose that the firm has the option to postpone the current investment. In this case the investment outlay is saved. If the firm decides to invest right now, the value of the firm will be the discounted present value of future profits generated from the investment. We denote the expected value of the firm if it invests now as \( V_{\text{now}}(\pi) \), then:

\[
V_{\text{now}}(\pi) = \frac{\pi}{\rho - \mu_\pi}
\]  

(A3)

This is the discounted present value of the firm if it starts to invest in the current year. However, if the firm decides to postpone the investment, the present value of the firm is the value of the investment opportunity. If we denote the present value of the firm in case of waiting as \( V_{\text{wait}}(\pi) \), it needs to satisfy the Bellman equation:

\[
\rho V_{\text{wait}}(\pi) dt = E[dV_{\text{wait}}(\pi)]
\]  

(A4)

As widely documented in the literature (Dixit and Pindyck, 1994), by Ito’s Lemma and using (A1), we can solve for \( E[dV_{\text{wait}}(\pi)] \) and (A4) becomes:

\[
\frac{1}{2} \sigma_\pi^2 \pi^2 V''_{\text{wait}}(\pi) + \mu_\pi \pi V'_{\text{wait}}(\pi) - \rho V_{\text{wait}}(\pi) = 0
\]  

(A5)

where \( V''_{\text{wait}}(\pi), V'_{\text{wait}}(\pi) \) are the first- and second-order derivatives of the value of the firm with respect to profits, respectively. The solution to the differential equation (A5) must satisfy the boundary condition:

\[
\lim_{\pi \to 0} V_{\text{wait}}(\pi) = 0
\]  

(A6)

As widely proved in the literature, the solution is:

\[
V_{\text{wait}}(\pi) = A \pi^k
\]  

(A7)

where \( A \) is a constant and \( k \) is the positive root of the characteristic equation of (A5):

\[
k = \frac{1}{2} - \frac{\mu_\pi}{\sigma_\pi^2} + \left[ \left( \frac{\mu_\pi}{\sigma_\pi^2} - \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma_\pi^2} \right]^{1/2}
\]  

(A8)

To solve the threshold level of profits, both the value matching and smooth pasting conditions have to be satisfied:

\[
V_{\text{wait}}(\pi^*) = V_{\text{now}}(\pi^*) - IC
\]  

(A9)

and

\[
V'_{\text{wait}}(\pi^*) = V'_{\text{now}}(\pi^*)
\]  

(A10)

where \( IC \) represents the investment cost. The value matching condition (A9) states that at optimal the firm is indifferent between investing right now and delaying the investment. The smooth pasting condition (A10) guarantees that the value function of the firm is continuous at the threshold value of profit (\( \pi^* \)) if \( \pi^* \) maximizes the value of the firm. We will measure the cost of investment by the observable gross fixed investment expenditures undertaken by the firm at time \( t \). By the value matching condition,

\[
A \pi^* = \frac{\pi^*}{\rho - \mu_\pi} - IC_t
\]  

(A11)

By the smooth pasting condition,

\[
A \kappa \pi^{k-1} - 1 = \frac{1}{\rho - \mu_\pi}
\]  

(A12)

Solving Equations (A11) and (A12) simultaneously, we have:

\[
\pi^* = \frac{\kappa}{\kappa - 1} (\rho - \mu_\pi) IC_t
\]  

(A13)

Equation (A13) is the threshold level of profits. \( \kappa/(\kappa - 1) \) is the so-called ‘option value multiple’.

**Appendix B: Data Description**

The data used in this paper is taken from the *Jaarboek van Nederlandse Ondernemingen*. The 55 manufacturing firms in the data set are listed on the Amsterdam stock exchange (AEX) over the period 1985–1997 in the Dutch economy. The set contains the following variables:

- **Net investment (NI)** the changes in the capital stock.
- **Capital stock (K)** the book value of the capital stock.
- **Profit (\( \pi \))** operating profits after tax and before interest payments.
- **Payout ratio (\( \omega \))** the ratio of dividend per share to net profit per share.
Sales (S) the product of the output price and the amount of products sold.
Cash flow (CF) the sum of net profit after tax before interest payments and depreciation

Average $Q$ ($Q$) (the end-of-year price of equity times the number of shares plus the book value of debt)/the book value of the capital stock.