University of Groningen

Integrated Passenger and Freight Train Planning on Shared-Use Corridors

Ursavas, Evrim; Zhu, Xiang

Published in:
Transportation Science

DOI:
[10.1287/trsc.2017.0796](https://doi.org/10.1287/trsc.2017.0796)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2018

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Ursavas, E., & Zhu, X. (2018). Integrated Passenger and Freight Train Planning on Shared-Use Corridors. *Transportation Science*, 52(6), 1376–1390. <https://doi.org/10.1287/trsc.2017.0796>

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Transportation Science

Publication details, including instructions for authors and subscription information:
<http://pubsonline.informs.org>

Integrated Passenger and Freight Train Planning on Shared-Use Corridors

E. Ursavas, Stuart X. Zhu

To cite this article:

E. Ursavas, Stuart X. Zhu (2018) Integrated Passenger and Freight Train Planning on Shared-Use Corridors. Transportation Science 52(6):1376-1390. <https://doi.org/10.1287/trsc.2017.0796>

Full terms and conditions of use: <https://pubsonline.informs.org/page/terms-and-conditions>

This article may be used only for the purposes of research, teaching, and/or private study. Commercial use or systematic downloading (by robots or other automatic processes) is prohibited without explicit Publisher approval, unless otherwise noted. For more information, contact permissions@informs.org.

The Publisher does not warrant or guarantee the article's accuracy, completeness, merchantability, fitness for a particular purpose, or non-infringement. Descriptions of, or references to, products or publications, or inclusion of an advertisement in this article, neither constitutes nor implies a guarantee, endorsement, or support of claims made of that product, publication, or service.

Copyright © 2017, INFORMS

Please scroll down for article—it is on subsequent pages

INFORMS is the largest professional society in the world for professionals in the fields of operations research, management science, and analytics.

For more information on INFORMS, its publications, membership, or meetings visit <http://www.informs.org>

Integrated Passenger and Freight Train Planning on Shared-Use Corridors

E. Ursavas,^a Stuart X. Zhu^a

^a Faculty of Economics and Business, University of Groningen, 9747 AE Groningen, Netherlands

Contact: e.ursavas@rug.nl, <http://orcid.org/0000-0001-7371-642X> (EU); x.zhu@rug.nl (SXZ)

Received: September 15, 2016

Revised: April 26, 2017

Accepted: July 12, 2017

Published Online in Articles in Advance:
October 16, 2017

<https://doi.org/10.1287/trsc.2017.0796>

Copyright: © 2017 INFORMS

Abstract. Our study involves the decision-making problems that railway infrastructure managers face in a rail network with dedicated tracks and shared-use corridors. We will analyze the consolidation strategy for shared-use corridors, where the track serves passenger and freight trains. In the stochastic demand case, we will provide an analytical model for the railway infrastructure manager to compute the expected long-term profit using a consolidation system. We will pinpoint the different characteristics of passenger and freight trains, and analytically derive the optimum track allocation and consolidation time, together with the optimum price, in all such cases, using two different model structures, i.e., the additive and the multiplicative forms. We will extend our model further to consider the due-date requirements and volume incentives for railway operators. Our experiments will use realistic parameter values, based on the Dutch railway system.

History: This paper has been accepted for the TRISTAN IX Special Section for *Transportation Science*.

Funding: This research was partly supported by the Netherlands Organisation for Scientific Research under [Grants 040.03.021 and 040.21.001], and the National Natural Science Foundation of China under [Grants 71571125, 7151130129, and 71711530046].

Keywords: shared-use corridors • freight consolidation • integrated passenger and freight train planning

1. Introduction

An increase in railway traffic of almost 32% has been reported in Europe in the past decade, with the increase in the total number of railway kilometers reported as just 5% for the same period (Sameni and Landex 2013). With the very high capital expenditures required for infrastructure construction, infrastructure managers are looking for solutions to cope with this increase in demand. Rail infrastructure managers handle the demand requests of railway operators, who are in charge of operating trains (freight or passenger), by allocating access to the railway tracks, and determining regularity, charges, prices, and incentives.

Rail tracks can be dedicated to operate as passenger-only or freight-only. It is also possible for rail tracks to be shared between passenger and freight traffic, along with the additional challenges this entails. Successful planning of these shared-use rail corridors enables better use of resources and creates a smooth transfer of passengers and freight. Integrated rail operations provide an efficient use of resources and maximize the use of existing infrastructure, while contributing to the sustainable development of logistics. Given the economic and environmental interests at stake, the interactions between passenger and freight trains must be carefully studied to take into account the different characteristics of both parties. The rail infrastructure manager must deal with the pressure of planning freight trains and

setting prices for the given service in the presence of passenger trains that must operate with set frequencies and, ideally, with no delays. Accordingly, working with such complex guidelines, it is essential to study strategies that will support the rail network within the given restrictions. In that sense, shipment consolidation is a promising practice offering benefits from the economies of scale associated with lower operating costs. This is especially promising in Europe with the liberalization of the railway market, which has led to many different cargo companies becoming involved in freight travel by rail. Since this development, freight traffic has generally become more fragmented. It is therefore beneficial for the orders of these small-sized companies to be consolidated before being dispatched.

Developing a suitable consolidation strategy is crucial. The analysis of shipment consolidation policies that consider the movement of freight and passenger trains with respect to their own characteristics is the focus of this paper. The allocation of tracks to the railway operators is complex. The allocated track can be a dedicated-use or a shared-use track. Together with this decision, the problem faced by the infrastructure manager is to determine the dispatching time of the trains, while setting the price of the service. In doing so, the infrastructure manager may need to adhere to the due-date limitations set by the railway operators. The departure time should incorporate these due-date requirements.

In addition to dispatching-time decisions, the price that infrastructure managers quote to the railway operators for the given service for use of the tracks must also be carefully computed. Price incentives are a tool used by railway infrastructure managers to offer price discounts for railway operators with higher volumes of traffic. These price discounts should also reflect consolidation policy. The price charged to the railway operator depends on several factors and differs among countries, based on the route, areas, market segments, type of product, weight, and the size of the goods (Nash 2005, Sanchez-Borras et al. 2010, Harrod 2013, ProRail 2016).

The railway infrastructure manager also incurs costs for providing the transportation service. These cost elements are associated with infrastructure, construction, maintenance, power, renewal, train planning and operations, congestion, accidents, and the environment (Nash 2005, Sanchez-Borras et al. 2010, ProRail 2016). The railway infrastructure manager can benefit from shared-use tracks by effective use of the current infrastructure. Use of shared tracks can also leave more capacity for dedicated tracks, leading to increased potential to serve more trains (Transit Cooperative Research Program 2009, Lai, Lin, and Cheng 2014).

This balance between revenues and costs, while simultaneously taking into account the characteristics of the railway operators, must be considered by the infrastructure manager. Therefore, we will study the profit-maximization job of the infrastructure manager, while recognizing the fact that infrastructure managers also need to serve passenger and freight trains with different characteristics. The rail infrastructure manager must unravel passenger and freight demand patterns so as to use them as a basis for consolidation decision-making. Passenger trains follow set frequencies, whereas freight train departures are a function of uncertain demand (Talebian and Zou 2015). This is also true for the Netherlands, where each railway operator submits requests to the infrastructure manager for a track and a time-slot suitable for transporting their freight cars. The infrastructure manager then allocates the track and the time-slot suitable for the shipment. The traffic can be directed to a dedicated track or a shared-use corridor, each with its own unique costs and challenges. Coupled with busy railway traffic, this presents a major challenge for the infrastructure manager in terms of providing the appropriate service with the resources available (Pouwels 2014).

With its practical and economic benefits, shipment consolidation, and its consequences, has naturally attracted the interest of academia over the past few decades (Mutlu, Çetinkaya, and Bookbinder 2010). To our knowledge, there is no one exact analytical model for computing the optimal time and price, based on policy parameters involving revenues and costs in

terms of passenger and freight trains. For this reason, we will provide various analytical means for evaluating profits generated through using such consolidation policies. We will show the optimal price and the consolidation time in terms of both demand structures, i.e., the additive and the multiplicative forms. We take into account that optimal solutions take shape based on price sensitivity. Depending on the functional form of the profit function, the optimal conditions differ in terms of whether the function is concave or convex. We further extended our model to take into account price discounts for high-volume customers. Moreover, we derived the optimal strategy for settings where railway operators require service with a due date. We applied our analytical model to a real-life rail infrastructure setting in the Netherlands and showed the applicability of our proposed approach.

The remaining parts of this paper are organized as follows. In Section 2, we introduce the relevant literature. In Section 3, the problem confronted in this paper is fully described, and a specific model for the profit optimization is created. We also present extensions to our model in terms of price incentives for different market segments and time-sensitive customers. Subsequently, we present our experiments for a particular case in the Netherlands. We then conclude our paper.

2. Literature Review

Rail transportation has been widely studied in academia and it remains a topic that continues to interest many researchers. Studies by Cordeau, Toth, and Vigo (1998); Caprara, Fischetti, and Toth (2002); and Cacchiani et al. (2014) provide excellent overviews of previous research. Our study addresses two streams of research, i.e., train scheduling and freight consolidation.

We will first review work on train-scheduling problems, where the focus is constructing a timetable for the trains and allocating the tracks. A wide range of studies are available, which formulate an optimization model and develop algorithms. Few studies, however, consider the uncertainty inherent in the environment; for these conditions, simulation and stochastic programming approaches are implemented.

Among the early research on timetabling, Higgins, Kozan, and Ferreira (1996) formulated a mathematical-programming model and used a branch and bound procedure that considers the priority of trains based on delays. Higgins and Kozan (1998) formulated a model that used an iterative refinement algorithm to address delays in urban networks. Chang and Kwan (2004) proposed three techniques based on genetic algorithms, particle swarm optimization, and differential evolution to solve the train-timetabling problem, and compared various performance indices. Zhou and Zhong (2007) worked on the train-timetabling problem

and developed branch-and-bound algorithms. Barrena et al. (2014) first formulated a mathematical programming model and then developed a metaheuristic to construct a timetable for passenger trains. Yin et al. (2017) studied the timetabling of passenger trains, with the aim of arriving at a balance between passenger waiting times and train energy consumption.

Petering, Heydar, and Bergmann (2015) formulated a mixed integer-programming (MIP) model that used a commercial solver for the train-timetabling and track-allocation problem, with a focus on passenger railway systems. Track allocation refers to the allocation of tracks to trains over time (Billionnet 2003, Abbink et al. 2004, Chakroborty and Vikram 2008, Chung, Oh, and Choi 2009, Lee and Chen 2009, Lusby et al. 2011, Xu, Li, and Yang 2015, Lamorgese, Mannino, and Piacentini 2016). Rescheduling because of deviations in the original plan is also studied, for example, by Corman et al. (2010), Veelenturf et al. (2015), Bešinović et al. (2016), and Samà, Corman, and Pacciarelli (2017). Hallowell (1993) developed a simulation model and an improved analytical line delay model to analyze a train dispatching process in scheduled rail operations under uncertain conditions. Dorfman and Medanic (2004) studied priority rules with a discrete event-simulation framework to solve train-scheduling problems. Meng and Zhou (2011) followed a stochastic-programming approach in the rolling horizon decision-making process to recoup train schedules after disturbances.

Focusing on freight trains, Salim and Cai (1995) formulated genetic algorithms tackling the scheduling of trains. Similarly, Mu and Dessouky (2011) investigated the scheduling of freight trains. They first developed exact methods to find a solution to the scheduling problem and proposed several heuristics. İċyüz et al. (2016) formulated an MIP model for the coal train reservation-planning problem solved through problem-specific heuristics.

Shared use of the railway network by passenger and freight transportation brings with it additional challenges. As Dingler, Lai, and Barkan (2009), Xu, Li, and Yang (2015), and Sogin, Barkan, and Saat (2011) noted, the heterogeneity in the type of trains results in more delays. Brännlund et al. (1998) studied the scheduling of freight trains and passenger trains in Sweden. They applied Lagrangian relaxation to find a solution and simultaneously incorporated the two types of trains by assigning higher priority to the passenger trains. Cacchiani, Caprara, and Toth (2010) investigated scheduling freight trains in a timetable that formerly included passenger trains.

The studies cited above, which address the train-scheduling problem, aimed to determine a time for each incoming order. These studies do not allow for consolidation of freight cars but rather set a schedule for each incoming order.

Various researchers have analyzed the practice of freight consolidation vis-à-vis different transportation methods (Nguyen, Dessouky, and Toriello 2014). Huang and Chi (2007) studied the consolidation problem of airfreight forwarders; Nemoto (1997) described the social effects of freight consolidation by road. In 1956, Beckmann, McGuire, and Winsten (1956) introduced two principal ways of determining the time to dispatch a freight train, i.e., wait until a certain number of freight cars have been collected or dispatch them according to a timetable. These ways form the basis for what, in later research, are called the quantity policy and the time policy. Çetinkaya, Tekin, and Lee (2008) used the renewal reward theory to study a quantity policy. Marklund (2011) and Mutlu and Çetinkaya (2010) studied cases where dispatch occurs after a predefined time. Ülkü (2012) proposed a discrete time-based policy where dispatch time is set to certain times of the day. Combined use of the two policies can also be observed in Jackson (1985). When practiced, this implies that a train is not dispatched until the time span has expired or the maximum quantity, which can be weight, length or the number of freight cars allowed, is reached. Bookbinder and Higginson (2002) used stochastic clearing theory to obtain such a time-and-quantity strategy. Çetinkaya and Bookbinder (2003) included a quantity policy and a time policy in their paper and analyzed the two policies for private and common carriage. Higginson and Bookbinder (1994) compared all three policies, based on cost and delay, using simulation. The quantity policy yielded the lowest cost, and the time-and-quantity policy resulted in the least delay. Mutlu, Çetinkaya, and Bookbinder (2010) developed an analytical model for a time-and-quantity-based strategy to compute the expected long-run average cost. These studies evaluate performance based on cost rather than also considering revenues and profit, which play a major role in the decision-making process.

From an inventory point of view, Hong and Lee (2013) and Ülkü and Bookbinder (2012) considered the price and delivery lead time for each customer. They assume that the orders arriving are of a standard size. The point of focus in these studies is to reduce inventory holding costs. However, in the transportation sector, we need to account for shipment costs. Moreover, based on the characteristics of the railway industry, we further need to consider a setting wherein passenger and freight trains together may need to share the corridor. Accordingly, unlike prior studies, our research studies consolidation practices for shared railway systems, where passenger and freight trains are considered together. We further extend the literature on consolidation by considering different price and time strategies in terms of volume incentives. A

discounted price that would maximize profit can be derived through the proposed model and offered to the railway operator. Our model also accounts for railway-operator due dates by incorporating these in the optimal consolidation time decision.

3. Model Formulation and Analysis

We consider a situation wherein the rail infrastructure manager allocates demand from railway operators for tracks and sets a time for dispatch. The railway operators run freight or passenger traffic. The objective of the infrastructure manager is to establish safe movement of the trains from the point of origin to the destination, taking into account revenues and costs, and acknowledging the limitations and service requirements. Passenger trains need to adhere to schedules, preferably with no delays. On the other hand, the freight to be forwarded to its destination arrives at the point of origin through a stochastic process. Each order also has an associated volume that affects the price calculated by the infrastructure manager. The infrastructure manager must adhere to the technical and safety limitations concerning carloads and maximum weight restrictions. Furthermore, depending on the size and weight of the order, the load per order may need to be distributed over a number of freight cars. In addition to passenger train information, other inputs of the problem are the corridor configuration (dedicated or shared-use tracks) and the associated costs for transportation, along with the holding costs for freight cars. The transportation costs include the overhead line fee, and the energy, personnel, setup, and usage fees. The holding costs account for the accumulation delay, that is, the time freight cars must wait before the train is scheduled to leave.

Corridors may be dedicated to passengers only or can be used in an integrated way between passenger and freight traffic. In the shared used corridors, passenger trains are given priority over freight trains. The arrival of freight, on the other hand, is not fixed but fragmented. The departure of the freight trains is determined based on the consolidation policy. This way the fragmented nature of the freight traffic can be handled. In that procedure, after a computed time threshold T , a freight train will be dispatched. The passenger train can use the same track as the freight train or use a dedicated track. We denote the usage cost of the shared-use track as c_0 . Using a dedicated track requires a higher cost factor of c_1 . Hence, the benefit accrued from sharing the track for passenger trains can be calculated with $c = c_1 - c_0$. Denote x as the dispatch frequency of passenger trains on the shared corridor in a cycle. The corresponding savings is calculated by cx . Denote T_p as the corridor duration occupied by a passenger train of one dispatch. The total occupied duration of the passenger trains in a cycle is xT_p .

Based on business practice, we assume that consolidation will be applied for small-size fragmented orders, which means that the freight train is only dispatched once per cycle. Denote T_f as the corridor duration occupied by the freight train for one dispatch. The cycle length T must be no less than $xT_p + T_f$, that is, $T \geq xT_p + T_f$.

For each arrival, a price P is charged for the request for freight delivery. Each consolidation cycle T consists of two time segments, one reserved for the passenger trains and one remaining for the freight train. The problem is to find the decision variables T , P , and x while maximizing the long-run average profit, denoted by $\pi(T, P, x)$. We used the renewal reward theory to compute this exact analytical expression of $\pi(T, P, x)$ given by

$$\pi(T, P, x) = T^{-1} \cdot [\mathbb{E}(\text{revenue incurred during a cycle}) - \mathbb{E}(\text{costs incurred during a cycle}) + cx]. \quad (1)$$

The optimal length of the cycle, equaling the optimal consolidation time T^* , extends the time slot reserved for passenger trains. So the shared use of the corridor is organized such that, following on the passenger trains, the consolidated freight cars are dispatched. The infrastructure manager computes the optimal price P^* , the optimal time T^* , and the optimal dispatch frequency of passenger trains on the shared-used tracks x^* by solving

$$\underset{T \geq xT_p + T_f, P > 0}{\text{maximize}} \pi(T, P, x). \quad (2)$$

By (1), it is clear that $\pi(T, P, x)$ is increasing in x . To achieve the maximum cost savings, it is optimal to satisfy $x^* = (T - T_f)/T_p$. Substituting x^* into (2), we can only consider the decisions of T and P , that is,

$$\underset{T > 0, P > 0}{\text{maximize}} \pi(T, P). \quad (3)$$

We assume that the arrival process of orders is modeled by a compound Poisson process, that is,

$$Y(t) = \sum_{n=1}^{N(t)} V_n,$$

where $Y(t)$ is the total volume of the freight by time t , $N(t)$ is the number of arrivals by time t , and V_n is the volume of the n th order. Denote λ as the arrival rate of this arrival process and μ_v as the mean of V_n . We assume that $N(t)$ is independent of V_n , that is, the interarrival time of an order is independent of the volume of the order. This is a plausible assumption, since it is not expected that the interarrival time will affect the volume of the order.

The profit function requires the computation of revenues and costs. Revenue per cycle can be calculated

by multiplying the price by the demand per cycle. The price is essentially controlled by the demand. Because the revenue depends on the volume of an order, the total amount of the volume in one cycle is required to calculate the revenue per cycle. This total amount of the volume is denoted by $\sum_{n=1}^{N(T)} V_n$. The expected revenues per cycle can thus be written as

$$\begin{aligned} \mathbb{E}(\text{revenue incurred during a cycle}) \\ = P\mathbb{E}(V_n)\mathbb{E}(N(T)), \end{aligned} \quad (4)$$

through independence property. The price P is the price per unit volume for the total distance to be traveled. Note that this volume can be based on weight or the number of freight cars depending on the strategy of the infrastructure manager.

The second part of the profit function in Equation (1) consists of the costs incurred during a cycle. These costs can be partitioned into shipment costs and holding costs, as in

$$\begin{aligned} \mathbb{E}(\text{costs incurred during a cycle}) \\ = \mathbb{E}(\text{shipment costs during a cycle}) \\ + \mathbb{E}(\text{holding costs during a cycle}). \end{aligned}$$

We first derive an expression for the expected shipment costs. We combine the overhead line fee, energy fee, personnel fee, and set-up fee in a parameter K . In addition to the fixed cost, we assume that there is a variable cost β that depends on the volume and the traveling distance of the order.

The total shipment fee of a train can be expressed as

$$\begin{aligned} \mathbb{E}(\text{shipment costs during a cycle}) \\ = K + d\beta \times \mathbb{E}(V_n)\mathbb{E}(N(T)), \end{aligned} \quad (5)$$

where d is the number of kilometers to be traveled.

Now we define the holding costs that depend on the volume of freight waiting to be dispatched. An illustration is given in Figure 1. Letting 0 be the start time of a cycle, S_n represents the time the n th order arrives. After T_1 , the interarrival time of the first order, the first order arrives at S_1 with a volume of V_1 . From time S_2 until time S_3 the total amount of volume waiting to be dispatched is thus $V_1 + V_2$, the sum of the first two orders. This continues until the time of dispatch, T , is reached. The time interval from the last arrival epoch before T to T is defined as $A(T) = T - S_{N(T)}$. Figure 1 shows an example with $N(T) = 4$. The holding costs can then be expressed as

$$\begin{aligned} \mathbb{E}(\text{holding costs during a cycle}) \\ = \mathbb{E}\left(h \times \left(\sum_{i=2}^{N(T)} T_i \sum_{j=1}^{i-1} V_j + A(T) \sum_{i=1}^{N(T)} V_i\right)\right), \end{aligned} \quad (6)$$

where h is the holding cost fee per volume per unit time.

Figure 1. Arrival Process of Orders

Based on the properties of the compound Poisson process, we arrive at

$$\begin{aligned} \mathbb{E}\left[\sum_{i=2}^{N(T)} T_i \sum_{j=1}^{i-1} V_j\right] &= \mu_v \mathbb{E}\left[\sum_{i=1}^{n-1} i \mathbb{E}[T_{i+1} | N(T) = n]\right] \\ &= \frac{\mu_v}{2\lambda} (\lambda^2 T^2 - 2\lambda T - 2e^{-\lambda T} + 2), \end{aligned}$$

and $\mathbb{E}A(T) = (1 - e^{-\lambda T})/\lambda$.

By substituting the above expressions into (6), we can simplify the expression of the expected holding cost during a cycle to $h\mu_v\lambda T^2/(2)$.

Thus, the optimization problem of Equation (3) becomes

$$\begin{aligned} \text{maximize}_{T>0, P>0} \frac{1}{T} &\left(P\mathbb{E}(V_n)\mathbb{E}(N(T)) - K - d\beta \times \mathbb{E}(V_n)\mathbb{E}(N(T)) \right. \\ &\left. - \frac{h\mu_v\lambda T^2}{2} + c \frac{T - T_f}{T_p} \right). \end{aligned}$$

To thoroughly analyze the impact of the price on demand, we characterize the optimal solution C in what follows in terms of two well known models, i.e., the additive model and the multiplicative model. The additive model assumes a linear relationship between the price and the demand rate, whereas the multiplicative model assumes a curvilinear relationship (Tellis 1988).

3.1. Additive Model

With the additive structure, we assume that the mean demand rate is given by $\lambda = a - bP$, where a is the market potential and b is the price sensitivity factor that indicates the price elasticity of the demand. Note that $a > 0$ and $b > 0$. Clearly, the demand rate decreases when the price becomes high.

The profit function can be written as

$$\pi(P, T) = (P - \beta d)\mu_v\lambda - \frac{h\mu_v\lambda T^2}{2} - \frac{K + cT_f/T_p}{T} + \frac{c}{T_p}. \quad (7)$$

Here we use a sequential solution procedure that can be shown to be valid for our problem. See the proof of Proposition 1 for details (Ha 1998). To solve (7), for a given P , we first obtain $T^*(P)$ as a function of P ; we then substitute $T^*(P)$ into π and change the decision problem to a single-variable problem in terms of P .

We have the following lemma.

Lemma 3.1. *For a given P , the optimal cycle length T^* is uniquely given by*

$$T^*(P) = \sqrt{\frac{2(K + cT_f/T_p)}{h\mu_v\lambda}}. \quad (8)$$

Proof of Lemma 3.1. For any given P , the second derivative of $\pi(P, T)$ with respect to T is given by

$$\frac{\partial^2 \pi(P, T)}{\partial T^2} = -2 \frac{K + cT_f/T_p}{T^3}.$$

Since the second derivative is negative, $\pi(T, P)$ is strictly concave in T . The optimal cycle length T^* is uniquely given by the first-order condition. Based on the first-order condition, we obtain (8). QED.

Denote P_0 as the solution of the equation, that is,

$$a + b\beta d - 2bP_0 + b\sqrt{\frac{h\mu_v(K + cT_f/T_p)}{2(a - bP_0)}} = 0.$$

The following theorem characterizes the global optimum that maximizes the profit by (7).

Theorem 3.1. *The unique optimum is given by $(P^*, T^*(P^*))$, where $P^* = P_0$ and $T^*(P^*)$ can be found from (8).*

Proof of Theorem 3.1. Substituting $T^*(P)$ into (7), we can rewrite (7) as

$$\pi(P, T^*(P)) = (P - \beta d)\mu_v\lambda - \sqrt{2h\mu_v(K + cT_f/T_p)}\lambda. \quad (9)$$

Checking the second derivative of $\pi(T^*(P), P)$ with respect to P , we obtain

$$\begin{aligned} \frac{d^2 \pi(P, T^*(P))}{dP^2} \\ = -2b\mu_v + \frac{b^2}{4}\sqrt{2h\mu_v(K + cT_f/T_p)}(a - bP)^{-3/2}. \end{aligned} \quad (10)$$

Since $0 \leq P \leq a/b$ and this second derivative is increasing in P , we can show that $d^2 \pi(P, T^*(P))/dP^2|_{P=P_u}$ is positive. Thus, $\pi(P, T^*(P))$ is first strictly concave in P and then convex in P . The interior solution P_0 is given by the first-order conditions, that is, (9). Note P is bounded by P_u , where P_u is the upper bound of the price, that is, $P_u = a/b$. When P approaches P_u , the expected profit approaches zero. Thus, it is clear that the optimal pricing solution is given by P_0 to yield a positive profit.

Then, we can substitute P^* into (8) and obtain the global optimum for the consolidation time. QED.

3.2. Multiplicative Model

For the multiplicative model, the mean demand rate is given by $\lambda = aP^{-b}$ and $a > 0, b > 0$. The profit function is given by (7). Similar to the analysis of the additive model, for any given P , we can prove that $\pi(P, T)$ is strictly concave in T . Thus, the optimal time $T^*(P)$ is also given by (8).

Denote P_1 as the solution to the following equation, i.e.,

$$P_1 - b(P_1 - \beta d) + \Delta P_1^{b/2} = 0, \quad (11)$$

where $\Delta = b\sqrt{(h\mu_v(K + cT_f/T_p))/(2a)}$. Denote P_u as the upper bound of the price.

The optimal solution is given by the following theorem.

Theorem 3.2. *The optimal solution is characterized by the following three cases.*

- (i) For $b < 2$, the optimal price P^* is uniquely given by P_1 .
- (ii) For $b = 2$, if $bc_0 - b + 1 \leq 0$, then the optimal price P^* is uniquely given by P_1 ; otherwise, $P^* = P_u$.
- (iii) For $b > 2$, if

$$b\beta d - (b - 1) \left[\frac{2(b - 1)}{b\Delta} \right]^{2/(b-2)} + \Delta \left[\frac{2(b - 1)}{b\Delta} \right]^{b/(b-2)} \geq 0,$$

then $P^* = P_u$; otherwise, the optimal solution is given by $P^* = P_1$.

Proof of Theorem 3.2. The detailed proof is as follows: First, for any given P , we can prove that $\pi(P, T)$ is strictly concave in T . Thus, the optimal time $T^*(P)$ is also given by (8). Substituting $T^*(P)$ into $\pi(P, T)$, we check the first derivative

$$\frac{d\pi(P, T^*(P))}{dP} = a\mu_v P^{-b-1} [P - b(P - \beta d) + \Delta P^{b/2}].$$

Denote $f(P) = P - b(P - \beta d) + \Delta P^{b/2}$. Now let us study the property of $f(P)$. Then, we need to consider three cases depending on the value of b .

Case (i). $b < 2$. It is clear that $f(P)$ is strictly concave in P .

Since $f(0) > 0$, the optimal solution is uniquely given by P_1 , where P_1 is the unique point satisfying $f(P_1) = 0$. Thus, P_1 is uniquely given by (11).

Case (ii). $b = 2$. It is clear that $f(P)$ is linear in P .

If $\Delta - b + 1 < 0$, we can prove that $\pi(T^*(P), P)$ is unimodal in P . The optimal solution is uniquely given by P_1 .

If $\Delta - b + 1 \geq 0$, we can prove that $\pi(T^*(P), P)$ is strictly increasing in P . Clearly, $P^* = P_u$, where P_u is the upper bound of the price.

Case (iii). $b > 2$. It is clear that $f(P)$ is strictly convex in P .

The first-order condition of $f(P)$ yields $\bar{P}$ satisfying $1 - b + (b\Delta/2)\bar{P}^{b/2-1} = 0$. Then, we have

$$\bar{P} = \left[\frac{2(b-1)}{b\Delta} \right]^{2/(b-2)}.$$

Under the condition

$$b\beta\delta - (b-1) \left[\frac{2(b-1)}{b\Delta} \right]^{2/(b-2)} + \Delta \left[\frac{2(b-1)}{b\Delta} \right]^{b/(b-2)} \geq 0,$$

we have $f(\bar{P}) \geq 0$, which means that $f(P) \geq 0$ for $P \geq 0$. Thus, $\pi(T, P)$ is always increasing in P . Then, we have $P^* = P_u$, where P_u is the upper bound of the price.

If the above condition does not hold, we have $f(\bar{P}) < 0$. Since the profit is almost zero when P approaches P_u , the optimal solution is P_1 , where P_1 is uniquely given by (11) due to the convexity of $f(P)$. QED.

Theorem 3.2 indicates that, when the railway operators are price-sensitive ($b < 2$), the infrastructure manager should offer a relatively low price, and vice versa, the manager could charge a higher price under certain circumstances depending on the shipment and holding costs. Competition in the railway sector plays a role in setting this price. Infrastructure managers cannot be free to set the price and should consider the competition among other transportation modes.

4. Extensions

We now consider two extensions to the base model, i.e., due-date considerations and volume incentives. We will use the additive model as an illustration. For the multiplicative model, we can follow a similar procedure. Numerical experiments in Section 5.3 provide further insight into the application of these two extensions.

4.1. Due-Date Consideration

We assume that railway operators are concerned about the due date of their freight. The due date follows a uniform distribution between $[L_0, L_1]$, where $\delta = L_1 - L_0$. If the consolidation time T is beyond the due date of an order, this order leaves the system. We assume that $T \leq L_1$. The effective arrival rate is $\lambda = a((L_1 - T)/\delta) - bP$.

For any given T , we can prove that $\pi(T, P)$ is strictly concave in P . The optimal pricing decision is given by

$$P^*(T) = \frac{a(L_1 - T) + b\beta\delta\delta}{2b\delta} + \frac{hT}{4}. \quad (12)$$

Because $0 \leq P \leq a(L_1 - T)/(b\delta)$, we require that

$$0 \leq T \leq \bar{T},$$

where $\bar{T} = (2aL_1 - 2b\beta\delta\delta)/(2a + bh\delta)$.

Substituting (12) into (7), we rewrite (7) as a single-variable function, i.e.,

$$\begin{aligned} \pi(T) = & \mu_v \left(\frac{a(L_1 - T) - b\beta\delta\delta}{2b\delta} - \frac{hT}{4} \right) \\ & \cdot \left(\frac{a(L_1 - T) - b\beta\delta\delta}{2\delta} - \frac{bhT}{4} \right) \\ & - \frac{K + cT_f/T_p}{T} + \frac{c}{T_p}. \end{aligned} \quad (13)$$

Lemma 4.1. For $T \leq \hat{T}$, $\pi(T)$ is concave in T and convex in T for $T > \hat{T}$.

$$\hat{T} = \left[\frac{2(K + cT_f/T_p)}{A_0} \right]^{1/3},$$

where $A_0 = \mu_v(a/(2b\delta)) + h/(64\lambda_1)(a/(2\delta) + bh/4)$.

Proof of Lemma 4.1. Let us first examine the property of $\pi(T)$ by (13).

Based on the second derivative of $\pi(T)$, we have

$$\frac{d^2\pi(T)}{dT^2} = \mu_v \left(\frac{a}{2b\delta} + \frac{h}{64\lambda_1} \right) \left(\frac{a}{2\delta} + \frac{bh}{4} \right) - 2 \frac{K + cT_f/T_p}{T^3}. \quad (14)$$

By (14), it is clear that $d^2\pi(T)/dT^2$ is strictly increasing in T . Denote $\hat{T} = [2(K + cT_f/T_p)/A_0]^{1/3}$, where $d^2\pi(T)/dT^2|_{T=\hat{T}} = 0$.

It is clear that $d^2\pi(T)/dT^2 \leq 0$ for $T \leq \hat{T}$ and $d^2\pi(T)/dT^2 > 0$ for $T > \hat{T}$. QED.

By Lemma 4.1, we find that there exist two positive roots to the first-order condition $d\pi(T, P(T))/dT = 0$. Denote these two roots as T_0 and T_1 . It is clear that $0 < T_0 < \hat{T} < T_1$.

We can characterize the optimal solution by the following theorem.

Theorem 4.1. The optimum is given by the following two cases:

(i) for $\bar{T} \leq \hat{T}$, we have

$$T^* = \begin{cases} T_0, & \text{if } T_0 \leq \bar{T}; \\ \bar{T}, & \text{otherwise;} \end{cases} \quad (15)$$

(ii) for $\bar{T} > \hat{T}$, we have

$$T^* = \begin{cases} T_0, & \text{if } \pi(T_0) \geq \pi(\bar{T}); \\ \bar{T}, & \text{otherwise} \end{cases} \quad (16)$$

Proof of Theorem 4.1. The results immediately follow from Lemma 4.1. QED.

By Theorem 4.1, we find that the optimum depends on two critical values, i.e., $\bar{T}$ and $\hat{T}$, where $\bar{T}$ guarantees that the optimal price is within the limit and $\hat{T}$ characterizes the property of the profit function. The optimal consolidation time is T_0 or $\bar{T}$. In sum, if the profit at $T = T_0$ is no less than that at $T = \bar{T}$, the optimum is given by T_0 ; otherwise $T^* = \bar{T}$.

4.2. Volume Incentive

Our results thus far have considered a unique price for different freight market segments. We now consider an extension of an offering price discount for different market segments in the rail freight business. Here, we assume that there are two types of market segments, i.e., high-volume and low-volume. Following this strategy, the infrastructure manager can provide the necessary incentives for preferable orders. For low-volume orders, we charge a regular price P . For high-volume orders, we offer a discount price θP , where θ is the discount factor. The pricing scheme is defined as

$$P = \begin{cases} P, & \text{if } \mu_v = \mu_v^L; \\ \theta P, & \text{if } \mu_v = \mu_v^H, \end{cases} \quad (17)$$

where μ_v^L is the demand rate of the low-volume orders and μ_v^H is the demand rate of the high-volume orders.

For the additive model, we can rewrite Equation (7)

$$\pi(T, P) = (P - \beta d)\mu_v^L \lambda^L + (\theta P - \beta d)\mu_v^H \lambda^H - \frac{h\mu_v \lambda T}{2} - \frac{K + cT_f/T_p}{T} + \frac{c}{T_p}, \quad (18)$$

where $\lambda = \lambda^H + \lambda^L$, $\lambda^L = a^L - b^L P$, and $\lambda^H = a^H - b^H \theta P$. For any given P , we first optimize T and obtain

$$T^*(P) = \sqrt{\frac{2(K + cT_f/T_p)}{h\mu_v \lambda}}.$$

Substituting $T^*(P)$ into $\pi(P, T)$, we can prove that $\pi(P, T^*(P))$ is strictly concave in P . The optimal solution P^* is given by the first-order condition, i.e.,

$$\mu_v^L(a^L + \beta db^L - 2b^L P^*) + \theta \mu_v^H(a^H + \beta db^H - 2b^H \theta P^*) + (b^L + \theta b^H) \sqrt{\frac{h\mu_v(K + cT_f/T_p)}{2(a^H + a^L - b^L P^* - b^H \theta P^*)}} = 0.$$

The above expression indicates that this optimal pricing could be treated as a weighted average of marketing parameters and logistics costs. The increase in logistics costs (e.g., K and h) results in the increase in the optimal price, while the increase in price elasticity (e.g., b^H and b^L) causes a decrease in the optimal price.

5. Numerical Study: The Brabantroute Case

This problem can be illustrated by a real-life case from the Netherlands. The case of the “Betuweroute” is a good example, since it is one of the most important transport routes in Europe. The “Betuweroute” connects the ports of Amsterdam and Rotterdam with the rest of Europe. The line is scheduled to be shut down at several points in time for maintenance and construction work to expand capacity between Zevenaar and Oberhausen. Construction work is planned for the period from 2015 until 2022 on the Dutch and German sides of the border. Consequently, traffic will have to be diverted from the “Betuweroute” to the “Brabantroute” (ProRail 2015). This route is used for

Figure 2. The “Brabantroute”

Source. Adapted from Cargo (2016).

passenger and freight transportation. Because of the additional freight trains to be run, the need for an appropriate policy arose. The route, with a total length of 140 kilometers, starts in Kijfhoek and ends in Venlo where it crosses the border into Germany. In Figure 2, the route, including distances, is highlighted. At the large yard in Kijfhoek, freight cars wait to be consolidated. Here the infrastructure manager tries to find the optimal consolidation time as well as the price levels for the incoming orders. Note that, for this case in the Netherlands, weight is given as a decisive factor for price calculations.

In this decision-making process, the different characteristics of the passenger and the freight cars need to be considered. Safety measures for passenger transportation are a high priority. To this end, the infrastructure manager needs to calculate the passenger train allocation for the tracks, the optimal values for the dispatching time, and the price in terms of the railway infrastructure.

5.1. Case Data

We have based our data and information on ProRail, the main infrastructure manager in the Netherlands. We extracted the passenger-train schedule, the interarrival time, the weight, and the length of an order in the “Brabantroute” corridor from historical data for the year 2014.

We initially let the indicator for price sensitivity of demand, b , be 0.007. A price increase results in a lower rate and hence less frequent orders. In light of other available modes of transportation, such as a truck, the price for rail transportation cannot be easily increased. The price must therefore be set carefully. By statistical analysis of the historical data, we find that the number of arrival orders follows a Poisson distribution with the average arrival rate of 5.98 per hour, which indicates the market potential. Considering that the distance from Kijfhoek to Venlo is 140 kilometers, and that the average speed of a freight train is 95 kilometers per hour, the time that the route is occupied by the freight train is about 88 minutes. Note that the speed of the passenger train is about 130 kilometers per hour on average and that this cycle also accounts for the passenger trains that use the corridor for 65 minutes (T_p). The deterministic part of the transportation costs is given by $K = €70.4$, c can be approximated as €25, and h is €0.4 per unit per unit time. Substituting the representative values into Equations (5) and (6), we can calculate the expected shipment and holding costs, respectively.

Through the additive model, we find $T^* = 31.14$ hours, $x^* = 27$ per cycle, and $P^* = €8.9$ per ton resulting in a representative profit of €187.9 per hour. For practical implementation, it may be more convenient for the manager to dispatch a freight train every 24 hours. For $T = 24$ hours, the expected profit is €165.9. If the

Figure 3. (Color online) Impact of Market Potential on the Optimal Solution

manager deviates from the optimal solution, the manager needs to be aware of the related consequence, that is, a reduction in profit of 11.7%. The manager then arrives at the best trade-off by balancing scheduling convenience vis-à-vis reduction in profit.

5.2. Sensitivity Analysis

Furthermore, it is useful to analyze the sensitivity of the results in terms of the changes in the parameters. There are two groups of parameters, i.e., market parameters and logistics parameters. The market parameters include market potential a and price elasticity b while the logistics parameters include fixed shipment cost K , unit holding cost h , and cost savings from the shared-use corridor c . For each parameter, we examine the impacts of that parameter on the optimal solution and the total profit as shown in Figures 3–12. Our findings are summarized as follows:

- (i) The price decision increases in a , K , h , and c , but decreases in b .

Figure 4. (Color online) Impact of Market Potential on the Total Profit

Figure 5. (Color online) Impact of Price Elasticity on the Optimal Solution

Figure 8. (Color online) Impact of Fixed Shipment Cost on the Total Profit

Figure 6. (Color online) Impact of Price Elasticity on the Total Profit

Figure 9. (Color online) Impact of Unit Holding Cost on the Optimal Solution

Figure 7. (Color online) Impact of Fixed Shipment Cost on the Optimal Solution

Figure 10. (Color online) Impact of Unit Holding Cost on the Total Profit

Figure 11. (Color online) Impact of Cost Saving on the Optimal Solution

(ii) The time decision increases in b , K , and c , but decreases in a and h .

(iii) The profit increases in a and c , but decreases in b , K , and h .

(iv) Market parameters usually have a more significant impact on the price decision than the time decision while logistics parameters usually have a more significant impact on the time decision than the price decision. For example, when the price elasticity is increased from 0.007 per hour to 0.009, the price is decreased by almost 20% while the length of the cycle time is only increased by 3%. When the fixed shipment cost is increased from 70 to 100, the cycle length is increased by 13.6% while the price is about the same.

(v) Market parameters usually have a more significant impact on profit than logistics parameters. For instance, when the market potential is increased by 33.33% from 6 per hour to 8 per hour, the average profit can be increased by 41.5%. When the cost saving is

Figure 12. (Color online) Impact of Cost Saving on the Total Profit

increased by 40% from 25 to 35, the average profit is only increased by 5%.

Based on the above findings, we obtain the following insights:

(i) Marketing decision p is sensitive to the change in the market parameter and relatively robust to the change in the logistics parameters.

(ii) Operational decision T is sensitive to the change in the logistics parameters and relatively robust to the change in the market parameter.

(iii) The increase in market potential and cost savings results in a positive effect on the profit, while when the incoming orders are more price-sensitive and the logistics costs increase the infrastructure manager incurs a profit loss.

(iv) When the number of incoming orders increases or becomes more expensive to hold stock, to reduce holding costs, the infrastructure manager needs to shorten the length of the consolidation cycle so as to increase the dispatch frequency of the freight trains. When K and c become high, the infrastructure manager needs to increase the length of the consolidation cycle to take advantage of economies of scale and assign passenger trains more frequently to the shared corridor. When the price elasticity increases, the number of actual orders per time unit will decrease. As a result, the infrastructure manager also needs to increase the length of the consolidation cycle to have enough orders for consolidation.

(v) When the price elasticity increases, the manager must lower the price to attract more orders. When the size of the market increases, the manager can still obtain a sufficient number of orders by charging a higher price. When K and h increase, the manager has to charge a higher price to cover the corresponding operational costs.

More interestingly, we can observe that the market potential has a significant impact on the price decision and the time decision. On one hand, a high market potential allows the manager to charge a higher price; meanwhile, the manager can still have a sufficient number of orders. On the other hand, an increase in incoming orders may result in a significant increase in holding costs and transportation costs, which means that the manager is obliged to reduce the length of the consolidation cycle. Furthermore, we perform the same sensitivity analysis with respect to the multiplicative demand function. We obtain similar observations as those of the additive demand function.

5.3. Due-Date and Volume Incentives

At this point, we perform numerical experiments for the two extensions discussed in Section 4. For the extension of the due-date consideration, we set $L_0 = 6$ hours and $L_1 = 48$ hours. The other parameters are the same as those in Section 5.1. We obtain the following

optimal solution: $P^* = €8.24$ per ton, $T^* = 12.58$ hours. The maximum profit is €157.19, $\bar{T} = 42.66$ hours, and $\hat{T} = 10.58$ hours. Compared with the original model, we observe that the manager sets the consolidation cycle close to the lower limit of the due date to maintain a large portion of the market potential.

For the extension of volume incentive, we set $\lambda^H = \lambda^L = \lambda/2$, $b^H = 0.007$ and $b^L = 0.005$, $\theta = 0.9$ for a 10% discount. We obtain the following optimal solution: $P^* = €8.35$ per ton and $T^* = 33.53$ hours. The maximum profit is €216.87. Compared with the original model, we observe that the manager intends to offer a lower price and extend the length of the consolidation cycle to attract more demand.

6. Conclusion

This paper contributes to the literature by developing an analytical model for integrated passenger and freight planning on shared-use rail corridors. We distinguish between the different characteristics of passenger and freight trains, and compute the optimal allocation, dispatching time, and pricing level. We characterize the optimal solution for the time and price strategy in terms of two different demand function structures, i.e., the additive and the multiplicative cases.

Using realistic parameter values representing the rail network in the Netherlands, we have formulated an optimal strategy for real-life practice. By using analytical formulations, we analyzed several scenarios for the case studied. We carried out sensitivity analysis, based on market and logistics parameters. We further enhanced our work by studying the optimal policy parameters for railway operators with due-date contracts. In such a setting, the dispatching of the freight cars must respect the deadline. Moreover, we extended our work by considering price discounts for high-volume railway operators and by obtaining the optimal policy for such initiatives.

The developed analytic models can be used by infrastructure managers to allocate tracks and compute the optimal pricing levels and dispatching times for shared-use corridors under different settings. Experiences with the model were received quite positively by ProRail by the capacity allocation planners and the personnel responsible with railway operators. The analytical models we developed can be used by infrastructure managers to allocate tracks and compute the optimal pricing levels and dispatching times for shared-use corridors according to different settings. ProRail used the model to decide which choice of track (dedicated or shared-rail track) was optimal by evaluating the benefits of shared track in terms of what freight traffic on the shared corridor would allow, while at the same time meeting the schedules of the passenger trains. In the Netherlands, where passenger traffic is

extremely dense, diverting some of the traffic toward shared-use corridors leaves more room for dedicated tracks to allow more passenger trains. The departure frequency for the passenger trains is preset, as is common in most countries. In this case, while adhering to a set departure frequency, the railway infrastructure manager can decide which track to use based on the model. In actuality, the number of passenger trains that are allocated to the shared-use track will be limited by the freight trains. In light of this, by using the model, ProRail can decide on the optimal allocation of passenger trains for passenger train departure. The freight trains will then use the remaining time left from the passenger trains within each cycle. At the consolidation time computed by the model the freight train will depart and the cycle will start again.

In the shared-use corridors, due to the density of passenger train departures in the Netherlands, there are very few time slots that can provide an unhindered route from the point of origin to the destination available for freight trains. Often time slots that only provide part of the route are allocated, and cargo trains are forced to wait at railway yards or sidetracks until additional capacity is available. For ProRail, what is of paramount importance is for freight trains to be organized in such a way that the smallest possible number of train routes is used. A reduction in the number of time slots used by freight trains by means of consolidation would then increase the available unused time slots. With more time slots available, the flexibility increases so that traffic control can assign capacity on the railway network to freight trains, which is especially important in the increasing rail traffic demand. Consolidation would decrease the likelihood that a train has to wait on a node and could ultimately lead to travel time reductions. This would reduce the probability of waiting times at railway yards and sidetracks, and possibly result in shorter travel times. This was deemed quite useful.

Another point that was discussed entailed prospects for lowering costs for the railway operators using the proposed model. By consolidating cargo trains, railway operators can share operational costs related to running the locomotive along with personnel expenditures such as the locomotive engineer. Moreover, the usage fees from ProRail can be spread over a longer train, thereby reducing costs per car for usage fees, personnel expenditures, and running costs of the locomotive.

Next, we point out some of the limitations of the study, together with the necessities and settings that should be considered in a real-life application of the model based on our discussions with ProRail. First, consolidation requires a sufficiently large shunting yard. On the Brabant route, only the shunting yards of Kijfhoek and Venlo are deemed sufficiently large.

Second, only shipments of freight cars that have the same destination and origin can be consolidated. Note also that consolidation can be implemented for a pre-determined segment of a journey. When the model is applied to a larger network, preferably an international network, improvements in performance can be achieved when freight remains consolidated over longer distances. Therefore, a rail network with sufficiently large shunting yards and a large flow of goods with a common destination should consider this strategy. Third, some cargo trains transport dangerous substances. Specified combinations of dangerous substances in one train are forbidden by law. Because of the complexity of the regulations and the absence of information on exact cargo loads, ProRail's suggestion was not to consolidate dangerous goods at all. Finally, a further point of discussion is which party should be responsible for consolidation initiatives and in what way. Railway operators should make their available train routes, train route requests, and train configurations readily available to each other. Many smaller railway operators have opted to outsource the route-request procedure. Rail operators seek time-slot availability and flexibility of departure times. Cost reductions are always welcome: Reduced travel time, or more efficient use of rolling stock and personnel, could result in cost reductions. Consolidation could result in reducing the number of time slots used and would thus provide room for future growth in cargo transport by rail, which is predicted to increase in the near future. Consolidation could also contribute to general cost reductions in terms of freight cars for rail operators. On the other hand, this would imply that the infrastructure manager would collect fewer usage fees in return for additional space for future growth in cargo traffic on the railways. Currently, ProRail, the infrastructure manager, is the sole party with the information available on all of the requested and assigned time slots, together with the configurations of trains. Because of the availability of this data and its primary task in assigning capacity to railway operators, it was discussed whether ProRail should be fulfilling such a role. ProRail occupies a unique position in terms of motivating rail operators to consolidate. To incentivize rail operators, the variable-costs structure of the usage fees could be made more attractive to attract longer and heavier trains. The extension of the model where we consider price discounts for high volume customers is regarded as useful for that purpose.

Generalization of the case study is possible. Europe-wide, similar structures and interactions between railway operators and infrastructure managers exist. In some countries, multiple passenger-railway operators operate on the same line. Consolidation between such companies might be possible as well, but this will most

likely be more controversial than consolidation among cargo companies.

Note also that this study provides an analytical model for strategic decisions in terms of uncertainty as to track allocation, optimal dispatch times, and pricing, considering the expected average of profits in the long run. For more detailed operational issues and in the case of general demand distribution, a simulation approach should be used. Several areas of interest exist for future research. The impact of consolidation on railway yards and shunting movements needs to be investigated further. For railway operators, cost allocation for the parties involved with the consolidated train is an important topic. Not all parties in the train benefit equally from the practice of consolidation. A more in-depth analysis of the impact of consolidation on different parties could clarify this.

Acknowledgments

The authors thank the referees and the associate editor for critical and insightful comments and suggestions that helped to significantly improve the models, results, and presentation. The authors also thank ProRail for all of the support and information.

Appendix. List of Symbols

c_0	usage cost of the shared-use track
c_1	usage cost of the dedicated track
K	combined overhead line fee, energy fee, personnel fee, and set-up fee
β	a variable cost that depends on the volume and the traveling distance of the order
d	traveling distance
$Y(t)$	total volume of the freight by time t
$N(t)$	number of arrivals by time t
V_n	volume of the n th order indicating weight or the number of wagons
λ	arrival rate
h	holding cost fee per volume (per weight unit or per wagon) per unit time
a	market potential
b	price elasticity
x	dispatch frequency of passenger trains on the shared corridor in a cycle
T_p	corridor duration occupied by a passenger train of one dispatch
T_f	corridor duration occupied by a freight train of one dispatch
T	consolidation cycle length
P	price for freight delivery.

References

- Abbink E, Van den Berg B, Kroon L, Salomon M (2004) Allocation of railway rolling stock for passenger trains. *Transportation Sci.* 38(1):33–41.
- Barrena E, Canca D, Coelho LC, Laporte G (2014) Single-line rail rapid transit timetabling under dynamic passenger demand. *Transportation Res. Part B: Methodological* 70:134–150.
- Beckmann M, McGuire C, Winsten CB (1956) *Studies in the Economics of Transportation* (Yale University Press, New Haven, CT).

- Bešinović N, Goverde RM, Quaglietta E, Roberti R (2016) An integrated micro-macro approach to robust railway timetabling. *Transportation Res. Part B: Methodological* 87:14–32.
- Billionnet A (2003) Using integer programming to solve the train-platforming problem. *Transportation Sci.* 37(2):213–222.
- Bookbinder JH, Higgins JK (2002) Probabilistic modeling of freight consolidation by private carriage. *Transportation Res. Part E: Logist. Transportation Rev.* 38(5):305–318.
- Brännlund U, Lindberg PO, Nou A, Nilsson JE (1998) Railway timetabling using Lagrangian relaxation. *Transportation Sci.* 32(4):358–369.
- Cacchiani V, Caprara A, Toth P (2010) Scheduling extra freight trains on railway networks. *Transportation Res. Part B: Methodological* 44(2):215–231.
- Cacchiani V, Huisman D, Kidd M, Kroon L, Toth P, Veenturf L, Wagenaar J (2014) An overview of recovery models and algorithms for real-time railway rescheduling. *Transportation Res. Part B: Methodological* 63:15–37.
- Caprara A, Fischetti M, Toth P (2002) Modeling and solving the train timetabling problem. *Oper. Res.* 50(5):851–861.
- Cargo R (2016) Infrabeheer. Rail Cargo Information Netherlands, <http://www.railcargo.nl/spoormarkt/railin/infrabeheer>.
- Çetinkaya S, Bookbinder JH (2003) Stochastic models for the dispatch of consolidated shipments. *Transportation Res. Part B: Methodological* 37(8):747–768.
- Çetinkaya S, Tekin E, Lee CY (2008) A stochastic model for joint inventory and outbound shipment decisions. *IIE Trans.* 40(3):324–340.
- Chakroborty P, Vikram D (2008) Optimum assignment of trains to platforms under partial schedule compliance. *Transportation Res. Part B: Methodological* 42(2):169–184.
- Chang CS, Kwan CM (2004) Evaluation of evolutionary algorithms for multi-objective train schedule optimization. Webb GI, Yu X, eds. *AI 2004: Advances in Artificial Intelligence*, Lecture Notes Comput. Sci., Vol. 3339 (Springer, Berlin Heidelberg), 803–815.
- Chung JW, Oh SM, Choi IC (2009) A hybrid genetic algorithm for train sequencing in the Korean railway. *Omega* 37(3):555–565.
- Cordeau JF, Toth P, Vigo D (1998) A survey of optimization models for train routing and scheduling. *Transportation Sci.* 32(4):380–404.
- Corman F, D'Ariano A, Pacciarelli D, Pranzo M (2010) A tabu search algorithm for rerouting trains during rail operations. *Transportation Res. Part B: Methodological* 44(1):175–192.
- Dingler M, Lai YC, Barkan C (2009) Impact of train type heterogeneity on single-track railway capacity. *Transportation Res. Record: J. Transportation Res. Board* 2117:41–49.
- Dorfman M, Medanic J (2004) Scheduling trains on a railway network using a discrete event model of railway traffic. *Transportation Res. Part B: Methodological* 38(1):81–98.
- Ha AY (1998) Incentive-compatible pricing for a service facility with joint production and congestion externalities. *Management Sci.* 44(12):1623–1636.
- Hallowell SF (1993) Optimal dispatching under uncertainty with application to railroad scheduling. Unpublished doctoral dissertation, University of Pennsylvania, Philadelphia.
- Harrod S (2013) Auction pricing of network access for North American railways. *Transportation Res. Part E: Logist. Transportation Rev.* 49(1):176–189.
- Higgins A, Kozan E (1998) Modeling train delays in urban networks. *Transportation Sci.* 32(4):346–357.
- Higgins A, Kozan E, Ferreira L (1996) Optimal scheduling of trains on a single line track. *Transportation Res. Part B: Methodological* 30(2):147–161.
- Higginson JK, Bookbinder JH (1994) Policy recommendations for a shipment-consolidation program. *J. Bus. Logist.* 15(1):87–112.
- Hong KS, Lee C (2013) Optimal time-based consolidation policy with price sensitive demand. *Internat. J. Production Econom.* 143(2):275–284.
- Huang K, Chi W (2007) A Lagrangian relaxation based heuristic for the consolidation problem of airfreight forwarders. *Transportation Res. Part C: Emerging Tech.* 15(4):235–245.
- İçyüz IE, Richard JPP, Eskigun E, Acharya D (2016) A two-model solution approach for the monthly coal train reservations planning problem. *Transportation Sci.* 50(3):926–946.
- Jackson G (1985) A survey of freight consolidation practices. *J. Bus. Logist.* 6(1):13–34.
- Lai YC, Lin YJ, Cheng YF (2014) Assessment of capacity charges for shared-use rail lines. *Transportation Res. Record: J. Transportation Res. Board* 2448:62–70.
- Lamorgese L, Mannino C, Piacentini M (2016) Optimal train dispatching by Benders-like reformulation. *Transportation Sci.* 50(3):910–925.
- Lee Y, Chen CY (2009) A heuristic for the train pathing and timetabling problem. *Transportation Res. Part B: Methodological* 43(8):837–851.
- Lusby R, Larsen J, Ryan D, Ehrgott M (2011) Routing trains through railway junctions: A new set-packing approach. *Transportation Sci.* 45(2):228–245.
- Marklund J (2011) Inventory control in divergent supply chains with time-based dispatching and shipment consolidation. *Naval Res. Logist.* 58(1):59–71.
- Meng L, Zhou X (2011) Robust single-track train dispatching model under a dynamic and stochastic environment: A scenario-based rolling horizon solution approach. *Transportation Res. Part B: Methodological* 45(7):1080–1102.
- Mu S, Dessouky M (2011) Scheduling freight trains traveling on complex networks. *Transportation Res. Part B: Methodological* 45(7):1103–1123.
- Mutlu F, Çetinkaya S (2010) An integrated model for stock replenishment and shipment scheduling under common carrier dispatch costs. *Transportation Res. Part E: Logist. Transportation Rev.* 46(6):844–854.
- Mutlu F, Çetinkaya S, Bookbinder JH (2010) An analytical model for computing the optimal time-and-quantity-based policy for consolidated shipments. *IIE Trans.* 42(5):367–377.
- Nash C (2005) Rail infrastructure charges in Europe. *J. Transport Econom. Policy* 39(3):259–278.
- Nemoto T (1997) Area-wide inter-carrier consolidation of freight in urban areas. *Transport Logist.* 1(2):87–101.
- Nguyen C, Dessouky M, Toriello A (2014) Consolidation strategies for the delivery of perishable products. *Transportation Res. Part E: Logist. Transportation Rev.* 69:108–121.
- Petering ME, Heydar M, Bergmann DR (2015) Mixed-integer programming for railway capacity analysis and cyclic, combined train timetabling and platforming. *Transportation Sci.* 50(3):892–909.
- Pouwels M (2014) Meer goederenvervoer per spoor. CBS, <https://www.cbs.nl/nl-nl/achtergrond/2014/19/meer-goederenvervoer-per-spoor>.
- ProRail (2015) Betere aansluiting betuweroute richting Duitsland. <https://www.prorail.nl/projecten/derde-spoor-duitsland>.
- ProRail (2016) Network statement 2016 combined network. Technical report, ProRail, Utrecht, Netherlands.
- Salim V, Cai X (1995) Scheduling cargo trains using genetic algorithms. *Evolutionary Comput.* 1995, *IEEE Internat. Conf.*, Vol. 1 (IEEE, New York), 224–227.
- Samà M, Corman F, Pacciarelli D (2017) A variable neighbourhood search for fast train scheduling and routing during disturbed railway traffic situations. *Comput. Oper. Res.* 78:480–499.
- Sameni MK, Landex A (2013) Capacity utilization in European railways: Who is the fairest of them all? *Transportation Res. Board* 92nd Annual Meeting, Washington, DC.
- Sanchez-Borras M, Nash C, Abrantes P, Lopez-Pita A (2010) Rail access charges and the competitiveness of high speed trains. *Transport Policy* 17(2):102–109.
- Sogin S, Barkan CP, Saat MR (2011) Simulating the effects of higher speed passenger trains in single track freight networks. *Proc. 2011 Winter Simulation Conf.*, Phoenix, 3679–3687.
- Talebian A, Zou B (2015) Integrated modeling of high performance passenger and freight train planning on shared-use corridors in the U.S. *Transportation Res. Part B: Methodological* 82:114–140.

- Tellis GJ (1988) The price elasticity of selective demand: A meta-analysis of econometric models of sales. *J. Marketing Res.* 25(4): 331–341.
- Transit Cooperative Research Program (2009) *Shared Use of Railroad Infrastructure with Noncompliant Public Transit Rail Vehicles: A Practitioner's Guide*, Vol. 130 (National Academies Press, Washington, DC).
- Ülkü MA (2012) Dare to care: Shipment consolidation reduces not only costs, but also environmental damage. *Internat. J. Production Econom.* 139(2):438–446.
- Ülkü MA, Bookbinder JH (2012) Optimal quoting of delivery time by a third party logistics provider: The impact of shipment consolidation and temporal pricing schemes. *Eur. J. Oper. Res.* 221(1):110–117.
- Veelenturf LP, Kidd MP, Cacchiani V, Kroon LG, Toth P (2015) A railway timetable rescheduling approach for handling large-scale disruptions. *Transportation Sci.* 50(3):841–862.
- Xu X, Li K, Yang L (2015) Scheduling heterogeneous train traffic on double tracks with efficient dispatching rules. *Transportation Res. Part B: Methodological* 78:364–384.
- Yin J, Yang L, Tang T, Gao Z, Ran B (2017) Dynamic passenger demand oriented metro train scheduling with energy-efficiency and waiting time minimization: Mixed-integer linear programming approaches. *Transportation Res. Part B: Methodological* 97:182–213.
- Zhou X, Zhong M (2007) Single-track train timetabling with guaranteed optimality: Branch-and-bound algorithms with enhanced lower bounds. *Transportation Res. Part B: Methodological* 41(3):320–341.