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Brief paper

Sliding mode observers for a network of thermal and hydroelectric power plants[☆]Gianmario Rinaldi^{a,*}, Michele Cucuzzella^b, Antonella Ferrara^a^a Department of Electrical, Computer and Biomedical Engineering, University of Pavia, Via Ferrata 5, 27100, Pavia, Italy^b Jan C. Willems Center for Systems and Control, ENTEG, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands

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ABSTRACT

This paper deals with the design of a novel sliding mode observer-based scheme to estimate and reconstruct the unmeasured state variables in power networks including hydroelectric power plants and thermal power plants. The proposed approach reveals to be flexible to topological changes to power networks and can be easily updated only where changes occur. The discussed numerical simulations validate the effectiveness of the proposed estimation scheme.

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1. Introduction

Nowadays, the design and validation of advanced and robust schemes to monitor and control power networks represent an important field of research (Bevrani, 2009). The installed capacity of distributed renewable power plants increases worldwide. The resulting growing intermittent power generation of this kind of plants can destabilize an entire power network (Gautam, Vittal, & Harbour, 2009). In order to solve this issue, on one side, advanced strategies have been designed to monitor and control distributed renewable power plants in a more intelligent and smarter way (Fang, Misra, Xue, & Yang, 2012). On the other side, big attention has been paid to conceive more robust and efficient control methods for conventional power plants (such as thermal, nuclear and hydroelectric power plants), dealing also with the uncertainties caused by the increasing distributed generation. Having in mind the aforementioned issues, sliding mode control techniques have been proposed for the Load Frequency Control (LFC) in power networks. In Prasad, Purwar, and Kishor (2017), a sliding mode control technique dealing with matched and unmatched uncertainties has been employed for LFC purposes. In Trip, Cucuzzella,

Ferrara, and De Persis (2017) and Trip, Cucuzzella, Persis, van der Schaft, and Ferrara (2018) passivity- and energy-based sliding mode control schemes have been proposed in order to regulate the frequency, and also minimize the generation costs or maintain the scheduled power flows, respectively. The use of estimation schemes, specifically the so-called state observers, can be seen as a way to enhance the monitoring and thus the control of a power network, in the sense that they represent an additional tool to check the validity of some measurements or to reconstruct the unmeasured states, especially when one deals with large-scale networks. Few relevant works have dealt with the design of sliding mode observers in power systems. For example, in Liu et al. (2017), an extended state observer (ESO) based second-order sliding-mode (SOSM) control has been designed for three-phase two-level grid-connected power converters. In Rinaldi, Menon, Edwards, and Ferrara (2017), a combination of original super-twisting-like sliding mode observers and algebraic observers have been proposed to robustly estimate the unmeasured state variables in power grids in a distributed fashion. In Rinaldi, Cucuzzella, and Ferrara (2017), a third order sliding mode observer-based approach has been designed for optimal load frequency control in power networks partitioned into control areas. In Mellucci, Menon, Edwards, and Ferrara (2017), a robust multi-variable super-twisting sliding mode observer has been employed to detect the position and to reconstruct the time evolution of load alterations in power networks.

The main contribution of the present paper is the design of a novel decentralized sliding mode observer-based estimation scheme with application to power networks comprising thermal

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power plants and hydroelectric power plants. In Mellucci et al. (2017), Rinaldi, Cucuzzella et al. (2017), Rinaldi, Menon et al. (2017), and Vrdoljak, Perić, and Petrović (2010), simplified mathematical models of the power networks have been used as a starting base to design the estimation schemes (neglecting both the hydrothermal turbine–governor dynamics and the nonlinearities in the synchronous generators dynamics). With respect to the aforementioned works, the mathematical model of the power network is significantly detailed in our approach. Specifically, the turbine+governor dynamics are considered for the two types of power plants (thermal and hydroelectric). Moreover, differently from Vrdoljak et al. (2010), in which Luenberger's state observers have been designed, in our paper sliding mode observers are proposed to robustly estimate all the unmeasured state variables for the turbine–governor dynamics, which can be considered as a perturbed linear system for both thermal and hydraulic plants. The recently proposed nonlinear swing equations (Monshizadeh, De Persis, Monshizadeh, & van der Schaft, 2016) are adopted to model the synchronous generators in a more realistic and accurate way in each plant. A sub-optimal sliding mode observer is designed to estimate the frequency deviation of each generator. All the observers for each plant require only local information and local measurements, so that the proposed estimation scheme results in being completely decentralized. In addition, the proposed solution is easily adaptable to topological changes affecting power network, such as the opening or the closing of power transmission line switches, or the plugging-in or -out of some plants. In such case, it is necessary only to re-tune the gains of the preexisting sub-optimal observers for the generators directly connected to the new neighboring plants, leaving the other observers gains untouched. These advantages are possible thanks to the robustness features of the adopted sliding mode approach. The observer-based scheme is also assessed in simulation to verify its effectiveness.

The rest of the present paper is structured as follows. In Section 2, the description of the hydro-thermal power network is recalled. In Section 3, the design procedure of the novel observer-based scheme is presented. In Section 4, the proposed observers are assessed in simulations, whilst in Section 5 the conclusions are reached. Table 1 shows the physical meanings and the measurement units of the states variables and the model parameters adopted in the present paper. In this paper, the following (standard) notation is adopted. For a given state variable x , $\hat{x}$ denotes the estimated value of x . For a given matrix or vector X , X^T denotes its transpose. Expression $\text{sgn}(\cdot)$ denotes the signum function.

2. Power network description

2.1. Graph theory recalls

A power network can be interpreted as an undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ (Kundur, Balu, & Lauby, 1994). Specifically, $\mathcal{V}$ represents the set of nodes of the graph (which are n_t thermal power plants and n_h hydroelectric power plants), and it consists of two subsets, i.e., $\mathcal{V} = \mathcal{V}_t \cup \mathcal{V}_h$. The set $\mathcal{V}_t$ denotes all the n_t thermal power plants, whilst $\mathcal{V}_h$ denotes all the n_h hydroelectric power plants. The set of edges $\mathcal{E} = \{1, \dots, k, \dots, m\}$ comprises all the power transmission lines linking the plants. Each k th edge is denoted as $k \triangleq [(i, j); X_{ij}]$, where (i, j) is the unordered pair of the distinct nodes linked by the k th power transmission line, and X_{ij} is the reactance of the k th power transmission line. The topology of the graph can be encapsulated in the Laplacian Matrix $\mathcal{X} \in \mathbb{R}^{N \times N}$, where $N = n_t + n_h$, and its elements are defined as follows (Kundur et al., 1994)

$$\mathcal{X} = \begin{cases} \mathcal{X}_{ii} = \sum_{j \in \mathcal{N}_i} X_{ij} \\ \mathcal{X}_{ij} = -X_{ij} & \text{if } \exists k = [(i, j); X_{ij}] \in \mathcal{E} \\ \mathcal{X}_{ij} = 0 & \text{otherwise,} \end{cases} \quad (1)$$

Table 1

State variables and model parameters adopted in the paper.

Symbols	Meanings	Units
$P_{m_{a_i}}, P_{m_{b_i}}, P_{m_{c_i}}$	a, b, c turbines powers	(p.u.)
P_{g_i}	Governor power	(p.u.)
P_{m_i}	Total mechanical power	(p.u.)
P_{d_i}	Electrical power demand	(p.u.)
u_i	Control input	(p.u.)
$T_{a_i} \in [0.1, 0.4]$	a -turbine time constant	(s)
$T_{b_i} \in [4, 11]$	b -turbine time constant	(s)
$T_{c_i} \in [0.3, 0.5]$	c -turbine time constant	(s)
$T_{g_i} \in [0.2, 0.3]$	Governor time constant	(s)
$\alpha_i, \beta_i, \gamma_i$	Power conversion constants	–
P_{c_i}	Transient compensator power	(p.u.)
W_i	Water speed	(p.u.)
$T_{c1_i} \approx 5$	Compensator time constant 1	(s)
$T_{c2_i} \approx 50$	Compensator time constant 2	(s)
$T_{h_i} \in [1, 2]$	Hydro turbine time constant	(s)
δ_i	Generator angle	(rad)
ω_i	Generator frequency deviation	(rad/s)
ω^*	Network nominal frequency	(rad/s)
J_i	Generator inertia	(kg m ²)
D_i	Generator damping	(N m s)
V_i	Voltage magnitude	(p.u.)
X_{ij}	Reactance of the line	(p.u.)

where $\mathcal{N}_i$ is the set of nodes directly connected to the i th node via power transmission lines.

Remark 1. From the point of view of the power network operations, it is reasonable to suppose that the use of power transmission lines changes with respect to time due to scheduled electricity trade among the plants. Therefore, the set of edges $\mathcal{E}$ represents all the possible interconnections among the plants in the most conservative situation, which means that all the available power transmission lines are used. Consequently, also the Laplacian Matrix in (1) encapsulates the power grid topology in the most conservative situation, as well as the set $\mathcal{N}_i$ for each node.

2.2. Steam turbines and governor dynamics

The so-called single tandem reheat arrangement is adopted in the present work. This comprises three steam turbines, denoted as a_i , b_i , and c_i , which are attached to the same shaft, and it represents the most common configuration used for large thermal power plants (Machowski, Bialek, & Bumby, 2011). The following dynamics yield:

$$\begin{aligned} \dot{P}_{m_{a_i}} &= -\frac{1}{T_{a_i}} P_{m_{a_i}} + \frac{1}{T_{a_i}} P_{g_i} \\ \dot{P}_{m_{b_i}} &= -\frac{1}{T_{b_i}} P_{m_{b_i}} + \frac{1}{T_{b_i}} P_{m_{a_i}} \\ \dot{P}_{m_{c_i}} &= -\frac{1}{T_{c_i}} P_{m_{c_i}} + \frac{1}{T_{c_i}} P_{m_{b_i}} \\ \dot{P}_{g_i} &= -\frac{1}{T_{g_i}} P_{g_i} + \frac{1}{T_{g_i}} u_i - \frac{1}{R_i T_{g_i}} \omega_i \\ y_{i1} &= \alpha_i P_{m_{a_i}} + \beta_i P_{m_{b_i}} + \gamma_i P_{m_{c_i}} = P_{m_i}. \end{aligned} \quad (2)$$

The reader can refer to Table 1 for the physical meanings and the measurement units of the introduced state variables and model parameters. Typical values for the constants are $\alpha_i = 0.3$, $\beta_i = 0.4$, $\gamma_i = 0.3$, and the fundamental relation $\alpha_i + \beta_i + \gamma_i = 1$ holds (Machowski et al., 2011). It is possible to compactly rewrite

Eq. (2):

$$\dot{x}_{p_{t_i}} = A_{t_i} x_{p_{t_i}} + B_{t_i} \left(u_i - \frac{\omega_i}{R_i} \right) \quad (3)$$

$$y_{t_{i_1}} = C_{t_i} x_{p_{t_i}},$$

where $x_{p_{t_i}}$, A_{t_i} , B_{t_i} , C_{t_i} are suitably defined vectors and matrices.

2.3. Hydraulic turbine and governor dynamics

The linearized hydraulic turbine and governor dynamics comprises a governor (similar to the one described for thermal power plants), a transient droop compensator (introduced to enhance the stability Kundur et al., 1994; Machowski et al., 2011), and the hydraulic turbine. The following dynamics yield:

$$\begin{aligned} \dot{P}_{g_i} &= -\frac{1}{T_{g_i}} P_{g_i} + \frac{1}{T_{g_i}} u_i - \frac{1}{R_i T_{g_i}} \omega_i \\ \dot{P}_{c_i} &= -\frac{1}{T_{c_{1i}}} P_{c_i} + P_{g_i} \\ \dot{W}_i &= \frac{T_{c_{1i}}}{T_{c_{2i}}} P_{g_i} + \frac{T_{c_{2i}} - T_{c_{1i}}}{T_{c_{2i}}^2} P_{c_i} - \frac{2}{T_{h_i}} W_i \\ y_{h_{i_1}} &= -2 \frac{T_{c_{1i}}}{T_{c_{2i}}} P_{g_i} + 2 \frac{T_{c_{1i}} - T_{c_{2i}}}{T_{c_{2i}}^2} P_{c_i} + \frac{6}{T_{h_i}} W_i = P_{m_i}. \end{aligned} \quad (4)$$

The reader can refer again to Table 1 for the physical meanings and the measurement units of the introduced state variables and model parameters. The following compact representation holds.

$$\dot{x}_{p_{h_i}} = A_{h_i} x_{p_{h_i}} + B_{h_i} \left(u_i - \frac{\omega_i}{R_i} \right) \quad (5)$$

$$y_{h_{i_1}} = C_{h_i} x_{p_{h_i}},$$

where, again, $x_{h_{i_1}}$, A_{h_i} , B_{h_i} , C_{h_i} are suitably defined vectors and matrices.

2.4. Generator dynamics

Both the steam turbines and the hydroelectric turbines are coupled with a generator (typically a synchronous machine). Several models have been adopted in the literature for the generator dynamics (see, e.g., the detailed description provided in Machowski et al., 2011). However, in the last few years it has been shown that linearized models are not accurate, even under small perturbations (Monshizadeh et al., 2016). Consequently, in the present work, the generator of each thermal and hydroelectric power plant is modeled by using the so-called nonlinear improved swing equations proposed in Monshizadeh et al. (2016) given by:

$$\begin{aligned} \dot{\delta}_i &= \omega_i \\ \dot{\omega}_i &= -\frac{\sum_{j \in \mathcal{N}_i} \frac{V_i V_j}{X_{ij}} \sin(\delta_i - \delta_j)}{J_i (\omega_i + \omega^*)} + \frac{P_{m_i} - P_{d_i}}{J_i (\omega_i + \omega^*)} - \frac{D_i \omega_i}{J_i} \\ y_{\bullet_{i_2}} &= \delta_i. \end{aligned} \quad (6)$$

Note that in (6) it is assumed to locally measure only the generator angle δ_i . This can be easily implemented in practice by equipping each generator with an encoder to measure the position of the rotor (Chowdhury, Haque, Das, Gargoom, & Negnevitsky, 2015). Moreover, the subscript $\bullet$ in $y_{\bullet_{i_2}}$ is equal to $\bullet = t$ in case of thermal power plant, and $\bullet = h$ in case of hydroelectric power plant.

Remark 2. Note that in (6) the only mutual interaction among the plants takes place at the level of electrical active power exchange, which can be modeled according to the power flow method (Machowski et al., 2011) as

$$P_{i_{\mathcal{N}_i}} \triangleq \sum_{j \in \mathcal{N}_i} \frac{V_i V_j}{X_{ij}} \sin(\delta_i - \delta_j), \quad (7)$$

where $P_{i_{\mathcal{N}_i}}$ is the total electrical active power transmitted by the i th plant to its neighbors.

Remark 3. Each thermal power plant node is governed by (2) together with (6), whilst each hydroelectric power plant is governed by (4) together with (6). It is assumed to measure at the node level only the mechanical power delivered by the turbine P_{m_i} and the generator angle δ_i both in the thermal and hydroelectric power plants. All the other state variables have to be estimated via observers. Moreover, the matrices and vectors in (3)–(5) are assumed to be known at each power plant node level.

3. Observers design

In this section, the design procedures of the sliding mode observers to estimate the unmeasured state variables of each node of the power network are presented. Specifically, a first-order sliding mode observer is proposed to robustly estimate the powers associated with the three turbines and the governor of the thermal power plant node. The same idea is applied also to robustly estimate the governor power, the transient compensator power and the water speed of the hydraulic turbine. Moreover, a sub-optimal sliding mode observer is proposed to robustly estimate the frequency deviation of each generator. This kind of observer is required to perform state estimation in nonlinear dynamical systems in the form of (6), as detailed in sequel. Specific rules to easily update the estimation scheme to topological changes affecting the power network are presented, such as the addition or the removal of edges and nodes, making also reference to the physical meaning of these changes.

3.1. Sub-optimal sliding mode observer for generator

For the sake of clarity, it is better to start to design the sub-optimal sliding mode observer to robustly estimate the frequency deviation of each plant.

Assumption 1. Given the signals

$$\phi_i \triangleq -\frac{\sum_{j \in \mathcal{N}_i} \frac{V_i V_j}{X_{ij}} \sin(\delta_i - \delta_j)}{J_i (\omega_i + \omega^*)} + \frac{P_{m_i} - P_{d_i}}{J_i (\omega_i + \omega^*)} - \frac{D_i \omega_i}{J_i}, \quad (8)$$

$$\Phi_i \triangleq \left| \frac{\sum_{j \in \mathcal{N}_i} \frac{V_i V_j}{X_{ij}} \sin(\delta_i - \delta_j)}{J_i (\omega_i + \omega^*)} \right| + \left| \frac{P_{m_i} - P_{d_i}}{J_i (\omega_i + \omega^*)} \right| + \left| \frac{D_i \omega_i}{J_i} \right|, \quad (9)$$

it is assumed that ϕ_i is a bounded disturbance, which means that $|\phi_i| \leq \Phi_i < \Lambda_i$, where Λ_i is a known positive constant which can be determined from the understanding of the power network.

Note that the term ϕ_i include all the neighboring plants in the most conservative situation specified by Remark 1. Moreover, in such approach, the interactions among the plants detailed in Remark 2 are treated as bounded disturbances, which give to the sub-optimal sliding mode observers a completely decentralized feature. Consider the following sub-optimal sliding mode observer

$$\begin{aligned} \hat{\delta}_i &= \hat{\omega}_i \\ \dot{\hat{\omega}}_i &= u_{\text{sub}_i}, \end{aligned} \quad (10)$$

where $\hat{\delta}_i$ is the estimate of δ_i , $\hat{\omega}_i$ is the estimate of ω_i , and u_{sub_i} is equal to (Bartolini, Ferrara, & Usai, 1998)

$$u_{\text{sub}_i} \triangleq -\mu_i V_i^{\max} \text{sgn} \left(e_{\delta_i} - \frac{1}{2} e_{\delta_i}^{\max} \right), \quad (11)$$

where $e_{\delta_i} \triangleq \hat{\delta}_i - \delta_i$. Moreover, the following relations are considered

$$\begin{cases} \mu_i^* & \in (0, 1] \\ V_i^{\max} & > \max \left(\frac{\Lambda_i}{\mu_i^*}, \frac{4\Lambda_i}{3 - \mu_i^*} \right). \end{cases} \quad (12)$$

The signal $e_{\delta_i}^{\max}$ and the design constant μ_i are computed by using the peak detection algorithm (Bartolini et al., 1998) originally proposed for control purposes and here used for the observer design. The following theorem can be proven.

Theorem 2. *Given the generator dynamics (6), Assumption 1, and the signal u_{sub_i} defined by (11), then, the sub-optimal sliding mode observer in the form of (10) leads to a correct estimation of the frequency deviation ω_i of each generator in a finite time.*

Proof. The error dynamics are obtained by subtracting the generator dynamics (6) to the sub-optimal observer dynamics (10) and are given by

$$\begin{aligned} \dot{e}_{\delta_i} &= e_{\omega_i} \\ \dot{e}_{\omega_i} &= \phi_i - \mu_i V_i^{\max} \text{sgn} \left(e_{\delta_i} - \frac{1}{2} e_{\delta_i}^{\max} \right), \end{aligned} \quad (13)$$

where $e_{\delta_i} \triangleq \hat{\delta}_i - \delta_i$, $e_{\omega_i} \triangleq \hat{\omega}_i - \omega_i$. Eq. (13) is in the standard form for the sub-optimal sliding mode controlled system (Bartolini et al., 1998). More precisely, if the signal u_{sub_i} is designed in such a way to fulfill the inequalities in (12), it follows that (13) converges to the origin in a finite time, guaranteeing a correct state estimation of the frequency deviation of each generator.

Remark 4. Note that Theorem 2 is still valid if additional bounded uncertainties, appearing in the matched channel of the system (6), are included in the term ϕ_i in (8). These additional uncertainties can be due to unmodeled dynamics, parameters variation and external disturbances. This fact confirms that the proposed observer (10) guarantees the robustness property typical of sliding mode observers.

3.2. Sliding mode observer for steam turbine and governor

In order to design a sliding mode observer to estimate the unmeasured variables P_{gi} , $P_{m_{gi}}$, $P_{m_{bi}}$, $P_{m_{ci}}$ it is first necessary to verify the detectability of the pair (A_{ti}, C_{ti}) . The following theorem can be proven by direct calculation.

Theorem 3. *The pair (A_{ti}, C_{ti}) in (3) is detectable.*

Consider now the following sliding mode observer

$$\dot{\hat{x}}_{p_{ti}} = A_{ti} \hat{x}_{p_{ti}} + B_{ti} \left(u_i - \frac{\hat{\omega}_i}{R_i} \right) - G_{ti} C_{ti} e_{p_{ti}} - B_{ti} \rho_{ti} \frac{F_i C_{ti} e_{p_{ti}}}{|F_i C_{ti} e_{p_{ti}}|}, \quad (14)$$

where $e_{p_{ti}} \triangleq \hat{x}_{p_{ti}} - x_{p_{ti}}$, $F_i \in \mathbb{R}$ (F_i is a scalar in our case), ρ_{ti} is a positive design constant, G_{ti} is a design matrix, and $\hat{\omega}_i$ is the estimated value of the frequency deviation ω_i , communicated by the sub-optimal observer for the generator of the same plant. From the development in Section 3.1, it is reasonable to assume that

$$\psi_i \triangleq (\hat{\omega}_i - \omega_i) / R_i = e_{\omega_i} / R_i \quad (15)$$

is a bounded disturbance, which means that its modulus is upper-bounded. Moreover, ψ_i converges to zero by virtue of Theorem 2. The following theorem holds.

Theorem 4. *Given the thermal turbine–governor dynamics (4), suppose that for a positive definite symmetric matrix P_i , one has*

$$P_i A_{0_{ti}} + A_{0_{ti}}^T P_i < 0, \quad (16)$$

where $A_{0_{ti}} \triangleq A_{ti} - G_{ti} C_{ti}$ and the following structural constraint (Edwards & Spurgeon, 1994) is fulfilled

$$P_i B_{ti} = C_{ti}^T F_i^T. \quad (17)$$

Then, the sliding mode observer in the form of (14) asymptotically leads to a correct state estimation of $x_{p_{ti}}$ provided that the positive design constant ρ_{ti} is chosen such that $\rho_{ti} > |\psi_i|$.

Proof. By subtracting (3) from (14), the so-called error system dynamics can be obtained as follows

$$\dot{e}_{p_{ti}} = A_{0_{ti}} e_{p_{ti}} - B_{ti} \psi_i - B_{ti} \rho_{ti} \frac{F_i C_{ti} e_{p_{ti}}}{|F_i C_{ti} e_{p_{ti}}|}. \quad (18)$$

Eq. (18) is in the standard form of the perturbed sliding mode observer error dynamics (Shtessel, Edwards, Fridman, & Levant, 2014), with the associated sliding surface $\sigma_i = F_i C_{ti} e_{p_{ti}} = 0$ to be reached in a finite time. The aim here is to show, exploiting ideas from Shtessel et al. (2014), that the function $V_{p_{ti}}(e_{p_{ti}}) \triangleq e_{p_{ti}}^T P_i e_{p_{ti}}$ is a Lyapunov Function for the system (18), ensuring that the point $e_{p_{ti}} = 0$ is an asymptotically stable equilibrium point. It is immediate to show that $V_{p_{ti}}(0) = 0$. Moreover, by differentiating with respect to time $V_{p_{ti}}$, it yields

$$\begin{aligned} \dot{V}_{p_{ti}} &= e_{p_{ti}}^T \left(P_i A_{0_{ti}} + A_{0_{ti}}^T P_i \right) e_{p_{ti}} - 2e_{p_{ti}}^T P_i B_{ti} \psi_i \\ &\quad - 2e_{p_{ti}}^T P_i P_i^{-1} C_{ti}^T F_i^T \rho_{ti} \frac{F_i C_{ti} e_{p_{ti}}}{|F_i C_{ti} e_{p_{ti}}|}. \end{aligned} \quad (19)$$

By exploiting Eq. (19), one has that $\dot{V}_{p_{ti}}(0) = 0$. Consider now $e_{p_{ti}} \neq 0$. By virtue of (16), it yields

$$e_{p_{ti}}^T \left(P_i A_{0_{ti}} + A_{0_{ti}}^T P_i \right) e_{p_{ti}} < 0, \quad \forall e_{p_{ti}} \neq 0. \quad (20)$$

By using (17) and (20), after few algebraic simplifications in (19), the following inequality yields

$$\dot{V}_{p_{ti}} \leq -2e_{p_{ti}}^T C_{ti}^T F_i^T \left(\frac{\psi_i}{F_i C_{ti} e_{p_{ti}}} + \frac{\rho_{ti}}{|F_i C_{ti} e_{p_{ti}}|} \right) F_i C_{ti} e_{p_{ti}}. \quad (21)$$

Considering the scalar nature of the terms ψ_i and ρ_{ti} , the right side of inequality (21) is strictly negative if

$$\rho_{ti} > |\psi_i|. \quad (22)$$

By virtue of the Lyapunov Theorem (Khalil, 1996), $e_{p_{ti}} = 0$ is an asymptotically stable equilibrium point, and therefore, a correct state estimation of the unmeasured state variables can be performed. It is worth noting that although $e_{p_{ti}} = 0$ is asymptotically reached, according to Shtessel et al. (2014), $C_{ti} e_{p_{ti}} = 0$ is reached in a finite time. In addition, detailed algorithm to numerically solve the Linear Matrix Inequality in (16) combined with the linear constraint (17) has been provided in the literature in Edwards and Spurgeon (1994), Xiang, Su, and Chu (2005), and is not reported here for the sake of simplicity.

Remark 5. Note that, in analogy with Theorem 2, Theorem 4 is still valid if additional bounded uncertainties, appearing in the matched channel of the system (18), are included in the term ψ_i in (15). These uncertainties can be due to unmodeled dynamics, parameters variation and external disturbances as mentioned before. The robustness property typical of sliding mode observer is guaranteed also for observer (14).

3.3. Sliding mode observer for hydraulic turbine and governor

In this section, a sliding mode observer is designed to estimate the unmeasured state variables for the hydraulic turbine–governor dynamics. Since the design procedure is similar to the one discussed in Section 3.2, only the relevant key-ideas are here reported. As above, it is necessary to verify the detectability of the pair (A_{h_i}, C_{h_i}) . The following theorem can be proven by direct calculation.

Theorem 5. *The pair (A_{h_i}, C_{h_i}) in (5) is detectable.*

In perfect analogy to Section 3.2, the following observer can be introduced

$$\begin{aligned} \dot{\hat{x}}_{p_{h_i}} = & A_{h_i} \hat{x}_{p_{h_i}} + B_{h_i} \left(u_i - \frac{\hat{\omega}_i}{R_i} \right) - G_{h_i} C_{h_i} e_{p_{h_i}} \\ & - B_{h_i} \rho_{h_i} \frac{F_i C_{h_i} e_{p_{h_i}}}{\left| F_i C_{h_i} e_{p_{h_i}} \right|}, \end{aligned} \quad (23)$$

where $e_{p_{h_i}} \triangleq \hat{x}_{p_{h_i}} - x_{p_{h_i}}$, $F_i \in \mathbb{R}$ (F_i is a scalar in our case), ρ_{h_i} is a positive design constant, G_{h_i} is a design matrix, and $\hat{\omega}_i$ is the estimated value of the frequency deviation ω_i . The following Theorem can be proven in perfect analogy to Theorem 4.

Theorem 6. *Given the hydraulic turbine–governor dynamics (5), suppose that for a positive definite symmetric matrix P_i , one has $P_i A_{0_{h_i}} + A_{0_{h_i}}^T P_i < 0$, where $A_{0_{h_i}} \triangleq A_{h_i} - G_{h_i} C_{h_i}$, and the following structural constraint is fulfilled $P_i B_{h_i} = C_{h_i}^T F_i^T$. Then, the sliding mode observer in the form of (23) asymptotically leads to a correct state estimation of $x_{p_{h_i}}$ provided that the positive design constant ρ_{h_i} is chosen such that $\rho_{h_i} > |\psi_i|$.*

Note that robustness feature highlighted by Remark 5 still holds in this case.

3.4. Scalability and resilience of observers

Opening or closing of a power transmission line. In case of an opening or closing of the power transmission line linking the i th and the j th plant, the magnitude of uncertainty in the signals Φ_i and Φ_j decreases (see Eq. (9) to this end) and the sliding motion cannot be lost, guaranteeing a correct frequency estimation in each plant.

Plugging-in of a plant. Suppose now that a new j th plant is linked to a given number of existing plants via power transmission lines. Let $\mathcal{N}_j$ be the set of the existing nodes directly connected to the new j th node (in the most in Remark 1). The proposed estimation scheme can be easily updated according to the following steps:

- (1) For the given j th new plant, design a sliding mode observer in the form of (14) in case of a new thermal power plant, or in the form of (23) in case of a new hydroelectric power plant.
- (2) Re-tune the gains of the preexisting sub-optimal observers for the generators of all the neighbors nodes $k \in \mathcal{N}_j$, by updating the terms Δ_k and fulfilling the tuning rules in (12). This is true because the source of uncertainty increases in the nodes directly connected to the new ones.
- (3) Design a new sub-optimal sliding mode observer to estimate the frequency deviation of the new j th plant.

The proposed observer-based estimation scheme has to be updated only where topological changes occur, i.e., at the level of the new node and of its neighborhood. All the other observers do not need to be updated. Therefore, one can conclude that the estimation scheme is scalable in case of adding new plants and is resilient in case of changing in the operation of the power transmission lines.

Fig. 1. Scheme of the considered power network comprising two thermal power plants (red circles) and two hydroelectric power plants (blue circles) in the most conservative operation (left), and with the power transmission line X_{14} open (right). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 2

Values of the parameters of the power network test case (base power of 1000 MVA).

	Node 1	Node 2	Node 3	Node 4
T_a (s)	–	0.30	–	0.35
T_b (s)	–	7.50	–	10.00
T_c (s)	–	0.40	–	0.45
T_{c_1} (s)	5.00	–	4.50	–
T_{c_2} (s)	50.00	–	40.00	–
T_h (s)	2.00	–	2.20	–
T_g (s)	0.50	0.25	0.45	0.27
J (p.u.)	0.0028	0.0038	0.0038	0.0042
D (p.u.)	1.4×10^{-4}	1.5×10^{-4}	1.3×10^{-4}	1.7×10^{-4}
ΔP_d (p.u.)	0.10	0.50	0.20	0.70

4. Simulation results

In this section, the observer-based scheme is assessed in simulation to verify its effectiveness. The proposed estimation scheme is compared with the well-established Unknown Input Luenberger state observer, originally proposed in Guan and Saif (1991) and applied to a large class of dynamical systems. In this framework, symbol $\tilde{x}$ denotes the estimate of the state variable x via Unknown Input (UI) Luenberger observer. A power network comprising two thermal power plants and two hydroelectric power plants linked via power transmission lines is considered (see Fig. 1). For the sake of simplicity, in this simulation case it is assumed that each power plant is controlled only via primary frequency controller (Kundur et al., 1994). This means that the control input u_i is set equal to zero in each plant. The simulation time interval is $T = 200$ (s), while the integration step size is $T_s = 1 \times 10^{-3}$ (s). The selected sub-optimal sliding mode observer parameters are $V^{\max} = 10$, $\rho_t = 1$, and $\rho_h = 10$ for each plant. The values of the reactances of the edges are: $X_{12} = 0.19$ (p.u.), $X_{23} = 0.20$ (p.u.), $X_{34} = 0.22$ (p.u.), $X_{14} = 0.19$ (p.u.). Power network dynamics have been numerically simulated in a Matlab-Simulink R2017b environment. Table 2 shows the numerical representation of the model parameters. The following three scenarios are considered:

- (1) **Scenario 1**, $0 \leq t < 20$ (s), during which the power network is at steady state, which means that there is a perfect balance between electrical active power generation and consumption;
- (2) **Scenario 2**, $20 \leq t < 100$ (s), during which, after a step variation of the active power demand in each node according to Table 2, the frequency decreases;
- (3) **Scenario 3**, $100 \leq t \leq 200$ (s), during which the power transmission line X_{14} in Fig. 1 is removed.

During Scenario 1 the power network is at steady-state. In such situation each sliding mode observer for turbine–governor dynamics and each sub-optimal sliding mode observer for the frequency deviation are capable of asymptotically estimating all

Fig. 2. Time evolution of the frequency deviation ω_i in each plant, its estimated value via the proposed sliding mode observer $\hat{\omega}_i$ and via UI Luenberger observer $\tilde{\omega}_i$.

Fig. 3. Time evolution of the governor power variation P_{g_i} in each plant, its estimated values $\hat{P}_{g_i}$ via proposed sliding mode observer and $\tilde{P}_{g_i}$ via UI Luenberger observer.

Fig. 4. Time evolution of transient compensator power variation P_{c_i} , the water speed variation W_i in each hydroelectric power plant and their estimated value $\hat{P}_{c_i}$ and $\hat{W}_i$ via proposed sliding mode observer and $\tilde{P}_{c_i}$ and $\tilde{W}_i$ via UI Luenberger observer.

the unmeasured states (see Figs. 2, 3, 4, 5). It is worth noting the presence of fast transients during the first seconds. These are due to the initial conditions of the observers which are set different from the actual states to be estimated (see again Figs. 2, 3, 4, 5). During Scenario 2, the sudden variation of electrical active power demand

Fig. 5. Time evolution of the turbine power variation $P_{m_{a_i}}$, $P_{m_{b_i}}$, and $P_{m_{c_i}}$ in each thermal power plant and their estimated values $\hat{P}_{m_{a_i}}$, $\hat{P}_{m_{b_i}}$, and $\hat{P}_{m_{c_i}}$ via proposed sliding mode observer and their estimate values $\tilde{P}_{m_{a_i}}$, $\tilde{P}_{m_{b_i}}$, and $\tilde{P}_{m_{c_i}}$ via UI Luenberger observer.

P_{d_i} in each plant causes a transient during which the frequency of each generator decreases (see also Eq. (6) for a mathematical justification). All the plants, governed by the primary frequency controllers, response by increasing the electrical active power generation. All the observers are capable of tracking the time evolution of all the unmeasured state variables (see again Figs. 2, 3, 4, 5). During Scenario 3, the power transmission line linking the 1-st plant and the 4-th plant is open. In such situation more relevant oscillations of the frequencies take place in these two plants (see the enlargements in Fig. 2). Also in such situation, all the observers track the unmeasured state without losing the induced sliding motion (see Figs. 2, 3, 4, 5). Note that in all the three Scenarios, the proposed sliding mode observers are capable of tracking the unmeasured state variables with higher accuracy with respect to the well-established UI Luenberger observers, particularly during transients. This is in accordance to Edwards and Tan (2006).

5. Conclusions

In this paper, a novel decentralized sliding mode observers scheme has been designed to estimate and track the unmeasured states of power networks comprising thermal and hydraulic power plants linked via power transmission lines. The flexibility of the proposed scheme to topological changes affecting the network has also been discussed. Moreover, the simulation performances in the discussed scenarios have validated the effectiveness of our proposal. Possible future works may involve the design of decentralized observers-based sliding mode control algorithms relying on the detailed mathematical models of the plants considered in this paper.

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