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CHILD SUBSIDIES AND THE CROSS-SECTIONAL FERTILITY PATTERN*

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This paper studies the effect of different types of child subsidies on the economic allocation and the cross-sectional fertility pattern. I construct a rich but tractable general equilibrium model with overlapping generations of heterogeneous individuals, two sectors of production and a government. I derive analytically how fertility choices are affected by changes in household resources and child subsidies and show the numerical results of two policy experiments. A subsidy on childcare favors the birth rates of educated, high-wage individuals. Parents work more so that the subsidy program is partly financed by a rise in labor earnings. In contrast, a lump-sum child allowance stimulates fertility of uneducated, low-wage parents the most. This policy has a larger effect on the average birth rate, but as parents perform most of the care themselves it comes at the expense of a lower supply of labor and a greater tax burden.

Keywords: Family Policy, Fertility, Childcare Subsidies

1. INTRODUCTION

Raising a child requires a significant amount of money and time. Most developed countries provide financial support to parents in the form of cash transfers or subsidies on the use of professional child care. There can be several reasons why policy makers do this. For example, they might wish to stimulate the labor force participation of women and ensure equality of opportunity for both sexes. Blau and Robins (1988) provide empirical evidence that the use of market care is responsive to its price and that subsidies, therefore, indeed have the intended effect of encouraging labor supply. Attanasio et al. (2008) show that declining costs of childcare can explain a large part of the rise in participation rates of married women in the United States as observed in recent decades. A second possible reason to subsidize parents is to increase fertility in order to avoid the

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TABLE 1. Statistics on formal childcare in selected OECD countries

Country	Childcare support per child (US\$ PPP)	Employment in childcare (% of total)
Finland	\$7,118	2.46
Norway	\$6,425	3.75
Denmark	\$6,376	3.26
Sweden	\$5,928	1.97
Iceland	\$5,733	2.68
United Kingdom	\$3,563	2.72
Belgium	\$2,333	1.31
France	\$2,858	2.73
Japan	\$2,683	0.80
Italy	\$1,558	0.23
Netherlands	\$1,092	1.51
Switzerland	\$1,129	0.42
Germany	\$860	0.39
United States	\$794	0.40

Childcare support: OECD Family Database, PF3.1 “Public spending on childcare and early education” for 2011.

Employment shares: Labour Force Survey 2013 (EU), ISCO-08 code 531, title “Childcare workers”; Bureau of Labor Statistics May 2014 (US), 2010 SOC code 39-9001, title “Childcare workers”; Population Census 2010 (Japan), occupation “Childcare workers.”

adverse effects associated with population aging, such as a low support ratio and a strained pension system. As a final example, policy makers might want to give children from disadvantaged backgrounds access to professional care in order to aid their development early in life [see, for example, Heckman (2006)]. Table 1 shows that the amount of childcare support provided differs greatly between OECD countries, with Scandinavia at the head of the list and the United States at the bottom. Greater public spending tends to reduce the cost that parents pay per hour of formal care and to increase the corresponding demand. This is reflected in the share of employment in childcare in each country, given in the second column of Table 1, which ranges from only 0.23% in Italy to 3.75% in Norway.

The aim of this paper is to study the implications of different types of child subsidies on the economic allocation and the cross-sectional fertility pattern. I construct a rich but tractable general equilibrium model with overlapping generations of households that has the following main features. First, individuals start their adult life as a single and are either educated or uneducated. They decide how much of their labor income to consume immediately and how much to save for when they are married. Second, married couples optimally decide about the number of children they want to have, taking into account that the consumption of their children is proportional to their own and that childcare requires time from either parents or professional caregivers. Third, educated and uneducated labor are imperfect substitutes in the production of goods, which implies that changes in their relative supply affect relative wages. Finally, the general equilibrium

framework requires individual decisions to be consistent at the aggregate level so that any child subsidies provided by the government have to be financed by (potentially distorting) taxes and the demand for formal childcare will have to be met by hours of uneducated labor.

In the benchmark economy, without any child subsidies, a couple consisting of an educated and an uneducated spouse has the most children, while parents who are both educated have the least. This result is mainly driven by the assumption that parents are gross substitutes in childcare (so that spouses with unequal wages can benefit from partial specialization into childcare and market work) and the equilibrium outcome that educated individuals save relatively less before marriage (so that they rely more heavily on their labor income and time spent on childcare is costly).

In the subsequent policy analysis, I distinguish between a childcare subsidy that lowers the price of an hour of formal childcare and a lump-sum transfer in the form of a child allowance. I show analytically that a subsidy on childcare favors the birth rates of educated, high-wage individuals as it reduces the time cost of a child the most for them. Despite having more children, parents also have more time left for work so that the subsidy program is partly financed by a rise in labor earnings. The increased demand for professional care draws uneducated labor into childcare and away from the production of goods. Relative wages adjust which reduces the fertility of educated relative to uneducated individuals. In contrast, the introduction of a child allowance stimulates fertility of uneducated, low-wage parents the most as the subsidy is largest for them compared to their own wage. For the same average amount of subsidy per child, this policy has a larger effect on the average birth rate, but as parents perform most of the care themselves it comes at the expense of a lower supply of labor and a greater tax burden.

This paper provides a contribution to two separate strands of research. The first consists of theoretical and quantitative studies on childcare subsidies. Most papers in this field focus on the implications for (female) labor supply decisions while keeping the number of children constant. For example, Domeij and Klein (2013) derive that in an economy with preexisting distortionary taxes on labor it can be efficiency improving to subsidize day care. In a numerical application with German data they show that under an optimal subsidy rate of 50% the labor supply of mothers with small children nearly doubles. A second example is the recent contribution by Guner et al. (2014). They develop a quantitative model with heterogeneous households and calibrate it to the U.S. economy in order to study the welfare effects of an expansion of current subsidy arrangements. Their conclusion is that childcare subsidies have a significant effect on female labor supply and mostly benefit poorer households. Bick (2016) does allow for fertility responses to changes in family policy and shows that this is important in order not to overestimate the effect on female labor supply. Fehr and Ujhelyiova (2013) conclude that it is possible to increase fertility and labor force participation of women at the same time by reducing the cost of childcare facilities.

The second strand of research to which this paper is related is the economics of fertility [see Hotz et al. (1997) for a survey]. The starting point of the

FIGURE 1. Life-cycle stages.

pioneering work by Becker (1960) is the observation that empirical studies tend to find a negative relationship between the number of children and a measure of income (usually a proxy for male wages), both in the cross-section and over time. Economic models of fertility have attempted to explain this “stylized fact” [see Jones et al. (2010)]. In a static setting, these explanations rely on the existence of time costs of childcare (which are higher for high-wage parents) or a trade-off between the quantity and “quality” of the offspring (the latter of which is assumed to be relatively cheap for high-wage parents). I add to this discussion by highlighting the role of intertemporal dynamics (such as premarital savings choices) and institutional features (such as taxes and subsidies) in explaining the cross-sectional fertility pattern.

Hazan and Zoabi (2015) have shown that in recent years the relationship between female education and fertility has become U-shaped. Their explanation relies on a quantity–quality trade-off (which yields the downward-sloping part) combined with the availability of formal care that is relatively cheap for highly educated females (which generates the upward-sloping part). I show that the model in this paper can also generate a U-shaped pattern, if I allow for three education types instead of two and is therefore consistent with this phenomenon.

The remainder of this paper is organized as follows. Section 2 describes in detail the model. In Section 3, I will discuss the model’s implications for the cross-sectional fertility pattern, followed by a numerical assessment of different child subsidy policies in Section 4. The last section concludes.

2. MODEL

In this section, I describe the behavior of households, firms, and the government in the model and provide a definition of the equilibrium.

2.1. Households

The population consists of an equal number of males (indicated with superscript $j = m$) and females ($j = f$). Every individual lives for four periods or “life-cycle stages”; see Figure 1. The first of these (stage 0) is spent in the parental household. The remaining three constitute the life span of an adult individual. In stage 1, an exogenous fraction θ of the cohort becomes educated ($E = 1$) while the remainder stays uneducated ($E = 0$). At the start of stage 2, everyone gets married and couples jointly decide about fertility and childcare. Their children

stay with them for exactly one period. The couple stays married in the final stage and works until retirement.

In every period, young adults and married couples make consumption and savings decisions. I assume that there are no bequests from parents, so that each individual enters adulthood with zero financial assets and leaves nothing behind after death. The share of the unit time endowment not spent in college, retirement, or childcare is sold on the labor market. Preferences over consumption c and the number of children b can be represented by the following felicity function:

$$u(c, b) = \varepsilon \ln c + (1 - \varepsilon) \ln(1 + b), \quad 0 < \varepsilon < 1, \quad (1)$$

where ε represents the weight of consumption. This specification is similar to that used by Baudin et al. (2015) and ensures that utility is well defined when b is equal to zero. This is relevant because individuals are assumed to be single and childless in stage 1. The optimal household allocations during each stage of life can be derived using backward induction, starting from the marriage phase.

Marriage and fertility. Married couples jointly decide how many children to raise and how much to consume during life-cycle stages 2 and 3, under the restriction that the level of consumption is the same for all household members.¹ There are economies of scale for a multiperson household (think of sharing a house, kitchen equipment, etc.) so that the total consumption expenditures of a household are proportional to the amount of “adult equivalents” that it consists of and not the number of family members. This is a very common assumption in household economics [see, for example, Attanasio et al. (2008) or Bick (2016)].

As children are a public good within the household and consumption is equal, it follows that felicity during marriage is the same for husband and wife. This means that there is no conflict of interest between married partners and the objective of the household is to maximize the welfare of its representative member. Consider a married couple of which the wife has education E^f and the husband has education E^m . If they enter the last stage of life with a stock of savings equal to a_3 and b grown-up children, they face the following problem:

$$\begin{aligned} \mathcal{M}_{3,t}(E^f, E^m, a_3, b) &= \max_{c_3} u(c_3, b) \\ \text{s.t. } 0 &= (1 + r_t)a_3 + (1 - \tau_t)[w_t(E^f) + w_t(E^m)](1 - \bar{R}) - 2\bar{\tau}_t - Q^a c_3, \quad (2) \end{aligned}$$

where $\bar{R}$ is the (exogenous) fraction of time spent in retirement and $Q^a \leq 2$ is the equivalence scale for two adults. Household resources in old age consist of savings a_3 inclusive of interest accrued at rate r_t , plus education-dependent wages $w_t(E)$ of husband and wife net of labor taxes τ_t , minus lump-sum taxes $\bar{\tau}_t$ and expenditures on consumption c_3 . The (trivial) solution to this problem is that the couple consumes all its resources.

One period before, in stage 2, couples decide how many children to raise. The cost of having a child is twofold. First, consumption expenditures increase as the

adult equivalence scale goes up with $Q^b \leq 1$ during this period. Second, there is a fixed amount of childcare N^b that has to be provided. This care can be created using time inputs of the parents n^p and professional caregivers o :

$$\Upsilon(n^p, o) = (n^p)^\psi (n^p + o)^{1-\psi}, \quad 0 < \psi \leq 1, \quad (3)$$

where n^p is a combination of mother time n^f and father time n^m :

$$n^p = \Gamma(n^f, n^m) = [(n^f)^{1-1/\xi} + (n^m)^{1-1/\xi}]^{\frac{1}{1-1/\xi}}, \quad \xi > 1. \quad (4)$$

This set-up has the following features. First, from (4) it follows that parents are equally productive but imperfect substitutes for each other as long as the substitution elasticity ξ is finite. Second, (3) implies that parental time is a necessary input in childcare while formal care is not. In addition, the marginal productivity of a professional caregiver is always lower than that of a parent. Other papers that allow for market inputs in childcare usually assume a constant and symmetric elasticity of substitution between the two types of care [see, for example, Hazan and Zoabi (2015)]. With such a specification, a corner solution without any formal childcare does not occur although this is quite common in practice.² Third, an hour spent with a parent yields more childcare than an hour spent with a professional caregiver, to reflect that only the former can provide the love that is an essential element of a child's needs. Fourth, in the absence of formal childcare it follows from (3) that the time requirement per child reduces to $n^p = N^b$. If parents decide to outsource part of the care, then the total time input $n^p + o$ goes up. This captures the fact that professional caregivers need to be paid by the hour, even when not actively minding a child. In addition, many childcare facilities charge by the day or half day so that parents might end up paying for more hours than they actually require. Finally, the specification in (3) implies that the less time parents spend with their own children, the harder it becomes to substitute market-bought childcare for parental time.

The decision problem of a couple from the perspective of stage 2 can be written as:

$$\begin{aligned} \mathcal{M}_{2,t}(E^f, E^m, a_2) &= \max_{c_2, a_3, b, n^f, n^m, o} \left[u(c_2, b) + \frac{1}{1+\rho} \mathcal{M}_{3,t+1}(E^f, E^m, a_3, b) \right] \\ \text{s.t. } a_3 &= (1+r_t)a_2 + (1-\tau_t) [w_t(E^f)(1-n^f) + w_t(E^m)(1-n^m)] \\ &\quad - 2\bar{\tau}_t - (Q^a + Q^b b)c_2 - (1-s_t)p_t o + \bar{s}_t b, \\ \Upsilon(\Gamma(n^f, n^m), o) &= N^b b, \quad b \geq 0, \quad 0 \leq n^f \leq 1, \quad 0 \leq n^m \leq 1, \quad o \geq 0, \end{aligned} \quad (5)$$

where ρ is the rate of time preference. The household decides how many children to have, in what way to arrange their care, and how to split resources between consuming in the current period and saving for the next. From the budget constraint

it follows that the cost of childcare consists of the foregone wages of the parents and the bill for formal care services, with p_t its relative price and s_t the corresponding subsidy rate. There is a child allowance of $\bar{s}_t$ per child while the child is living in the parental household.

Since by assumption parents do not get any direct utility from caring for their children, it follows that the optimal allocation of total childcare over parents and professional caregivers for a given number of children b is the one that minimizes the total associated costs. As the childcare production functions (3) and (4) both feature constant returns to scale this equals the lowest possible cost per unit of childcare times the total amount of childcare required $N^b b$. The unit cost function can be derived in two steps. First, I let w_t^p denote the minimum before-tax cost of a unit of parent time. Assuming that an interior solution exists that satisfies the time constraints of both parents I find:

$$w_t^p \equiv \left\{ \min_{n^f, n^m} [w_t^f n^f + w_t^m n^m] \text{ s.t. } \Gamma(n^f, n^m) = 1 \right\} = \left[(w_t^f)^{1-\xi} + (w_t^m)^{1-\xi} \right]^{\frac{1}{1-\xi}}, \quad (6)$$

where $w_t^j = w_t(E^j)$ denotes the wage rate earned by spouse j . Because the constant elasticity of substitution (CES) production function (4) is self-dual, the price of parent time is a CES aggregate of parental wages with substitution elasticity $1/\xi$. For a given total production of parent time n^p , the optimal input quantities of father and mother time are $n^j = n^p (w_t^j / w_t^p)^{-\xi}$. As long as parents are imperfect substitutes both will contribute a positive amount of time to childcare, but the parent with the lower wage does most. In the limiting case with $\xi \rightarrow \infty$ all care is performed by the spouse with the lowest opportunity cost of time.

In the next step, I define v_t to be the minimum cost of a unit of childcare:

$$v_t \equiv \left\{ \min_{n^p, o} [(1 - \tau_t) w_t^p n^p + (1 - s_t) p_t o] \text{ s.t. } \Upsilon(n^p, o) = 1 \right\}. \quad (7)$$

There is a trade-off between the (after-tax) value of parental time and the (after-subsidy) price of formal childcare. Two cases can be distinguished, depending on whether there is an interior solution to the problem expressed in (7) or not. First, if formal childcare is relatively cheap in the sense that $(1 - s_t) p_t \leq (1 - \psi)(1 - \tau_t) w_t^p$, then the care burden is shared between parents and professional caregivers. The optimal allocation is given by:

$$n^p = N^b b \frac{\psi v_t}{(1 - \tau_t) w_t^p - (1 - s_t) p_t}, \quad o = N^b b \frac{(1 - \psi) v_t}{(1 - s_t) p_t} - n^p, \quad (8)$$

with the following minimum cost function:

$$v_t = \left[\frac{(1 - s_t) p_t}{1 - \psi} \right]^{1-\psi} \left[\frac{(1 - \tau_t) w_t^p - (1 - s_t) p_t}{\psi} \right]^\psi. \quad (9)$$

FIGURE 2. Minimum cost of childcare and demand for formal childcare.

Note that it is not sufficient that the net price of an hour of formal childcare is lower than the after-tax value of parental time. It has to be even lower than that in order to compensate for the fact that professional caregivers do not provide the same type of care as a parent. If this condition is not satisfied, then all care is performed by the parents ($o = 0$ and $n^p = N^b b$) and the unit cost is $v_t = (1 - \tau_t)w_t^p$. These two cases are illustrated in panel (a) of Figure 2. The upper diagram depicts the minimum unit cost of childcare as a function of the price of formal care net of subsidies, keeping the tax rate fixed. It is increasing up to the point where it becomes optimal to leave all care to the parents, after that it stays flat. By Shephard's Lemma, the demand for formal childcare equals the slope of the minimum cost function, which is positive in region (i) but zero in (ii); see the lower diagram in panel (a). If the subsidy on childcare goes up then, ceteris paribus, the minimum cost weakly decreases and the demand for professional caregivers weakly increases (a movement along the respective curves to the left). However, if the subsidy is paid for by higher taxes on labor income then the opportunity cost of parental time decreases as well. This is illustrated in panel (b). Both the minimum cost function and the demand schedule shift down (vertical arrows) and

the line of demarkation between regions (i) and (ii) moves to the left (horizontal arrows). As a consequence, there is a further drop in the cost of childcare but also a reduction in the demand for formal care in favor of parental time.

Assuming that the number of children need not be an integer and that an interior solution exists, the remaining first-order conditions of the household problem in (5) can be written as:

$$\frac{Q^a c_3}{(Q^a + Q^b b) c_2} = \frac{1 + r_{t+1}}{1 + \rho}, \quad (10a)$$

$$\frac{2 + \rho}{1 + \rho} \frac{1 - \varepsilon}{\varepsilon} \frac{c_2}{1 + b} = \frac{N^b v_t + Q^b c_2 - \bar{s}_t}{Q^a + Q^b b}, \quad (10b)$$

$$W_{2,t} = Q^a c_2 + \frac{Q^a c_3}{1 + r_{t+1}} + (N^b v_t + Q^b c_2 - \bar{s}_t) b, \quad (10c)$$

where $W_{2,t}$ is household wealth at the start of stage 2, which consists of asset income and discounted wages net of taxes:

$$W_{2,t} = (1 + r_t) a_2 + (1 - \tau_t)(w_t^f + w_t^m) - 2\bar{\tau}_t \\ + \frac{(1 - \tau_{t+1})(w_{t+1}^f + w_{t+1}^m)(1 - \bar{R}) - 2\bar{\tau}_{t+1}}{1 + r_{t+1}}. \quad (11)$$

Equation (10a) is the couple's Euler equation for total consumption expenditures. The number of adult equivalents present in the household in each period can be seen as the price of the corresponding consumption good. The second equation captures the trade-off between fertility and consumption. The left-hand side represents the marginal ratio of substitution and the right-hand side the relative prices. The "price" of a child (numerator) consists of the minimum time cost $N^b v_t$ and the consumption expenditures $Q^b c_2$ net of the child allowance $\bar{s}_t$. It is increasing in c_2 , while at the same time the price of second-period consumption (denominator) is increasing in the number of children b . This makes the trade-off that the household faces between consuming and raising children different from the standard problem of allocating a budget over two different goods. Equation (10c) restates the household budget constraint with the minimum cost per child substituted in.

Young adults. Moving back yet another stage in life, the felicity from marriage is perfectly anticipated by a young adult in life-cycle stage 1 and he or she makes decisions accordingly. For example, consider a female of education type E who has just left the parental household at the start of period t . She chooses consumption c_1 and savings a_2 , taking as given the probability of meeting a future spouse with education level E^m and corresponding financial assets $a_{2,t}^m(E^m)$. Her expected

lifetime utility is given by:

$$\begin{aligned} S_{1,t}^f(E) = \max_{c_1, a_2} & \left(u(c_1, 0) + \frac{1}{1 + \rho} \left[\pi^{m|f}(0|E) \mathcal{M}_{2,t+1}(E, 0, a_2 + a_{2,t}^m(0)) \right. \right. \\ & \left. \left. + \pi^{m|f}(1|E) \mathcal{M}_{2,t+1}(E, 1, a_2 + a_{2,t}^m(1)) \right] \right) \\ \text{s.t. } & a_2 = (1 - \tau_t) w_t(E) (1 - \bar{e}E) - \bar{\tau}_t - c_1. \end{aligned} \quad (12)$$

The first term in curly brackets is the immediate felicity from consumption in the absence of any children. The remaining ones capture the expected discounted utility from stage 2 onward. There is a probability $\pi^{m|f}(0|E)$ of being matched to an uneducated male and a probability $\pi^{m|f}(1|E)$ of finding an educated spouse in the next period. Importantly, these probabilities are conditional on her own educational attainment. The amount of savings that she brings to the marriage consists of labor income net of taxes and consumption expenditures, with $\bar{e}$ the time cost of a college degree. In equilibrium, the expectations that individuals have about the premarital savings of others coincide with the actual savings decisions. As there are no differences between men and women this implies that $a_{2,t}^f(E) = a_{2,t}^m(E)$.

Matching probabilities. By assumption, everyone gets married at the start of stage 2 and there is no divorce. I denote by $\pi(E^f, E^m)$ the fraction of couples in which the wife has education E^f while the husband has education E^m . Bayes' Rule implies that the conditional probability that a woman with education E^f is matched to a man with education E^m is given by:

$$\pi^{m|f}(E^m|E^f) = \frac{\pi(E^f, E^m)}{\theta}. \quad (13)$$

The expressions for males are similar. If there is positive assortative matching, so that individuals are likely to marry someone with a similar level of education, then $\pi^{m|f}(1|1) > \pi^{m|f}(1|0)$ so that a woman has a greater probability to be matched to an educated husband if she is herself educated.

Demography and aggregation. At the start of each period, a new cohort with an equal number of males and females is born. This means that four different cohorts (or "overlapping generations") are alive at the same time. I let $P_{s,t}$ denote the size of the cohort that is in stage $s \in \{0, 1, 2, 3\}$ at time t and define $P_t = P_{1,t} + P_{2,t} + P_{3,t}$ as the adult population. Despite an average birth rate per woman of less than two there is still a positive population growth rate in most OECD countries due to immigration. I allow for this by assuming that the cohort of young adults grows at a constant rate η over time even if there are fluctuations in the number of births so that $P_{1,t} = (1 + \eta)P_{1,t-1}$.³ Since there is no mortality risk it follows that $P_{3,t+2} = P_{2,t+1} = P_{1,t}$.

Given the demographic structure of the population and its marital composition, I can aggregate all household choices to find total financial asset holdings A_t , labor supply by education type $L_t(0)$ and $L_t(1)$, consumption C_t , and demand for formal care O_t . The optimal fertility rates of each couple determine the average birth rate $\bar{b}_t$.

2.2. Firms

There are two types of commodities in the economy: a good that is used for consumption and investment (the numeraire) and formal childcare. These are produced in separate sectors using two production factors: physical capital and labor. I assume that capital is only used in goods production and that labor is perfectly mobile between the two sectors.

Goods sector. The production of a homogeneous good for consumption and investment purposes proceeds according to a constant returns to scale technology:

$$Y_t = \Phi \left(K_t \right)^\phi \left(N_t^Y \right)^{1-\phi}, \quad \Phi > 0, \quad 0 < \phi < 1, \quad (14)$$

where K_t is the capital stock at the start of period t and N_t^Y is a composite of labor supplied by educated workers $N_t^Y(1)$ and uneducated workers $N_t^Y(0)$:

$$N_t^Y = \left[N_t^Y(0)^{1-1/\alpha} + \beta N_t^Y(1)^{1-1/\alpha} \right]^{\frac{1}{1-1/\alpha}}, \quad \alpha > 1, \quad \beta > 0. \quad (15)$$

The parameter α captures the constant elasticity of substitution between the two types of labor. I denote by w_t^Y the minimum unit cost of the labor composite. Assuming that both types of labor are employed it follows that $w_t^Y = [w_t(0)^{1-\alpha} + \beta^\alpha w_t(1)^{1-\alpha}]^{\frac{1}{1-\alpha}}$. The stock of capital increases with firm investment I_t and depreciates at rate δ such that $K_{t+1} = (1 - \delta)K_t + I_t$. The representative profit-maximizing firm in the goods sector chooses capital and total labor in such a way that the following conditions are satisfied:

$$r_t + \delta = \phi \Phi \left(\frac{K_t}{N_t^Y} \right)^{-(1-\phi)}, \quad w_t^Y = (1 - \phi) \Phi \left(\frac{K_t}{N_t^Y} \right)^\phi. \quad (16)$$

That is, each factor of production earns exactly its marginal product. The linear homogeneity of the production function implies that firms in the goods sector do not earn a profit. The corresponding demand for uneducated and educated labor follows from:

$$w_t(0) = w_t^Y \left[\frac{N_t^Y(0)}{N_t^Y} \right]^{-1/\alpha}, \quad w_t(1) = w_t^Y \beta \left[\frac{N_t^Y(1)}{N_t^Y} \right]^{-1/\alpha}. \quad (17)$$

Childcare sector. The production function for formal childcare is as follows:

$$Z_t = \Psi N_t^Z, \quad \Psi \geq 1, \quad (18)$$

where N_t^Z is the total of uneducated and educated labor employed.⁴

$$N_t^Z = N_t^Z(0) + N_t^Z(1). \quad (19)$$

If $\Psi > 1$, then each professional caregiver can provide care for more than one child at the same time. In contrast to the goods sector, educated and uneducated workers are assumed to be perfect substitutes in creating childcare. As a consequence, only uneducated labor will be hired since it is less expensive. Perfect labor mobility between sectors implies that the wage rate is equal to that earned in the goods sector so that the minimum unit cost is $w_t^Z = w_t(0)$ as determined in (17). The competitive price of formal childcare relative to the consumption good is $p_t = w_t^Z / \Psi$.

2.3. Government

The government levies taxes to finance a general level of government spending G_t , subsidies on childcare and a child allowance. I assume that there is no debt financing so that in every period the government has to maintain a balanced budget:

$$T_t \equiv \tau_t [w_t(0)L_t(0) + w_t(1)L_t(1)] + \bar{\tau}_t P_t = G_t + s_t p_t O_t + \bar{s}_t P_{0,t} \equiv G_t + S_t. \quad (20)$$

The left-hand side is the total revenue from the proportional labor income tax plus the proceeds from lump-sum taxation of the adult population. The right-hand side consists of government spending, subsidies paid out for every hour of professional childcare demanded by households and the allowance for each child.

2.4. General Equilibrium

The equilibrium of the model is a sequence of prices and allocations such that in every period

- i. Each young adult and every couple maximizes utility subject to a budget constraint taking prices and the behavior of everyone else as given.
- ii. Firms maximize profits taking prices as given.
- iii. The government budget is balanced.
- iv. All markets clear.
 - Capital market:

$$K_t = A_t.$$

- Labor market:

$$N_t^Y(1) = L_t(1), \quad N_t^Y(0) + N_t^Z(0) = L_t(0).$$

- Goods market:

$$Y_t = C_t + I_t + G_t.$$

- Childcare market:

$$Z_t = O_t.$$

The circular flow diagram in Figure 3 provides a graphic visualization of the equilibrium. It shows how the three actors in the economy (households, firms, and the government) interact on four markets (capital, labor, goods, and childcare). The solid lines capture the flow of goods, services, and labor hours, the dashed lines represent payments in terms of the numeraire commodity.

The steady state of the model is a balanced growth path along which all variables grow at a constant rate. In particular, prices (such as p_t) are constant while macroeconomic aggregates (such as K_t) grow at the same rate as the population and remain stationary when scaled by the size of the adult population (for example, K_t/P_t). The average birth rate $\bar{b}_t$ is constant in the long run as well.

3. THE CROSS-SECTIONAL FERTILITY PATTERN

In this section, I discuss the model's implications regarding optimal fertility choices. Three assumptions are crucial in this respect. First, parents derive utility from the quantity of children only. There is no quantity–quality trade-off à la Becker (1960). Second, the consumption expenditures for children are proportional to parental consumption. This prevents these costs from becoming negligible as wages increase and automatically ensures that parents with more income provide their offspring with a higher standard of living. Third, the specification of the utility function ensures that fertility remains constant if the cost per child and household wealth change by the same proportion.

I can derive comparative static effects on the optimal fertility choice by analyzing the system of first-order conditions given in (10a)–(10c). Ignoring time subscripts from now on, I substitute out for c_2 and c_3 and write the first-order condition for fertility as:

$$F(b) = f_1(b)W_2 - f_2(b)(N^b v - \bar{s}) = 0, \quad (21)$$

where $f_1(b)$ and $f_2(b)$ are positive functions of b :

$$f_1(b) = 1 - \frac{1 + \rho}{2 + \rho} \frac{\varepsilon}{1 - \varepsilon} \frac{Q^b(1 + b)}{Q^a + Q^b b}, \quad (22)$$

$$f_2(b) = \frac{b + \varepsilon}{1 - \varepsilon} - \frac{1 + \rho}{2 + \rho} \frac{\varepsilon}{1 - \varepsilon} \frac{Q^b(1 + b)}{Q^a + Q^b b} b. \quad (23)$$

Note that the consumption cost of a child $Q^b c_2$ does not explicitly feature in (21) because the level of household consumption c_2 is endogenous. The optimal number of children b^* is such that $F(b^*) = 0$. It is straightforward to show that the following proposition holds.

FIGURE 3. Circular flow diagram.

FIGURE 4. Trade-offs in the choice of fertility.

PROPOSITION 1. *The optimal number of children is increasing in household wealth and the child allowance and decreasing in the time cost per child.*

Proof. See Appendix A.

In the special case that $Q^b = 0$ so that there are no consumption costs of children it follows that $f_1(b) = 1$ and $f_2(b) = (b + \varepsilon)/(1 - \varepsilon)$. It is then possible to either write down the optimal number of children in closed form or to derive it graphically as the intersection between two straight lines, see panel (a) of Figure 4.⁵ The results of Proposition 1 follow readily as both an increase in W_2 (an upward shift of the horizontal line) and a decrease in $N^b v - \bar{s}$ (a downward shift of the upward sloping line) increase fertility. In the more general case with $Q^b > 0$, the function $f_1(b)$ is decreasing and convex in b , while $f_2(b)$ is increasing and concave in b , so that the determination of the optimal number of children can be represented by the diagram in panel (b). In this case the effect of a change in wealth, time costs or the child allowance on fertility is dampened, because they also affect the optimal choice of consumption and thereby the total cost of a child.

3.1. Parental Wages

Proposition 1 cannot be directly applied to understand how fertility varies with parental wages. If the wage of one of the parents goes up then this increases both household wealth and the opportunity cost of time, which means that the overall effect is ambiguous in general. Note that with $Q^b > 0$, it is not possible to discuss these opposing forces in terms of income and substitution effects in the traditional sense. The reason for this is that the “price” of a child depends on second-period consumption, while the cost of consumption is increasing in the number of children

(as mentioned in Section 2.1). Additional insights can be derived by dividing the first-order condition by the after-tax wage of the wife:

$$\frac{F(b)}{(1-\tau)w^f} = f_1(b) \left[(1+r) \frac{a_2}{(1-\tau)w^f} + \left(1 + \frac{w^m}{w^f} \right) \left(1 + \frac{1-\bar{R}}{1+r} \right) - 2 \frac{\bar{\tau}}{(1-\tau)w^f} \right] - f_2(b) \left[N^b \frac{v}{(1-\tau)w^f} - \frac{\bar{s}}{(1-\tau)w^f} \right] = 0, \quad (24)$$

where I have used the definition of household wealth from (11) and assume that factor prices remain constant over time. The minimum unit cost of childcare v is homogeneous of degree 1 in the after-tax value of parental time $(1-\tau)w^p$ and the net price of formal childcare $(1-s)p$ so that:

$$\frac{v}{(1-\tau)w^f} = \begin{cases} \frac{w^p}{w^f} = \left[1 + \left(\frac{w^m}{w^f} \right)^{1-\xi} \right]^{\frac{1}{1-\xi}} & \text{if } (1-s)p > (1-\psi)(1-\tau)w^p \\ v \left(\frac{w^p}{w^f}, \frac{(1-s)p}{(1-\tau)w^f} \right) & \text{if } (1-s)p \leq (1-\psi)(1-\tau)w^p \end{cases}. \quad (25)$$

As a starting point, suppose that there are no premarital savings ($a_2 = 0$), there is no lump-sum tax ($\bar{\tau} = 0$) or child allowance ($\bar{s} = 0$), and formal childcare is either not available or prohibitively expensive ($v = (1-\tau)w^p$). Under these assumption it follows that (24) depends on the wages of the spouses only through the ratio w^m/w^f .

PROPOSITION 2. *Assume there is no premarital saving, no formal childcare, and no lump-sum tax or child allowance. Then the optimal number of children depends on the relative wages of husband and wife and attains a minimum when wages are equal.*

Proof. See Appendix A.

The optimal number of children as a function of the ratio of parental wages is illustrated in Figure 5. If both the female and male wage grow at the same rate then the positive effect of higher wealth and the negative effect of a greater cost of parental time exactly cancel out and the optimal number of children remains constant. The fact that fertility is lowest when wages are equal hinges crucially on the existence of time costs for which parents are substitutes. It only holds if $N^b > 0$ and $\xi > 0$. Intuitively, couples in which the spouses have the same wage rate miss out on the gains from (partial) specialization that can be reaped when a low-wage parent takes on most of the childcare while the high-wage spouse can spend more time on market work. The higher is the elasticity of substitution between parents ξ , the greater are these gains from specialization and the higher will be the fertility of unequal-wage couples relative to those with equal wages. Since there is no asymmetry between the sexes it is irrelevant whether the wife

FIGURE 5. Illustration of Proposition 2.

earns more than her husband or vice versa. In terms of Figure 5, the optimal number of children is the same for w^m/w^f equal to γ and $1/\gamma$ for any $0 < \gamma \leq 1$.

The prediction that fertility choices depend on relative wages only will change if one of the premises of Proposition 2 is relaxed. From (25) it follows that, all else equal, an increase in household wealth relative to parental wages promotes fertility, while an increase in the cost of a child relative to parental wages lowers fertility. I will discuss the relaxation of each assumption in turn. First, the availability of (subsidized) formal childcare lowers the time cost of children relative to the opportunity cost of time of the mother v/w^f . The reduction is greatest for high-wage parents and hence their fertility increases relative to low-wage couples. If the price is not sufficiently low to make formal childcare an attractive alternative for parental care for low-wage earners, then it affects the fertility choice of the high-wage earners only. A lump-sum tax, on the other hand, has a greater impact on the wealth of low-wage individuals as for them $\bar{\tau}/w^f$ is higher. In case of a transfer (a negative tax), the fertility of low-wage parents increases relative to high-wage parents. A similar argument can be made for a child allowance: it lowers the cost of having a child relatively more for uneducated couples as it is $\bar{s}/w^f$ that features in (24).

The stock of household savings a_2 is predetermined at the time of marriage and therefore has a pure income effect on fertility.⁶ It is likely that educated men and women save relatively less before marriage because they forego wages by going to college and they face a better prospect of marrying a high-wage spouse. If so then

a_2/w^f is lower for high-wage couples, which reduces their fertility compared to low-wage parents. Finally, under the premises of Proposition 2 the labor tax rate does not affect the fertility choice because the term $(1 - \tau)$ drops out. All else equal, either the presence of positive financial assets ($a_2 > 0$), lump-sum transfers ($\bar{\tau} < 0$), or a child allowance ($\bar{s} > 0$) ensures that a higher tax rate increases fertility by lowering the opportunity cost of parental time spent on childcare. The opposite is true when parents make use of formal childcare, as a higher tax rate increases its relative price in (25).

3.2. Fertility by Education

In the model, I distinguish two education levels $E \in \{0, 1\}$ with corresponding wages $w(E)$. I write $b^*(E^f, E^m)$ for the optimal fertility choice of a couple where the wife has education E^f and the husband E^m . As alluded to in the introduction, the main empirical stylized fact about fertility is the negative relationship between the number of children and income. Due to data limitations, income is usually captured by a proxy for the male wage or education level [see, for example, Jones et al. (2010)]. Hence, in the context of the model a negative income–fertility relationship would correspond to $\bar{b}^m(0) < \bar{b}^m(1)$ with $\bar{b}^m(E)$ the average number of births to men with education E :

$$\bar{b}^m(E) = \pi^{f|m}(0|E)b^*(0, E^m) + \pi^{f|m}(1|E)b^*(1, E). \quad (26)$$

It is clear that the correlation between fertility and male education will depend both on cross-sectional fertility choices and the marital-sorting process. If there is a high degree of positive assortative matching, then $\pi^{f|m}(E|E) \approx 1$ and $\bar{b}^m(E)$ is close to $b^*(E, E)$. In that case, the education–fertility relationship is mainly determined by the fertility of couples with the same level of education.

Figure 6 illustrates how the model can generate a negative (or positive) relationship between education and fertility. Panel (a) is the benchmark described by Proposition 2 in which only relative wages matter. The difference with Figure 5 is that the male wage features along the horizontal axis instead of the wage ratio so that a change in the wage earned by the female shifts the entire fertility profile. The solid line shows how the optimal number of children varies with the male wage if w^f is fixed at $w(0)$, the dashed line does the same for $w^f = w(1)$. The optimal number of children is at a minimum when parental wages are equal, from which it follows that $b^*(0, 0) = b^*(1, 1) < b^*(1, 0) = b^*(0, 1)$. In this case, $\bar{b}^m(0) = \bar{b}^m(1)$. In panel (b), I introduce a childcare subsidy. As explained above, this kind of subsidy has no or only a small impact on the fertility of uneducated, low-wage parents. As a consequence, the solid line is virtually unchanged compared to panel (a). The dashed profile, which corresponds to a higher w^f , tilts upwards as w^m increases. It follows that $b^*(1, 1) > b^*(0, 0)$ and fertility is increasing in the education level of the husband. Panel (c) shows that a child

FIGURE 6. Optimal number of children with two education types.

allowance tends to increase fertility for all household types, but the most for uneducated parents with low wages so that $b^*(0, 0) > b^*(1, 1)$. The effect of a lump-sum transfer in panel (d) is similar but less pronounced as it constitutes an income gain that is unrelated to the number of children. If this lump-sum transfer is financed by a proportional labor tax then fertility increases even further for all types of couples through a reduction in the opportunity cost of time spent on childcare.

In Appendix C, I show how the model can account for a U-shaped relationship between female education and fertility [as in Hazan and Zoabi (2015)] while maintaining a negative relationship between male education and fertility [as in Jones et al. (2010)]. To do so, I need to allow for three different education types and an asymmetry between the sexes. In particular, I assume that the educational attainment of women exceeds that of men as is the case for recent cohorts in developed countries. This implies that some women will have to marry husbands who are less educated than them and earn a lower wage. Due to the benefits of partial specialization in childcare and market work for couples with unequal wages this raises their fertility. Cherchye et al. (2012) show that the time that a husband allocates to childcare is increasing in the wife's wage, which provides support

that this channel is indeed operative. The U-shaped pattern in fertility does not show up in the numerical results discussed below because I only distinguish two education types and do not allow for asymmetries between men and women.

4. POLICY EXPERIMENTS

To show the effect of child subsidies on the economic allocation and the cross-sectional fertility pattern, I parameterize the model and calculate the resulting long-run equilibria; see Appendix B for computational details. This is not intended as a calibration exercise for a specific economy, as the model is by construction much too stylized for that purpose. Nevertheless, I use reasonable parameter values that are as much as possible in line with the existing literature and data for OECD countries.

4.1. Parameterization

The length of each period is 18 years. Obtaining a college degree takes about 4 years ($\bar{e} = 0.25$), while retirement starts around age 65 ($\bar{R} = 0.4$). The impatience discount factor is set at 1% per annum, which translates into $\rho = (1.01)^{18} - 1$. I assume that each child requires $N^b = 0.25$ of care when measured in parental time. Using the OECD scale to translate household members into adult equivalents gives $Q^a = 1.7$ and $Q^b = 0.5$.

The elasticity of substitution between father and mother time is $\xi = 4$, which is in line with the estimate by Abbott (2014). It implies that parents are rather good but not perfect substitutes in childcare. There is no direct evidence on the parameter ψ in the childcare production function, so I set it equal to 0.5 and discuss how the results are affected by changes in this parameter in Section 4.3. The National Association for the Education of Young Children (NAEYC) in the United States reports the required number of childcare providers depending on the age and number of children in its accreditation criteria. These range from one per three children under the age of 15 months to one per nine children of 3 years and older. Taking into account that this is a minimum requirement and that there is additional staff required for administration and for replacement during break times and illness, I set $\Psi = 3$ in the childcare production function. This ensures that childcare costs are at a realistic level, as for OECD countries the gross childcare fees amount to about 30% of the average wage.⁷

I start from an initial steady state where the wage rate of an uneducated worker is normalized to $w_t(0) = 1$ and the college wage premium is 70% [see, for example, Hubbard (2011) for the United States]. The annual net return to capital r^a equals 5%. The elasticity of substitution between educated and uneducated labor in the production of goods is taken from Katz and Murphy (1992) and is equal to $\alpha = 1.41$. Childcare is not subsidized ($s = 0$), and there is no child allowance ($\bar{s} = 0$). The marginal tax rate is set at $\tau = 0.40$ or 40%, which is on the high end of the schedule of the United States but low compared to most

TABLE 2. Calibrated parameters

Parameter(s)	Target(s)
Utility function	Average birth rate
ε 0.692	$\bar{b}$ 1.7
Production function	Factor prices
Φ 0.808	$w(0)$ 1
ϕ 0.239	r $(1 + 0.05)^{18} - 1$
Labor aggregate	Relative wages
β 0.885	$w(1)/w(0)$ 1.7

European countries. To allow for some progressivity in taxes, I set $\bar{\tau} = -0.10$ (a lump-sum transfer), which implies that the average tax rate faced by a full-time employed uneducated worker is 30%. Government spending is set equal to the revenue from this preexisting labor tax.

The percentage of men and women in the United States that have at least a Bachelor’s degree is around 30% so I take $\theta = 0.3$. The matching probabilities are chosen such that the correlation coefficient between the education level of the wife and the husband equals 0.6 as in Fernández and Rogerson (2001), who calculate this value from panel data for the United States. It follows that more than 61% of marriages are between two uneducated individuals and in 21.6% of the cases both spouses are educated. I target an average birth rate of 1.7 in the initial equilibrium, which corresponds to the OECD average for 2014. The population growth rate is set at an annual value of 0.5% and held constant throughout the policy experiments below. Table 2 lists the calibrated parameters and their corresponding targets. As the number of targets is exactly the same as the number of free parameters, the model can match them perfectly.

4.2. Numerical Results

The first column in Table 3 is the benchmark steady state without any subsidies. The price of an hour of childcare is then $p = w(0)/\Psi = 0.333$. Parents compare this to the after-tax value of their own time, taking into account the difference in the quality of care between a parent and a professional caregiver. The threshold value of $(1 - \psi)(1 - \tau)w^p$ equals 0.238 for uneducated couples, 0.282 for mixed households, and 0.405 when both parents are educated. As a consequence, only the last make some use of formal care. The childcare sector is therefore small and provides only 0.203% of total employment in the economy. A couple in which the spouses have different education levels has the most children [$b^*(0, 1) = b^*(1, 0) = 1.774$], while parents who are both educated have the least [$b^*(1, 1) = 1.502$]. The fact that educated men and women save less than uneducated individuals combined with the lump-sum tax transfer to households outweighs the opposing effect of the availability of formal care so

TABLE 3. Long-run equilibria

		Care subsidy			Child allowance		
		FT	PE	GE	FT	PE	GE
<i>Subsidies</i>							
$s \times 100\%$	0.000	50.000	50.000	50.000			
$\bar{s}$	0.000				0.035	0.035	0.035
<i>Taxes</i>							
$\tau \times 100\%$	40.000		40.436	40.438		42.151	42.225
$\bar{\tau}$	−0.100						
<i>Prices</i>							
$w(0)$	1.000			1.007			1.002
$w(1)/w(0)$	1.700			1.669			1.683
$(1-s)p$	0.333	0.167	0.167	0.168	0.333	0.333	0.334
$r^a \times 100\%$	5.000			5.004			5.029
<i>Fertility</i>							
$b^*(0, 0)$	1.749	1.820	1.818	1.818	2.289	2.321	2.329
$b^*(1, 0) = b^*(0, 1)$	1.774	1.913	1.910	1.897	2.205	2.232	2.242
$b^*(1, 1)$	1.502	1.786	1.781	1.772	1.776	1.786	1.790
$\bar{b}^m(0)$	1.752	1.831	1.829	1.828	2.279	2.310	2.319
$\bar{b}^m(1)$	1.578	1.821	1.817	1.807	1.897	1.911	1.917
$\bar{b}$	1.700	1.828	1.826	1.822	2.164	2.190	2.198
<i>Premarital savings</i>							
$a_2^j(0)$	0.114		0.113	0.114		0.109	0.110
$a_2^j(1)$	0.094		0.092	0.091		0.089	0.090
<i>Labor</i>							
$L(1)/P$	0.226		0.229	0.229		0.224	0.224
$L(0)/P$	0.570		0.581	0.581		0.556	0.555
$N^Y(1)/N^Y(0)$	0.398			0.409			0.404
$(N^Z/L) \times 100\%$	0.203			2.588			0.189

Notes: FT is the direct effect on fertility, PE is the partial equilibrium, GE is the general equilibrium.

that $b^*(1, 1) < b^*(0, 0)$ (see Section 3). There is a negative correlation between male wages and fertility as uneducated fathers have on average more children [$\bar{b}_t^m(0) = 1.752$] than educated ones [$\bar{b}_t^m(1) = 1.578$].

Care subsidy. Suppose that the government introduces a childcare subsidy of $s = 0.5$ or half of the price of formal childcare, financed by means of an increase in the labor tax rate. To highlight the separate mechanisms present in the model the comparison between the steady states is split up into three steps and only those figures are reported that differ relative to the benchmark scenario.

Fertility. Initially, I focus on the direct effect of the subsidy on fertility. I keep factor prices, premarital savings, and tax rates fixed and ignore the balanced budget requirement of the government. The results are reported in the column labeled “FT.” It is clear that the lower price of formal care increases fertility, in particular for educated (high-wage) parents. The birth rate for couples that are both educated increases the most, by 0.284 children. All household types make some use of formal childcare, ranging from about 52% of total care time for uneducated parents to more than 78% for educated parents. Average fertility is still higher for uneducated males than for educated males, but this could easily reverse if the subsidy would increase above 50%.

Partial equilibrium. In the next step, I require that the labor tax rate adjusts to balance the government budget and let individuals adjust their savings before marriage. In the column “PE,” it is clear that the use of formal childcare allows parents to work more hours compared to the benchmark. The effect on the total labor supply by education type, $L_t(0)$ and $L_t(1)$, is nevertheless limited because fertility rates have increased as well (and thereby the total care requirement) and the hours supplied by young adults and those of old age are exogenously fixed. The tax rate increases by a modest 0.436 percentage points as the subsidy is partially financed through higher labor earnings. Fertility decreases somewhat compared to the previous column because higher taxes increase the relative price of formal childcare for parents (recall the discussion in Section 3.1).

General equilibrium. Finally, I let factor prices adjust in order to obtain the new general equilibrium “GE.” The marginal productivity conditions of the firm given by (17) imply that relative wages depend negatively on the ratio of educated to uneducated labor hours employed in goods production:

$$\frac{w_t(1)}{w_t(0)} = \beta \left[\frac{N_t^Y(1)}{N_t^Y(0)} \right]^{-1/\alpha}. \quad (27)$$

Compared to the benchmark, the supply of educated labor has increased by 1.3% and that of uneducated labor by almost 2%. Nevertheless, the demand for formal childcare has drawn uneducated workers from the goods sector into the childcare sector, which now accounts for 2.6% of total hours. As a consequence, the labor ratio in (27) increases from 0.398 to 0.409 and the college wage premium falls to 66.9%. The fertility of households in which at least one of the spouses is educated goes down (but is still higher than in the benchmark).

Child allowance. I now consider an economy that provides a child allowance instead of a subsidy on childcare. To facilitate the comparison, I set $\bar{s}$ equal to the average amount of subsidy paid out per child under the previous scenario.

Fertility. I start again by keeping the labor tax rate and the factor prices fixed at their benchmark level. As the allowance reduces the cost of a child, the desired number of children goes up. This is especially true for uneducated couples because

for them the allowance is largest relative to the wage they earn as explained in Section 3. Their fertility rate increases by 0.540 and is now highest among the different household types. The increase in the average birth rate (from $\bar{b} = 1.7$ to 2.164) is much larger than in the previous policy experiment where the subsidy was paid on hours of formal care.

Partial equilibrium. As the price of formal care is unchanged only educated parents make some use of formal care. All other households rely only on parental time in providing for a larger number of children so that labor supply drops by 2.5% for uneducated and 0.9% for educated individuals. To finance government spending and the subsidization program, the labor tax rate has to go up by 2.151 percentage points. As I have shown in Section 3.1, in combination with a child allowance this leads to a further increase in the fertility rates, which is only partly offset by a decrease in premarital savings.

General equilibrium. Since the reduction in labor hours is greater for uneducated than for educated individuals and the demand for formal childcare has hardly changed, it follows that educated labor becomes relatively abundant in goods production and the college wage premium decreases by 1.7 percentage points. The childcare sector shrinks a little as the increase in the labor tax rate combined with a higher price of uneducated labor imply that educated persons demand less childcare despite the fact that they have more children. Overall, I conclude that a child allowance has a bigger effect on fertility rates than a childcare subsidy but at the expense of lower parental labor supply and consequently a greater tax burden.

4.3. Discussion

In this section, I briefly discuss how the results obtained above depend on certain model assumptions and parameter choices. First, the parameter that was most difficult to choose is ψ in the childcare production function as there is no direct evidence on its value. An increase in this parameter reduces the extent to which parents can substitute their own time for formal childcare. This will dampen the effect that a subsidy has on the macroeconomic outcome, but does not alter the qualitative predictions of the model. Similarly, a lower value of ψ makes it easier to substitute parent time for formal childcare. If $\psi = 0$, then parents perform either all care or none at all (depending on the value of parental time relative to the price of formal care).

Second, the fraction of educated individuals is assumed to be exogenous and does not respond to changes in family policy. In an earlier version of this paper, I did consider endogenous education choices [see Reijnders (2015)]. With a realistic elasticity of substitution between the two types of labor in goods production the negative effect of a decrease in the college wage premium dominates the education-inducing effect of childcare subsidies (which favor high-wage parents) so that college graduation rates drop relative to the benchmark.

Finally, I have not allowed for any differences between men and women. If there is a gender wage gap, so that women earn less than men with the same education level, then *ceteris paribus* the amount of childcare performed by mothers goes up. Guner et al. (2014), Bick (2016), and Fehr and Ujhelyiova (2013) assume that fathers do not provide any childcare and always work fulltime. In that case only the female wage will play a role in the trade-off between formal care and parental care and the wage of the male has a pure income effect on fertility.

5. CONCLUSION

I have constructed a rich but tractable general equilibrium model with overlapping generations of heterogeneous individuals, two sectors of production and a government to study the implications of different child subsidy schemes on the economic allocation and the cross-sectional fertility pattern. I find that a subsidy on childcare favors the birth rates of educated, high-wage individuals as it reduces the time cost of a child the most for them. This is partly offset by the change in relative wages as the demand for professional care draws uneducated labor into the childcare sector and away from goods production. Despite having more children, parents also have more time left for work so that the subsidy program is partly financed by a rise in labor earnings. In contrast, the introduction of a child allowance stimulates fertility of uneducated, low-wage parents the most because the subsidy is largest for them compared to their own wage. For the same average amount of subsidy per child this policy has a larger effect on the average birth rate, but as parents perform most of the care themselves it comes at the expense of a lower supply of labor and a greater tax burden.

NOTES

1. In a comment on Becker (1960) published in the same chapter, James S. Duesenberry argues that: [...] there is no area in which the sociological limitations of freedom of choice apply more strongly than to behavior in regard to bringing up children [...] in many respects the standard of living of the children is mechanically linked to that of the parents (pp. 233–234). Similarly, according to Bernard Okun: “It is almost impossible to conceive of a child who is raised at a much lower level of living than that of his parents” (p. 236).

2. For example, the enrolment rate for children aged 0 to 2 ranges from 20% in Austria to 67% in Denmark according to the OECD Family Database for 2013.

3. Other studies make similar assumptions. For example, Fehr and Ujhelyiova (2013) assume that there is an exogenous population growth rate from immigration. Guner et al. (2014) impose a positive population growth rate despite their assumption that women have either two children or none. In both cases, this is needed to avoid a balanced growth path with negative population growth.

4. Naturally, I assume that $N_t^Z(0) \geq 0$ and $N_t^Z(1) \geq 0$.

5. If $Q^b = 0$, then the analytical solution is given by

$$b = (1 - \varepsilon) \frac{W_2}{N^b \nu - \bar{s}} - \varepsilon.$$

6. At the time of marriage, a_2 can be seen as a source of nonlabor income. Jones et al. (2010) show that the existence of nonlabor income can generate a negative relationship between wages and fertility in a setup where there are no consumption costs of children.

7. Source: OECD Family Database, PF3.4 “Childcare support” for 2012.

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APPENDIX A: PROOFS

The system of first-order conditions for the couple as given in (10a)–(10c) can be reduced to a single equation in the number of children b only by substituting out for c_2 and c_3 . This yields:

$$F(b) = f_1(b)W_2 - f_2(b)(N^b v - \bar{s}) = 0, \quad (\text{A.1})$$

where the functions $f_1(b)$ and $f_2(b)$ are defined as:

$$f_1(b) = 1 - \frac{1+\rho}{2+\rho} \frac{\varepsilon}{1-\varepsilon} \frac{Q^b(1+b)}{Q^a + Q^b b},$$

$$f_2(b) = \frac{b+\varepsilon}{1-\varepsilon} - \frac{1+\rho}{2+\rho} \frac{\varepsilon}{1-\varepsilon} \frac{Q^b(1+b)}{Q^a + Q^b b} b.$$

Both are strictly positive because for any $b > 0$ satisfying the first-order condition it holds:

$$\frac{\varepsilon}{1-\varepsilon} \frac{Q^b(1+b)}{Q^a + Q^b b} \frac{1+\rho}{2+\rho} = \frac{W_2 - (N^b v - \bar{s})^{\frac{b+\varepsilon}{1-\varepsilon}}}{W_2 - (N^b v - \bar{s})b} < 1.$$

under the assumption that $N^b v - \bar{s} > 0$. By differentiation it follows that f_1 is decreasing and convex in b while f_2 is increasing and concave.

PROPOSITION 1. *The optimal number of children is increasing in household wealth and the child allowance and decreasing in the time cost.*

Proof. Define:

$$F(b, W_2, v, \bar{s}) = f_1(b)W_2 - f_2(b)(N^b v - \bar{s}). \quad (\text{A.2})$$

The optimal choice b^* is such that $F(b^*, W_2, v, \bar{s}) = 0$. This implicitly defines b^* as a function of $(W_2, v, \bar{s})$. By the Implicit Function Theorem:

$$\frac{\partial b^*}{\partial W_2} = -\frac{\partial F(b^*, W_2, v, \bar{s})/\partial W_2}{\partial F(b^*, W_2, v, \bar{s})/\partial b} > 0,$$

$$\frac{\partial b^*}{\partial v} = -\frac{\partial F(b^*, W_2, v, \bar{s})/\partial v}{\partial F(b^*, W_2, v, \bar{s})/\partial b} < 0,$$

$$\frac{\partial b^*}{\partial \bar{s}} = -\frac{\partial F(b^*, W_2, v, \bar{s})/\partial \bar{s}}{\partial F(b^*, W_2, v, \bar{s})/\partial b} > 0,$$

as the second-order condition for a maximum implies $\partial F(b^*, W_2, v, \bar{s})/\partial b < 0$. ■

PROPOSITION 2. *Assume there is no premarital saving, no formal childcare, and no lump-sum tax or child allowance. Then the optimal number of children depends on the relative wages of husband and wife and attains a minimum when wages are equal.*

Proof. Let $a_2 = 0$, $\psi = 1$, $\bar{\tau} = 0$, and $\bar{s} = 0$. Write $X = w^m/w^f$. Define:

$$\tilde{F}(b, X) = f_1(b)\tilde{W}_2(X) - f_2(b)N^b \tilde{v}(X),$$

where:

$$\begin{aligned}\tilde{F}(b, X) &\equiv \frac{F(b)}{w^f} \\ \tilde{W}_2(X) &\equiv \frac{W_2}{w^f} = (1 - \tau)(1 + X) \left(1 + \frac{1 - \bar{R}}{1 + r} \right), \\ \tilde{v}(X) &\equiv \frac{v}{w^f} = N^b (1 + X^{1-\xi})^{\frac{1}{1-\xi}}.\end{aligned}$$

The optimal choice b^* is such that $\tilde{F}(b^*, X) = 0$. This implicitly defines b^* as a function of X . The partial derivatives of $\tilde{F}$ satisfy:

$$\begin{aligned}\frac{\partial \tilde{F}(b^*, X)}{\partial b} &< 0, \\ \frac{\partial \tilde{F}(b^*, X)}{\partial X} &= f_1(b) \frac{\tilde{W}_2(X)}{1 + X} - f_2(b) \frac{X^{-\xi} \tilde{v}(X)}{1 + X^{1-\xi}}, \\ \frac{\partial^2 \tilde{F}(b^*, X)}{\partial X^2} &= f_2(b) \frac{\xi X^{-(1+\xi)} \tilde{v}(X)}{(1 + X^{1-\xi})^2} > 0,\end{aligned}$$

where the sign of the first follows from the second-order condition for a maximum. In particular, when evaluated at $X = 1$:

$$\begin{aligned}\frac{\partial \tilde{F}(b^*, 1)}{\partial X} &= \frac{1}{2} \tilde{F}(b^*, 1) = 0, \\ \frac{\partial^2 \tilde{F}(b^*, 1)}{\partial X^2} &= \frac{2^{\frac{1}{1-\xi}}}{4} f_2(b^*) \xi N^b > 0.\end{aligned}$$

The Implicit Function Theorem then implies that b^* attains a minimum at $X = 1$:

$$\begin{aligned}\left. \frac{\partial b^*}{\partial X} \right|_{X=1} &= - \frac{\partial \tilde{F}(b^*, 1) / \partial X}{\partial \tilde{F}(b^*, 1) / \partial b} = 0, \\ \left. \frac{\partial^2 b^*}{\partial X^2} \right|_{X=1} &= - \frac{\partial^2 \tilde{F}(b^*, 1) / \partial X^2}{\partial \tilde{F}(b^*, 1) / \partial b} > 0.\end{aligned}$$

■

APPENDIX B: COMPUTATIONAL DETAILS

B.1. HOUSEHOLD DECISIONS

Given a set of prices, I can solve for the household decisions using backward induction.

1. For each level of education of both spouses (E^f, E^m) and stock of joint savings a_2 , I find the household choices $(c_2, c_3, b, n^f, n^m, o, a_3)$ and corresponding value function at the start of the second stage $\mathcal{M}_2$ by solving the first-order conditions of a married couple.

2. I solve numerically for the optimal level of savings in the first stage for each type of individual $j \in \{f, m\}$ and $E \in \{0, 1\}$ given the choice of the other types. The intersection of these best responses determines the actual savings decisions $a_2^j(0)$ and $a_2^j(1)$.

B.2. GENERAL EQUILIBRIUM

The general equilibrium is obtained by the following iterative procedure.

1. Make a guess for the interest rate r_t and the wage rates $w_t(0)$ and $w_t(1)$.
2. Calculate the implied price of formal childcare:

$$p = \frac{w_t(0)}{\Psi}.$$

3. Find the household decisions given the prices by the method described in B.1.
4. Aggregate the household decisions.
 - The average fertility rate $\bar{b}_t$.
 - Financial assets per adult A_t/P_t , labor supply by education $L_t(0)/P_t$ and $L_t(1)/P_t$, consumption C_t/P_t , and demand for formal childcare O_t/P_t .
5. Determine the capital stock and labor allocations under the assumption that factor markets clear:

$$\frac{K_t}{P_t} = \frac{A_t}{P_t}, \quad \frac{N_t^Z}{P_t} = \frac{1}{\Psi} \frac{O_t}{P_t}, \quad \frac{N_t^Y(1)}{P_t} = \frac{L_t(1)}{P_t}, \quad \frac{N_t^Y(0)}{P_t} = \frac{L_t(0)}{P_t} - \frac{N_t^Z}{P_t}.$$

6. Calculate investment I_t/P_t , output in the goods sector Y_t/P_t , and output in the childcare sector Z_t/P_t :

$$\frac{I_t}{P_t} = (\eta + \delta) \frac{K_t}{P_t}, \quad \frac{Y_t}{P_t} = \Phi \left(\frac{K_t}{P_t} \right)^\phi \left(\frac{N_t^Y}{P_t} \right)^{1-\phi}, \quad \frac{Z_t}{P_t} = \Psi \frac{N_t^Z}{P_t}.$$

7. Change one of the taxes or subsidies so that the government budget is balanced:

$$\tau_t \left[w_t(0) \frac{L_t(0)}{P_t} + w_t(1) \frac{L_t(1)}{P_t} \right] + \bar{\tau}_t = \frac{G_t}{P_t} + s_t p_t \frac{O_t}{P_t} + \bar{s}_t \frac{P_{0,t}}{P_t}.$$

8. Calculate the factor prices from the marginal productivity conditions of the firms.
 - The interest rate r_t and the return to effective labor in the goods sector w_t^Y :

$$r_t + \delta = \phi \Phi \left(\frac{K_t/P_t}{N_t^Y/P_t} \right)^{-(1-\phi)}, \quad w_t^Y = (1 - \phi) \Phi \left(\frac{K_t/P_t}{N_t^Y/P_t} \right)^\phi,$$

where

$$\frac{N_t^Y}{P_t} = \left\{ \left[\frac{N_t^Y(0)}{P_t} \right]^{1-1/\alpha} + \beta \left[\frac{N_t^Y(1)}{P_t} \right]^{1-1/\alpha} \right\}^{\frac{1}{1-1/\alpha}}.$$

- The wage rates of uneducated workers $w_t(0)$ and educated workers $w_t(1)$:

$$w_t(0) = w_t^Y \left[\frac{N_t^Y(0)/P_t}{N_t^Y/P_t} \right]^{-1/\alpha}, \quad w_t(1) = w_t^Y \beta \left[\frac{N_t^Y(1)/P_t}{N_t^Y/P_t} \right]^{-1/\alpha}.$$

9. Check whether the new factor prices are equal to the initial guess. If not, then partially update the guess and start again from (1).

APPENDIX C: THREE EDUCATION TYPES

In this appendix, I show how the model can account for a U-shaped relationship between female education and fertility [as in Hazan and Zoabi (2015)] while maintaining a negative relationship between male education and fertility [as in Jones et al. (2010)].

Instead of two education types, I have to allow for three: low ($E = 0$), middle ($E = 1$), and high ($E = 2$). For simplicity, I assume that initially there are an equal number of men and women in every group and that each is married to their own type. Panel (a) of Figure A.1 shows optimal fertility as a function of the male wage conditional on different wages for the female (either $w(0)$, $w(1)$, or $w(2)$). Each of the profiles is U-shaped because there are gains from partial specialization into childcare and market work for unequal-wage couples. Parents with lower wages tend to have higher fertility, for example, due to a lump-sum transfer or a child allowance. Because matching is perfectly assortative it follows that average fertility by gender and education type is given by $\bar{b}^j(E) = b^*(E, E)$. For both men and women it follows that $\bar{b}^j(0) > \bar{b}^j(1) > \bar{b}^j(2)$ so that fertility is decreasing in education.

Now suppose that I allow for an asymmetry between men and women. In particular, there are more women than men who have education $E = 2$ and fewer who have education $E = 1$. This captures the recent trend in most developed countries that the educational attainment of women exceeds that of men. It follows that some women with $E = 2$ have to marry men with $E = 1$ and they have $b^*(2, 1)$ children. As a result of returns to specialization this exceeds the fertility level of the equal-education couple $b^*(2, 2)$. It follows that $\bar{b}^m(1)$ and $\bar{b}^f(2)$ increase. Provided that there are sufficiently many educated women who marry down, it is now possible that $\bar{b}^f(0) > \bar{b}^f(1) < \bar{b}^f(2)$ (a U-shaped relation between female education and fertility) while at the same time $\bar{b}^m(0) > \bar{b}^m(1) > \bar{b}^m(2)$ (a negative relation between male education and fertility).

FIGURE A.1. Optimal number of children with three education types.

As in Hazan and Zoabi (2015), the model can also generate a U-shape as a result of the marketization of childcare. Panel (b) shows how the profiles change if formal childcare becomes available at a price that makes it most attractive to highly educated parents. In this case $b^*(2, 2) > b^*(1, 1)$ so that with perfect assortative matching it follows that $\bar{b}^j(0) > \bar{b}^j(1) < \bar{b}^j(2)$ and there is a U-shaped pattern for both sexes.