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Wei, Yanji; Barradas Berglind, Jose de Jesus; Yu, Zhiheng; van Rooij, Marijn; Prins, Wouter; Jayawardhana, Bayu; Vakis, Antonis I.

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Frequency-domain hydrodynamic modelling of dense and sparse arrays of wave energy converters

Y. Wei ^a, J.J. Barradas-Berglind ^b, Z. Yu ^a, M. van Rooij ^a, W.A. Prins ^a, B. Jayawardhana ^b, A.I. Vakis ^{a,*}

^a Advanced Production Engineering, Engineering and Technology Institute Groningen, Faculty of Science & Engineering, University of Groningen, Nijenborgh 4, Groningen 9747AG, the Netherlands

^b Discrete Technology & Production Automation, Engineering and Technology Institute Groningen, Faculty of Science & Engineering, University of Groningen, Nijenborgh 4, Groningen 9747AG, the Netherlands

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ABSTRACT

In this work, we develop a frequency-domain model to study the hydrodynamic behaviour of a floater blanket (FB), i.e., an array of floater elements individually connected to power take-off (PTO) systems, which constitutes the core technology of the novel Ocean Grazer (OG) wave energy converter (WEC). The boundary element method open-source code NEMOH is used to solve the scattering and radiation problem. The aforementioned floater elements that comprise the FB are mechanically interconnected via (cylindrical, revolutionary or spring) joints, which add extra constraint equations to the multibody problem. Various scenarios are investigated to understand the hydrodynamic response of the FB. The variation of the capture factor, PTO damping coefficients, q -factor and response amplitude operator (RAO) of each scenario are analysed, in order to quantify the device performance. A new concept based on a negative-stiffness spring joint is proposed to increase the energy output of the FB. Attention is also paid to the anti-resonance that is found in the numerical simulations. This study provides further insight into the hydrodynamic behaviour of dense or sparse interconnected arrays of WECs, which is fundamental for the design and optimisation of the OG-WEC.

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1. Introduction

The recently proposed novel wave energy converter (WEC) that constitutes the core technology of the Ocean Grazer (OG), a hybrid ocean energy collection and storage platform, has the potential to be an effective contender in the challenge of extracting the latent energy from ocean waves. The OG-WEC utilises an adaptable multi-pump multi-piston power take-off (MP²PTO) system that consists of a grid of interconnected floater elements—termed a floater blanket (FB)—, with each floater element being connected to a piston-type hydraulic pumping system—a multi-piston pump (MPP) [1]. Each MPP consists of three differently-sized engageable pistons, which allow for seven different pumping combinations.

Recent work by Wei et al. [2] has demonstrated that the adaptable MPP unit, through a simple control algorithm, can extract more energy than constant piston combinations; furthermore, the overall energy extraction of the MP²PTO system can be improved by optimising the combinations of MPPs according to the incident waves. Nevertheless, due to the fact that the possible combinations of MPPs increase dramatically as the number of floater elements increases, it is computationally costly to identify the optimal configurations of the MP²PTO system via our time-domain-based numerical model. Hence, prior to the development of a real-time model-based control of the PTO configuration of the MP²PTO system in the face of a wide variety of sea conditions, a number of issues must already be tackled. The first one, which is addressed in the present paper, is to develop a frequency-domain (FD) model to investigate the hydrodynamics of the FB in a numerically tractable fashion. The dynamics of the interconnected floater elements are mainly dominated by two physical phenomena: (i) the interaction between the multibody system and the incident wave; and, (ii) the dynamics of the multibody system itself. There has been extensive

* Corresponding author.

E-mail addresses: y.wei@rug.nl (Y. Wei), jj.barradas.b@gmail.com (J.J. Barradas-Berglind), yu.zhiheng0305@hotmail.com (Z. Yu), m.van.rooij@rug.nl (M. van Rooij), w.a.prins@rug.nl (W.A. Prins), b.jayawardhana@rug.nl (B. Jayawardhana), a.vakis@rug.nl (A.I. Vakis).

work dealing with these two problems, particularly for wave energy applications, using various numerical [3,4] and analytical models [5,6].

A series of WEC devices can be deployed into an array to create a wave farm, giving rise to a multibody and wave interaction problem. Therefore, the extracted energy of a WEC in a farm is not only determined by the incident wave, but also by the radiated/diffracted waves from neighbouring WECs. The latter may become more relevant as the distances between the WECs become smaller, which may alter their dynamic behaviour, affecting the overall extracted energy of the wave farm. For example, Garnaud and Mei [5] compared the wave power extraction of a large buoy with a compact array of small buoys. Results indicate that the latter system with many degrees of freedom had a higher efficiency for a much broader bandwidth. The impact of wave interaction effects on energy absorption in a WEC array was investigated by Borgarino et al. [7]; where it was found that the superposition of diffracted and radiated waves in the array may be either constructive or destructive to the energy absorption. Sammarco et al. [6] gave a mathematical solution for a gate farm WEC with an infinitely long channel; it was found that employing a larger number of much smaller devices in trapped resonant modes had a beneficial effect on the system performance. Furthermore, Sinha et al. [8] investigated the performance of heaving point absorber arrays under various spatial arrangements. Their results indicate that optimising the PTO damping and mass of the individual floaters can lead to more energy extraction.

For a single device, there have been several WEC concepts consisting of arrays of joined hydrodynamic elements, which take advantage of their relative motion to extract energy, e.g., the Hagen-Cockereil WEC [9], the Pelamis WEC [10] and the hinge-barge WEC [3,4]. Accordingly, to solve the dynamic response of the joined elements, additional constraint equations need to be introduced into the problem, e.g., to satisfy the continuity at the joints. In Newman [11]; an approximate analysis of a flexible or hinged raft was provided, which could be applied to estimate the performance of any elongated floating element. In Haren and Mei [12]; a hydrodynamic model to investigate the wave extraction by a train of rafts was proposed. In addition, Haren and Mei [9] employed the previous approach to investigate the performance of a parallel array of rectangular pontoons in a channel. A similar concept is the Pelamis WEC, which was the first offshore device to provide electricity to the grid in 2004, and consists of a series of semi-submerged cylindrical sections linked by hinged joints. This concept has been extensively modelled using various numerical techniques [10], but there are few open reports available. Recently, researchers proposed a joint-coordinate approach in the time-domain, aiming to represent the fully non-linear joints of the hinge-barge WEC [4]; this approach was applied to study a three-hinged-barge system.

Although the FB of the OG is conceptually similar to the WEC arrays of a wave farm or single WECs with joined floating components, it has some unique features. It is essentially different from the WEC array, since for such arrangements there are no physical connections between the WECs and, in addition, the distance between the WECs is generally large with respect to their dimensions and wavelength. The FB concept is also different from the multi-element single WECs, whose elements are significantly fewer in comparison. Moreover, the FB is projected to be mounted on the OG platform through hundreds of cables. Thus, the coupling dynamics of the FB and platform may affect the overall performance of the device, requiring careful investigation.

The main purpose of the present paper is to investigate the hydrodynamic behaviour of a FB located at the open sea via numerical simulations. For this purpose, a frequency domain model is

developed, based on the boundary element method (BEM), which is described in Section 2. A series of numerical simulations, which include various single floater and floater array WECs, is undertaken, in order to ascertain the dynamic characteristics of the FB. The performance of the FB is evaluated by the capture factor of the WECs and the body-to-body interaction in floater arrays is quantified by the q -factor; these are presented in Section 3.1. In Section 3.2, the excitation force and response amplitude operator (RAO) of the floater elements in a FB are analysed. The effects of various joints on the performance of the FBs are investigated in Section 4, with special attention given to the negative-stiffness spring joint. The interesting finding from the simulations –that anti-resonance may occur at specific frequencies which lead to decreased performance for the FB– is analysed in Section 5. Lastly, conclusions and further research points are given in Section 6.

2. Numerical modelling

A linear frequency-domain numerical model of the FB is developed to evaluate its hydrodynamic interactions and energy extraction capabilities. In this model, the hydrodynamic coefficients are calculated by using NEMOH, an open-source BEM code based on linear potential flow theory, which enables the solution of the scattering and radiation problem for an array of floating bodies [13]. The dynamics of the floater elements are handled by solving a set of motion equations corresponding to the multibody system, together with additional constraint equations introduced by the coupling with various joints. The assumptions of the present model are summarised as follows: the fluid is an irrotational, inviscid, incompressible ideal fluid and potential theory is applicable; the wave amplitude is small such that it satisfies linear wave theory; and lastly, the displacement amplitude is small such that the small-angle approximation can be adopted for the constraint equations of the FB.

2.1. Scenarios and considerations

In the OG concept, a FB consists of hundreds of interconnected floater elements and is mounted on a massive platform, which introduces a great number of degrees of freedom (DOFs) into the system. Unfortunately, there is not yet any existing mature device that can guide the design of such a complex system. Hence, it is decided to firstly perform the preliminary investigation on a single column of simple box-shaped floaters, aiming to establish the basic understanding of the hydrodynamics of continuous floating bodies in waves. Subsequently, a comprehensive model of a multi-column FB with various joints is developed. In addition, the floater arrays with one DOF in heave are included in the comparison. Following this idea, seven scenarios are proposed for the present investigation –labeled as *case1* to *case7*, and with three variants of *case5*– as listed in Table 1 and sketched in Fig. 1. For the sake of clarity, in the present paper we refer to the compact floater array as the FB, in order to distinguish it from the common sparse floater array (FA).

Although some of the scenarios under consideration in Table 1 are clearly not optimal designs for harvesting energy and will not be applied in practice, they serve as a baseline for comparison and exemplify situations that are relevant in answering the following questions of concern:

- What is the hydrodynamic behaviour of a FB compared to the single floaters or other FAs?
- Is the idea of using joined floater elements to form a FB beneficial for extracting wave energy from ocean waves?
- How do the joint constraints of the FB influence the overall performance of the device?

Table 1
Configurations of various scenarios.

Cases	MC	MR	$L \times W \times H$	D_c	DOFs	J&C
case1	1	1	$7 \times 7 \times 2 \text{ m}^3$	7	[0 0 1 0 0 0]	C
case2	1	1	$70 \times 70 \times 2 \text{ m}^3$	70	[0 0 1 0 0 0]	C
case3	1	10	$7 \times 7 \times 2 \text{ m}^3$	7	[1 0 1 0 1 0]	R
case4	1	10	$7 \times 70 \times 2 \text{ m}^3$	70	[1 0 1 0 1 0]	R
case5a	10	10	$7 \times 7 \times 2 \text{ m}^3$	70	[1 1 1 1 1 1]	S
case5b	10	10	$7 \times 7 \times 2 \text{ m}^3$	70	[1 0 1 0 1 0]	R
case5c	10	10	$7 \times 7 \times 2 \text{ m}^3$	70	[0 0 1 0 0 0]	C
case6	5	10	$7 \times 7 \times 2 \text{ m}^3$	70	[1 0 1 0 1 0]	R
case7	5	5	$7 \times 7 \times 2 \text{ m}^3$	70	[0 0 1 0 0 0]	C

MC: column number of a FA/FB.

MR: row number of a FA/FB.

$L \times W \times H$: dimensions of the floater element (length $\times$ width $\times$ height).

D_c : characteristic dimension of the FA/FB.

DOFs: degrees of freedom in surge, sway, heave, roll, pitch and yaw, respectively; 1 = free, 0 = restrained.

J&C: type of joint and constraint between the floaters; C = cylindrical joint, which allows translation along a single standard axis, i.e., heaving motion; R = revolute joint, which allows rotation about a single standard axis; and S = spring joint, which allows translation and rotation about any 3-D axis, but the linked elements have an internal spring force.

Fig. 1. Sketch of the floater (array) for various scenarios.

These scenarios are briefly described below:

- *case1* corresponds to a single small box-shaped floater with a single DOF in heave, which can be considered as a heave point absorber.
- *case2* is similar to *case1*, but its dimensions (length & width) are ten times those of *case1*. Accordingly, it cannot be considered as a point absorber anymore because its dimensions are not small with respect to the wavelength of the incident wave.
- *case3* describes a column array of floaters consisting of ten individual floaters. These floaters are interconnected with hinges that allow three DOFs: surge, heave and pitch.
- *case4* is also a hinged column array of floaters like *case3*, however the width of its floaters is ten times that of *case3*.
- *case5* corresponds to a compact FB with 10×10 elements, numbering a total of 100 floaters. These floaters are connected to their neighbours with joints. The number of DOFs in a FB is determined by the type of joints. In order to understand the constraint effects of a FB, three variant sub-scenarios with different joints are discussed:

- *case5a* is a general case in which the floater elements are connected into a FB with spring joints between both rows and columns. Note that this is a free-floating FB, meaning that it is not anchored.
- *case5b* uses the revolute joints within individual columns, but removes the connection between the columns. In this way, the sway, roll and yaw motions of the floaters are restricted, but each floater has dependent surge, heave and pitch motions about the hinges, in what is essentially a FB consisting of multiple columns. We remark that connecting floaters with a revolute joint along both rows and columns will result in over-constraining their motion, hence we do not explore this option.
- *case5c* restricts five DOFs of the floater elements, thus only allowing the floaters to move in the heave direction. This can be considered as an array of heave absorbers.
- *case6* removes five floater columns from *case5*, each column array being the same as that of *case3*. In this case, we remove the alternating columns, which results in spaces being left between each column array that is not joined to the neighbouring columns.
- *case7* further reduces the number of the floaters in rows, resulting in a 5×5 sparse FA. Furthermore, all floaters are constrained such that they only have one DOF in heave.

The standard floater element we use for this investigation is a rectangular cuboid with dimensions of $7 \times 7 \times 2 \text{ m}^3$. The properties of the non-standard floater, e.g., the mass of the floaters used in *case2* and *case4*, are proportional to their volumes. The floaters are considered as lightweight bodies with an internal ballast. Hence, the initial mass of the floater used here is much smaller than that of other floating WECs. However, it is worth noting that, since the operating principle of the OG-WEC is to pump working fluid from a lower to a higher pressure state, forces resulting from heavy added mass may be exerted on the floaters due to the significantly large water column mass moved within the hydraulic system. In addition, the ballast function allows the mass of the floater itself to be adjusted by pumping water into or out of the floater. Although the draft will vary according to the mass of the floaters, there is existing technology to apply a supplementary mass to the system without changing the draft [14]. It is also possible in the OG-WEC to adjust the inertia of the system while keeping the draft constant: this can be achieved by changing the length of the horizontal piping in its pumping system. Hence, we do not investigate the draft effects in the present paper but assume that the inertia of the system can vary while the draft is constant, so that the hydrodynamic coefficients will be the same in all the scenarios. The hydrodynamic coefficients are calculated when floaters are submerged by half their height. Furthermore, the effects of directionality are not considered in this paper, because in the original OG concept, the FB would be mounted on a huge platform that could be rotated towards the dominant incident wave direction; in the latest OG concept, currently under investigation, the floater blanket would be symmetric and, thereby, omnidirectional. In the present model, the FB rows are defined perpendicular to the wave propagation direction and columns along it. The relevant parameters of the model are summarised in Table 2.

The joint stiffness will be further discussed in Section 2.3 where we present the constraint equations.

In the FA/FB, a uniform linear PTO system is connected to the floater elements. Since the OG-WEC can conceptually adapt the pumping system to deal with various sea states, we vary the damping configuration to obtain the maximum energy output in frequency; this is described in Section 2.4. As we aim to develop the OG-WEC to efficiently extract energy for a wide range of wave

Table 2
Parameters of the numerical model.

Description	Value
Floater dimensions [m ³]	7 × 7 × 2
Floater mass [kg]	50225
Draft [m]	1
Water depth [m]	600
Water density ρ [kg/m ³]	1025
Joint stiffness K_s [N/m]	1 × 10 ⁶

periods, the hydrodynamic responses of the FB—and, more precisely, the scenarios in Table 1—corresponding to waves with periods between 4 s and 20 s (≈ 0.2 to 1.6 rad/s in the frequency domain) are considered in the present paper.

2.2. Motion equations

Consider the motion equations for a FB comprised of M floater elements, where we use the index m to describe the m^{th} element, such that

$$\mathbf{M}^{(m)} \ddot{\mathbf{X}}^{(m)} = \mathbf{F}_e^{(m)} + \mathbf{F}_r^{(m)} + \mathbf{F}_{hs}^{(m)} + \mathbf{F}_{pto}^{(m)} + \mathbf{F}_j^{(m)}, \quad (1)$$

where $\mathbf{M}^{(m)} \in \mathbb{R}^{6 \times 6}$ is a diagonal matrix of masses and moments of inertia for the general case of a floater with 6 DOFs. We denote $\mathbf{X}^{(m)} = [x_1^{(m)}, \dots, x_6^{(m)}]^T$ as the displacement vector of the m^{th} floater, whose elements represent the surge, sway, heave, roll, pitch and yaw displacements, respectively. Furthermore, $\mathbf{F}_e^{(m)} \in \mathbb{R}^6$ is the wave excitation force vector; $\mathbf{F}_r^{(m)} \in \mathbb{R}^6$ is the radiation force vector composed of the radiation forces due to the motion of the floater m itself as well as other floater elements; $\mathbf{F}_{hs}^{(m)} \in \mathbb{R}^6$ is the restoring force vector, which arises when the floater is perturbed away from its equilibrium; $\mathbf{F}_{pto}^{(m)} \in \mathbb{R}^6$ is the PTO force vector; and $\mathbf{F}_j^{(m)} \in \mathbb{R}^6$ is the composite of the internal force vector of the joints on the floater m .

The radiation force $\mathbf{F}_r^{(m)}$ includes added-mass and wave damping terms, which can be written as

$$\mathbf{F}_r^{(m)} = - \sum_{n=1}^M (\mathbf{A}^{(m,n)} \ddot{\mathbf{X}}^{(n)} + \mathbf{B}^{(m,n)} \dot{\mathbf{X}}^{(n)}) \quad (2)$$

where $\mathbf{A}^{(m,n)}, \mathbf{B}^{(m,n)} \in \mathbb{R}^{6 \times 6}$ are the frequency-dependent added-mass and damping coefficient matrices, respectively. Note that the superscripts m and n denote the radiation force of floater m as a result of the motion of floater n .

In addition, $\mathbf{F}_{hs}^{(m)}$ is a linear function of the heave, pitch and roll perturbations, which is expressed in terms of the floater's motion about the centre of gravity as

$$\mathbf{F}_{hs}^{(m)} = -\mathbf{K}_{hs}^{(m)} \mathbf{X}^{(m)} \quad (3)$$

where $\mathbf{K}_{hs} \in \mathbb{R}^{6 \times 6}$ corresponds to the hydrostatic stiffness matrix that specifies the variation of the net weight and buoyancy load with respect to the changes in position from equilibrium.

A simple PTO is considered in the present paper, whose mechanism is represented as a linear spring-damper system. Since the FB extracts energy from the wave heave, we assume that the PTO force in the heave direction is linearly proportional to the heave motion of the floaters, namely that

$$\mathbf{F}_{pto,3}^{(m)} = -K_{pto}^{(m)} x_3^{(m)} - C_{pto}^{(m)} \dot{x}_3^{(m)} \quad (4)$$

where $F_{pto,3}^{(m)}$ is the third entry of $\mathbf{F}_{pto}^{(m)}$, corresponding to heave motion; $K_{pto}^{(m)}$ is the stiffness of the m^{th} PTO; and, $C_{pto}^{(m)}$ is the damping of the m^{th} PTO. Consequently, we consider all other elements of $\mathbf{F}_{pto}^{(m)}$ to be zero. We acknowledge that the PTO forces exerted by the MP²PTO system are highly non-linear and cannot be explicitly described by a linear PTO. This non-linearity stems from the engaging and disengaging of piston combinations, which allows the OG-WEC to adapt to different sea states; details of the MP²PTO system can be found in Vakis and Anagnostopoulos [1]. The inclusion of a non-linear PTO force in a FD model is not straightforward [15], but a higher order model could be developed in order to incorporate directly the pumping force of the MP²PTO system in the future.

Substituting the force vectors in Eqs. (2)–(4) into Eq. (1) and assuming that the forces and displacements are harmonic with a frequency ω , we obtain the motion equation in terms of complex amplitudes, which is

$$\begin{aligned} & \left[-\omega^2 (\mathbf{M}^{(m)} + \mathbf{A}^{(m,m)}) + i\omega (\mathbf{B}^{(m,m)} + \mathbf{C}_{pto}^{(m)}) + (\mathbf{K}_{hs}^{(m)} + \mathbf{K}_{pto}^{(m)}) \right] \\ & \times \hat{\xi}^{(m)} + \sum_{n=1, n \neq m}^M \left(-\omega^2 \mathbf{A}^{(m,n)} + i\omega \mathbf{B}^{(m,n)} \right) \hat{\xi}^{(n)} - \hat{\mathbf{F}}_j^{(m)} \\ & = \hat{\mathbf{F}}_e^{(m)} \end{aligned} \quad (5)$$

where $\hat{\mathbf{F}}_e$, $\hat{\mathbf{F}}_j$ and $\hat{\xi} \in \mathbb{C}^6$ are the complex amplitude vectors of the excitation force, joint force and floater displacement, respectively; $\mathbf{F}_e = \Re\{\hat{\mathbf{F}}_e e^{i\omega t}\}$, $\mathbf{F}_j = \Re\{\hat{\mathbf{F}}_j e^{i\omega t}\}$ and $\mathbf{X} = \Re\{\hat{\xi} e^{i\omega t}\}$ represent their real part. On the left-hand side of Eq. (5), the first term includes the general free floating element forces with PTO, the second term represents the body-to-body interaction via radiation, and the third term incorporates the joint constraints which are described in the following section.

2.3. Constraint equations

The floater elements are considered as rigid bodies, which are connected to their neighbours by means of various joints to restrict their 6-DOF motion, as described in Table 1—in the DOFs column. When applying different joints to the system, additional equations that describe the connectivity at the joints and the internal force of the joints must be introduced into the system equations, i.e., by means of the joint force $\hat{\mathbf{F}}_j$ in Eq. (5).

When a floater is restrained by a cylindrical joint, it has only 1-DOF in heave. The displacement of the floater and the force of the joint can be described as:

$$\hat{\xi}_i^{(m)}|_{i \neq 3} = 0 \quad \text{and} \quad \hat{\mathbf{F}}_{j,3}^{(m)} = 0. \quad (6)$$

For a floater column that is hinged with revolute joints, the displacements of sway, roll and yaw are set to zero. A sketch showing the convention of the displacements and forces on the floater column is depicted in Fig. 2 (A).

The local coordinate of each floater $O^{(m)}$ is located at its centre of gravity. We number the right and left joints of the m^{th} floater as joint m and joint $m+1$, respectively. Based on the assumption of the small displacement amplitude, the displacements at the joints satisfy the following:

(A) Displacements and forces in a floater column (side view)

(B) Floater blanket with spring joints (top view)

Fig. 2. Sketch of (A) a floater column with joints and (B) a FB with joints. The dashed-line floaters represent the original position of the respective floaters on the still water surface.

$$\begin{aligned}
 \hat{\xi}_1^{(m)} - \hat{\xi}_1^{(m+1)} &= 0, \\
 \hat{\xi}_2^{(m)} - \hat{\xi}_2^{(m+1)} &= 0, \\
 \hat{\xi}_3^{(m)} - \frac{\hat{\xi}_5^{(m)} L^{(m)}}{2} - \hat{\xi}_3^{(m+1)} - \frac{\hat{\xi}_5^{(m+1)} L^{(m+1)}}{2} &= 0, \\
 \hat{\xi}_4^{(m)} - \hat{\xi}_4^{(m+1)} &= 0 \quad \text{and} \\
 \hat{\xi}_6^{(m)} - \hat{\xi}_6^{(m+1)} &= 0.
 \end{aligned} \quad (7)$$

Consequently, the composition of the joint forces on the m^{th} floater can be conveniently expressed in matrix form as:

$$\hat{\mathbf{F}}_{j,1}^{(m)} = K_s \begin{bmatrix} -\hat{F}_{jc,1}^{(m)} + \hat{F}_{jc,1}^{(m+1)} \\ -\hat{F}_{jc,2}^{(m)} + \hat{F}_{jc,2}^{(m+1)} \\ -\hat{F}_{jc,3}^{(m)} + \hat{F}_{jc,3}^{(m+1)} \\ -\hat{F}_{jc,4}^{(m)} + \hat{F}_{jc,4}^{(m+1)} \\ \frac{\hat{F}_{jc,3}^{(m)} L^{(m)}}{2} - \frac{\hat{F}_{jc,3}^{(m+1)} L^{(m+1)}}{2} \\ -\hat{F}_{jc,6}^{(m)} + \hat{F}_{jc,6}^{(m+1)} \end{bmatrix}, \quad (8)$$

where $\hat{\mathbf{F}}_{jc}^{(m)} \in \mathbb{C}^6$ is the complex vector force of the joint m in a

column, and $\hat{F}_{jc,5}^{(m)} = 0$.

The spring joints are used to assemble the multiple-row-and-column floater elements into a FB (case5a), as shown in Fig. 2(B). For the floater m (associated with 1-DOF, i.e., surge/sway), the joint forces on the floater are equal to the sum of the force from all springs attached to it, which are expressed as:

$$\begin{aligned}
 \hat{F}_{j,1}^{(m)} &= K_x^m \left(\hat{\xi}_1^{(m-1)} - \hat{\xi}_1^{(m)} \right) + K_x^{m+1} \left(\hat{\xi}_1^{(m)} - \hat{\xi}_1^{(m+1)} \right) \quad \text{and} \\
 \hat{F}_{j,2}^{(m)} &= K_y^m \left(\hat{\xi}_2^{(m-MR)} - \hat{\xi}_2^{(m)} \right) + K_y^{m+1} \left(\hat{\xi}_2^{(m)} - \hat{\xi}_2^{(m+MR)} \right)
 \end{aligned} \quad (9)$$

where K_x^m and K_y^m are the stiffness of the spring along the rows and columns; it should be noted that uniform stiffness is assumed in a FB, $K_x^{(m)} = K_y^{(m)} = K_s$ (see Table 2). Finally, $\hat{F}_{j,1}^{(m)}$ and $\hat{F}_{j,2}^{(m)}$ are the surge and sway components of $\hat{\mathbf{F}}_{j,i}^{(m)}$, and $\hat{F}_{j,i}^{(m)}|_{i=3,4,5,6} = 0$.

The above equations describe the joint constraints inside a FA/FB. The whole system may float freely on the water surface or be anchored to a fixed structure. Thus, extra constraint equations are included into the system equations to represent the moorings, where applicable. For cases 3, 5b and 6, we assume that the moorings are attached to the first floater of each column in a FB and that the surge motion of this floater is zero; both previous restrictions can then be written as

$$\hat{\xi}_1^{((i-1)MR+1)} = 0 \quad \text{and} \quad \hat{F}_{jc,3}^{((i-1)MR+1)} = 0, \quad (10)$$

where i is the column of the FB.

The motion equations in Eq. (5) together with the constraint equations in Eqs. (6), (7) and (10) constitute a system of algebraic equations with ξ , $\hat{\mathbf{F}}_{jc}$ being unknown. Such a system of equations in matrix form can be efficiently solved numerically using tools such as Matlab.

2.4. Performance evaluation

The performance of the WEC is generally evaluated by the capture power of the PTOs. According to Falnes [16]; the time-averaged capture power of a linear PTO can be calculated from the displacement

$$P^{(m)} = \frac{1}{2} \omega^2 C_{pto}^{(m)} |\hat{\xi}_3^{(m)}|^2, \quad (11)$$

such that the overall capture power is obtained by summing up the power of the individual PTOs

$$P_s = \sum_{m=1}^M P^{(m)}. \quad (12)$$

In this model, each floater can vary its PTO setting to respond to the incident waves by suitably adjusting $C_{pto}^{(m)}$ and/or $K_{pto}^{(m)}$ in Eq. (4). Theoretically, there are optimal PTO configurations in the FB, which maximise the energy extraction under the incident wave for a specific frequency. For an isolated floater with one DOF, there exists an optimal PTO damping which can maximise the capture power $\bar{P}$ [16]:

$$C_{pto}^{opt} = \left[B^2 + \omega^2 \left(M + A - \frac{K_{hs}}{\omega^2} \right) \right]^{\frac{1}{2}}. \quad (13)$$

However, Eq. (13) is not directly applicable for a FA/FB comprised of multiple bodies and, therefore, with multiple modes of motion. Determining such optimal PTO configurations is challenging and would require the solution of a complicated optimisation problem of maximising Eq. (12) subject to the motion equation in Eq. (5) and the constraints in Eqs. (6)–(10), which is outside the scope of this paper. Alternatively, we apply the uniform PTO damping coefficient on all the floaters. Hence, $C_{pto} = C_{pto}^{(1)} = \dots = C_{pto}^{(M)}$ is chosen in order to maximise the overall capture power of the system for each given wave frequency, implemented numerically by using Matlab's *fminbnd* function. For further simplicity, we neglect the spring effect of the PTO in the model, i.e., we set $K_{pto} = K_{pto}^{(1)} = \dots = K_{pto}^{(M)} = 0$ for all simulations.

To compare the performance of the WECs with different dimensions, it is convenient to use the capture factor for such an assessment. The capture factor is defined as the ratio between the capture power of the device and the power of the incident wave per unit width of the device, i.e.,

$$CF = \frac{P_s}{\frac{1}{2} \rho g A_w^2 c_g D_c}, \quad (14)$$

where ρ is the water density, which can be found in Table 2; A_w is the amplitude of the incident wave; c_g is the group velocity of the incident wave; and D_c is the characteristic dimension of the device, which is shown in Table 1 for each scenario.

The constructive or destructive body-to-body interaction of a WEC array can significantly impact the overall capture power of the system, which can be quantified by the q -factor [17]. The q -factor expresses the average capture power of a FA/FB compared to the capture power of an isolated floater in open sea,

$$q = \frac{P_s}{M P_l}, \quad (15)$$

where M is the number of the floaters in an array; and P_l is the capture power by an isolated floater. Accordingly, we use the power capture of *case1* as P_l in order to perform the calculation of Eq. (15) for all the other scenarios.

In order to verify the proposed numerical model, the results of the FD model are compared against the results produced by our recently developed time-domain model [2], which is based on the open-source WEC-Sim code [18], a block diagram model developed based on Simulink® and Simscape Multibody™. The hydrodynamic coefficients required for the model — $\mathbf{A}$, $\mathbf{B}$ and $\hat{\mathbf{F}}_e$ — are numerically obtained from NEMOH. For validation purposes, a case of a floater column consisting of ten hinged floater elements is simulated and good agreement is obtained from the comparison of the RAOs and capture power, which can be found in Wei et al. [19]. Hence, the present model is verified and can be further employed to investigate the more complicated cases, e.g., multiple column and row FAs and FBs.

3. Results

To understand the hydrodynamic behaviours of the FB, we investigate the series of scenarios described in Section 2.1. The numerical results of the capture power, optimal damping coefficient and RAO for each scenario are presented in this section.

3.1. Energy extraction

The main determinant of a WEC's viability is its performance with respect to harvesting energy. The capture factor corresponding to various scenarios of single floater/FA/FB are presented in Fig. 3. Since the capture factor defined in Eq. (14) is based on the width of the device, this may be an improper representation of reality for specific FB configurations; hence, we excluded *cases3*, 6 and 7 from the analysis. The single floater device has the lowest capture factor for either low frequency waves (*case1*) or high frequency waves (*case2*). For *case1* and *case2*, due to the small inertia and large buoyancy restoring force, the floaters' resonant frequency is quite high, thereby exhibiting a heaving point absorber behaviour. The large floater in *case2* can be resonated at low frequency, but it is hardly excited by the high frequency waves because of its large inertia (note that such a large floater cannot be considered as a point absorber, if reasoning similar to that applied to the Oyster

Fig. 3. Comparison of the capture factor in various scenarios.

WEC in Renzi et al. [20] is to be followed). If a large floater—such as the one in *case2*—is separated into a FB with ten wide floaters—as in *case4*—, then the energy extraction can be improved considerably. When the wide floaters in *case4* are further decomposed into square floaters as shown in *case5a*, the overall capture power can be slightly increased. This suggests that the FB significantly outperforms the single floater of equal volume. It can be observed that there is little difference between the capture factors of the various FB configurations with different constraints (*cases5a*, *b* and *c*) at low frequency. The deviation increases with frequency, and the case of hinges within a column only (*case5b*) can capture slightly more energy than the case connected with springs (*case5a*); in contrast, the case where only heave is allowed (*case5c*) captures the least amount of energy. Nonetheless, for a wide range of wave frequencies, the overall performance of a FB is very promising, since the capture factor is greater than unity and increases to a value of 1.35.

The optimal uniform damping coefficients in various cases are shown in Fig. 4. Clearly, the larger floater requires a more powerful PTO system. The damping coefficients in *case2* are two orders of magnitude greater than all the cases with a small-size floater. The minimum damping coefficient of *case2* is found at $\omega \approx 0.72$ rad/s, which is its natural frequency. Since the natural frequency of a small single floater in *case1* is beyond the range of frequency of interest, the optimal damping coefficients decrease monotonically with frequency. Similarly to *case1*, the optimal damping coefficients in FAs/FBs at low frequency are one order of magnitude greater than that at high frequency. An exception to this is at the occurrence of the jumps, which correspond to the drops observed in Fig. 3. This implies that the adaptability of a PTO system is very important for a FA/FB, in order to extract energy from ocean waves possessing a wide frequency range.

In order to make offshore wave energy harvesting economically feasible, the common approach is to install an array of WEC devices to increase the total installed capacity of a wave farm. A dense WEC array is generally more economical for operation and maintenance than a sparse one. However, the wave interaction between the WECs may have a destructive impact to the energy output, which is demonstrated in Fig. 5 by means of the q -factor for some cases. The body-to-body interaction generally exhibits a negative effect on the overall performance, except for *case3* at high frequency, where the q -factor is greater than 1. For the other cases, the q -factor is close to 1 at the low frequency range, but decreases for high frequencies. This negative impact becomes more significant for denser arrays. The q -factor of the FA consisting of 25 floaters—*case7*—is approximately 0.8, whereas the q -factor of a compact array like the

Fig. 5. Comparison of q -factor of various FAs and FBs (the results of the single floater in *case1* is employed as the reference).

FB consisting of 100 floaters—*case5*—decreases to approximately 0.3. It is emphasised that the present model uses a uniform damping coefficient for all the PTOs. This suggests that the PTO configuration can be optimised in the future to improve the FA/FB performance, i.e., increase the q -factor. Another interesting finding is that there are notches in the magnitude plot in the cases of FAs and FBs at high frequency, e.g., at $\omega \approx 1.5$ rad/s for *cases5a* and *b*. Such notches may be caused by the body-to-body interaction cancelling the motion of several floaters, resulting in the negative performance of the system. This phenomenon is further discussed in Section 5.

3.2. Hydrodynamic response

To understand the hydrodynamics of the FB, this section analyses the hydrodynamic forces and displacement of the floaters in FAs/FBs. In Fig. 6, the variations of the excitation forces (left) and RAOs (right) in the heave direction of floaters are depicted, corresponding to *case5a*, *case6* and *case7*.

In this figure, the x-axis corresponds to the wave frequency, the y-axis is the row/column number, and the colour represents the average of the results on the floaters in each row/column. For *case5a*, one can see that the excitation forces at low frequency are almost equal along both rows and columns. This happens because the presence of the FB does not sufficiently disturb the pressure field due to the incident waves, and the excitation force is mainly attributed to the Froude-Krylov force, which is the same for all floater elements. As the wave frequency increases, the excitation force presents a deviation, because of the relative importance of diffraction effects under high frequency waves. The force acting on the first row is the maximum of the ten rows, due to the fact that the floaters in the first row are directly facing the incident wave. The following floaters are protected by the first row, and their force declines gradually. The force variation within columns is symmetric, due to the symmetry of the FA/FB. It can be seen that the floaters at the sides (columns 1 and 10) endure a greater excitation force, compared to the force at the centre for low frequencies. However, this effect is reversed at high frequencies. The results of *case6* and *case7* present similar variation characteristics as those of *case5a*, but their excitation forces are stronger because of the sparse layout of the floaters in those two cases, which allow the wave to propagate through the gaps between them.

The left side of Fig. 6 presents the RAOs of the floaters, which give an intuitive impression of the dynamics of the FAs/FBs with wave frequency for various scenarios. In *case5a*, due to the stronger

Fig. 4. Comparison of the optimal damping coefficients for various scenarios.

Fig. 6. Average excitation force (left) and RAO (right) in the heave direction of the floaters along a row/column.

excitation force in the first row and the uniform damping applied on all the floaters, the floaters in the first row have greater displacements than the latter rows, while the RAOs of the central columns are smaller than those of the sides. The RAOs of the last few rows become small at high frequencies, implying that they rarely absorb energy at those frequencies. Although the RAOs of *case6* and *case7* are somewhat similar to those of *case5a*, their variation is less diverse. It is unexpected to see that the RAOs suddenly become close to zero for most of the floaters at $\omega = 1.5$ rad/s, resulting in the poor performance shown in Fig. 3. The same situation occurs for *case6*, with an offset of the frequency at $\omega \approx 1.35$ rad/s. According to these results, it is remarked that, due to the inequality of the excitation force in a FB, there are two disadvantages to applying uniform damping on all the floaters. On one hand, the large displacements in the first row radiate energy, which translates into a loss of harvested energy, while the following floaters are hard to be excited under a weak excitation force and strong PTO damping. On the other hand, the deviation of the RAOs in a FB implies strong internal forces between the floaters that may be harmful to the survivability of the structure. Therefore, it is recommended to perform an optimisation study on the damping—as well as the stiffness—configuration in the FB, although the first priority would be to implement the realistic nonlinear PTO force into the FD model to further examine this behaviour.

In the investigated scenarios, additional constraints which represent the moorings are applied on the first row of the floaters to restrain their surge motion, with the exception of *case5a*, which is a fully free floating FB. Since the floaters are connected into a FB as one body, the surge motion of the floaters is synchronous. The mean RAO in the surge direction of *case5a* is shown in Fig. 7. In this figure, it can be seen that the surge motion is close to one at low frequencies, but its influence on the capture power is insignificant, compared to the cases with surge constraint, i.e., *cases5b* and *c*. Under high frequency waves, its RAO is less than 0.2. Therefore, a light-weight mooring can be adopted for a FB, which may reduce some of the construction costs. Although the q -factor in Fig. 5 shows that the sparse FAs outperform the dense FBs, the sparse ones generally require a complex mooring and anchoring system, whose costs may counteract the gain of the increased total power output. A compact array like a FB can reduce such costs and, hence, may be an economically competitive solution with respect to a wave farm.

4. Effects of constraints

As stated in Section 2.1, the floater elements are connected with

Fig. 7. Mean RAO (surge) of the floaters in a FB.

joints in a FB, which restrain the motion of the floaters in several DOFs, thereby changing the dynamic response of the FB. This section discusses the effects of these constraints—particularly the spring joints—on the energy extraction.

Three scenarios with different constraints—*cases5a*, *b* and *c*—have been discussed in Section 3.1, and the results in Fig. 3 show that the FB with the hinges (or cylindrical joints) within each column outperforms the one connected with springs. We tested a wide range of the spring stiffness K_s in the model, and it was found that using spring joints in a FB always has a negative effect on the overall performance, as the results with a larger spring stiffness get closer to the case where the floaters are joined with hinges, i.e., *case5b*.

On the other hand, increasing the mass of the floaters exhibits positive effects on the overall performance, as shown in Fig. 8 that the capture factor increases significantly when the mass matrix $\mathbf{M}$ is varied from $0.5\mathbf{M}_0$ to $1.5\mathbf{M}_0$. On the other hand, increasing the inertia of the system exhibits positive effects on the overall performance, as shown in Fig. 8, where the capture factor increases significantly when the mass matrix $\mathbf{M}$ is varied from $0.5\mathbf{M}_0$ to $1.5\mathbf{M}_0$. Note that the draft of the floater is assumed to be constant and the hydrodynamic coefficients are the same in these three scenarios.

To physically explain these results, we recall the well-known natural frequency equation for a free floating body

$$\omega_{0,i} = \sqrt{\frac{K_{hs,i,i} + K_i}{M_i + A_{i,i}}}, \quad (16)$$

where i represents the mode of the motion; and K_i is the additional stiffness, usually representing the spring effects of the PTO system, which in our case is zero. Note that the components of the hydrostatic stiffness matrix $\mathbf{K}_{hs,i,i}$ for the surge, sway and yaw directions are all zero, and a revolute joint can be considered as a spring with infinite stiffness between two floaters. As a result, all the floaters in a FB will surge/sway synchronously. Thus, the natural frequency of the FB in surge/sway with the revolute joint is zero, based on Eq. (16). When applying a spring joint in both the surge and sway directions of the floater, each floater comprises a spring-damper system with non-zero natural frequency, based on Eq. (16). Such a system definitely radiates away more energy than the one with zero natural frequency under the same wave condition. This explains why using the spring joints in a FB always results in poorer performance.

A heaving point absorber, e.g., the single floater in *case1*, is

Fig. 8. Comparison of the capture factor of *case5a* with various masses.

Fig. 9. Comparison of the capture factor of case5a with various spring stiffness value.

expected to tune to resonance to enhance the power output. Based on Eq. (16), the natural frequency of the single floater in the present study is far beyond the wave frequency of interest, due to the large hydrostatic stiffness ($K_{hs,3,3} = \rho g A$, where A is the cross section of the floater). Increasing the inertia of the system can explicitly lower the natural frequency, thus increasing the energy extraction. When increasing the dimensions of the floater, as is done, for instance, in case2, both K_{hs} and M increase proportionally with its cross-sectional area but the added mass A grows faster, which dominates the denominator in Eq. (16). Consequently, its natural frequency becomes lower ($\omega \approx 0.72$ rad/s) and varying mass has little influence on its dynamics. A FB can be thought of as a large flexible floater and, therefore, it presents a dynamic characteristic in-between case1 and case2. Thus, increasing the inertia of the system in a FB can improve its overall performance.

The findings above suggest that lowering the natural frequency can increase the energy output for a FB. Mathematically, instead of increasing the inertia of the system, there is another way to lower the resonant frequency: reducing the numerator $K_{hs} + K$ in Eq. (16). Physically, one can submerge the buoys to reduce the heaving hydrostatic stiffness $K_{hs,3,3}$; a successful example of this is the CETO WEC, which is developed by Carnegie Clean Energy Limited. For a floating FB, $K_{hs,3,3}$ is quite large and hard to be adjusted technically.

An alternative may be to introduce a negative stiffness into the system. The results depicted in Fig. 9, demonstrate that using a negative stiffness K_z in the heaving direction results in similar behaviour as increasing the inertia of the system, presented in Fig. 8. The difference between these two settings is that the capture factor increases with a small amplitude oscillation in the case of using negative-stiffness springs.

Although numerical results support our hypothesis about the usage of negative stiffness to enhance the energy extraction, the question arises on how we can design a spring with negative stiffness in a FB. A negative-stiffness spring is not a new concept in the industry, and has been widely used in vibration isolators [21]. Fig. 10(A) shows a typical vertical-motion isolator. It consists of two bars and a conventional spring. The bars are connected to the spring at their inner ends, supported at their outer ends, and loaded in compression by force F . A weight load W deflects the spring to the operating position of the isolator. The resulting stiffness of the isolator can be expressed as $K = K_S - K_N$, where K_S is the spring stiffness and K_N is the magnitude of a negative stiffness which is a function of the properties of the bars and the load F . The negative stiffness can be adapted by changing the load F . Following this idea, the negative-stiffness spring joint can be obtained by connecting two floaters with a compression spring $S1$ at their inner ends and a tension spring $S2$ at their outer ends, as sketched in Fig. 10(B). The two floaters have a relative angle in the unloaded state. When the floaters are placed on the still water, the two-floater system will be loaded with a tension force F applied to the outer ends and a compression force exerted on the inner ends. The hydrostatic stiffness $K_{hs,3,3}$ is equivalent to the stiffness of the conventional spring K_S in Fig. 10(A). Physically, the compression/tension spring stores the potential energy, so that it can actively react to the incident waves, and consequently alter the response of the FB. Finally, a FB can be assembled with the spring system as sketched in Fig. 10(C). One can adjust F to change the system stiffness to accommodate varying sea states, instead of adapting the inertia.

5. Anti-resonance

In general, it is believed that a WEC can absorb maximum energy at its resonant frequency. However, it is reported in the literature that an oscillating system, e.g., a flap-type WEC array [22] and

Fig. 10. Sketch of a FB assembled with negative-stiffness springs.

a modular concept of flap-type WECs [17], may rarely harvest energy from ocean waves at some peculiar frequencies, but such an unexpected phenomenon has not been clearly explained. Sinha et al. [8] investigated tuned arrays of heaving point absorbers and found that destructive interference may be observed at some specific sea state. In the results presented in the previous sections, the significant notches or drops in capture factor are observed in several scenarios in Fig. 3 and the corresponding jumps of optimal damping coefficients at the same frequencies are found in Fig. 4. This section focuses on these phenomena.

To understand such events, we carry out simulations with refinement at the vicinity of those frequencies, and examine the dynamics of the floaters in the FA/FB, as presented in Fig. 11 and Fig. 12. In these figures, plots(A)–(F) show the RAO of all the floaters in the FA/FB at the specific frequencies; plot(G) is the variation of RAO of the floaters with frequency; plot(H) shows the variation of the excitation force; plot(I) presents the optimal damping coefficient; and plot(J) is the capture factor of the FA/FB. In plots(G) and

(H), the floaters at the corners and the centre in the first row are highlighted with thicker lines, and $f(i,j)$ represents the floater in row i and column j . The circles in plots(I) and (J) correspond to the frequencies in plots(A)–(F), respectively.

For the 5×5 FA in case7, it can be observed from plots(A) to (D) that the RAO in the first row decreases, while the RAO in the last row increases. This situation is inverted in plots(E) and (F). Note that plot(D) corresponds to the trough of the capture factor in plot(J). Special attention is paid to the floater $f(1,3)$, which is located at the centre of the first row. Its RAO varies from the local maximum to the local minimum and climbs to the maximum again. A similar behaviour is also observed in its excitation force in plot(G), implying that the dynamics of $f(1,3)$ are dominated by its diffractive properties in this range of the frequency. Because of the uniform PTO configuration, the main contribution of the overall energy extraction is made by those floaters with large RAO. The RAO of the floaters in the first row is generally higher than for those in the latter rows, so their variation has a significant impact on the

Fig. 11. Variation of the RAO, excitation force, optimal damping and capture factor at the frequencies with bad performance (case7).

overall energy extraction. As a result, the capture factor curve of plot(J) also behaves very similarly to the RAO and excitation force curves of those floaters in plots(G) and (H), e.g., $f(1, 1)$ and $f(1, 3)$. In conclusion, the drop of the capture factor in this case is associated with the dramatic change of the excitation force on a portion of the FB, which may be averted by optimising the geometry of the floaters and/or the layout of the array to obtain preferable excitation force distribution.

The RAO variation of the floaters in *case5a* during a drop is analogous to *case7*, as presented in plots(A)–(F) of Fig. 12. Nonetheless, the variation of the RAO in plot(G) is quite messy, compared to that in *case7*. This is caused by the optimal damping searching strategy resulting in a non-smooth jump of the damping coefficient, and is consequently translated into a strong damping force. For instance, a spike in the optimal damping at $\omega = 1.54$ rad/s is seen in plot(I), corresponding to the abrupt drop of the RAO in plot(G) at the same frequency. However, the excitation force declines steadily, as shown in plot(H), which is completely different

from that of *case7*; hence, the sudden fall of the capture factor in plot(J) cannot be easily explained. It is clear that there should be another cause behind this behaviour.

Physically, there are three different mechanisms that govern the energy transformation in a FB: (i) excitation, (ii) radiation, and (iii) connection. The excitation is excluded as the reason, as explained previously. The latter two depend on the motion of the floaters. Mathematically, their forces are represented on the left hand side of Eq. (5), with regard to the motion resonance in the theory of dynamical systems. The motion resonance of a system of oscillators in a wave has been investigated by Renzi and Dias [22]. The authors calculated the natural frequency of the system by solving the system of equations of motion when the damping terms (both radiation damping and PTO damping) and the excitation force are set to zero. They found that the strong out-of-phase oscillations were triggered when the incident wave frequency was close to some special resonance frequency, which enhances the power loss by radiation. Hence, the motion resonance is to be avoided in the

Fig. 12. Variation of the RAO, excitation force, optimal damping and capture factor at the frequencies with bad performance (*case5*).

Fig. 13. RAOs of the floaters in the first row (with radiation damping but no applied PTO damping).

design of a WEC array of closely spaced floaters. Unfortunately, their approach is difficult to apply to the present case for two reasons: (i) constraint equations are introduced into the system equations of a FB, thus it is not straightforward to obtain the characteristic equation and determine its eigenvalues; (ii) BEM-based simulation uses discrete frequencies, and it will be time-consuming to accurately find the eigenvalues that satisfy the implicit non-linear characteristic equation in ω , since this would require recalling the BEM code to calculate the hydrodynamic coefficients. Alternatively, we consider a FA/FB without PTO damping (but still with radiation damping). The reason to do so is that the fall of capture factor in Renzi and Dias [22] and Sarkar et al. [17] is observed to not occur at the exact frequency of the motion resonance, but in-between two resonant frequencies. At that frequency, one of the oscillators settled down with its RAO dropping almost to zero. This phenomenon has been extensively discussed in field of dynamics and vibrations of multi-DOF systems, generally known as an “anti-resonance”, although it is not commonly mentioned in the context of WEC arrays. Since the dynamics of the floaters in the first row have great impact on the system performance (as observed in Figs. 11 and 12), the main focus of our concern is their dynamics, as presented in Fig. 13.

Interestingly, the RAOs in *case7*—Fig. 13(A)— and *case5c*—Fig. 13(D)— oscillate around 1, whereas explicit drops (almost to zero on some floaters) are found at $\omega \approx 1.5$ rad/s in other two cases. Reviewing the results in Fig. 3, three in four of these cases encounter the drops of capture factor, except *case5c*. In broad terms, anti-resonances are caused by destructive interference. In a FA/FB, this can be regarded as being due to cancellation of forces acting on the floaters, for example, external driving forces (excitation), and interaction with other oscillators (radiation and connection). There are no anti-resonances present in the simulations, either with or without PTO damping for *case7*; this further confirms that the drop in the capture factor is caused by the inhibition of excitation at that specific frequency. Conversely, the drop in *case5a* is attributed to the complex body-to-body interaction by means of radiation and interconnection, cancelling out the forces

exerted on those floaters with strong excitation, i.e., in the first row. Note that *case5b* behaves similarly to *case5a*, where the stiffness difference between the two cases (using different joints) results in a visible shift of the anti-resonant frequency in plots(B) and (C). The absence of the drop in *case5c* can be explained due to the different constraints that may alter the system behaviour, i.e., the frequencies of the resonance and anti-resonance.

To nullify or cancel out the effects introduced by the anti-resonance, it is required to alter the inherent properties of the system. Several feasible ways may be applied for such a purpose, e.g., adapting the inertia of the system and/or the stiffness of the joints so that the anti-resonance shifts to an appropriate frequency. Moreover, one could apply a non-uniform damping to the PTO over a FB, to alleviate the negative effects introduced by the anti-resonance phenomenon, either by compensating it with other floaters or by shifting the anti-resonance to a frequency out of the operating range. Finding optimal values for inertia, stiffness and damping coefficients might require solving a complex optimisation problem. Finally, cancellation effects may be smoothed in a realistic sea state. All these aspects require careful consideration in the design of a FB.

6. Conclusions

This paper develops a frequency-domain numerical model of the FB of the OG-WEC. Accordingly, we investigate the hydrodynamic behaviour of the FB described by this model in the open sea. Based on the numerical results, we can draw the following conclusions:

- The performance of a FB consisting of 100 floaters is promising: its capture factor is greater than one (up to 1.35) in a wide range of frequencies, using uniform floaters and PTO settings. The adaptability of the PTO system is very important in achieving such a high performance.
- The excitation force behaves quite unevenly in a FB, which results in the deviation of the RAOs, when applying uniform

damping coefficient on all the floaters. A further optimisation study of damping configurations in a FB is necessary.

- The conventional spring joints in the FB have negative impact on its performance, whereas applying negative-stiffness springs has a positive impact. This effect is analogous to increasing the inertia of the system. The negative-stiffness may become a novel parameter for the optimisation and control of a FB.
- One disadvantageous behaviour of a FB is the occurrence of anti-resonance phenomena, which translates into destructive effects on the energy extraction, particularly when it appears on the floaters with strong excitation. Proper configuration and calibration of the system could eliminate the anti-resonant behaviour.

This work only investigated the hydrodynamics of a FB consisting of 10×10 floaters in the open sea. The present model has its inherent limitations: it is based on linear wave theory and solves the motion equations in the frequency domain; hence, the nonlinearity is hardly accounted for. Particularly, the linear PTO in the present model is significantly different from the MP²PTO system in the OG-WEC concept. The recently proposed non-linear frequency domain model [23] or our time-domain port-Hamiltonian model [24] seems to be a promising approach to account for such nonlinearity and will be the focus of further work. The studied FB configurations employ several simplifications and assumptions, namely: constant mass of the floaters, constant stiffness of the spring joints, and uniform damping coefficients of the PTOs. As these parameters are adaptable during operation, it is worth performing an optimisation study of the FB based on these in the future to explore the upper bound of the energy extraction of a FB.

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