University of Groningen

Direct Control Design via Controller Unfalsification

Battistelli, Giorgio; Mari, Daniele; Selvi, Daniela; Tesi, Pietro

Published in:
International Journal of Robust and Nonlinear Control

DOI:
[10.1002/rnc.3778](https://doi.org/10.1002/rnc.3778)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2018

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Battistelli, G., Mari, D., Selvi, D., & Tesi, P. (2018). Direct Control Design via Controller Unfalsification. *International Journal of Robust and Nonlinear Control*, 28(12), 3694-3712. <https://doi.org/10.1002/rnc.3778>

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Direct Control Design via Controller Unfalsification

Giorgio Battistelli^{1,*}, Daniele Mari², Daniela Selvi¹ and Pietro Tesi³

¹*Dipartimento di Ingegneria dell'Informazione (DINFO), Università di Firenze, 50139 Firenze, Italy*

²*Positech Consulting S.r.l., Milan 20122, Italy*

³*Institute for Technology and Management, Faculty of Science and Engineering, University of Groningen, 9747 AG Groningen, The Netherlands*

SUMMARY

This paper proposes a *non-iterative direct* approach for controller design from experimental data; the parameters of a controller of a prescribed order and structure are optimized with respect to a relevant performance criterion. The proposed design method enjoys the following features: (i) It does not involve the identification of the process to be controlled; (ii) it only requires a single experiment; (iii) in the case of stable plants, no initial controller is needed even when the process to be controlled is non-minimum phase; and (iv) it provides sufficient conditions for the resulting closed-loop system to be stable. The approach builds upon the so-called unfalsified control theory; this key point makes it possible to derive simple and intuitive relations between the choice of the performance criterion to be optimized and closed-loop stability conditions. The analysis is supported by numerical examples. Copyright © 2017 John Wiley & Sons, Ltd.

Received 30 October 2015; Revised 28 November 2016; Accepted 24 January 2017

KEY WORDS: control design; data-driven optimization; unfalsified control

1. INTRODUCTION

One of the topics of major interest in control design is the unavoidable trade-off between the complexity of the model of the plant to be controlled and the efforts needed to obtain a reliable mathematical description of it. In general, finding models from physical laws (first principles) that are simple, yet reliable for control design, can indeed be difficult and time-consuming. For this reason, design techniques based on experimental data have been studied and much attention is still devoted to improvements of existing solutions and the development of new ones. These methods can be essentially divided in two main classes, that is, *indirect* (often also referred to as *model-based*) and *direct* approaches (often also referred to as *model-free*).

Indirect approaches employ the collected data for the identification of a model of the process to be controlled, which is then used to synthesize a controller according to some prescribed design objectives. Therefore, this class of techniques heavily rely on *system identification* tools [1–3]. In contrast, direct approaches do not aim at deriving an accurate model of the process to be controlled. In direct approaches, the identification step is limited to basic plant properties (such as static gain, time constant, and delay) typically not sufficient for control design purposes. These properties are used to infer achievable closed-loop maps, usually expressed in terms of sensitivity functions, and to select an adequate parametric controller family in terms of both structure and order. A controller is then obtained by minimizing, over the controller parameter space, a suitable cost function that reflects the discrepancy between actual and desired closed-loop maps.

*Correspondence to: Giorgio Battistelli, Dipartimento di Ingegneria dell'Informazione (DINFO), Università di Firenze, Via S. Marta 3, 50139 Firenze, Italy.

†E-mail: giorgio.battistelli@unifi.it

Direct approaches have been extensively studied in the last years, and they can be classified into *iterative* [4–7] and *non-iterative* [8–12] approaches. While the former fine-tune the controller parameters by performing multiple experiments on the process, the latter are *one-shot* in the sense that a single batch of experimental data can be used for the controller optimization. In this respect, non-iterative solutions are extremely appealing from a practical viewpoint. Moreover, at least for linearly parametrized controllers, they provide a global solution via simple least-squares techniques [8, 9].

One of the main reasons for considering direct approaches (either iterative and non-iterative) is to overcome the so-called model-mismatch problem, that is, the problem of closed-loop instability arising from plant unmodeled dynamics (see [13, 14] for an extensive discussion on this topic). Yet ascertain stability in direct approaches is far from straightforward. In fact, because direct schemes do not make explicit use of a plant model, mechanisms to validate/falsify a controller before its actual implementation should be devised. A number of such mechanisms have been proposed in the literature, using either open-loop data or closed-loop data [10, 12, 15, 16].

In this paper, we propose a non-iterative data-driven control design approach that incorporates stability tests and makes use of concepts and tools originally introduced by [13, 17] within the so-called *unfalsified control* framework. In unfalsified control, the concept of *fictitious reference* [13, 17, 18] makes it possible to identify those control laws that are consistent with performance objectives and past experimental data (see also [19–24] for an application of this concept in the context of adaptive switching control). More recently [25–27], it has been shown that the fictitious reference can be used also as a convenient tool to define suitable cost functions in terms of discrepancy between desired and potential input/output behavior, which allow one to evaluate the potential performance of a given controller and exhibit simple and intuitive relations with closed-loop stability conditions. In the present work, we show that, when applied to the direct control design problem, these ideas make it possible to derive a data-driven controller tuning procedure incorporating simple stability tests.

Most of the analysis is carried out for the case of stable plants and open-loop data. Unstable plants can be dealt with by resorting to a simple cascade control architecture. For the case of stable plants, no initial stabilizing controller is required even when the plant to be controlled is non-minimum phase and the plant unstable zeros are unknown. This represents an important feature of the proposed approach because, in this case, other stability-oriented approaches require either that the unstable zeros are perfectly known [10] or that an initial stabilizing controller is available [12].

The paper is organized as follows. Section 2 describes the framework under consideration. In Section 3, we introduce and analyze the basic controller tuning procedure, and provide a preliminary stability analysis. The detailed controller tuning procedure incorporating stability tests is presented in Section 4. In Section 5, numerical examples are discussed to support the analysis. Section 6 provides concluding remarks.

Notation. We denote by $\mathbb{Z}_+ := \{0, 1, 2, \dots\}$ the set of nonnegative integers. Given a vector v , we denote by $\|v\|$ its Euclidean norm. Given a positive integer N and a scalar-valued sequence $\{x(t) : t = 0, 1, \dots, N\}$, we define $\|x\|^2 := \sum_{t=0}^{N-1} x(t)^2$. Given a linear time-invariant (LTI) dynamic system with input u and output y , by the notation $y(t) = W(d)u(t)$, we mean the response of the system to u under zero initial conditions, that is,

$$\sum_{k=0}^{n_G} g_k y(t-k) = \sum_{k=0}^{n_H} h_k u(t-k)$$

$$y(k) = 0, \quad u(k) = 0, \quad k = -1, -2, \dots$$

where $W(d) = H(d)/G(d)$ is the system input/output operator in the unit backward shift operator d , where $H(d) = \sum_{k=0}^{n_H} h_k d^k$ and $G(d) = \sum_{k=0}^{n_G} g_k d^k$ are polynomials. Because we will always consider systems under zero initial conditions, $W(d)$ will be referred to as transfer function.

Throughout the paper, transfer functions are meant after pole-zero cancelations, that is, if $H(d) = \overline{H}(d)\psi(d)$ and $G(d) = \overline{G}(d)\psi(d)$ for some polynomial $\psi(d)$, then $W(d) = \overline{H}(d)/\overline{G}(d)$.

Given a transfer function $W(d)$, we denote by $\|W(d)\|_\infty$ its H_∞ norm.

2. PROBLEM FORMULATION

Consider a discrete-time plant described by the input-output causal behavior

$$y(t) = \mathcal{P}(u, t) + n(t), \quad (1)$$

where $t \in \mathbb{Z}_+$, $y(t), u(t) \in \mathbb{R}$ are respectively the output and the input of the plant at time t , while $n(t) \in \mathbb{R}$ is a bounded disturbance acting on the output. The plant $\mathcal{P}$ is supposed to be unknown but stable,[‡] and the only available information is provided by a finite data set $\mathcal{D} = \{u(t), y(t) : t = 0, 1, \dots, N\}$ collected by means of a single open-loop experiment on the plant. While the methodology described in the following sections is well-suited for being applied also in a more general setting, the theoretical analysis will hinge on the assumption that the plant is LTI and described by the transfer function

$$P(d) = \frac{B(d)}{A(d)} \quad (2)$$

where $A(d) = 1 + a_1 d + \dots + a_{n_a} d^{n_a}$ and $B(d) = b_0 + b_1 d + \dots + b_{n_b} d^{n_b}$ are polynomials in the unit backward shift operator d . Both $A(d)$ and $B(d)$ are unknown, but $A(d)$ is supposed to be strictly Schur.

We consider a family of candidate LTI controllers $\mathcal{C} := \{C(d, \theta), \theta \in \Theta \subset \mathbb{R}^{n_\theta}\}$, where the transfer function $C(d, \theta)$ of the generic controller depends on a vector of design parameters θ , that is,

$$C(d, \theta) = \frac{S(d, \theta)}{R(d, \theta)}, \quad (3)$$

where $R(d, \theta)$ and $S(d, \theta)$ are polynomials in d whose coefficients are affine functions of θ , with $R(0, \theta) = 1$ for any θ . The compact set Θ denotes the region characterizing the controller model structure.

The problem that we address is that of tuning the controller parameter vector θ on the basis of the available data set $\mathcal{D}$ in order to achieve stabilization as well as satisfactory performance. In particular, the control specifications are defined in terms of two reference models $\mathcal{Q}$ and $\mathcal{W}$ reflecting the desired *input and complementary sensitivities*, that is, the behaviors of the plant input and, respectively, output in response to any bounded reference signal. The reference models $\mathcal{Q}$ and $\mathcal{W}$ are supposed to be described by the transfer functions

$$\mathcal{Q}(d) = \frac{N_Q(d)}{D_Q(d)}, \quad \mathcal{W}(d) = \frac{N_W(d)}{D_W(d)}, \quad (4)$$

which have to be *a priori* chosen by the designer. Both $\mathcal{Q}(d)$ and $\mathcal{W}(d)$ must be stable; in addition, $\mathcal{W}(d)$ must be strictly proper.

The reason for considering two reference models is that $\mathcal{W}(d)$ accounts for output performance, while $\mathcal{Q}(d)$ accounts for both input performance and stability requirements. In fact, as will be clarified in the following sections, making the input sensitivity suitably close to the reference model $\mathcal{Q}(d)$ ensures internal stability of the feedback loop, while in general $\mathcal{W}(d)$ is not able to account for the stability requirement. This makes the design problem inherently multi-objective as, in general, it is not possible to minimize simultaneously the two discrepancies

$$\mathcal{J}(\theta) = D \left(\frac{C(\theta, d)}{1 + P(d)C(d, \theta)}, \mathcal{Q}(d) \right), \quad \mathcal{V}(\theta) = D \left(\frac{C(\theta, d)P(d)}{1 + P(d)C(d, \theta)}, \mathcal{W}(d) \right)$$

[‡]The extension to unstable plants will be discussed later on in Remark 7.

where $D(\cdot, \cdot)$ stands for any measure of distance between the behaviors. Hence, we look for a *Pareto optimal solution*, that is, a solution that cannot be improved in one of the objectives without degrading the other one. Specifically, we pursue this goal by following a *linear scalarization* strategy that amounts to considering a single objective of the form

$$(1 - \delta)\mathcal{J}(\theta) + \delta\mathcal{V}(\theta) \quad (5)$$

with $\delta \in [0, 1]$. Clearly, for different values of δ , different Pareto optimal solutions are obtained and the choice of δ is critical for achieving a desirable tradeoff between the desired behaviors in terms of input/output response as well for ensuring closed-loop stability.

Summing up, the control objective consists of tuning the vector θ in order to (i) minimize the total discrepancy (5) and (ii) guarantee internal stability of the closed loop system. In the following, such a problem will be addressed by following a *data-driven paradigm* and exploiting concepts of unfalsified control theory.

Notice that an alternative approach to achieve Pareto optimal solutions would be the so-called *ε -constraint scalarization* that amounts to minimizing one of the two objectives while ensuring that the other does not exceed a given threshold ε . This approach would lead either to a complementary sensitivity matching problem with stability constraint or to an input sensitivity matching problem with output performance constraint.

Before entering into the details of the proposed approach, it is important to point out that, for the design problem to be well defined, the two transfer functions $Q(d)$ and $W(d)$ have to be consistent with the controller family $\mathcal{C}$. Further, the adopted choices should reflect as much as possible the control objectives. For instance, in most situations, one may want to keep a part of the controller fixed in order to fulfill some rejection/tracking objectives. This can be accomplished by considering a parametric controller family of the form

$$C(d, \theta) = \frac{\bar{S}(d, \theta)S^*(d)}{\bar{R}(d, \theta)R^*(d)}, \quad (6)$$

where $R^*(d)$ and $S^*(d)$ are fixed coprime polynomials with all unstable roots, $R^*(0) = 1$. A typical choice is to let $R^*(d) = 1 - d$ in order to equip the controller with an integral action. Then, the parameter vector θ will simply consist of all the coefficients of the polynomials $\bar{R}(d, \theta) = 1 + r_1d + \dots + r_{n_r}d^{n_r}$ and $\bar{S}(d, \theta) = s_0 + s_1d + \dots + s_{n_s}d^{n_s}$, that is, $\theta = [s_0 \ s_1 \ \dots \ s_{n_s} \ r_1 \ \dots \ r_{n_r}]'$. As well known, such a choice imposes some constraints on the achievable sensitivity functions. Accordingly, it is supposed that the controller family and the reference models are chosen so as to satisfy the following matching conditions.

Design Condition 1

The control sensitivity function $Q(d)$ can be factorized as $Q(d) = S^*(d)\bar{Q}(d)$ with $\bar{Q}(d)$ stable and minimum phase. Further, all the roots of $S^*(d)$ are zeros of $W(d)$, while all the roots of $R^*(d)$ are zeros of $1 - W(d)$ (here, it is understood that also the multiplicity is preserved).

Remark 1

As a matter of fact, some prior knowledge about the process is invariably needed, as in any direct approach, in order to select reference models $W(d)$ and $Q(d)$ corresponding to closed-loop behaviors that are close to what can be actually achieved. In this respect, guidelines on the choice of the reference models will be provided in the following (see Remark 6 as well as the examples of Section 5). However the required knowledge, which may be the result of physical considerations about the process to be controlled and/or experimental data, is limited to very basic features of the plant such as static gain and bandwidth. Hence, while such a coarse knowledge could in principle be used to derive a very simple (first/second order) process model, a controller designed only on the basis of such information would work only in very limited cases (i.e., when the process dynamics are indeed very simple).

3. OPTIMIZATION CRITERION

According to the assumption of considering stable plants, the data set $\mathcal{D}$ can be recorded by means of an open-loop experiment, where the process is directly excited by a suitable input u and the corresponding output y can be measured. In this respect, the following assumption is considered.

Assumption 1

The input u is a given stationary signal independent of the additive output disturbance n .

Let us now recall the concept of *fictitious reference*, as introduced in [17]. Given a controller $C(d, \theta)$ and a data set $\mathcal{D}$, the fictitious reference r_θ is the signal obtained by inverting the controller dynamics as follows:

$$r_\theta(t) = C(d, \theta)^{-1} u(t) + y(t), \quad t = 0, 1, \dots, N. \quad (7)$$

In other words, r_θ represents the fictitious reference that, if injected into the feedback system made up of the interconnection of the controller $C(d, \theta)$ with the unknown plant $\mathcal{P}$, would reproduce the data set $\mathcal{D}$. The fictitious reference r_θ turns out to be a particularly convenient tool for assessing the potential performance of a given candidate controller $C(d, \theta)$. In particular, the definition of r_θ as in (7) makes it possible to associate to each design vector θ a *fictitious closed-loop experiment*, wherein the plant is connected in feedback with the controller $C(d, \theta)$ and the resulting closed loop is driven by the reference signal r_θ .

In particular, by virtue of (7), we have

$$\begin{aligned} u(t) &= \frac{C(d, \theta)}{1 + P(d)C(d, \theta)} r_\theta(t) - \frac{C(d, \theta)}{1 + P(d)C(d, \theta)} n(t) \\ y(t) &= \frac{P(d)C(d, \theta)}{1 + P(d)C(d, \theta)} r_\theta(t) + \frac{1}{1 + P(d)C(d, \theta)} n(t) \end{aligned}$$

so that the mapping from r_θ to (u, y) provides a measure of the performance achievable by $C(d, \theta)$. Hence, a natural optimization criterion consists in selecting θ so as to minimize (in some suitable sense) the discrepancy between (u, y) and the signals

$$u_\theta^\circ(t) := \mathcal{Q}(d) r_\theta(t), \quad (8)$$

$$y_\theta^\circ(t) := \mathcal{W}(d) r_\theta(t), \quad (9)$$

with $t = 0, 1, \dots, N$, as schematically represented in Figure 1. This can be achieved by defining the following cost functions

$$\begin{aligned} J_N(\theta) &:= \| (u - u_\theta^\circ) |^N \|^2 \\ V_N(\theta) &:= \| (y - y_\theta^\circ) |^N \|^2. \end{aligned}$$

Figure 1. Discrepancy between (u, y) and $(u_\theta^\circ, y_\theta^\circ)$.

In fact, $J_N(\theta)$ and $V_N(\theta)$ provide a natural way for taking into account specifications on the control and, respectively, complementary sensitivity functions. Hence, a possible design criterion consistent with (5) amounts to finding the parameter vector that minimizes the weighted sum $(1 - \delta)J_N(\theta) + \delta V_N(\theta)$ with $\delta \in [0, 1]$.

Notice now that the input/output discrepancies can be written as

$$u(t) - u_\theta^\circ(t) = u(t) - Q(d) y(t) - C^{-1}(d, \theta) Q(d) u(t), \quad (10)$$

$$y(t) - y_\theta^\circ(t) = [1 - W(d)] y(t) - C^{-1}(d, \theta) W(d) u(t). \quad (11)$$

Hence, in practice, in order to evaluate the cost functions $J_N(\theta)$ and $V_N(\theta)$ for a given θ , it is not necessary to directly compute the fictitious reference $r_\theta(t)$. Moreover, in view of the Design Condition 1, it can be seen that the computation of the input/output discrepancies (10)–(11) is numerically stable whenever

$$C^{-1}(d, \theta) Q(d) = \overline{Q}(d) \frac{\overline{R}(d, \theta) R^*(d)}{\overline{S}(d, \theta)}$$

is stable. In fact, by Design Condition 1: (i) $\overline{Q}(d)$ is stable and minimum phase, and (ii) $W(d)$ can be factorized as $S^*(d) \overline{W}(d)$ with $\overline{W}(d)$ stable. Accordingly, $C^{-1}(d, \theta) W(d) = \overline{W}(d) \frac{\overline{R}(d, \theta) R^*(d)}{\overline{S}(d, \theta)}$ and stability of $C^{-1}(d, \theta) Q(d)$ also implies stability of $C^{-1}(d, \theta) W(d)$.

Let us denote by $\Theta_S \subseteq \Theta$ the set of all the parameter vectors θ such that $C^{-1}(d, \theta) Q(d)$ is stable. Then, the following design criterion can be considered

$$\theta_N^*(\delta) = \arg \min_{\theta \in \Theta_S} \{ (1 - \delta) J_N(\theta) + \delta V_N(\theta) \}. \quad (12)$$

We point out that imposing the stability of $C^{-1}(d, \theta) Q(d)$ is consistent with the control objectives represented by the sensitivity functions Q and W . In fact, as well known, a necessary condition for internal stability of a closed loop is that all the unstable zeros of the controller are also zeros of Q and W . Notice also that, as far as computation of the signals $u(t) - u_\theta^\circ(t)$ and $y(t) - y_\theta^\circ(t)$ is concerned, (10) and (11) take the same form of a predictor in an output error (OE) method [1, 2] (even if the problem of identifying the inverse controller $C^{-1}(d, \theta)$ is not an OE one because the best matching error in both (10) and (11) is not a white noise). Then, the gradients and Hessians of the two cost functions can be computed in a standard way by suitably filtering the collected data $\mathcal{D}$. Further, it can be seen that the problem of ensuring the numerical stability of the computation of $u(t) - u_\theta^\circ(t)$ and $y(t) - y_\theta^\circ(t)$ is analogous to the problem of ensuring the stability of the predictor in the OE method. In this respect, we note that, when the experiment is sufficiently informative, $u - u_\theta^\circ$ and $y - y_\theta^\circ$ (i.e., the data) is almost surely unbounded as N goes to infinity for any value of θ corresponding to an unstable $C^{-1}(d, \theta) Q(d)$ (e.g., see lemma 1 in [28]). In contrast, $u - u_\theta^\circ$ and $y - y_\theta^\circ$ always remain bounded when $C^{-1}(d, \theta) Q(d)$ is stable. Hence, when the number of collected data N is sufficiently large, $J_N(\theta)$ increases rapidly as θ approaches the boundary of Θ_S and the constraint on the stability of $C^{-1}(d, \theta) Q(d)$ can be easily obeyed during optimization (see also an analogous discussion in [1], section 10.2).

Remark 2

The idea of evaluating the potential performance of a candidate controller by making use of suitable fictitious (or virtual) signals has been investigated also outside the unfalsified control framework (see for instance the *Virtual Reference Feedback Tuning* (VRFT) approach of [8]). However, there are some important differences. For instance, in VRFT, the virtual reference is obtained in terms of the reference model $W(d)$, while here the fictitious reference r_θ is obtained by inverting the candidate controller $C(d, \theta)$.

In this respect, our approach is more similar to the VRFT with inverse controller identification of [10] that makes use of a concept of virtual reference borrowed from the unfalsified control literature.

Nevertheless, the two approaches are quite different because in [10] only the output sensitivity $W(d)$ is taken into account and used to define a test functional that approximates an H_2 model-reference criterion, while here the test functional is defined in terms of discrepancy between the potential and the desired performance in response to such a r_θ , in terms of both input and output behavior. As a result, the two test functionals are different even when $\delta = 1$ (i.e., only the output behavior is considered), because they reflect different goals.

This alternative choice is motivated by the fact that, as discussed by [25, 26] in the context of supervisory switching control, this kind of test functionals can exhibit simple and intuitive relations with closed-loop stability conditions. This latter feature will be also exploited in the following sections, in order to develop a data-driven tuning algorithm that accounts also for the closed-loop stability requirement. In contrast with [10], our approach turns out to be effective also in dealing with non-minimum phase plants with unknown unstable zeros.

Remark 3

The disturbance $n(t)$ enters the test functionals as an additive term that is independent of θ , in that

$$\begin{aligned} u(t) - u_\theta^\circ(t) &= [1 - Q(d)P(d)]u(t) - C^{-1}(d, \theta)Q(d)u(t) - Q(d)n(t) \\ y(t) - y_\theta^\circ(t) &= [1 - W(d)]P(d)u(t) - W(d)C^{-1}(d, \theta)u(t) + [1 - W(d)]n(t). \end{aligned}$$

Hence, under Assumption 1, when the number N of available data tends to infinity, the solution of (12) tends to the noise-free one.

3.1. Stability considerations

The presence of the cost function $J_N(\theta)$ associated to the input sensitivity reference model $\mathcal{Q}$ is crucial for taking into account the stability requirement in the proposed scheme. In fact, in case only $V_N(\theta)$ were present, internal stability would not be guaranteed even in the most favorable conditions (i.e., infinite-length experiment and $W(d)$ exactly achievable). This follows from the straightforward consideration that the controller exactly achieving $W(d)$ internally stabilizes the plant if and only if all the unstable zeros of $P(d)$ are also zeros of $W(d)$, with at least the same multiplicity. Because the plant is supposed to be unknown, an arbitrary choice of $W(d)$ will fulfill such a condition only under particular circumstances (e.g., when the plant numerator $B(d)$ is strictly Schur).

On the other hand, because the plant is stable, the controller

$$C_Q(d) = \frac{Q(d)}{1 - Q(d)P(d)} \quad (13)$$

exactly achieving $Q(d)$ always guarantees internal stability of the feedback system. In fact, the main motivation for taking into account the input discrepancy $u(t) - u_\theta^\circ(t)$ and the related criterion is that they exhibit simple and intuitive relations with closed loop stability conditions, which, as will be shown in the next section, hold even in the non-ideal case when the controller $C_Q(d)$ (exactly achieving $Q(d)$) does not belong to the controller family $\mathcal{C}$.

As a consequence, when $\delta = 0$, criterion (12) is consistent with the stability requirement. In most settings, however, it is not possible to express all the control objectives only in terms of the input sensitivity $\mathcal{Q}$ because also the complementary sensitivity $\mathcal{W}$ has to be taken into account. Nevertheless, by continuity arguments, it is possible to show that, when the parameter vector $\theta_N^*(0)$ corresponds to a stabilizing controller, then stability is preserved also in a neighborhood of $\delta = 0$. In particular, the following result holds.

Theorem 1

Let $\theta_N^*(0)$ be an isolated global minimum of $J_N(\theta)$ on $\bar{\Theta}_S$ (the closure of Θ_S) and let the corresponding controller $C(d, \theta_N^*(0))$ be stabilizing. Then, there exists $\bar{\delta} > 0$, such that for any $\delta \leq \bar{\delta}$ the controller $C(d, \theta_N^*(\delta))$ is stabilizing.

Proof

Notice first that the objective function in (12) is a smooth function of θ and δ . Consider now the modified criterion

$$\theta_N^*(\delta) = \arg \min_{\theta \in \Theta_S} \{(1 - \delta)J_N(\theta) + \delta V_N(\theta)\}. \quad (14)$$

with Θ_S replaced by its closure $\overline{\Theta}_S$. Because the set $\overline{\Theta}_S$ is compact, we can invoke the *Maximum Theorem* and conclude that, for any δ , the set of solutions of (14) is upper hemi-continuous[§] in δ . Thus, by definition, for any $\rho > 0$ there exists $\bar{\delta}(\rho)$ such that, for any $\delta \leq \bar{\delta}(\rho)$, all the solutions of (14) lie in the set $\{\theta \in \Theta_S : \|\theta - \theta_N^J\| \leq \rho\}$. Because the closed-loop characteristic polynomial has fixed order and depends affinely on the parameter vector θ , it follows that its roots will also depend continuously on θ . This, in turn, implies that there exists $\bar{\rho} > 0$ such that all controllers in the set $\{\theta \in \Theta_S : \|\theta - \theta_N^J\| \leq \bar{\rho}\}$ are stabilizing and hence belong to Θ_S . Then, the proof can be concluded by taking $\bar{\delta} = \bar{\delta}(\bar{\rho})$. $\square$

The importance of Theorem 1 is that it suggests a possible strategy for taking into account the stability requirement within the proposed data-driven controller tuning procedure. Specifically, the idea is that of addressing, in a first phase, the minimization of $J_N(\theta)$ by letting $\delta = 0$ and verify that it leads indeed to a stabilizing controller. Then, in a second phase, one can take into account also the contribution of the output discrepancy, in terms of the cost function $V_N(\theta)$, for example, by finding the largest value of $\delta \in [0, 1]$ for which stability is preserved. In fact, when the first phase succeeds, Theorem 1 ensures that such a δ can always be found by starting from $\delta = 1$ (or any other desired value) and then gradually decreasing δ until the stability requirement is met. On the other hand, the failure of the first phase provides useful insights because it indicates that either the experiment is not informative enough or the controller family is not rich enough (see also the discussion after Algorithm 1 subsequently). Of course, in order to apply the ideas discussed previously, it is necessary to develop a data-driven test able to infer whether a given candidate controller is stabilizing or not without inserting it in feedback with the plant. This issue will be addressed in the next section.

4. STABILITY TEST AND TUNING PROCEDURE

Let us start by discussing the relationship between the input discrepancy $u(t) - u_\theta^\circ(t)$ and closed loop stability. In this respect, the following result holds.

Proposition 1

The input discrepancy $u(t) - u_\theta^\circ(t)$ can be written as

$$u(t) - u_\theta^\circ(t) = \Delta_Q(d, \theta) Q(d) u(t) - Q(d) n(t) \quad (15)$$

where

$$\Delta_Q(d, \theta) = C_Q^{-1}(d) - C^{-1}(d, \theta). \quad (16)$$

Proof

Recall that $y(t) = P(d)u(t) + n(t)$. Equation (10) then implies

$$\begin{aligned} u(t) - u_\theta^\circ(t) &= [1 - Q(d)P(d)]u(t) \\ &\quad - C^{-1}(d, \theta)Q(d)u(t) - Q(d)n(t), \end{aligned}$$

which is equivalent to (15) because $1 - Q(d)P(d) = C_Q^{-1}(d)Q(d)$. $\square$

[§]Notice that the mapping from θ to the set of solutions of (14) is, in general, multi-valued. In this respect, we recall that a multi-valued mapping $f : \mathcal{X} \rightarrow \mathcal{Y}$ is said to be upper hemi-continuous at the point $\bar{x} \in \mathcal{X}$ if for any open neighborhood $\mathcal{V}$ of $f(\bar{x})$ there exists a neighborhood $\mathcal{U}$ of $\bar{x}$ such that for all x in $\mathcal{U}$, $f(x)$ is a subset of $\mathcal{V}$.

Proposition 1 is the starting point to build a stability test consistent within the considered framework. In fact, the next result follows from standard small gain arguments.

Theorem 2

Let $\theta \in \Theta_S$. Then, if

$$\|Q(d) \Delta_Q(d, \theta)\|_\infty < 1, \quad (17)$$

the controller $C(d, \theta)$ internally stabilizes the unknown plant $\mathcal{P}$.

Proof

Consider first the case in which both $C(d, \theta)$ and $C_Q(d)$ do not have unstable zeros. In this case, $\Delta_Q(d, \theta)$ is stable and $C(d, \theta)$ can be represented in terms of $C_Q(d)$ and of the stable perturbation $\Delta_Q(d, \theta)$ as depicted in the block scheme of Figure 2. It is straightforward to see that, as far as stability is concerned, such a scheme is equivalent to the one of Figure 3. Accordingly, by standard small gain stability arguments, it follows that $\|Q(d) \Delta_Q(d, \theta)\|_\infty < 1$ is a sufficient condition for the internal stability of the feedback system.

Suppose now that $C(d, \theta)$ and $C_Q(d)$ have some unstable zeros. By Design Condition 1, one can factorize $Q(d)$ as $Q(d) = S^*(d)\bar{Q}(d)$ where $\bar{Q}(d)$ is stable and minimum phase. Moreover, one can factorize the two controllers as $C(d, \theta) = S^*(d)\bar{C}(d, \theta)$ and $C_Q(d) = S^*(d)\bar{C}_Q(d)$, where

$$\bar{C}_Q(d) = \frac{\bar{Q}(d)}{1 - S^*(d)\bar{Q}(d)P(d)}, \quad \bar{C}(d, \theta) = \frac{\bar{S}(d, \theta)}{\bar{R}(d, \theta)R^*(d)},$$

Accordingly, we can write

$$\begin{aligned} Q(d)\Delta_Q(d, \theta) &= S^*(d)\bar{Q}(d)(\bar{C}_Q^{-1}(d) - \bar{C}^{-1}(d, \theta)) \\ &= \bar{Q}(d)(\bar{C}^{-1}(d) - \bar{C}^{-1}(d, \theta)) \end{aligned}$$

Because $P(d)$ is stable by hypothesis and $\bar{Q}(d)$ is minimum phase, it follows that $\bar{C}_Q^{-1}(d)$ is stable. As for $\bar{C}^{-1}(d, \theta)$, recall that Θ_S is the set of parameters for which $C^{-1}(d, \theta)Q(d) = \bar{C}^{-1}(d, \theta)\bar{Q}(d)$ is stable. Because $\bar{Q}(d)$ is minimum phase, it follows that $\bar{C}^{-1}(d, \theta)$ is

Figure 2. Controller perturbation.

Figure 3. Equivalent feedback interconnection.

stable. By exploiting such definitions, we can obtain an equivalent representation of the feedback loop associated with $C(d, \theta)$, which is similar to that of Figure 2, with $C_Q(d)$ replaced by $\bar{C}_Q(d)$, $P(d)$ replaced by $P(d)S^*(d)$, and $\Delta_Q(d, \theta)$ replaced by the stable perturbation $\bar{\Delta}_Q(d, \theta) = \bar{C}_Q^{-1}(d) - \bar{C}^{-1}(d, \theta)$. Hence, the proof can be concluded by means of the same small-gain arguments as before. $\square$

Notice that the H_∞ norm in (17) can be estimated from data by means of the previously defined input discrepancy $u - u_\theta^\circ$. To see this, suppose that a noise-free infinite-length data set $\mathcal{D}$ is available and let $\hat{u}(\omega)$ and $\hat{u}_\theta^\circ(\omega)$ denote the discrete Fourier transforms of $u(t)$ and $u_\theta^\circ(t)$, respectively. Suppose also that $|\hat{u}(\omega)| > 0$ for any $\omega \in [-\pi, \pi]$. Then, by virtue of (15), we have

$$\|Q(d) \Delta_Q(d, \theta)\|_\infty = \sup_{\omega \in [-\pi, \pi]} \frac{|\hat{u}(\omega) - \hat{u}_\theta^\circ(\omega)|}{|\hat{u}(\omega)|}. \quad (18)$$

In practice, for a given θ , the norm $\|Q(d) \Delta_Q(d, \theta)\|_\infty$ can be estimated from the data set $\mathcal{D}$ by computing the input discrepancy $u - u_\theta^\circ$ and then applying standard non-parametric identification techniques [1] such as the windowed Empirical Transfer Function Estimate (EFTE).

Summing up, all the foregoing considerations lead to the following tuning algorithm.

Algorithm 1 Data-driven controller tuning with stability test

Given the data set $\mathcal{D}$, the reference models Q and W , the controller family $\mathcal{C}$, and two positive scalars α, δ :

1. determine $\theta_N^*(0)$ by solving (12) with $\delta = 0$;
 2. compute the input discrepancy $u - u_{\theta_N^*(0)}^\circ$ and estimate $\|Q(d) \Delta_Q(d, \theta_N^*(0))\|_\infty$;
 3. if the estimate obtained at step 2 is less than $1 - \alpha$ then go to step 4, otherwise terminate;
 4. determine $\theta_N^*(\delta)$ by solving (12) using $\theta_N^*(0)$ as starting point for the optimization algorithm;
 5. compute the input discrepancy $u - u_{\theta_N^*(\delta)}^\circ$ and estimate $\|Q(d) \Delta_Q(d, \theta_N^*(\delta))\|_\infty$;
 6. if the estimate obtained at step 2 is less than $1 - \alpha$ then return $\theta_N^*(\delta)$ and terminate, otherwise decrease δ and go back to step 4.
-

The scalar α in steps 3 and 6 is needed in order to account for the error in the estimation of $\|Q(d) \Delta_Q(d, \theta)\|_\infty$ because of the finite data length and the presence of the disturbance n . As previously discussed, when the experiment is sufficiently informative, the failure of the test in step 3 may indicate that the controller family is not rich enough. In this case, one can re-run the algorithm by considering an augmented controller family, for example, by increasing the order of $\bar{R}(d, \theta)$ or $\bar{S}(d, \theta)$. In fact, by augmenting the controller family, one can make the optimal cost $J_N(\theta_N^*(0))$ (and hence the discrepancy $Q(d) \Delta_Q(d, \theta_N^*(0))$) suitably small so that the stability test is eventually passed.

Remark 4

The idea of using the data set $\mathcal{D}$ in order to assess whether a given candidate controller is stabilizing or not has already been exploited in the literature [12]. However, the stability test we propose here hinges upon different small-gain arguments. In fact, the condition in [12] is expressed in terms of the discrepancy between the controller $C(d, \theta)$ and a stabilizing controller $C_s(d)$, while our condition is expressed in terms of the discrepancy between the inverse controller $C^{-1}(d, \theta)$ and the inverse of the stabilizing controller $C_Q^{-1}(d)$. In [12], when the plant is stable and minimum-phase, the stabilizing controller $C_s(d)$ need not be known, as it can be inferred implicitly from the data and from the reference model $W(d)$. However, in the case of unstable or stable but non-minimum-phase plants, $C_s(d)$ is the controller used for collecting the data in a closed-loop experiment. Hence, the stability condition in [12] constrains the optimized controller to be close to the one used in the data collection experiment. This means that for stable but non-minimum-phase plants, the approach of [12] provides a refinement of an already available controller. On the contrary, in our approach, the controller $C_Q(d)$ is always implicitly defined from the data and from the reference model $Q(d)$.

Figure 4. Cascade control architecture.

For instance, this means that when the plant is stable but non-minimum phase, we do not need an initial controller to be refined.

Remark 5

A possible variant to Theorem 2 (and Algorithm 1) consists in replacing condition (17) with the requirement that the Nyquist plot of $Q(d) \Delta_Q(d, \theta)$ does not encircle the point $(-1, 0)$ in the complex plane. This variant has the advantage of being less conservative than (17). On the other hand, (17) does only require to look at the peak rather than to the whole frequency response.

Remark 6

The satisfiability of the stability condition in step 3 is crucial for the proposed tuning algorithm. In turn, this property depends on the choice of the reference model $Q(d)$, which should be defined by taking into account the available information on the plant, as discussed in Remark 1. However, the needed information is typically quite basic. For instance, supposing that an integral action is inserted in the controller by letting the fixed part of the denominator be $R^*(d) = 1 - d$, the only information that is critical for the success of the design algorithm concerns the plant static gain. In fact, any controller $C(d, \theta)$ equipped with an integrator is such that $C^{-1}(1, \theta) = 0$ so that, for $d = 1$, the small gain condition $|Q(1)(C_Q^{-1}(1) - C^{-1}(1, \theta))| < 1$ becomes $|1 - P(1)Q(1)| < 1$. Then, if we choose $Q(d)$ such that $Q(1) = 1/\hat{P}(1)$, with $\hat{P}(1)$ the available estimate of the plant static gain, we see that the condition for the satisfiability of the small-gain stability constraint is simply

$$|\hat{P}(1) - P(1)| < |\hat{P}(1)|,$$

which requires only a rough estimate of the plant static gain. Hence, when $Q(d)$ is sensibly chosen, the small gain condition in step 3 can be satisfied even when the set $\mathcal{C}$ does not contain the controller $C_Q(d)$ exactly achieving $Q(d)$.

Remark 7

The application of the proposed tuning algorithm to unstable plants is straightforward by resorting to a cascade control architecture (Figure 4). In this case, it is unavoidable to assume that the data set is collected by means of a closed-loop experiment by putting the plant in feedback with a given stabilizing controller $C_0(d)$, which however need not be performing. Then, one can treat the stable feedback interconnection of the plant $P(d)$ and the starting controller $C_0(d)$ as a new plant $P_0(d)$ with input $u_0(t)$ and output $y(t)$ and, accordingly, apply Algorithm 1 to the data set $\mathcal{D} = \{u_0(t), y(t) : t = 0, 1, \dots, N\}$. The only difference with respect to the stable case is that, in the cascade architecture of Figure 4, we assign the transfer function from $r(t)$ to $u_0(t) = u(t) + C_0(d)y(t)$. An example of application of Algorithm 1 to an unstable plant is provided in Section 5.

5. SIMULATION EXAMPLES

As previously pointed out, for minimum-phase stable plants, many effective techniques for direct control design are available in the literature [8, 10, 12]. On the other hand, the presence of unstable zeros poses more challenges, in that existing techniques either cannot provide any stability guarantee [10] or can only be used to optimize in a neighborhood of a stabilizing controller used for data

collection [12]. The aim of this section is to test the proposed algorithm in this latter, more challenging setting. To this end, simulation examples are proposed, which take into account different case studies, namely, a non-minimum phase plant, a non-minimum phase plant equipped with an integrator, and the flexible transmission system proposed by [29]. In each case, it is shown how the proposed algorithm can be applied straightforwardly to obtain a performing controller ensuring also closed-loop stability.

5.1. Example 1: Non-minimum phase plant

Consider the problem of controlling an unknown system with continuous-time transfer function given by

$$P(s) = \frac{s - 0.5}{s^2 + 2s + 1}, \quad (19)$$

where s denotes the Laplace operator. Assume to run an open-loop experiment of length 1000 s and to collect the input and output signals with sampling time $T_s = 0.1$ s, thus accordingly the length of the collected data is $N = 10^4$. The plant input u is a white noise with zero mean and standard deviation $\sigma_u = 1$, while the output y is affected by a disturbance n consisting of a white noise with zero mean and standard deviation $\sigma_n = 0.01$.

The control requirements are expressed by the reference models

$$Q(s) = \frac{1}{\hat{P}(0)} \frac{s^2 + 0.05s + 1}{s^2 + 2\xi s + 1}, \quad W(s) = \frac{1}{s^2 + 2\xi s + 1}, \quad (20)$$

where $\xi = 0.55$. Notice that the denominator is the same in both the reference models. Further, notice that $Q(s)$ and $W(s)$ cannot be simultaneously achieved because $W(s)/Q(s) \neq P(s)$ (of course this is consistent with the fact that the plant is unknown). The quantity $\hat{P}(0)$ is an estimate of the plant static gain (for the specific case, $\hat{P}(0) = -0.5034$) retrieved from the data set and used in $Q(s)$ so as to ensure that $Q(0)P(0) \approx W(0)$. Further, the numerator of $Q(s)$ is chosen by accounting for the additional specification that the response time of the control signal is similar to the one of the desired output (see black lines in Figure 5).

The reference models are then discretized by means of the step response invariance method with sampling time T_s .

Figure 5. Example 1. Step response obtained by solving problem (12) with $\delta = 0$ for $n_s = n_r = 2$ as compared with the response of the reference models: (up) output behavior; (down) input behavior.

Because the reference model $W(d)$ has an unitary static gain, it is reasonable to consider controllers of the form

$$C(d, \theta) = \frac{1}{1-d} \bar{C}(d, \theta), \quad (21)$$

with a fixed part equal to a discrete-time integrator and a variable part $\bar{C}(d, \theta) = \bar{S}(d, \theta)/\bar{R}(d, \theta)$ that is free to be designed. In particular, $\bar{C}(d, \theta)$ is parametrized by letting $n_s = n_r = 2$.

We first note that, if $\delta = 1$, criterion (12) is unable to provide a resulting controller with guaranteed stability irrespective of the complexity of the controller family. This is an inherent limitation because the zeros of $W(d)$ do not include the unstable zeros of the discrete-time transfer function of the plant. On the other hand, a stabilizing controller can be found by considering only the contribution given by the input sensitivity, that is, by solving the problem in (12) with $\delta = 0$. In fact, in this case, a controller $C(d, \theta_N^*(0))$ is obtained, which satisfies the stability test, because the corresponding estimate of $\|Q(d) \Delta_Q(d, \theta_N^*(0))\|_\infty$, obtained by smoothing its ETFE with a 19-point moving average, turns out to be equal to 0.8754. Indeed $C(d, \theta_N^*(0))$ stabilizes the plant and the spectral radius of the closed loop system is equal to 0.9827.

Figure 5 shows the unit step responses of the output and input plant signals for the resulting feedback system as compared with those of the reference models. It can be seen that the output settling time is about 13 s (considering a range of $\pm 10\%$ around the steady state value). Because it is known from the stability test that controller $C(d, \theta_N^*(0))$ is stabilizing, we resort to (12) with $\delta \neq 0$ in order to improve the step response performance.

The choice of the weight δ can significantly affect performance, as it can be seen from Figures 6 and 7. In fact, when δ is too large, the resulting controller is not stabilizing, while small values of δ lead to behaviors similar to the case $\delta = 0$. Indeed, for $\delta = 1/11$, the step response is almost identical to the behavior exhibited in the case $\delta = 0$. A satisfactory behavior is obtained for $\delta = 1/2$. In fact, the resulting controller is able to achieve a settling time of about 7 s.

In Figure 7, the conservativeness of the proposed stability test is analyzed by showing the behaviors of the estimate of $\|Q(d) \Delta_Q(d, \theta_N^*)\|_\infty$ as well as of the closed-loop spectral radius as a function of δ . It can be seen that, as expected, $\|Q(d) \Delta_Q(d, \theta_N^*)\|_\infty$ is an increasing function of δ and the maximum value for which the stability test is passed is about $\delta = 0.75$. On the other hand, the controllers resulting from the solution of problem (12) are stabilizing for any $\delta \leq 0.9$. Hence, there is an unavoidable gap between the truly achievable values of δ and the values of δ that are deemed suitable according to the stability test.

Figure 6. Example 1. Step responses obtained by solving problem (12) with $n_s = n_r = 2$ for different values of δ . [Colour figure can be viewed at wileyonlinelibrary.com]

Figure 7. Example 1. Behaviors of (left) estimate of $\|Q(d) \Delta_Q(d, \theta_N^*)\|_\infty$ and (right) closed-loop spectral radius as a function of δ , obtained by solving problem (12) with $n_s = n_r = 2$. [Colour figure can be viewed at wileyonlinelibrary.com]

5.2. Example 2: Non-minimum phase plant equipped with integrator

Consider now the problem of controlling an unknown system with continuous-time transfer function given by

$$P(s) = \frac{s - 0.5}{s(s^2 + 2s + 1)}. \quad (22)$$

The process has a pole in $s = 0$. In order to collect the data, we adopt a simple proportional controller $C_0(d) = K$, with $K = -0.35$, which ensures closed-loop stability but does not achieve satisfactory performance (recall Remark 7). We thus refer to the feedback interconnection of $P(s)$ and $C_0(s)$ as a new plant $P_0(s)$ to be controlled, having input $u_0(t)$ and output $y(t)$. The experiment is carried out over a time interval of 1000 s and the sampling time is set to $T_s = 0.1$ s (thus $N = 10^4$). The plant input u_0 is a white noise with zero mean and standard deviation $\sigma_{u_0} = 1$, while the output y is affected by a disturbance n consisting of a white noise with zero mean and standard deviation $\sigma_n = 0.01$. Then, after collecting the data set $\mathcal{D} = \{u_0(t), y(t) : t = 0, 1, \dots, N\}$, we apply Algorithm 1 by expressing the control requirements in terms of the reference models

$$Q(s) = \frac{1}{\hat{P}_0(0)} \frac{1}{\omega_n^2} \frac{s^2 + 0.5s + 1}{s^2 + \frac{2\xi}{\omega_n}s + \frac{1}{\omega_n^2}}, \quad W(s) = \frac{1}{\omega_n^2} \frac{1}{s^2 + \frac{2\xi}{\omega_n}s + \frac{1}{\omega_n^2}}, \quad (23)$$

where $\xi = 0.4$ and $\omega_n = 4.5$. As in the previous example, we denote by $\hat{P}_0(0)$ an estimate of the plant static gain retrieved from the data; specifically in this case, $\hat{P}_0(0) = 0.98$ (quite close to the real value $P_0(0) = 1$). The reference models are discretized by means of the *zero-order hold* method with sampling time T_s . The controller form is selected as in (21), that is, $S^*(d) = 1$ and $R^*(d) = 1 - d$, thus the fixed part is again a discrete-time integrator. With respect to the polynomials $\bar{S}(d, \theta)$ and $\bar{R}(d, \theta)$, their order is set to $n_s = 4$ and $n_r = 2$, respectively. As in the previous example, we underline that $Q(s)$ and $W(s)$ cannot be simultaneously achieved (because $W(s)/Q(s) \neq P(s)$); further, the problem in (12) with $\delta = 1$ cannot provide a controller with guaranteed stability, because $W(d)$ does not include the non-minimum phase zeros of the discrete-time transfer function of the plant.

The first test is performed by solving criterion (12) with $\delta = 0$; Figure 8 shows the unit step responses of the output and input plant signals for the resulting feedback system as compared with those of the reference models. Notice that, in this case, the response for the input signal obtained for $\delta = 0$ is very close to the response of the reference model. Then, as shown in Algorithm 1, we account for the case $\delta \neq 0$. The step responses of the closed-loop system obtained with $\delta = 0$,

Figure 8. Example 2. Step response obtained by solving problem (12) with $\delta = 0$ for $n_s = 4$ and $n_r = 2$ as compared with the response of the reference models: (up) output behavior; (down) input behavior.

Figure 9. Example 2. Step responses obtained by solving problem (12) with $n_s = 4$ and $n_r = 2$ for different values of δ .

$\delta = 3/13$ and $\delta = 1/2$, respectively, are depicted in Figure 9. As for the conservativeness of the stability test, the maximum value for which the stability test is passed is about $\delta = 0.55$, while the controllers resulting from (12) are stabilizing for $\delta \leq 0.9$.

5.3. Example 3: The flexible transmission system

As an additional illustrative example, we consider the flexible transmission system proposed by [29] as a benchmark for robust digital control, and typically considered in the context of model-free control design [8, 11, 12, 30].

In accordance with [29], the discrete-time linear model (obtained with sampling frequency $f_s = 20 \text{ Hz}$) is given by

$$P(d) = d^q \frac{B(d)}{A(d)}, \quad q = 2 \quad (24)$$

$$A(d) = 1 - 1.41833 d + 1.58939 d^2 - 1.31608 d^3 + 0.88642 d^4 \quad (25)$$

$$B(d) = 0.28261 d + 0.50666 d^2 \quad (26)$$

The system is stable but non-minimum phase and has delay $q = 2$. The exact model (24)–(26) is assumed to be unknown. The control objective is to design a feedback controller that achieves internal stability and offset-free tracking for step inputs.

We perform an open loop experiment of 500 s, where the input and output signals are collected with sampling frequency $f_s = 20 \text{ Hz}$, thus $N = 10^4$. The input u is a white noise with zero mean and standard deviation $\sigma_u = 1$, while the output y is affected by a white noise n with zero mean and standard deviation $\sigma_n = 0.01$.

In connection with the choice of the reference models $W(d)$ and $Q(d)$, the following information on the plant is extracted from the data set $\mathcal{D}$:

- (a) Estimated delay $\hat{q} = 2$;
- (b) Estimated static gain $\hat{P}(1) = 1.05$;
- (c) ETFE.

The estimated delay coincides with the true one, while the estimated static gain is close to the true value $P(1) = 1.0646$. As detailed next, the ETFE is not used for deriving a parametric plant model. Instead, it is just used for determining the frequency intervals where the plant gain is higher.

In accordance with (a) and the requirement of set-point tracking, the complementary sensitivity function $W(d)$ is chosen as

$$W(d) = d^q \frac{N_W(d)}{D_W(d)}, \quad q = 2 \quad (27)$$

$$D_W(d) = (1 - a d)^2, \quad N_W(d) = (1 - a)^2 d \quad (28)$$

The dominant pole is located at $a = e^{-\omega_b/f_s}$, where $\omega_b = 10$ is the desired closed-loop bandwidth, and the gain is selected so as to satisfy $W(1) = 1$ (see also Figure 10).

The controller family is again chosen as in (21) with $n_r = 1$ and $n_s = 4$. Because of the choice of $W(d)$ and the fact that the plant is non-minimum phase, the optimization problem (12) with

Figure 10. Example 3. (Up) Rough ETFE magnitude of $P(d)$ (black) and magnitude Bode plot of $W(d)$ (grey); (down) noisy step response of $P(d)$ (black) and noise-free step response of $W(d)$ (grey).

$\delta = 1$ is unable to achieve closed-loop stability. Hence, in order to obtain a stabilizing controller, Algorithm 1 has to be applied with $\delta < 1$.

To this end, we exploit the information available on the plant so as to make the two reference models $Q(d)$ and $W(d)$ consistent with each other. Of course, ideally, one should let $W(d) = P(d)Q(d)$, but this is not possible because $P(d)$ is unknown. However, we can let such relationship approximately hold by choosing $Q(d)$ so that $Q(1) = W(1)/\hat{P}(1) = 1/\hat{P}(1)$ and $Q(e^{-j\omega})\hat{P}(e^{-j\omega}) \ll 1$ when ω exceeds the desired bandwidth ω_b (here $\hat{P}(e^{-j\omega})$ denotes the ETFE of $P(e^{-j\omega})$). Accordingly, we let

$$Q(d) = \frac{1}{\hat{P}(1)} \frac{(1-\beta)^4}{(1-\alpha)^8} \frac{(1-\alpha d)^8}{(1-\beta d)^4}, \quad (29)$$

with $\alpha = e^{-20/f_s}$ e $\beta = e^{-4/f_s}$.

The solution to problem (12) with $\delta = 0$ provides a controller $C(d, \theta_N^*(0))$ that satisfies the stability test, because the estimate of $\|Q(d)\Delta_Q(d, \theta_N^*(0))\|_\infty$, obtained by smoothing its ETFE with a 19-point moving average, is equal to 0.3495. Indeed $C(d, \theta_N^*(0))$ is stabilizing and the spectral radius of the closed loop system is equal to 0.9818.

Then, a test is performed by putting $C(d, \theta_N^*(0))$ in feedback with the plant; the reference signal is a unit step and the plant output y is affected by a white noise n with zero mean and $\sigma_n = 0.01$. Figure 11 compares the measured output and input plant signals with the model-free step responses of the reference models $W(d)$ and $Q(d)$, respectively.

Because it is known from the stability test that controller $C(d, \theta_N^*(0))$ is stabilizing, one can resort to (12) with $\delta > 0$ in order to improve the step response performance. As in the previous examples, we notice that the choice of the weight δ can significantly affect the performance. In fact, when δ is too large, it is possible that the resulting controller does not stabilize the plant, while small values of δ lead to behaviors similar to the case $\delta = 0$. We notice that a satisfactory behavior is obtained for $\delta = 1/6$. In fact, the resulting controller improves the output performance, as shown in Figure 12. Further, the obtained controller satisfies the stability test, being the smoothed estimate of $\|Q(d)\Delta_Q(d, \theta_N^*)\|_\infty$ equal to 0.8961, while the corresponding closed-loop spectral radius is 0.9825. As for the conservativeness of the stability test, the maximum value for which the stability test is passed is about $\delta = 0.20$, while the controllers resulting from (12) are stabilizing for $\delta \leq 0.45$.

Figure 11. Example 3. Step response obtained by solving problem (12) with $\delta = 0$ for $n_s = 4$ and $n_r = 1$ as compared with the response of the reference models: (up) output behavior; (down) input behavior.

Figure 12. Example 3. Step responses obtained by solving problem (12) with $n_s = 4$ and $n_r = 1$ for different values of δ .

6. CONCLUSIONS

In this paper, we propose a *non-iterative direct* approach for the design of controllers from experimental data with asymptotic stability guarantees. The approach is inspired by the *unfalsified control* theory, which allows one to derive simple and intuitive relations between the cost functions to optimize and closed-loop stability conditions. Thanks to the apparent link between cost functions and closed-loop stability, a controller tuning procedure incorporating simple stability tests is derived. Simulation results show the effectiveness of the proposed approach when applied to stable non-minimum phase plants, and provides guidelines to address also the case of unstable open-loop plants. Future investigations will include the analysis of tuning procedures where the H_∞ -norm constraint is directly incorporated into the optimization routine.

REFERENCES

1. Ljung L. *System Identification - Theory for the User* (2nd edn). Prentice Hall: Upper Saddle River, NJ, USA, 1999.
2. Söderström T, Stoica P. *System Identification*. Prentice Hall: Upper Saddle River, NJ, USA, 1988.
3. Hjalmarsson H. From experiment design to closed-loop control. *Automatica* 2005; **41**:393–438.
4. Hjalmarsson H, Gunnarson S, Gevers M. Iterative restricted complexity control design scheme. *Proceedings of the 33th IEEE Conference on Decision and Control*, Lake Buena Vista, FL, USA, 1994.
5. Hjalmarsson H, Gevers M, Gunnarson S, Lequin O. Iterative feedback tuning: theory and applications. *IEEE Control Systems Magazine* 1998; **18**:26–41.
6. Kosut RL. Uncertainty model unfalsification for robust adaptive control. *Annual Reviews in Control* 2001; **25**:65–76.
7. Karimi A, Miskovic L, Bonvin D. Iterative correlation-based controller tuning. *International Journal of Adaptive Control and Signal Processing* 2004; **18**:645–664.
8. Campi M, Lecchini A, Savaresi S. Virtual reference feedback tuning: a direct method for the design of feedback controllers. *Automatica* 2002; **38**:1337–1346.
9. Campi M, Savaresi S. Direct nonlinear control design: the virtual reference feedback tuning (VRFT) approach. *IEEE Transactions on Automatic Control* 2006; **51**:14–27.
10. Sala A, Esparza A. Extensions to virtual reference feedback tuning: a direct method for the design of feedback controllers. *Automatica* 2005; **41**:1473–1476.
11. Formentin S, Karimi A, Savaresi S. Optimal input design for direct data-driven tuning of model-reference controllers. *Automatica* 2013; **49**:1874–1882.
12. Van Heusden K, Karimi A, Bonvin D. Data-driven model reference control with asymptotically guaranteed stability. *International Journal of Adaptive Control and Signal Processing* 2011; **25**:331–351.
13. Safonov MG, Tsao TC. The unfalsified control concept and learning. *IEEE Transactions on Automatic Control* 1997; **42**:843–847.
14. Fekri S, Athans M, Pascoal AM. Robust multiple model adaptive control (RMMAC): a case study. *International Journal of Adaptive Control and Signal Processing* 2007; **21**:1–30.

15. Van Heusden K, Karimi A, Bonvin D. Data-driven estimation of the infinity norm of a dynamical system. *Proceedings of the 46th IEEE Conference on Decision and Control*, New Orleans, USA, 2007.
16. Dehghani A, Lecchini A, Lanzon A, Anderson BDO. Validating controllers for internal stability utilizing closed-loop data. *IEEE Transactions on Automatic Control* 2009; **54**:2719–2725.
17. Safonov MG. Focusing on the knowable: controller invalidation and learning. In *Control Using Logic-based Switching*, Morse AS (ed.). Berlin: Springer-Verlag, 1996; 224–233.
18. Mosca E, Agnoloni T. Inference of candidate loop performance and data filtering for switching supervisory control. *Automatica* 2001; **37**(4):527–534.
19. Jin H, Safonov MG. Unfalsified adaptive control: controller switching algorithms for nonmonotone cost functions. *International Journal of Adaptive Control and Signal Processing* 2012; **26**:692–704.
20. Wang R, Paul A, Stefanovic M, Safonov MG. Cost detectability and stability of adaptive control systems. *International Journal of Robust and Nonlinear Control* 2007; **17**:549–561.
21. Battistelli G, Mosca E, Safonov MG, Tesi P. Stability of unfalsified adaptive switching control in noisy environments. *IEEE Transactions on Automatic Control* 2010; **55**:2424–2429.
22. Jun M, Safonov MG. Controller parameter adaptation algorithm using unfalsified control theory and gradient method. *Proceedings of IFAC World Congress*, Barcelona, Spain, 2002.
23. Stefanovic M, Safonov MG. Safe adaptive switching control: stability and convergence. *IEEE Transactions on Automatic Control* 2008; **53**:2012–2021.
24. Jin H, Chang MW, Safonov MG. Unfalsifying pole locations using a fading memory cost function. *Asian Journal of Control* 2014; **16**:1583–1591.
25. Baldi S, Battistelli G, Mosca E, Tesi P. Multi-model unfalsified adaptive switching supervisory control. *Automatica* 2010; **46**:249–259.
26. Baldi S, Battistelli G, Mosca E, Tesi P. Multi-model unfalsified adaptive switching control: test functionals for stability and performance. *International Journal of Adaptive Control and Signal Processing* 2011; **25**:593–612.
27. Battistelli G, Mosca E, Tesi P. Adaptive memory in multi-model switching control of uncertain plants. *Automatica* 2014; **50**:874–882.
28. Mosca E, Agnoloni T. Closed-loop monitoring for early detection of performance losses in feedback-control systems. *Automatica* 2003; **39**:2071–2084.
29. Landau ID, Rey D, Karimi A, Voda A, Franco A. A flexible transmission system as a benchmark for robust digital control. *European Journal of Control* 1995; **1**:77–96.
30. Hjalmarsson H, Gunnarsson S, Gevers M. Model-free tuning of a robust regulator for a flexible transmission system. *European Journal of Control* 1995; **1**:148–156.