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Published in:
Journal of Physics-Condensed Matter

DOI:
[10.1088/0953-8984/21/17/176002](https://doi.org/10.1088/0953-8984/21/17/176002)

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Document Version
Publisher's PDF, also known as Version of record

Publication date:
2009

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Nenert, G., & Palstra, T. T. M. (2009). Magnetoelectric and multiferroic properties of ternary copper chalcogenides (Cu₂MMS₄)-M-II-S-IV. *Journal of Physics-Condensed Matter*, 21(17), 176002-1-176002-4. [176002]. <https://doi.org/10.1088/0953-8984/21/17/176002>

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Magnetoelectric and multiferroic properties of ternary copper chalcogenides $\text{Cu}_2\text{M}^{\text{II}}\text{M}^{\text{IV}}\text{S}_4$

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Received 13 January 2009

Published 24 March 2009

Online at stacks.iop.org/JPhysCM/21/176002

Abstract

We investigate theoretically the ternary copper chalcogenides with the general formula $\text{Cu}_2\text{M}^{\text{II}}\text{M}^{\text{IV}}\text{S}_4$. This family of compounds can crystallize in two different non-centrosymmetric structures, $I\bar{4}2m$ or $Pnm2_1$. We show that all the reported members of $\text{Cu}_2\text{M}^{\text{II}}\text{M}^{\text{IV}}\text{S}_4$ having the $Pnm2_1$ symmetry exhibit a large spontaneous polarization. This result suggests that several of these materials are likely to be multiferroics since they order magnetically at low temperature. We discuss in detail in the framework of Landau theory the members $\text{Cu}_2\text{MnSnS}_4$ and $\text{Cu}_2\text{MnGeS}_4$ which should present both a linear magnetoelectric effect and multiferroic behavior.

1. Introduction

In recent years, the coupling between magnetic and dielectric properties in transition metal oxides attracted a lot of attention [1–3]. This interest is governed by the emerging of new fundamental physics and potential technological applications [2, 3].

The search for a strong interplay between magnetic and dielectric properties lead to the discovery of a new class of materials based on the perovskite structure RMnO_3 ($R = \text{Tb}, \text{Dy}$) and RMn_2O_5 ($R = \text{rare-earth}$). More recently Sergienko *et al* [4] demonstrated that E-type antiferromagnetic ordering results in a ferroelectric state below the Néel temperature of orthorhombic HoMnO_3 and RNiO_3 ($R = \text{rare-earth}$). These series of new materials are called magnetically induced ferroelectrics. The use of symmetry arguments in order to predict new multiferroic or magnetoelectric materials is not new [5, 6]. The recent investigations for new multiferroic and magnetoelectric materials is largely based on oxides [1–3]. While in the old literature the search for multiferroic and magnetoelectric materials has not been dedicated to a particular family of compounds [7], the recent revival is mostly dedicated to

perovskite related materials [8]. Since multiferroic and magnetoelectric materials are scarce, there is a need to look for other materials than oxides. We have predicted already several new magnetoelectric materials among the fluorides [9]. Several of them are likely to be multiferroic compounds.

In this contribution, we discuss the symmetry properties of the non-centrosymmetric ternary copper chalcogenides $\text{Cu}_2\text{M}^{\text{II}}\text{M}^{\text{IV}}\text{S}_4$. We use symmetry arguments and structural considerations to show that among this piezoelectric family, several of the compounds are possible multiferroics with high polarization. In addition, we use the results of group and Landau theory to discuss in detail the compounds $\text{Cu}_2\text{MnSnS}_4$ and $\text{Cu}_2\text{MnGeS}_4$. We demonstrate the possibility for a magnetic order induced polarization in $\text{Cu}_2\text{MnSnS}_4$. Moreover both compounds possess a polarization which can be switched by a magnetic field (linear magnetoelectric effect).

2. Possible new multiferroic materials in $\text{Cu}_2\text{M}^{\text{II}}\text{M}^{\text{IV}}\text{S}_4$

Polar structures having atomic displacements smaller than 1 Å with respect to a hypothetical non-polar configuration are considered materials with a high probability of having a phase transition into this configuration at higher temperatures. This concept gave rise to the prediction of a large number

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Table 1. Estimated ferroelectric transition temperatures for the candidate multiferroic materials belonging to the $\text{Cu}_2\text{M}^{\text{II}}\text{M}^{\text{IV}}\text{S}_4$ family. **H** is the room temperature space group and **G** is the minimal centrosymmetric supergroup of **H**.

Compound	H	G	$d_{\text{max}}^{\text{rel}}$ (Å)	T_c^{estim} (K)
$\text{Cu}_2\text{MnSiS}_4$	$Pmn2_1$	$Pmmn$	1.02	2600(500)
$\text{Cu}_2\text{MnGeS}_4$	$Pmn2_1$	$Pmmn$	0.942	2250(450)
Cu_3SiS_4	$Pmn2_1$	$Pmmn$	0.926	2200(450)
Cu_3GeS_4	$Pmn2_1$	$Pmmn$	0.935	2200(450)
$\text{Cu}_2\text{FeSiS}_4$	$Pmn2_1$	$Pmmn$	1.11	3000(600)

Table 2. Calculated high symmetry of the paraelectric state of $\text{Cu}_2\text{MnSiS}_4$ with **G** = $Pmmn$ and $a = 7.543$ Å, $b = 6.193$ Å and $c = 6.446$ Å.

Atom	Wyckoff	x	y	z
Cu	4f	1.0024	0.2501	−0.6780
Mn	2b	1/4	3/4	−0.8448
Si	2a	1/4	1/4	−0.1773
S ₁	4f	0.9790	0.749	−0.6645
S ₂	2a	1/4	1/4	−0.8628
S ₃	2b	1/4	3/4	−0.1913

of displacive ferroelectrics [10, 11]. If in addition, these candidates order magnetically, we can predict new multiferroic materials. Various materials among the ternary copper chalcogenides crystallize in the polar symmetry $Pmn2_1$ (No. 31). Among the $\text{Cu}_2\text{M}^{\text{II}}\text{M}^{\text{IV}}\text{S}_4$ family, $\text{Cu}_2\text{FeSiS}_4$, $\text{Cu}_2\text{NiGeS}_4$, $\text{Cu}_2\text{Cu}^{\text{I}}\text{Cu}^{\text{II}}\text{SiS}_4$, $\text{Cu}_2\text{Cu}^{\text{I}}\text{Cu}^{\text{II}}\text{GeS}_4$, $\text{Cu}_2\text{MnSiS}_4$ and $\text{Cu}_2\text{MnGeS}_4$ crystallize in this polar structure [12–16]. If these compounds present a pseudosymmetry with a non-polar symmetry, they are likely to be considered as possible multiferroic materials. We use the program PSEUDO [11] for the search of pseudosymmetry in these materials. A detailed description of the principle of this program can be found in [11, 17].

The procedure for the pseudosymmetry search has been applied to the compounds with symmetry **H** = $Pmn2_1$ in the $\text{Cu}_2\text{M}^{\text{II}}\text{M}^{\text{IV}}\text{S}_4$ family. The search for pseudosymmetry has been performed among all the non-polar minimal supergroups **G** of **H** = $Pmn2_1$ space group with a tolerance limit $\Delta_{\text{tol}} = 1.52$ Å, which restricts the maximal atomic displacement d_{max} to be less than 0.76 Å for supergroups of index 2. We present in table 1 our results. All the pseudosymmetry searches give rise to the non-polar **G** = $Pmmn$ space group as hypothetical high temperature phase. We show in table 2 the description of the structure of $\text{Cu}_2\text{MnSiS}_4$ with **G** = $Pmmn$. We give in figure 1 the relationship between the high temperature hypothetical space group **G** = $Pmmn$ and the room temperature polar **H** = $Pmn2_1$ symmetry.

It is possible to estimate the transition temperature towards the hypothetical high temperature paraelectric phase. For this purpose, we used the relation proposed by Kroumova *et al* [17]:

$$T_c = \alpha (d_{\text{max}}^{\text{rel}})^2 + \beta. \quad (1)$$

In equation (1), $\alpha = 0.22(4) \times 10^4 \text{ K } \text{Å}^2$, $\beta = 300(100) \text{ K}$ and $d_{\text{max}}^{\text{rel}} = d_{\text{max}}^z - d_{\text{min}}^z$. d_{max}^z and d_{min}^z are the values of the maximal and the minimal displacements along the polar axis

Figure 1. Chain of minimal supergroup that relate $Pmn2_1$ to $Pmmn$. We indicate the relation between the basis (**a**, **b**, **c**) of $Pmn2_1$ and the basis of the supergroup $Pmmn$ and the shift of origin **p**.

taken with their corresponding signs. The estimated values of the transition temperature for the candidates for proper ferroelectrics with symmetry $Pmn2_1$ among the $\text{Cu}_2\text{M}^{\text{II}}\text{M}^{\text{IV}}\text{S}_4$ family are given in table 1. The high values for the transition temperature corresponding to the compounds suggest that the paraelectric phase will not be reached before melting or decomposition. This is the case for instance for $\text{Cu}_2\text{MnGeS}_4$ where the melting point has been reported to be $T_m = 1287 \text{ K}$ [18].

The estimation of the transition temperature is made assuming a ferroelectric–paraelectric phase transition described by a displacive type transition. This hypothetical transition can be also of order–disorder type in which case we can not estimate the transition temperature. Consequently, the transition temperature can be lower than the one estimated in the hypothesis of a displacive type transition. If we estimate the transition temperature for the system LiNbO_3 , we find a transition temperature of about 2100 K. This is above the melting point which is of 1530 K. This discrepancy can be explained by the fact that LiNbO_3 is known to exhibit a order–disorder phase transition towards the ferroelectric phase [19]. Moreover, irrespective of the mechanism for the emergence of spontaneous electrical polarization, the possibility of twinning and domain switching may exist.

Additionally, one of the most studied families among magnetoelectric/multiferroic materials is BaXF_4 ($X = \text{Mn, Co; Fe, Ni}$). One of the main characteristics of this family is that its members melt before to reach the transition temperature [20]. Despite the absence of phase transition towards a paraelectric phase, ferroelectric switching has been demonstrated for several members [21]. Moreover several members exhibit rather strong magnetoelectric effect [22]. In light of this family, we can conclude that the melting prior to reach the paraelectric phase does not preclude of the interest and strength of the magnetoelectric coupling. Consequently, the search and investigations of new families of magnetically ordered pyroelectric materials is of prime importance in

Table 3. Estimated spontaneous polarization using an ionic model for the pyroelectric materials for the compounds $\text{Cu}_2\text{M}^{\text{II}}\text{M}^{\text{IV}}\text{S}_4$.

Compound	Spontaneous polarization P_s ($\mu\text{C cm}^{-2}$)
$\text{Cu}_2\text{MnSiS}_4$	76
$\text{Cu}_2\text{MnGeS}_4$	65
Cu_3SiS_4	85
Cu_3GeS_4	81
$\text{Cu}_2\text{FeSiS}_4$	86

the search for new magnetoelectric/multiferroic materials irrespective of their mechanism for electrical polarization.

One of the most important features for large coupling between the dielectric and magnetic properties in multiferroics is the value of the spontaneous polarization displayed by these materials. The spontaneous polarization can be estimated using an ionic model by simplifying the electric charge of each ion with a point charge [23].

$$P_s = \left(\frac{e}{V} \right) \sum_i m_i Q_i \Delta z_i. \quad (2)$$

e is the elementary charge, V the unit-cell volume, m_i the multiplicity of the ion, Q_i the ionic charge and Δz_i the displacement ion along the polar axis. Having discussed already the hypothetical high temperature paraelectric phase, we can estimate the displacement Δz_i for each ion of each material reported in table 1. We present in table 3 the results of these derivations.

The estimation of the spontaneous polarization for the compounds reported in table 3 is very high. These calculated spontaneous polarizations are upper bounds. The ionic model used is a very crude picture which does not take into account the covalency of the bonds. Additionally, the calculation is strongly dependent of the accuracy of the z coordinate for which only reasonably good data exist for $\text{Cu}_2\text{MnSiS}_4$ and $\text{Cu}_2\text{MnGeS}_4$ [14]. The crystal structure of the other materials is just a calculated one as reported in the inorganic crystal structure database [15]. For BaAl_2O_4 , the ionic model gives an overestimation of P_s by a factor of six compared to the experimental value [24]. Sulfur is a softer Lewis base than oxygen, and covalency effects are more dominant than for oxides. This should reduce further the polarization of a ionic point charge model. Taking into account the overestimation, the calculated spontaneous polarizations remain high and can be compared with the best ferroelectrics such as BaTiO_3 ($P_s = 26 \mu\text{C cm}^{-2}$) [25].

3. Phenomenological description of the antiferromagnetic ordering in $\text{Cu}_2\text{MnSnS}_4$

3.1. Introduction

Various members of the $\text{Cu}_2\text{M}^{\text{II}}\text{M}^{\text{IV}}\text{S}_4$ family exhibit long range magnetic order at low temperature. Among the materials having the $I\bar{4}2m$ symmetry, $\text{Cu}_2\text{FeSnS}_4$ (Stannite) orders below $T_N = 6.1(2)$ K [26] and $\text{Cu}_2\text{CoGeS}_4$ below $T_N = 25$ K [27]. For the pyroelectric members of this

Figure 2. Magnetic structure of $\text{Cu}_2\text{MnSnS}_4$ in the (a, c) plane. Arrows indicate the magnetic moments on the Mn^{2+} ions.

family we have, for instance, $\text{Cu}_2\text{NiGeS}_4$ which orders below $T_N = 36$ K [27]. However, only two members of the $\text{Cu}_2\text{M}^{\text{II}}\text{M}^{\text{IV}}\text{S}_4$ family have been investigated in detail both for their magnetic and their structural properties. These members are $\text{Cu}_2\text{MnSnS}_4$ and $\text{Cu}_2\text{MnGeS}_4$ [28, 12, 13]. We will discuss in detail in this section the magnetic properties of $\text{Cu}_2\text{MnSnS}_4$.

$\text{Cu}_2\text{MnGeS}_4$ crystallizes in an orthorhombic wurtzite superstructure type ($Pmn2_1$) while $\text{Cu}_2\text{MnSnS}_4$ crystallizes in a tetragonal sphalerite superstructure type ($I\bar{4}2m$). Wurtzite (ZnS) is based on a hexagonal closed packed structure of S, while sphalerite (ZnS) is based on a cubic closed packed structure of S. In both structures the metal ions occupy half of the tetrahedral sites. It has been shown that the relation of the sizes of the different tetrahedra plays the most important role with respect to the resulting structure type [14]. In the wurtzite superstructure type, the largest polarization contribution originates from the off-centering of the Cu^+ ion from the barycenter of the coordinating sulfur tetrahedron.

Several authors have investigated the magnetic structure of $\text{Cu}_2\text{MnSnS}_4$ [28, 12, 13]. The compound $\text{Cu}_2\text{MnSnS}_4$ crystallizes in the space group $I\bar{4}2m$ (No. 121) where the Mn^{2+} ions occupy the Wyckoff position 2a. This compound presents an antiferromagnetic structure characterized by a propagation wavevector $\vec{k} = (1/2, 0, 1/2)$ below $T_N = 8.8$ K. A representation of the magnetic structure is presented in figure 2.

Since the Mn^{2+} ions occupy the Wyckoff position 2a, there are two Mn atoms per unit cell: the spin at $(0, 0, 0)$ (carrying moment $\vec{S}_1$) and at $(1/2, 1/2, 1/2)$ (carrying moment $\vec{S}_2$). Consequently, we can define two magnetic vectors:

$$\vec{M} = \vec{S}_1 + \vec{S}_2 \quad \vec{L} = \vec{S}_1 - \vec{S}_2. \quad (3)$$

The little group of the $\vec{k}$ -vector can be derived using the program REPRES of the Bilbao Crystallographic Server [29]. The little group contains only two symmetry elements: the identity 1 and a two fold axis along the y axis. We denote them $\mathbf{h}_1$ and $\mathbf{h}_2$, respectively. The expressions of these two

Table 4. Irreducible representations of the little group where R represents the time-inversion operator.

	h_1	h_2	Rh_1	Rh_2
τ_1	1	1	-1	-1
τ_2	1	-1	-1	1

Table 5. Basis vectors for the atoms of the 2a site (Mn^{2+} ion site).

Basis vectors	x	y	z
τ_1	L_x	L_z	M_y
τ_2	L_y	M_x	M_z

symmetry elements are

$$\mathbf{h}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad \mathbf{h}_2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

The little group of the $\vec{k}$ -vector has two allowed irreducible representations. We notice that they are the same as the ones of point group 2 (see table 4).

In order to write down the general expression of the free energy of our system, we investigate the transformation properties of $\vec{L}$ and $\vec{M}$ under the elements of the little group. We present the results of our derivations in table 5.

3.2. Magnetically induced polarization in $\text{Cu}_2\text{MnSnS}_4$

3.2.1. Magnetic order induces spontaneous electrical polarization. The results of table 5 allow us to construct the Landau free energy [30]. We use here the notations for collinear structures described in equation (3) and the magnetic modes defined in table 5. The magnetic structure below T_N is described by the irreducible representation τ_1 . This irreducible representation is characterized by the magnetic modes L_x , L_z , and M_y . The various coupling terms between the various order parameters are derived from symmetry considerations. Every product of the axial vector components belonging to the same irreducible representation is invariant by time reversal and the crystal symmetry [30]. We call η , the order parameter which describes the antiferromagnetic ordering below T_N . This order parameter η has the same transformation properties of the components L_x and L_z . From these considerations, we can write the free energy in its simplest form as

$$F = F_0 + \frac{a}{2}\eta^2 + \frac{b}{4}\eta^4 + \sigma\eta M_y + \frac{P^2}{2\alpha} + \beta\eta^2 P_y + \frac{c}{2}M^2 + \lambda_1\eta M_y P_y. \quad (4)$$

In equation (4) the coefficient a becomes zero at T_N . Its temperature dependence is assumed to be $a = a_0(T - T_N)$. The only non-zero magnetization and polarization components are M_y and P_y . By minimizing F , one can work out the expression of these components. We can replace P_y by its expression in function of M_y and vice versa. We find

$$P_y = \frac{\alpha\eta^2(\beta c - \sigma\lambda_1)}{c - \alpha\lambda_1^2\eta^2} \quad M_y = \frac{-\eta(\sigma - \alpha\beta\lambda_1\eta^2)}{c - \alpha\lambda_1^2\eta^2}. \quad (5)$$

We notice that M_y varies as η as function of temperature while P_y varies as η^2 . In addition, we can check that there is no polarization above T_N in agreement with the non-polar structure of the compound. While a ferromagnetic component is allowed along y , this has not been observed in the neutron diffraction experiment and neither by magnetometry. It has been shown that the magnetization along the c axis is lower than perpendicular to it. This result is in agreement with a weak ferromagnetic component perpendicular to the c axis (ferromagnetic component M_y is expected to be along the b axis).

Now that we obtained the expressions for M_y and P_y , we can find the expression of η . For this purpose, we use the results of $\frac{\partial F}{\partial \eta} = 0$ and the expressions of M_y and P_y given in equation (5). We find a complicated expression:

$$\eta[-\alpha\lambda_1^2\eta^4(b - \alpha\beta^2c + \alpha\beta\lambda_1) + \eta^2(bc + 2\alpha\beta^2c - \alpha\lambda_1(a\lambda_1 + \beta\sigma + \beta\sigma c - \lambda_1\sigma^2)) + ac - \sigma^2] = 0. \quad (6)$$

From equation (6), we can describe the paramagnetic high temperature phase by $\eta = 0$. Otherwise, we have to solve an equation of the 2nd degree by substituting $x = \eta^2$.

If we assume that λ_1^2 are small in the equation (5), one can simplify the expression of equation (6). In this hypothesis, we obtain

$$\eta[ac - \sigma^2 + c(b + 2\alpha\beta^2)\eta^2] = 0$$

$$\eta = 0 \quad \text{or} \quad \eta^2 = \frac{\sigma^2 - ac}{c(b + 2\alpha\beta^2)}. \quad (7)$$

In this case (assuming the λ_1 coupling term negligible), the expressions for the associated polarization P_y and magnetization M_y are

$$P_y = \frac{\alpha\beta(\sigma^2 - ac)}{b + 2\alpha\beta^2} \quad M_y = \frac{-\sigma}{c} \sqrt{\frac{\sigma^2 - ac}{c(b + 2\alpha\beta^2)}}. \quad (8)$$

We see from equations (7) and (8) that η varies as $\sqrt{T_N - T}$, like M_y while P_y varies as $(T_N - T)$. We have been describing here the magnetic ordering of CuMnSnS_4 using symmetry analysis. We have shown that below T_N a polarization along y is allowed. In addition, symmetry allows a ferromagnetic component along y which has not been observed experimentally likely due to a too weak coupling between the order parameter η and M_y .

3.2.2. Linear magnetoelectric effect in $\text{Cu}_2\text{MnSnS}_4$. In addition to magnetic order, due to the magnetic point group 2, this compound presents magnetoelectric properties. To study theoretically this effect, several coupling terms have to be considered. In particular, one must take into account the possible magnetoelectric coupling terms which are of the type $\eta M_j P_k$. These terms are the couplings between the magnetic order parameter η , the total magnetization M_j and the electrical polarization P_k . For this purpose, one needs to know the transformation properties of the various components of the electrical polarization. It can be shown that P_x and P_z transform like the irreducible representation τ_2 while P_y transforms like the irreducible representation τ_1 . In addition, we need to add the term $-\mathbf{M} \cdot \mathbf{H}$ (application of a magnetic

field). Consequently, the expression for the free energy in a magnetic field is

$$F = F_0 + \frac{a}{2}\eta^2 + \frac{b}{4}\eta^4 + \sigma\eta M_y + \frac{P^2}{2\alpha} + \beta\eta^2 P_y + \frac{c}{2}M^2 + \lambda_1\eta M_y P_y + \lambda_2\eta M_x P_z + \lambda_3\eta M_x P_x + \lambda_4\eta M_z P_z + \lambda_5\eta M_z P_x - \mathbf{M} \cdot \mathbf{H}. \quad (9)$$

Due to the term $-\mathbf{M} \cdot \mathbf{H}$, this time we need to take into account the different $\eta M_i P_j$ terms since they are different from zero. While the expression for the various components of polarization remain unchanged compared to the case where there is no magnetic field, the expressions for the components of the magnetization change. First we will start by treating the case of the polarization components P_x and P_z . We will demonstrate that under the application of a magnetic field, these components become non-zero. For this purpose, we write down the new expressions for M_x and M_z as function of P_x and P_z . We find for M_x and M_z :

$$M_x = \frac{H_x - \eta(\lambda_2 P_z + \lambda_3 P_x)}{c} \quad (10)$$

$$M_z = \frac{H_z - \eta(\lambda_4 P_z + \lambda_5 P_x)}{c}.$$

After careful derivations, using the results of equation (10) and the expressions of the minima of P_x and P_z ($\frac{\partial F}{\partial P_j} = 0$), we find the rather complicated relationship between these two components:

$$P_x = \frac{-\alpha\eta[\lambda_3 H_x + \lambda_5 H_z - \eta P_z(\lambda_2 \lambda_3 + \lambda_4 \lambda_5)]}{c - \alpha\eta^2(\lambda_3^2 + \lambda_5^2)} \quad (11)$$

$$P_z = \frac{-\alpha\eta[\lambda_2 H_x + \lambda_4 H_z - \eta P_x(\lambda_2 \lambda_3 + \lambda_4 \lambda_5)]}{c - \alpha\eta^2(\lambda_3^2 + \lambda_5^2)}.$$

Using the expressions of equation (11), we find the non-trivial expressions for P_x and P_z :

$$P_x = \{-\alpha\eta[(\lambda_3 H_x + \lambda_5 H_z)(c - \alpha\eta^2(\lambda_3^2 + \lambda_5^2)) - \alpha\eta^2(\lambda_2 H_x + \lambda_4 H_z)(\lambda_2 \lambda_3 + \lambda_4 \lambda_5)]\} \times \{[c - \alpha\eta^2(\lambda_3^2 + \lambda_5^2)][c - \alpha\eta^2(\lambda_2^2 + \lambda_4^2)] - \alpha^2\eta^4(\lambda_2 \lambda_3 + \lambda_4 \lambda_5)^2\}^{-1} \quad (12)$$

$$P_z = \{-\alpha\eta[(\lambda_2 H_x + \lambda_4 H_z)(c - \alpha\eta^2(\lambda_3^2 + \lambda_5^2)) - \alpha\eta^2(\lambda_3 H_x + \lambda_5 H_z)(\lambda_2 \lambda_3 + \lambda_4 \lambda_5)]\} \times \{[c - \alpha\eta^2(\lambda_2^2 + \lambda_4^2)][c - \alpha\eta^2(\lambda_3^2 + \lambda_5^2)] - \alpha^2\eta^4(\lambda_2 \lambda_3 + \lambda_4 \lambda_5)^2\}^{-1}.$$

Despite of the complicated expressions for the polarization components P_x and P_z , there are several features that are important. First of all, if there is no applied magnetic field ($H_x = H_z = 0$), P_x and P_z are zero. This demonstrates that these induced polarization components arise from the linear magnetoelectric effect. Second, these components depend on both H_x and H_z . Let us consider first the P_x component. If we apply a magnetic field along x ($H_z = 0$), we will have a

Figure 3. Temperature behavior of the induced electrical polarization in the hypothesis of a strong magnetoelectric coupling (in black) and in the hypothesis of a weak coupling (in gray).

polarization along the x axis (term α_{11} in the magnetoelectric tensor [7]). If we apply a magnetic field along z , we will have also a polarization along x . This is the α_{13} term of the magnetoelectric tensor for the magnetic point group 2. On the other hand, let us consider now the P_z component. If we apply a magnetic field along z , we have an induced polarization along z . This is the α_{33} term. We get an extra term α_{31} resulting from the application of a magnetic field along x . This is the α_{31} term. We have treated here the polarization components P_x and P_z but not yet the P_y component. We recall that it is along this direction that a spontaneous polarization may arise below T_N . For this purpose, we can take the same approach than for P_x and P_z . After simplification and rearranging the different terms, we find

$$P_y = \frac{\alpha\eta^2(\sigma\lambda_1 - \beta c)}{c - \alpha\lambda_1^2\eta^2} - \frac{\alpha\lambda_1\eta}{c - \alpha\lambda_1^2\eta^2}H_y. \quad (13)$$

The temperature dependence of P_y can be modeled using equation (13). Knowing that η is proportional to $\sqrt{T - T_C}$, we are able to find the temperature dependence of P_y . This is given in figure 3 in the cases of a strong and a weak magnetoelectric coupling.

We notice two features in the expression of P_y . First, the spontaneous part (1st term) is unchanged under magnetic field. Second, the application of a magnetic field along y creates some additional polarization along y (2nd term). This second term is the α_{22} term of the magnetoelectric tensor. We show that the magnetic point group below T_N is 2 which is described by the magnetoelectric tensor given in equation (14).

$$\alpha_{ij} = \begin{pmatrix} \alpha_{11} & 0 & \alpha_{13} \\ 0 & \alpha_{22} & 0 \\ \alpha_{31} & 0 & \alpha_{33} \end{pmatrix}. \quad (14)$$

While $\text{Cu}_2\text{MnSnS}_4$ exhibits the magnetic space group $P_{2a}2_1$ and thus the magnetic point group 2, $\text{Cu}_2\text{MnGeS}_4$ orders below $T_N = 8.25 \pm 0.3$ K [31] with the magnetic space group $P_{2a}c$ [12]. Consequently the magnetic point group describing the pyroelectric material $\text{Cu}_2\text{MnGeS}_4$ is m . By inspection of the table 1.5.8.1 of [6], we see that the magnetic point group m gives rise to the complementary magnetoelectric tensor as in

$\text{Cu}_2\text{MnSnS}_4$ having the expression given in equation (15).

$$\alpha'_{ij} = \begin{pmatrix} 0 & \alpha_{12} & 0 \\ \alpha_{21} & 0 & \alpha_{23} \\ 0 & \alpha_{32} & 0 \end{pmatrix}. \quad (15)$$

We have shown in section 3.2.2 that terms like $\eta M_x P_x$, $\eta M_x P_z$, $\eta M_y P_y$, $\eta M_z P_z$ and $\eta M_z P_x$ were responsible for the linear magnetoelectric effect in $\text{Cu}_2\text{MnSnS}_4$. If we call ξ the magnetic order parameter describing the antiferromagnetic ordering in $\text{Cu}_2\text{MnGeS}_4$, the terms responsible for the linear magnetoelectric effect given by equation (15) are $\xi M_x P_y$, $\xi M_y P_x$, $\xi M_y P_z$ and $\xi M_z P_y$. We stress that the spontaneous polarization existing along z at room temperature can be controlled below the magnetic ordering temperature through the term $\xi M_y P_z$.

Using a Landau theoretical approach we are able to describe the magnetic phase transition of CuMnSnS_4 below T_N . We predict a ferromagnetic component along the y axis which has not been observed experimentally. This is likely due to weak coupling to the order parameter η . More importantly, we show that a spontaneous polarization along the y axis may arise below T_N . This polarization can be tuned via the linear magnetoelectric effect. We gave the expressions of the various components of the polarization with and without magnetic field.

4. Conclusion

We show using symmetry arguments that several members of the $\text{Cu}_2\text{M}^{\text{II}}\text{M}^{\text{IV}}\text{S}_4$ family are promising multiferroic candidates. In addition, we discuss in detail $\text{Cu}_2\text{MnSnS}_4$ which can be considered as a new magnetically induced ferroelectric. Moreover we demonstrate that $\text{Cu}_2\text{MnSnS}_4$ and $\text{Cu}_2\text{MnGeS}_4$ are good magnetoelectric candidates. This means that the polarization in these materials, magnetically induced and spontaneous respectively, can be tuned by this effect. We expect that this contribution will stimulate further experimental investigations of the $\text{Cu}_2\text{M}^{\text{II}}\text{M}^{\text{IV}}\text{S}_4$ family and in particular of the dielectric properties of $\text{Cu}_2\text{MnSnS}_4$ and $\text{Cu}_2\text{MnGeS}_4$.

Acknowledgments

We thank M Mostovoy for stimulating discussions. The work was supported by the Dutch National Science Foundation NWO by the breedtestrategieprogramma of the Materials Science Center, MSC⁺.

References

- [1] Fiebig M 2005 *J. Phys. D: Appl. Phys.* **38** R123
- [2] Eerenstein W, Mathur N D and Scott J F 2006 *Nature* **442** 759
- [3] Cheong S-W and Mostovoy M 2007 *Nat. Mater.* **6** 13
- [4] Sergienko I A, Sen C and Dagotto E 2006 *Phys. Rev. Lett.* **97** 227204
- [5] Goshen S, Mukamel D, Shaked H and Shtrikman S 1969 *J. Appl. Phys.* **40** 1590
- [6] Goshen S, Mukamel D, Shaked H and Shtrikman S 1970 *Phys. Rev. B* **12** 4679
- [6] Dzialoshinskii I E 1960 *Sov. Phys.—JETP* **10** 628
- [7] Authier A (ed) 2003 *International Tables for Crystallography D Physical Properties of Crystals* (Dordrecht: Kluwer Academic)
- [8] Kimura T, Ishihara S, Shintani H, Arima T, Takahashi K T, Ishizaka K and Tokura Y 2003 *Phys. Rev. B* **68** 060403(R)
- [8] Kimura T, Goto T, Shintani H, Ishizaka K, Arima T and Tokura Y 2003 *Nature* **426** 55
- [8] Goto T, Kimura T, Lawes G, Ramirez A P and Tokura Y 2004 *Phys. Rev. Lett.* **92** 257201
- [8] Kimura T, Lawes G, Goto T, Tokura Y and Ramirez A P 2005 *Phys. Rev. B* **71** 224425
- [8] Higashiyama D, Miyasaka S, Kida N, Arima T and Tokura Y 2004 *Phys. Rev. B* **70** 174405
- [8] Hur N, Park S, Sharma P A, Ahn J S, Guha S and Cheong S-W 2004 *Nature* **429** 392
- [9] Nénert G and Palstra T T M 2007 *J. Phys.: Condens. Matter* **19** 406213
- [9] Nénert G 2008 *J. Phys.: Condens. Matter* **20** 335229
- [10] Abrahams S C 2006 *Acta Crystallogr. B* **62** 26
- [10] Abrahams S C 2003 *Acta Crystallogr. B* **59** 541
- [10] Abrahams S C 2000 *Acta Crystallogr. B* **56** 793
- [10] Abrahams S C 1999 *Acta Crystallogr. B* **55** 494
- [10] Abrahams S C, Mirsky K and Nielson R M 1996 *Acta Crystallogr. B* **52** 806
- [10] Abrahams S C 1996 *Acta Crystallogr. B* **52** 790
- [10] Abrahams S C 1990 *Acta Crystallogr. B* **46** 311
- [10] Abrahams S C 1988 *Acta Crystallogr. B* **44** 585
- [11] Kroumova E, Aroyo M I, Perez-Mato J M, Ivantchev S, Igartua J M and Wondratschek H 2001 *J. Appl. Crystallogr.* **34** 783
- [12] Allemand J and Wintenberger M 1970 *Bull. Soc. Fr. Minéral. Cristallogr.* **93** 141
- [13] Allemand J and Wintenberger M 1970 *Bull. Soc. Fr. Minéral. Cristallogr.* **93** 14
- [14] Bernert T and Pfitzner A 2005 *Z. Kristallogr.* **220** 968
- [15] Files PDF 04-005-0277 (Cu_3SiS_4); PDF 04-005-0274 (Cu_3GeS_4); PDF 04-003-8930 ($\text{Cu}_2\text{FeSiS}_4$)
- [16] Nitsche R, Sargent D F and Wild P 1967 *J. Cryst. Growth* **1** 52
- [17] Kroumova E, Aroyo M I and Perez-Mato J M 2002 *Acta Crystallogr. B* **58** 921
- [18] Bernert T, Zabel M and Pfitzner A 2006 *J. Solid State Chem.* **179** 849
- [19] Boysen H and Altorfer F 1994 *Acta Crystallogr. B* **50** 405
- [20] DiDomenico M Jr, Eibschütz M, Guggenheim H J and Camlibel I 1969 *Solid State Commun.* **7** 1119
- [21] Eibschütz M, Guggenheim H J, Wemple S H, Camlibel I and DiDomenico M Jr 1969 *Phys. Lett. A* **29** 409
- [22] Fox D L, Tilley D R, Scott J F and Guggenheim H J 1980 *Phys. Rev. B* **21** 2926
- [23] Abrahams S C 1978 *Mater. Res. Bull.* **13** 1253
- [24] Huang S-Y, Von Der Mühl R, Ravez J, Chaminade J-P, Hagenmüller P and Couzi M 1994 *J. Solid State Chem.* **109** 97
- [25] Wieder H H 1955 *Phys. Rev.* **99** 1161
- [25] Merz W J 1952 *Phys. Rev.* **88** 421
- [26] Caneschi A, Cipriani C, Di Benedetto F and Sessoli R 2004 *Phys. Chem. Miner.* **31** 190
- [26] Bernardini G P, Borriani D, Caneschi A, Di Benedetto F, Gatteschi D, Ristori S and Romanelli M 2000 *Phys. Chem. Miner.* **27** 453
- [27] Guen L and Glaunsinger W S 1980 *J. Solid State Chem.* **35** 10
- [28] Fries T, Shapira Y, Palacio F, Carmen Morón M, McIntyre G J, Kershaw R, Wold A and McNiff E J Jr 1997 *Phys. Rev. B* **56** 5424
- [29] Aroyo M I, Perez-Mato J M, Capillas C, Kroumova E, Ivantchev S, Madariaga G, Kirov A and Wondratschek H 2006 *Z. Kristallogr.* **221** 15
- [29] Aroyo M I, Kirov A, Capillas C, Perez-Mato J M and Wondratschek H 2006 *Acta Crystallogr. A* **62** 115
- [30] Tolédano J-C and Tolédano P 1987 *The Landau Theory of Phase Transitions* (Singapore: World Scientific)
- [31] Shapira Y, McNiff E J, Oliveira N F, Honig E D, Dwight K and Wold A 1988 *Phys. Rev. B* **37** 411