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Published in:
Physical Review D

DOI:
[10.1103/PhysRevD.98.084009](https://doi.org/10.1103/PhysRevD.98.084009)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2018

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Buoninfante, L., Harmsen, G., Maheshwari, S., & Mazumdar, A. (2018). Nonsingular metric for an electrically charged point-source in ghost-free infinite derivative gravity. *Physical Review D*, 98(8), [084009]. <https://doi.org/10.1103/PhysRevD.98.084009>

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Nonsingular metric for an electrically charged point-source in ghost-free infinite derivative gravity

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(Received 31 July 2018; published 5 October 2018)

In this paper we construct a linearized metric solution for an electrically charged system in a *ghost-free* infinite derivative theory of gravity which is valid in the entire region of spacetime. We show that the gravitational potential for a point-charge with mass m is nonsingular, the Kretschmann scalar is finite, and the metric approaches conformal flatness in the ultraviolet regime where the nonlocal gravitational interaction becomes important. We show that the metric potentials are bounded below one as long as two conditions involving the mass and the electric charge are satisfied. Furthermore, we argue that the cosmic censorship conjecture is not required in this case. Unlike in the case of the Reissner-Nordström metric in general relativity, where $|Q| \leq m/M_p$ must always be satisfied, in ghost-free infinite derivative gravity $|Q| > m/M_p$ is also allowed, such as for an electron.

DOI: [10.1103/PhysRevD.98.084009](https://doi.org/10.1103/PhysRevD.98.084009)

I. INTRODUCTION

Einstein’s theory of general relativity (GR) has been tested to a very high precision in the infrared (IR) regime, i.e., at large distances and late times [1], including the recent detections of gravitational waves due to binary mergers which match excellently with the predictions of GR [2]. However, as soon as one approaches short distances, the theory presents several problems, such as e.g., black-hole and cosmological singularities at the classical level, and fails to be perturbatively renormalizable at the quantum level. Indeed, GR turns out to be incomplete in the ultraviolet (UV) regime, i.e., at short distances and small time scales.

It is well known that by adding quadratic terms in the curvature to the Einstein-Hilbert action one obtains a gravitational theory which becomes power-counting renormalizable [3]. In the UV, the theory becomes conformal and in the infrared Einstein’s GR is recovered, except that being a higher-derivative theory of gravity, it suffers from instability due to the presence of a massive spin-2 ghost-like degrees of freedom (d.o.f.) in the physical spectrum, whose emergence becomes evident by computing the graviton propagator around the Minkowski background [3].

Recently, it has been shown that the *infinite derivative gravity* (IDG) can resolve this ghost problem and, at the same time, it can also resolve the short-distance singularity for a point-like object, regularizing the $1/r$ behavior of the metric potential in the linearized limit, therefore ameliorating the UV aspects of Einstein and

Newtonian gravity [4,5].¹ Furthermore, the time-dependent case yields a vacuum solution which is devoid of any cosmological singularity [4,10,11]. It was also shown that the theory has a mass gap at the linear level [12], and never develops a singularity even in a dynamical setup [13,14]. At the linearized level, the nontrivial gravitational solution, in the UV, gives rise to a constant Ricci scalar, Ricci tensor and Riemann tensor, and the Weyl tensor approaches zero at the origin (quadratically in distance from $r = 0$ at best). The Kretschmann scalar remains constant and never blows up at the origin [15]. Also, the static solution has now been extended to a nonsingular rotating metric which resolves the Kerr-type ring singularity in Einstein’s GR [16]. In Ref. [17], the authors studied gravitational potentials in extended objects within *ghost-free* IDG, and found them to be free of singularities.

Very recently, it has been shown that the full nonlinear equations of motion for ghost-free infinite derivative gravity, given in Ref. [18], do not provide a $1/r^\alpha$ behavior, with $\alpha > 0$, as a full vacuum solution [19], where the Weyl term plays a crucial role. Furthermore, in the time-dependent context the infinite derivative gravity does not give rise to the Kasner-type metrics as vacuum solutions, and therefore provides a way to avoid the Belinskii-Khalatnikov-Lifshitz singularity which describes an anisotropic collapse of a

¹Prior to Ref. [5], there were discussions on resolving singularities in infinite derivative gravity in Refs. [6–9].

time-dependent metric solution in the case of the Einstein-Hilbert action [20].

As pointed out in Ref. [21], the usual notion of a *vacuum* solution that we use in GR, for example for the Schwarzschild solution, does not apply to the case of infinite derivative gravity. Indeed, the delta source distribution at $r = 0$, which is imposed as a boundary condition, is smeared out by the infinitely many derivatives, generating an extended source whose size is given by the scale of nonlocality, $r_{NL} \sim 2/M_s$ [22]. This point is crucial in order to obtain a full metric solution which is regular at $r = 0$.

Infinite derivative gravity has also been discussed in the context of field theory. It is expected that infinite derivatives will ameliorate the quantum aspects as well, and it has been argued that such a class of theories will be super-renormalizable; see Refs. [6,7,23]. Moreover, the high-energy scattering of gravitons in this class of theories does not necessarily lead to the formation of a black hole as shown in Ref. [24]. The scattering amplitude decreases exponentially, if the propagator and vertex corrections are correctly taken into account [23,24]. Furthermore, in quantum field theory, infinite derivatives can be useful to understand the UV completion of the Standard Model [25], and also the stability of the Higgs within the Abelian Higgs model [26].

In spite of all these interesting results, there are many important challenges and unanswered riddles regarding IDG and the nonlocal regime of gravity, such as renormalizability, the concept of spacetime, unitarity, causality structure, etc. Indeed some of these issues were partly discussed in Refs. [27–31], but a clear quantum picture would be more desirable, which goes beyond the scope of the current paper. Among other things, the formulation of the initial value problem within the context of infinite-derivative theories remains an interesting point of discussion [32].

The aim of this paper is to show how infinite derivative gravity will modify the gravitational potential generated by a charged source; we will produce a nonsingular generalization of the Reissner-Nordström metric. We will briefly review the infinite derivative ghost-free gravity; then we will discuss the linearized metric of an electrically charged source. We study various spacetime properties and discuss the consequences for the *cosmic censorship* hypothesis [33–35].

II. INFINITE DERIVATIVE GHOST-FREE GRAVITY

The most general quadratic curvature gravity in four dimensions, which is parity invariant and torsion free around a constant curvature background can be written as [5,36]

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [\mathcal{R} + \alpha(\mathcal{R}\mathcal{F}_1(\Box_s)\mathcal{R} + \mathcal{R}_{\mu\nu}\mathcal{F}_2(\Box_s)\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{F}_3(\Box_s)\mathcal{R}^{\mu\nu\rho\sigma})], \quad (1)$$

where $G = 1/M_p^2$ is Newton's constant and $\alpha \sim 1/M_s^2$ is a dimensionful coupling. We are working with mostly positive

metric signature $(-, +, +, +)$, and $\Box_s \equiv \Box/M_s^2$, where the d'Alembertian is given by $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$, and $\mu, \nu = 0, 1, 2, 3$. The new scale of gravity is given by $M_s \leq M_p$.² The three gravitational form factors $\mathcal{F}_i$ are reminiscent of any massless theory, and are constrained by the tree-level unitarity and ghost-free condition derived in Ref. [5]. Around the Minkowski background we can avoid the presence of other dynamical d.o.f. other than the spin-2 massless graviton by first demanding

$$2\mathcal{F}_1(\Box_s) + \mathcal{F}_2(\Box_s) + 2\mathcal{F}_3(\Box_s) = 0. \quad (2)$$

Thus, the propagator of this theory is given by [5,37,38]

$$\Pi(-k^2) = \frac{1}{a(-k^2)} \left(\frac{\mathcal{P}^{(2)}}{k^2} - \frac{\mathcal{P}_s^{(0)}}{2k^2} \right), \quad (3)$$

where $\Pi_{GR} = \mathcal{P}^{(2)}/k^2 - \mathcal{P}_s^{(0)}/2k^2$ is the graviton GR propagator and $\mathcal{P}^2, \mathcal{P}_s^0$ are the two spin projection operators along the spin-2 and spin-0 components, respectively, while³

$$a(\Box_s) = 1 - \frac{1}{2}\mathcal{F}_2(\Box_s)\Box_s - 2\mathcal{F}_3(\Box_s)\Box_s. \quad (4)$$

Moreover, in order to have *no* ghost-like d.o.f., the form of $a(\Box)$ is restricted to be an *entire function*, so that *no* new poles are introduced in the propagator in Eq. (3). One simple choice is indeed

$$a(\Box_s) = e^{-\Box/M_s^2}. \quad (5)$$

Other choices for this *entire function* have been explored in Ref. [39], which illustrates universal behavior in the Newtonian potential from IR all the way up to the UV regime.

In Ref. [5] it was shown that the ghost-free condition, like in Eq. (5), also yields regular static solutions in the linear regime, where the metric potential satisfies $2|\Phi| < 1$. Furthermore, in Ref. [21], it was argued that this inequality can always hold true when the quadratic part of the action in Eq. (1) dominates over the Einstein-Hilbert term at the horizon scales.⁴ In this paper we will show that a similar scenario also holds when we consider the spacetime metric for a charged point-source, but in this case an additional inequality, involving the electric charge, will be required in order to have bounded metric potentials.

²We work in natural units: $\hbar = 1 = c$.

³Around the Minkowski background, up to quadratic order in the metric perturbation, the form factor $\mathcal{F}_3(\Box_s)$ can be set to zero without any loss of generality.

⁴Indeed, by considering a characteristic length scale L , such that $dx \sim L$, $\partial_x \sim 1/L$ and $\mathcal{R} \sim 1/L^2$, and assuming $L \sim 2Gm$, where m is the mass of the object in the static geometry, we obtain $S \sim \frac{m^2}{M_p^2} (1 + \frac{M_p^4}{m^2 M_s^2})$, which tells us that the quadratic part of the action dominates over the Einstein-Hilbert term if and only if $mM_s < M_p^2$ [21]. Moreover, as long as this condition holds, the static metric potential is always bounded by one.

III. LINEARIZED METRIC SOLUTION FOR AN ELECTRICALLY CHARGED SOURCE

We now want to determine the spacetime metric for an electrically charged source in infinite derivative gravity, in the linear regime. The linearized field equations corresponding to the action in Eq. (1), with the ghost-free choice in Eq. (5) are given by [5]

$$e^{-\Box/M_s^2}[\Box h_{\mu\nu} + (\eta_{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma} + \partial_\mu\partial_\nu h) - \partial_\sigma(\partial_\nu h_\mu^\sigma + \partial_\mu h_\nu^\sigma) - \eta_{\mu\nu}\Box h] = -16\pi G\tau_{\mu\nu}. \quad (6)$$

The rank-2 tensor $\tau_{\mu\nu}$ stands for the electromagnetic energy-momentum tensor which is defined by

$$\tau_{\mu\nu} = \frac{1}{4\pi} \left(\eta_{\rho\nu} F_{\mu\sigma} F^{\rho\sigma} - \frac{1}{4} \eta_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \right), \quad (7)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength, with A_μ being the potential vector. We will consider the case in which the source is static and spherically symmetric, and no magnetic monopole is present; thus the only nonvanishing contributions of the electromagnetic field strength will come from

$$F_{10} = -F_{01} = E_r \quad \text{and} \quad E_r = \frac{Q}{r^2}, \quad (8)$$

where E_r is the radial component of the electric field, with the Coulomb constant $k_e = 1$ in natural (Gaussian) units. We consider the case of a weak gravitational field and static source, so that the metric solution can be written as

$$ds^2 = -(1 + 2\Phi(r))dt^2 + (1 - 2\Psi(r))(dr^2 + r^2 d\Omega^2), \quad (9)$$

where (t, r, θ, φ) are the isotropic coordinates, $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$, and Φ and Ψ are the two unknown gravitational fields that need to be found. By looking at the 00-component and trace of the linearized field equations in Eq. (6) we obtain, respectively

$$e^{-\nabla^2/M_s^2}[\nabla^2 h_{00} - \partial_i\partial_j h^{ij} + \nabla^2 h] = -16\pi G\tau_{00},$$

$$e^{-\nabla^2/M_s^2}[-2\nabla^2 h + 2\partial_i\partial_j h^{ij}] = -16\pi G\tau. \quad (10)$$

From the metric form in Eq. (9) we note that $h = 2(\Phi - 3\Psi)$, $h_{00} = -2\Phi$ and $h_{ii} = -2\Psi$; thus we obtain two differential equations for the two metric potentials Φ and Ψ :

$$e^{-\nabla^2/M_s^2}\nabla^2\Phi = 4\pi G(\tau + 2\tau_{00}),$$

$$e^{-\nabla^2/M_s^2}\nabla^2\Psi = 4\pi G\tau_{00}. \quad (11)$$

In the case of an electric charged static source the energy-momentum tensor in Eq. (7) turns out to be traceless, $\tau = 0$,

while the 00-component is given by $\tau_{00} = Q^2/8\pi r^4$, so that Eq. (11) reads

$$e^{-\nabla^2/M_s^2}\nabla^2\Phi = \frac{GQ^2}{r^4}, \quad e^{-\nabla^2/M_s^2}\nabla^2\Psi = \frac{GQ^2}{2r^4}. \quad (12)$$

The details of solving the differential equations above are laid out in Appendix B. Here, we state the final result for the two metric potentials:

$$\Phi(r) = -\frac{Gm}{r}\text{Erf}\left(\frac{M_s r}{2}\right) + \frac{GQ^2 M_s}{2r}\text{F}\left(\frac{M_s r}{2}\right),$$

$$\Psi(r) = -\frac{Gm}{r}\text{Erf}\left(\frac{M_s r}{2}\right) + \frac{GQ^2 M_s}{4r}\text{F}\left(\frac{M_s r}{2}\right), \quad (13)$$

where

$$\text{Erf}(x) := \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt, \quad (14)$$

is the error function and

$$\text{F}(x) := e^{-x^2} \int_0^x e^{t^2} dt, \quad (15)$$

is the Dawson function. Note that $\Phi \neq \Psi$, as it also happens in the GR case (see Appendix A). In the case $Q = 0$ we recover the linearized IDG metric for a static neutral point-source derived in Ref. [5], as expected. In the IR regime, $M_s r \gg 2$, the potentials in Eq. (13) reduce to those of the linearized Reissner-Nordström metric in GR [Eq. (A14)], as the asymptotic expansions for the error function and the Dawson function are $\text{Erf}(x) \sim 1$ and $\text{F}(x) \sim 1/2x$. More interestingly, in the nonlocal regime, $M_s r \ll 2$, the metric potentials $\Phi(r)$ and $\Psi(r)$ approach nonsingular constant values given by

$$\lim_{M_s r \rightarrow 0} \Phi(r) \equiv -A = -\frac{GmM_s}{\sqrt{\pi}} + \frac{1}{4}GQ^2 M_s^2,$$

$$\lim_{M_s r \rightarrow 0} \Psi(r) \equiv -B = -\frac{GmM_s}{\sqrt{\pi}} + \frac{1}{8}GQ^2 M_s^2, \quad (16)$$

because for $x \ll 1$ one has $\text{Erf}(x) \sim 2x/\sqrt{\pi}$ and $\text{F}(x) \sim x$. Note that when $Q = 0$, we recover the case of a neutral source, $A = B = GmM_s/\sqrt{\pi}$.

In Fig. 1 the potential Φ is plotted as a function of $M_s r$ and we compare ghost-free IDG and GR. We have chosen the values $G = 1$, $m = 1$, $M_s = 0.5$ and $Q = 0.5$. It is also worth mentioning that the force per unit mass, $F_g(r) = -\partial\Phi/\partial r$, goes to zero linearly as a function of r . This vanishing force shows the classical aspect of asymptotic freedom in ghost-free IDG [5].

Since we are working in the linearized regime, we need to satisfy the following two inequalities: $2|\Phi| < 1$ and

FIG. 1. The behavior of the potential Φ in IDG (blue line) and a comparison with the corresponding one in the case of the Reissner-Nordström metric in GR (orange line). We set $G = 1$, $m = 1$, $M_s = 0.5$, and $Q = 0.5$.

$2|\Psi| < 1$. First of all, note that it is sufficient to focus on just one of the two inequalities, as the two metric potentials are almost the same apart from a factor of $1/2$ appearing in the gravitational contribution due to the charge; see Eqs. (13) and (16). Moreover, for simplicity, we can drop all numerical factors and focus on just the essential parameters m , $G = 1/M_p^2$, M_s and Q .

Mathematically, the weak-field inequality can be also satisfied when the neutral and charged contributions to the gravitational potential are both very large but such that the modulus of their sum is still less than one. However, from a physical point of view, in order to satisfy the weak-field inequalities, we have to require that both contributions are smaller than one⁵ [5]:

$$mM_s < M_p^2, \quad (17)$$

$$Q^2 M_s^2 < M_p^2 \Leftrightarrow |Q| M_s < M_p. \quad (18)$$

We can immediately notice that, with respect to the neutral case, we now have the additional inequality in Eq. (18); together they will play a crucial role. In fact, we can make the following important observations.

- (1) In the case of the Reissner-Nordström metric in Einstein's GR one has to demand that the inequality

$$|Q| \leq \frac{m}{M_p} \quad (19)$$

always holds true in order to avoid the formation of a *naked singularity* during a collapsing phase. This is one example of the weak formulation of the *cosmic*

censorship conjecture; see Refs. [33–35]. In the case of a charged source in ghost-free IDG there is *no* need to require a cosmic censorship conjecture as the theory turns out to be singularity free and devoid of any horizon as long as the inequalities in Eqs. (17) and (18) hold true.

- (2) Thus, in Einstein's GR one cannot describe the gravitational field generated by objects for which

$$|Q| > \frac{m}{M_p}, \quad (20)$$

such as for an electron. In ghost-free IDG, both inequalities in Eqs. (19) and (20) are allowed. Indeed, consistently with the weak-field inequalities in Eqs. (17) and (20), we can have *two* possible scenarios:

$$|Q| < \frac{m}{M_p} < \frac{M_p}{M_s} \quad (21)$$

or

$$\frac{m}{M_p} < |Q| < \frac{M_p}{M_s}. \quad (22)$$

The most interesting case is the one described by the second inequality, Eq. (22), which says that in ghost-free IDG we *can* study the gravitational field of an object whose charge is bigger than its mass (in units of Planck mass), allowing us to include, e.g., an electron in IDG.⁶

- (3) Finally, note that in the short-distance regime, $r < 2/M_s$, one can in principle also have repulsive gravity, when

$$Q^2 M_s > m,$$

and it is only compatible with the scenario in Eq. (22), and not with Eq. (21), as can be easily checked. However, such a repulsive contribution would be confined within the region of nonlocality, while outside, gravity would still be attractive as in standard GR.

IV. CURVATURE TENSORS

We have computed all curvature tensors for the linearized IDG metric in Eqs. (9) and (13). All curvature expressions are evaluated up to first order in G , as we are working in the linear regime ($h_{\mu\nu} \sim G$). The full expressions are shown in Appendix C; instead, in this

⁵As pointed out in Ref. [21], the inequality $mM_s < M_p^2$ always holds in order for the quadratic part of the gravitational action to dominate over the Einstein-Hilbert term in the UV regime. If the inequality was not satisfied, we would not be able to modify the Reissner-Nordström geometry and avoid the metric singularity at the origin.

⁶Note that for an elementary particle, like an electron, the Compton wavelength is much larger than its Schwarzschild radius, so in such a regime, quantum effects are not negligible and a classical theory of gravity alone cannot account for its gravitational effects.

section we wish to emphasize that the presence of a nonlocal gravitational interaction is such that all the curvature tensors and invariants are regularized at the origin, i.e., at $r = 0$. Therefore, we will focus on their forms in the region of nonlocality, $r < 2/M_s$.

In the nonlocal-dominated regime, many components of the curvature tensors vanish, while the remaining tend to *finite* constant values at the origin. The nonzero components of the Riemann tensor are given by

$$\mathcal{R}_{0101} \sim \frac{1}{12} GM_s^3 \left(\frac{2m}{\sqrt{\pi}} - Q^2 M_s \right), \quad (23)$$

the nonzero components of the Ricci tensor are

$$\begin{aligned} \mathcal{R}_{00} &\sim \frac{1}{4} GM_s^3 \left(\frac{2m}{\sqrt{\pi}} - Q^2 M_s \right), \\ \mathcal{R}_{11} &\sim \frac{1}{12} GM_s^3 \left(\frac{6m}{\sqrt{\pi}} - Q^2 M_s \right), \end{aligned} \quad (24)$$

and the Ricci scalar is

$$\mathcal{R} \sim \frac{GmM_s^3}{\sqrt{\pi}}. \quad (25)$$

Finally, we note that all components of the Weyl tensor tend to zero at the origin, as we take $M_s r \rightarrow 0$:

$$\mathcal{C}_{\mu\nu\rho\sigma} \sim 0. \quad (26)$$

It is important to note that with decreasing distance, different components of the Weyl tensor approach zero with different rates, particularly $O(r^2)$, $O(r^4)$ and $O(r^6)$ for small r , such that very close to the origin, we can imagine a constant valley, and the metric becomes exactly conformally flat at $r = 0$. This implies that the static metric of a charged source approaches *conformal flatness* in the UV regime, $r < 2/M_s$. In fact, in the short-distance regime, the metric in Eqs. (9) and (13) can be approximated as

$$ds^2 \approx -(1 - 2A)dt^2 + (1 + 2B)(dr^2 + r^2 d\Omega^2), \quad (27)$$

which can be put in a conformally flat form by introducing a *conformal time*, τ , through the coordinate transformation

$$\tau = \sqrt{\frac{1 - 2A}{1 + 2B}} t, \quad (28)$$

such that the metric in the nonlocal region reads

$$ds^2 = (1 + 2B)[-d\tau^2 + dr^2 + r^2 d\Omega^2] = F^2 \eta, \quad (29)$$

where η is the Minkowski metric and $F^2 \equiv 1 + 2B > 0$ is the conformal factor. In Fig. 2 we plot the component $\mathcal{C}_{0101}$

(a) Weyl component $\mathcal{C}_{0101}$

(b) Kretschmann invariant $\mathcal{K}$

FIG. 2. The spacetime properties of ghost-free IDG (blue line) and GR (orange line) as functions of $M_s r$: (a) $\mathcal{C}_{0101}$ component of the Weyl tensor and (b) the Kretschmann scalar $\mathcal{K}$. We set $M_p = 1$, $m = 1$, $M_s = 0.5$ and $Q = 0.5$. In these units the Schwarzschild radius is $r_{sch} = 2$. Note that, unlike in GR where the linearized approximation would break down for $r \leq 2$, in the case of ghost-free IDG we can smoothly approach $r = 0$, provided $mM_s < M_p^2$ and $|Q|M_s < M_p$. From these two plots it is obvious that the presence of nonlocality helps to avoid the curvature singularities at the origin.

of the Weyl tensor for both the charged case in ghost-free IDG and Reissner-Nordström in GR.

We can note that, by setting $Q = 0$ we recover the curvature tensors for the ghost-free IDG neutral case obtained in Ref. [15], as expected. Furthermore, we have computed all the curvature invariants squared. Their full expressions are shown in Appendix C, while in this section we will present their values in the nonlocal region. In the nonlocal regime, $r < 2/M_s$, the Kretschmann scalar $\mathcal{K} = \mathcal{R}_{\mu\nu\rho\sigma} \mathcal{R}^{\mu\nu\rho\sigma}$ tends to the following finite constant value as $M_s r \rightarrow 0$:

$$\mathcal{K} \sim \frac{G^2 M_s^6 (10m^2 - 6\sqrt{\pi}mQ^2 M_s + \pi Q^4 M_s^2)}{6\pi}. \quad (30)$$

In Fig. 2 we see explicitly that the Kretschmann invariant for a charged source in ghost-free IDG is regularized at the origin,

unlike the case of Reissner Nordström in GR where we have a curvature singularity.

Moreover, in the nonlocal region, as $M_s r \rightarrow 0$, the nonzero components of the Ricci scalar squared and the Ricci tensor squared are given by the finite constant values

$$\mathcal{R}^2 \sim \frac{G^2 m^2 M_s^6}{\pi}, \quad (31)$$

$$\mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} \sim \frac{G^2 M_s^6 (12m^2 - 6\sqrt{\pi} m Q^2 M_s + \pi Q^4 M_s^2)}{12\pi}, \quad (32)$$

while the Weyl tensor squared vanishes at the origin,

$$C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \sim 0. \quad (33)$$

For completeness, it can be checked that the squared curvatures satisfy the following identity:

$$C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} = \frac{\mathcal{R}^2}{3} - 2\mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} + \mathcal{K}. \quad (34)$$

Needless to say, all the curvature invariants for the case of a point-charge source in ghost-free IDG reduce to those of the uncharged case, $Q = 0$, obtained in Ref. [15], as expected.

V. CONCLUSIONS

In this paper we have found a linearized static metric solution for an electric point-charge in ghost-free IDG whose action is governed by the results of Ref. [5]. We have noticed that all the features of the case of a neutral source are still present; indeed the metric turns out to be regular at the origin and all curvature invariants are singularity free. Moreover, all components of the Weyl tensor tend to zero as $M_s r \rightarrow 0$, meaning that the spacetime metric approaches conformal flatness in the nonlocal regime.

We have also argued that in ghost-free IDG there is no need to require the cosmic censorship conjecture as the spacetime is singularity free and devoid of any horizon as long as the inequalities in Eqs. (17) and (18) hold true. Moreover, unlike the case of Reissner-Nordström in GR, in ghost-free IDG *no* constraint has to be imposed on the relation between the mass and charge of the source. Indeed, we can have cases where $|Q| > m/M_p$ is satisfied, allowing us to include in the theory objects such as an electron. Furthermore, we would expect that massive compact charged systems, devoid of singularities and event horizons can be found in this case too, similar to the case of a neutral source studied in Ref. [22].

ACKNOWLEDGMENTS

The authors would like to thank Alan S. Cornell and Gaetano Lambiase for discussions. G. H. is supported in part by the National Research Foundation of South Africa. A. M.'s research is financially supported by Netherlands Organisation for Scientific Research (NWO) Grant No. 680-91-119.

APPENDIX A: REISSNER-NORDSTRÖM METRIC IN EINSTEIN'S GENERAL RELATIVITY

In this Appendix we wish to briefly review the main aspects of the Reissner-Nordström metric in GR; in particular we will introduce its form in the isotropic coordinates which is usually not studied in the standard textbooks. The Reissner-Nordström metric is a nonvacuum solution of the full nonlinear Einstein equations:

$$\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = 8\pi G \tau_{\mu\nu}, \quad (A1)$$

where $\tau_{\mu\nu}$ is the electromagnetic energy-momentum tensor

$$\tau_{\mu\nu} = \frac{1}{4\pi} \left(\eta_{\rho\nu} F_{\mu\sigma} F^{\rho\sigma} - \frac{1}{4} \eta_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \right). \quad (A2)$$

Assuming that no magnetic monopole is present, the only components of the field strength $F_{\mu\nu}$ will be given by the electric field part; then, due to spherical symmetry there will be a radial dependence so that the only nonvanishing components are $F_{10} = -F_{01} = Q/r^2$. In the Schwarzschild coordinates (t, R, θ, φ) , where R is the polar radial coordinate, the metric reads

$$ds^2 = - \left(1 - \frac{2Gm}{R} + \frac{GQ^2}{R^2} \right) dt^2 + \left(1 - \frac{2Gm}{R} + \frac{GQ^2}{R^2} \right)^{-1} dR^2 + R^2 d\Omega^2, \quad (A3)$$

where $d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\varphi^2$, and m and Q are the mass and the charge of the source, respectively.

One can immediately notice that the Reissner-Nordström metric has *two* horizons that are defined as

$$r_{\pm} = Gm \pm \sqrt{G^2 m^2 - GQ^2}, \quad (A4)$$

where r_+ is called the *outer* horizon, while r_- is the *inner* horizon.

We are now interested in finding the corresponding weak-field approximation for the metric in Eq. (A3) and to do so, as it happens for the Schwarzschild metric, it is more convenient to use the *isotropic coordinates* (t, r, θ, φ) , where r is the isotropic radial coordinate that in the case of the Reissner-Nordström metric is defined by the following transformation:

$$R = r \left[\left(1 + \frac{Gm}{2r} \right)^2 - \frac{GQ^2}{4r^2} \right]. \quad (A5)$$

In isotropic coordinates the metric in Eq. (A3) becomes

$$ds^2 = - \left[\frac{1 - \left(\frac{Gm}{2r} \right)^2 + \frac{GQ^2}{4r^2}}{\left(1 + \frac{Gm}{2r} \right)^2 - \frac{GQ^2}{4r^2}} \right]^2 dt^2 + \left[\left(1 + \frac{Gm}{2r} \right)^2 - \frac{GQ^2}{4r^2} \right]^2 (dr^2 + r^2 d\Omega^2); \quad (A6)$$

it is evident that for $Q = 0$ we recover the Schwarzschild metric in isotropic coordinates.

We can now make an expansion for a weak gravitational field and stop at the linear order in G ; thus the metric in Eq. (A6) in the linearized regime reads

$$ds^2 = -\left(1 - \frac{2Gm}{r} + \frac{GQ^2}{r^2}\right)dt^2 + \left(1 + \frac{2Gm}{r} - \frac{GQ^2}{2r^2}\right)(dr^2 + r^2 d\Omega^2). \quad (\text{A7})$$

It is very important to stress that the metric in Eq. (A7) can also be obtained with a different procedure, i.e., by perturbing the metric around a Minkowski background and working in the linear regime,

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x), \quad (\text{A8})$$

so that the Einstein field equations in Eq. (A1) in the linear regime are given by

$$\square h_{\mu\nu} + (\eta_{\mu\nu} \partial_\rho \partial_\sigma h^{\rho\sigma} + \partial_\mu \partial_\nu h) - \partial_\sigma (\partial_\nu h_\mu^\sigma + \partial_\mu h_\nu^\sigma) - \eta_{\mu\nu} \square h = -16\pi G \tau_{\mu\nu}. \quad (\text{A9})$$

By working in the weak-field regime and considering the case of a static source, the spacetime metric can be written as

$$ds^2 = -(1 + 2\Phi(r))dt^2 + (1 - 2\Psi(r))(dr^2 + r^2 d\Omega^2), \quad (\text{A10})$$

where r is the isotropic radial coordinate; thus the only two unknowns we need to find are the two gravitational potentials Φ and Ψ . By looking at the 00-component and the trace of the linearized field equations in Eq. (A9) we obtain

$$\begin{aligned} \nabla^2 h_{00} - \partial_i \partial_j h^{ij} + \nabla^2 h &= -16\pi G \tau_{00}, \\ -2\nabla^2 h + 2\partial_i \partial_j h^{ij} &= -16\pi G \tau. \end{aligned} \quad (\text{A11})$$

Using $h = 2(\Phi - 3\Psi)$, $h_{00} = -2\Phi$ and $h_{ii} = -2\Psi$ we obtain two differential equations for the two gravitational fields Φ and Ψ :

$$\nabla^2 \Phi = 4\pi G(\tau + 2\tau_{00}), \quad \nabla^2 \Psi = 4\pi G \tau_{00}. \quad (\text{A12})$$

In the case of a charged static source, the energy-momentum tensor turns out to be traceless, $\tau = 0$, while the 00-component is given by $\tau_{00} = Q^2/8\pi r^4$. We can now solve the two differential equations in Eq. (A12) and obtain

$$\begin{aligned} \Phi(r) &= -\frac{C_1}{r} + \frac{GQ^2}{2r^2} + C_2, \\ \Psi(r) &= -\frac{C_1}{r} + \frac{GQ^2}{4r^2} + C_2, \end{aligned} \quad (\text{A13})$$

where C_1 and C_2 are two integration constants whose values can be fixed by imposing suitable physical boundary conditions. First of all, for $r = \infty$ we want Φ and Ψ to be zero (asymptotic flatness) which means $C_2 = 0$, while C_1 can be found by imposing that for $Q = 0$ we have the usual Newtonian potential, and thus $C_1 = Gm$,

$$\Phi(r) = -\frac{Gm}{r} + \frac{GQ^2}{2r^2}, \quad \Psi(r) = -\frac{Gm}{r} + \frac{GQ^2}{4r^2}. \quad (\text{A14})$$

Notice that in the case of a charged source $\Phi \neq \Psi$. It is clear that the linearized metric in Eq. (A10) with the gravitational fields derived in Eq. (A14) coincides with the metric in Eq. (A7) which we derived instead by starting from the full nonlinear Reissner-Nordström metric. It is very important to note that in the weak-field approximation one has $2|\Phi|, 2|\Psi| < 1$, which means that the radial coordinate has to always be greater than the outer horizon radius, $r > r_+$. Thus, in Einstein's GR it so happens that the linear approximation breaks down once one approaches the horizon. As we have seen in the paper, this is not the case for infinite derivative gravity, where (for both charged and neutral sources) the linear regime holds all the way up to $r = 0$.

APPENDIX B: WORKING WITH INFINITE-ORDER DIFFERENTIAL EQUATIONS

We can solve the differential equations in Eq. (12) in several ways, for example by making the temporary field redefinitions

$$\bar{\Phi} := e^{-\nabla^2/M_s^2} \Phi, \quad \bar{\Psi} := e^{-\nabla^2/M_s^2} \Psi, \quad (\text{B1})$$

so that the equations in Eq. (A8) become

$$\nabla^2 \bar{\Phi}(r) = \frac{GQ^2}{r^4}, \quad \nabla^2 \bar{\Psi}(r) = \frac{GQ^2}{2r^4}. \quad (\text{B2})$$

The structure of these equations is similar to the one we have in the case of a charged source in Einstein's GR, i.e., Reissner-Nordström, as also shown in Appendix A in Eq. (A12); thus the solution will also have the same structure as the one in Eq. (A13):

$$\begin{aligned} \bar{\Phi}(r) &= -\frac{C_1}{r} + \frac{GQ^2}{2r^2} + C_2, \\ \bar{\Psi}(r) &= -\frac{C_1}{r} + \frac{GQ^2}{4r^2} + C_2. \end{aligned} \quad (\text{B3})$$

By requiring the boundary conditions of asymptotic flatness one gets $C_2 = 0$; while C_1 can be fixed that for $Q = 0$ one recovers the case of neutral point-source, which means $C_1 = Gm$. In terms of the real fields Φ and Ψ , Eq. (B1) will give

$$\begin{aligned}\Phi(r) &= -Gme^{\nabla^2/M_s^2} \left(\frac{1}{r} \right) + \frac{GQ^2}{2} e^{\nabla^2/M_s^2} \left(\frac{1}{r^2} \right), \\ \Psi(r) &= -Gme^{\nabla^2/M_s^2} \left(\frac{1}{r} \right) + \frac{GQ^2}{4} e^{\nabla^2/M_s^2} \left(\frac{1}{r^2} \right).\end{aligned}\quad (\text{B4})$$

We now need to compute the action of the exponential $e^{-\nabla^2/M_s^2}$ on the functions $1/r$ and $1/r^2$. Notice that⁷

$$\begin{aligned}e^{\nabla^2/M_s^2} \left(\frac{1}{r} \right) &= e^{\nabla^2/M_s^2} \int \frac{d^3k}{(2\pi)^3} \frac{4\pi}{k^2} e^{i\vec{k}\cdot\vec{r}} \\ &= \int \frac{d^3k}{(2\pi)^3} \frac{4\pi}{k^2} e^{-k^2/M_s^2} e^{i\vec{k}\cdot\vec{r}} \\ &= \frac{2}{\pi} \int_0^\infty dk \frac{\sin(kr)}{kr} e^{-k^2/M_s^2} \\ &= \frac{1}{r} \text{Erf} \left(\frac{M_s r}{2} \right),\end{aligned}\quad (\text{B5})$$

where we have used the fact that $4\pi/k^2$ is the Fourier transform of $1/r$. We can proceed in the same way to compute the second contribution; indeed by noting that the Fourier transform of $1/r^2$ is $\frac{2\pi^2}{k} \text{sign}(k)$, one has

$$\begin{aligned}e^{\nabla^2/M_s^2} \left(\frac{1}{r^2} \right) &= e^{\nabla^2/M_s^2} \int \frac{d^3k}{(2\pi)^3} \frac{2\pi^2}{k} \text{sign}(k) e^{i\vec{k}\cdot\vec{r}} \\ &= \int \frac{d^3k}{(2\pi)^3} \frac{2\pi^2}{k} \text{sign}(k) e^{-k^2/M_s^2} e^{i\vec{k}\cdot\vec{r}} \\ &= \int_0^\infty dk \frac{\sin(kr)}{r} e^{-k^2/M_s^2} \\ &= \frac{M_s}{r} \text{F} \left(\frac{M_s r}{2} \right),\end{aligned}\quad (\text{B6})$$

where $\text{F}(M_s r/2)$ is the Dawson function.

Thus, by using Eqs. (B5) and (B6), we can now obtain the expressions for the two metric potentials in Eq. (13).

APPENDIX C: FULL EXPRESSIONS OF THE CURVATURE TENSORS

In this Appendix we show the full expressions for all curvature tensors and invariants for the IDG linearized static metric for a charged point-source derived in Eqs. (9) and (13).

The Ricci scalar is given by

$$\mathcal{R} = \frac{GmM_s^3 e^{-\frac{1}{4}r^2 M_s^2}}{\sqrt{\pi}}, \quad (\text{C1})$$

the nonzero components of the Riemann tensor are

$$\begin{aligned}\mathcal{R}_{0101} &= \frac{1}{8} G \left[M_s^3 \left(\frac{2Q^2 \text{F}(\frac{rM_s}{2})}{r} + \frac{4me^{-\frac{1}{4}r^2 M_s^2}}{\sqrt{\pi}} \right) + \frac{8M_s(Q^2 \text{F}(\frac{rM_s}{2}) + \frac{2mre^{-\frac{1}{4}r^2 M_s^2}}{\sqrt{\pi}})}{r^3} \right. \\ &\quad \left. + Q^2 r M_s^5 \text{F} \left(\frac{rM_s}{2} \right) - \frac{16m \text{Erf}(\frac{rM_s}{2})}{r^3} - \frac{4Q^2 M_s^2}{r^2} - Q^2 M_s^4 \right], \\ \mathcal{R}_{0303} &= \mathcal{R}_{0202} \sin^2(\theta) \\ &= \frac{1}{4} G \sin^2(\theta) \left[M_s \left(-\frac{2Q^2 \text{F}(\frac{rM_s}{2})}{r} - \frac{4me^{-\frac{1}{4}r^2 M_s^2}}{\sqrt{\pi}} \right) - Q^2 r M_s^3 \text{F} \left(\frac{rM_s}{2} \right) + \frac{4m \text{Erf}(\frac{rM_s}{2})}{r} + Q^2 M_s^2 \right], \\ \mathcal{R}_{1313} &= \mathcal{R}_{1212} \sin^2(\theta) \\ &= \frac{1}{16} G \sin^2(\theta) \left[M_s \left(\frac{4Q^2 \text{F}(\frac{rM_s}{2})}{r} + \frac{16me^{-\frac{1}{4}r^2 M_s^2}}{\sqrt{\pi}} \right) + Q^2 r^3 M_s^5 \text{F} \left(\frac{rM_s}{2} \right) - \frac{16m \text{Erf}(\frac{rM_s}{2})}{r} \right. \\ &\quad \left. + \frac{8mr^2 M_s^3 e^{-\frac{1}{4}r^2 M_s^2}}{\sqrt{\pi}} - Q^2 r^2 M_s^4 - 2Q^2 M_s^2 \right], \\ \mathcal{R}_{2323} &= \frac{1}{4} G r \sin^2(\theta) \left[M_s \left(-2Q^2 \text{F} \left(\frac{rM_s}{2} \right) - \frac{8mre^{-\frac{1}{4}r^2 M_s^2}}{\sqrt{\pi}} \right) - Q^2 r^2 M_s^3 \text{F} \left(\frac{rM_s}{2} \right) + 8m \text{Erf} \left(\frac{rM_s}{2} \right) + Q^2 r M_s^2 \right],\end{aligned}\quad (\text{C2})$$

⁷We can also consider other choices of entire functions; for example we can generalize Eq. (4) to $a(\square_s) = e^{(-\square_s)^n}$. In such a general case the factor e^{-k^2/M_s^2} in the third equality of Eqs. (B5) and (B6) is replaced by $e^{-k^{2n}/M_s^{2n}}$. For instance, for $n = 2$, the potentials evaluate to generalized hypergeometric functions, and as expected, they have nonsingular constant values in the UV and recover GR in the IR.

the nonzero components of the Ricci tensor are

$$\begin{aligned}
\mathcal{R}_{00} &= \frac{1}{8} GM_s^3 \left[\frac{Q^2(r^2 M_s^2 - 2)F(\frac{rM_s}{2})}{r} + \frac{4me^{-\frac{1}{4}r^2 M_s^2}}{\sqrt{\pi}} - Q^2 M_s \right], \\
\mathcal{R}_{11} &= \frac{1}{4} GM_s \left[M_s \left(\frac{2mM_s e^{-\frac{1}{4}r^2 M_s^2}}{\sqrt{\pi}} + \frac{Q^2}{r^2} \right) - \frac{Q^2(r^2 M_s^2 + 2)F(\frac{rM_s}{2})}{r^3} \right], \\
\mathcal{R}_{22} &= \frac{1}{16} GM_s \left[\frac{Q^2(r^4 M_s^4 + 4)F(\frac{rM_s}{2})}{r} + M_s \left(\frac{8mr^2 M_s e^{-\frac{1}{4}r^2 M_s^2}}{\sqrt{\pi}} - Q^2 r^2 M_s^2 - 2Q^2 \right) \right], \\
\mathcal{R}_{33} &= \frac{1}{16} G \sin^2(\theta) M_s \left[\frac{Q^2(r^4 M_s^4 + 4)F(\frac{rM_s}{2})}{r} + M_s \left(\frac{8mr^2 M_s e^{-\frac{1}{4}r^2 M_s^2}}{\sqrt{\pi}} - Q^2 r^2 M_s^2 - 2Q^2 \right) \right], \tag{C3}
\end{aligned}$$

and the nonzero components of the Weyl tensor are

$$\begin{aligned}
\mathcal{C}_{0101} &= \frac{1}{48} G \left[M_s^3 \left(\frac{12Q^2 F(\frac{rM_s}{2})}{r} + \frac{16me^{-\frac{1}{4}r^2 M_s^2}}{\sqrt{\pi}} \right) + \frac{M_s(36Q^2 F(\frac{rM_s}{2}) + \frac{96mr e^{-\frac{1}{4}r^2 M_s^2}}{\sqrt{\pi}})}{r^3} \right. \\
&\quad \left. + 3Q^2 r M_s^5 F\left(\frac{rM_s}{2}\right) - \frac{96m \text{Erf}\left(\frac{rM_s}{2}\right)}{r^3} - \frac{18Q^2 M_s^2}{r^2} - 3Q^2 M_s^4 \right], \\
\mathcal{C}_{0303} &= -\mathcal{C}_{1313} = \mathcal{C}_{0202} \sin^2(\theta) = -\mathcal{C}_{1212} \sin^2(\theta) \\
&= \frac{1}{96} G \sin^2(\theta) \left[4r M_s^3 \left(-3Q^2 F\left(\frac{rM_s}{2}\right) - \frac{4mr e^{-\frac{1}{4}r^2 M_s^2}}{\sqrt{\pi}} \right) + M_s \left(-\frac{36Q^2 F(\frac{rM_s}{2})}{r} - \frac{96me^{-\frac{1}{4}r^2 M_s^2}}{\sqrt{\pi}} \right) \right. \\
&\quad \left. - 3Q^2 r^3 M_s^5 F\left(\frac{rM_s}{2}\right) + \frac{96m \text{Erf}\left(\frac{rM_s}{2}\right)}{r} + 3Q^2 r^2 M_s^4 + 18Q^2 M_s^2 \right], \\
\mathcal{C}_{2323} &= \frac{1}{48} G r \sin^2(\theta) \left[4r^2 M_s^3 \left(-3Q^2 F\left(\frac{rM_s}{2}\right) - \frac{4mr e^{-\frac{1}{4}r^2 M_s^2}}{\sqrt{\pi}} \right) + M_s \left(-36Q^2 F\left(\frac{rM_s}{2}\right) - \frac{96mr e^{-\frac{1}{4}r^2 M_s^2}}{\sqrt{\pi}} \right) \right. \\
&\quad \left. - 3Q^2 r^4 M_s^5 F\left(\frac{rM_s}{2}\right) + 96m \text{Erf}\left(\frac{rM_s}{2}\right) + 3Q^2 r^3 M_s^4 + 18Q^2 r M_s^2 \right]. \tag{C4}
\end{aligned}$$

We will now show the expressions for the curvature invariants. The Ricci scalar squared is given by

$$\mathcal{R}^2 = \frac{G^2 m^2 M_s^6 e^{-\frac{1}{2}r^2 M_s^2}}{\pi}, \tag{C5}$$

the Ricci tensor squared is

$$\begin{aligned}
\mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} &= \frac{G^2 M_s^2}{128\pi r^6} \left[F\left(\frac{rM_s}{2}\right) (32\sqrt{\pi} m Q^2 r^7 M_s^6 e^{-\frac{1}{4}r^2 M_s^2} - 64\sqrt{\pi} m Q^2 r^5 M_s^4 e^{-\frac{1}{4}r^2 M_s^2} \right. \\
&\quad \left. - 6\pi Q^4 r^7 M_s^7 + 4\pi Q^4 r^5 M_s^5 - 24\pi Q^4 r^3 M_s^3 - 48\pi Q^4 r M_s) \right. \\
&\quad \left. + \pi Q^4 (3r^8 M_s^8 - 8r^6 M_s^6 + 24r^4 M_s^4 + 32r^2 M_s^2 + 48) F\left(\frac{rM_s}{2}\right)^2 \right. \\
&\quad \left. + M_s^4 (128m^2 r^6 e^{-\frac{1}{2}r^2 M_s^2} + 4\pi Q^4 r^4) - 32\sqrt{\pi} m Q^2 r^6 M_s^5 e^{-\frac{1}{4}r^2 M_s^2} + 3\pi Q^4 r^6 M_s^6 + 12\pi Q^4 r^2 M_s^2 \right], \tag{C6}
\end{aligned}$$

the Weyl tensor squared is

$$\begin{aligned}
 C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} = & \frac{G^2 e^{-\frac{1}{2}r^2 M_s^2}}{192\pi r^6} \left[-4r^2 M_s^3 \left(3\sqrt{\pi} Q^2 e^{\frac{1}{4}r^2 M_s^2} F\left(\frac{rM_s}{2}\right) + 4mr \right) \right. \\
 & - 12M_s \left(3\sqrt{\pi} Q^2 e^{\frac{1}{4}r^2 M_s^2} F\left(\frac{rM_s}{2}\right) + 8mr \right) \\
 & - 3\sqrt{\pi} Q^2 r^4 M_s^5 e^{\frac{1}{4}r^2 M_s^2} F\left(\frac{rM_s}{2}\right) + 96\sqrt{\pi} m e^{\frac{1}{4}r^2 M_s^2} \text{Erf}\left(\frac{rM_s}{2}\right) \\
 & \left. + 18\sqrt{\pi} Q^2 r M_s^2 e^{\frac{1}{4}r^2 M_s^2} + 3\sqrt{\pi} Q^2 r^3 M_s^4 e^{\frac{1}{4}r^2 M_s^2} \right]^2, \quad (C7)
 \end{aligned}$$

and the Kretschmann invariant is

$$\begin{aligned}
 \mathcal{K} = & \frac{e^{-\frac{1}{2}r^2 M_s^2} G^2}{32\pi r^6} \left[3e^{\frac{1}{2}r^2 M_s^2} \pi Q^4 r^8 F\left(\frac{rM_s}{2}\right)^2 M_s^{10} - 6e^{\frac{1}{2}r^2 M_s^2} \pi Q^4 r^7 F\left(\frac{rM_s}{2}\right) M_s^9 \right. \\
 & + \left(32e^{\frac{1}{4}r^2 M_s^2} m \sqrt{\pi} Q^2 F\left(\frac{rM_s}{2}\right) r^7 + 3e^{\frac{1}{2}r^2 M_s^2} \pi Q^4 r^6 + 8e^{\frac{1}{2}r^2 M_s^2} \pi Q^4 F\left(\frac{rM_s}{2}\right)^2 r^6 \right) M_s^8 \\
 & - 4e^{\frac{1}{4}r^2 M_s^2} \sqrt{\pi} Q^2 r^5 \left(7e^{\frac{1}{4}r^2 M_s^2} \sqrt{\pi} F\left(\frac{rM_s}{2}\right) Q^2 + 8mr \right) M_s^7 \\
 & + 4r^4 \left(18e^{\frac{1}{2}r^2 M_s^2} \pi F\left(\frac{rM_s}{2}\right)^2 Q^4 + 5e^{\frac{1}{2}r^2 M_s^2} \pi Q^4 + 32e^{\frac{1}{4}r^2 M_s^2} m \sqrt{\pi} r F\left(\frac{rM_s}{2}\right) Q^2 + 24m^2 r^2 \right) M_s^6 \\
 & - 24e^{\frac{1}{4}r^2 M_s^2} \sqrt{\pi} Q^2 r^3 \left(8mr + e^{\frac{1}{4}r^2 M_s^2} \sqrt{\pi} F\left(\frac{rM_s}{2}\right) \left(5Q^2 + 4mr \text{Erf}\left(\frac{rM_s}{2}\right) \right) \right) M_s^5 \\
 & + 4r^2 \left(40e^{\frac{1}{2}r^2 M_s^2} \pi F\left(\frac{rM_s}{2}\right)^2 Q^4 + 15e^{\frac{1}{2}r^2 M_s^2} \pi Q^4 + 144e^{\frac{1}{4}r^2 M_s^2} m \sqrt{\pi} r F\left(\frac{rM_s}{2}\right) Q^2 \right. \\
 & \left. + 24e^{\frac{1}{2}r^2 M_s^2} m \pi r \text{Erf}\left(\frac{rM_s}{2}\right) Q^2 + 128m^2 r^2 \right) M_s^4 \\
 & - 16e^{\frac{1}{4}r^2 M_s^2} \sqrt{\pi} r \left(3e^{\frac{1}{4}r^2 M_s^2} \sqrt{\pi} F\left(\frac{rM_s}{2}\right) \left(5Q^2 + 8mr \text{Erf}\left(\frac{rM_s}{2}\right) \right) Q^2 \right. \\
 & \left. + 4mr \left(9Q^2 + 8mr \text{Erf}\left(\frac{rM_s}{2}\right) \right) \right) M_s^3 + 48 \left(5e^{\frac{1}{2}r^2 M_s^2} \pi F\left(\frac{rM_s}{2}\right)^2 Q^4 \right. \\
 & \left. + 24e^{\frac{1}{4}r^2 M_s^2} m \sqrt{\pi} r F\left(\frac{rM_s}{2}\right) Q^2 + 4mr \left(3e^{\frac{1}{2}r^2 M_s^2} \pi \text{Erf}\left(\frac{rM_s}{2}\right) Q^2 + 8mr \right) \right) M_s^2 \\
 & \left. - 384e^{\frac{1}{4}r^2 M_s^2} m \sqrt{\pi} \left(3e^{\frac{1}{4}r^2 M_s^2} \sqrt{\pi} F\left(\frac{rM_s}{2}\right) Q^2 + 8mr \right) \text{Erf}\left(\frac{rM_s}{2}\right) M_s + 1536e^{\frac{1}{2}r^2 M_s^2} m^2 \pi \text{Erf}\left(\frac{rM_s}{2}\right)^2 \right]. \quad (C8)
 \end{aligned}$$

In the case $Q = 0$, we would recover all curvature tensors and invariants for the case of a neutral point-source obtained in Ref. [15], as expected.

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