Technical note

The force resulting from the action of mono- and biarticular muscles in a limb

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Abstract

Human and animal limbs can be modelled as a chain of segments connected at joints. For a static limb, the force exerted at the endpoint due to the force of a single muscle has been calculated. It turns out that there are marked differences in the action of mono- vs. biarticular muscles. Monoarticular muscles produce an endpoint force that is directed in the lengthwise direction of the limb, i.e. in the direction of one of the segments. The force from biarticular muscles can have a marked transverse component. The 'principal direction' of this endpoint force is also the movement direction of the endpoint which is the most favourable for the muscle to do work. The reasoning presented can explain e.g. the differences in the activity of mono- and biarticular muscles in cycling. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

In biomechanics and functional anatomy, more attention has been paid to the question, why there are biarticular muscles in the body, next to monoarticular ones. This discussion has markedly been revived by the challenging papers of the late Ingen Schenau and coworkers (Ingen Schenau, 1989; Bolhuis and Gielen, 1997). Several mechanisms have been revealed since: the transport of energy from one joint to the other (Bobbert and Ingen Schenau, 1988) and the efficient handling of movements in which force and movement are not in the same direction (Gielen and Ingen Schenau, 1992).

In this paper, we seek attention to an effect that may provide additional insight. It pertains to the force that is exerted at the end of a closed kinematic chain by single muscles, mono- or biarticular. The model is not purely static, but the assumption throughout will be that the inertial forces and moments can be neglected.

It is easiest to explain the principle at hand of a concrete example, for which we will consider the human leg (Figs. 1a–f). The kinematic chain consists of three segments, thigh, shank and foot, connected at the ankle joint A, the knee joint K and with the hip joint H to the rest of the body. The foot is on the floor, not necessarily flat, as in Fig. 1. The action of the muscles results in a force $F$ that is exerted on the floor, originating in the centre of pressure $P$. In the figures to follow, we will not show the ground reaction force, as measured by a force plate, but its opposite, the action force, exerted by the leg on the ground.

When the masses of the leg segments can be neglected, the moments $M_H$, $M_K$ and $M_A$ around the three joints can be found as the magnitude of $F$ times the perpendicular distances from the joint to the line of action of the force vector, $r_H$, $r_K$ and $r_A$, respectively. In the following, extensor moments will be counted positive.

In the reasoning to follow, it is assumed that only one muscle works at a time. For each muscle, the action force is determined and finally the total action force is the vector sum of the action forces of the separate muscles, now simultaneously active. The situation with only one active muscle is rather

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artificial, of course. In many cases, it would even lead to unstable configurations. In the Appendix it is shown, however, that the action force can also be determined at once for more simultaneously active muscles, so that the instability issue is no matter of concern.

The origin and direction of the $F_H$ originating from a mono-articular hip extensor like gluteus maximus can be found by considering that it generates a moment about the hip joint, but that the moments of $F_H$ with respect to ankle and knee are zero: the line of action of $F_H$ should thus pass through $A$ and $K$ (Fig. 1a). The direction of $F_H$ will be called the ‘principal direction’ of the muscle in the given limb geometry. The magnitude of $F_H$ follows from:

$$F_H = \frac{M_H}{r_H}.$$  (1)

When the hip extensor $m$ has a moment arm $d_{hm}$ with respect to the hip joint, $F_H$ can be expressed in the muscle force $F_m$:

$$F_{Hm} = \frac{d_{Hm}}{r_H}F_m.$$  (2)

**Mono-articular hip flexors** have the same line of action, but now $d_{H}$ should be counted negative, and $F_H$ is directed upward, again in a direct line with AK.

Following the same argument, **mono-articular knee extensors** and -flexors have a line of action through HA (Fig. 1b), and the magnitude of $F_K$ follows from:

$$F_{Km} = \frac{d_{Km}}{r_K}F_m.$$  (3)

Similarly, **mono-articular ankle extensors/flexors** have a line of action through HK (Fig. 1c) and the magnitude of $F$ follows from:

$$F_{Am} = \frac{d_{Am}}{r_A}F_m.$$  (4)

The force due to the action of more than one mono-articular muscle can be found from the vectorial addition of the forces from the participating muscles.

Fig. 1. Action force due to force of: (a) monoarticular hip extensors (e.g. psoas); (b) monoarticular knee extensors (vasti group); (c) monoarticular ankle extensor (soleus); (d) biarticular rectus femoris (hip flexor and knee extensor); (e) biarticular hamstrings (hip extensors and knee flexors), and (f) biarticular gastrocnemius (knee flexor and ankle extensor). The moment arms used in d–f have been given in Table 1.
The same holds for biarticular muscles: the components due to their action around both joints can be added vectorially. For the hip flexor/knee extensor rectus femoris, approximate moment arms as found in the literature are given in Table 1. Applying these, we find an $F$ directed obliquely forward (Fig. 1d), with a line of action passing through the ankle. In a similar way we can find the $F$ for the biarticular hamstrings. It gives an $F$ directed aft and upward (Fig. 1e). The ground force due to the action of the gastrocnemius has a centre of pressure far in front of the foot (Fig. 1f). The implications will be discussed below.

At this stage, already a conclusion can be drawn regarding the different actions of biarticular and monoarticular muscles. With the limb extended, the lines of action of the monoarticular muscles are all directed more or less lengthwise (direction AK, AH or KH). In contrast, the action forces from the biarticular muscles can have a considerable transverse component.

### Table 1

Approximate moment arms for the major human leg muscles (cm). Data are approximate for an average size subject in an erect position. Moment arms for flexion are negative. In reality, moment arms change with joint angle and may be expected to show considerable inter-individual differences. Average values have been chosen for the moment arms of muscles belonging to the same group, like hamstrings and gastrocnemius.

<table>
<thead>
<tr>
<th>Muscle</th>
<th>Hip $d_H$</th>
<th>Knee $d_K$</th>
<th>Ankle $d_A$</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iliopsoas</td>
<td>–2</td>
<td></td>
<td></td>
<td>(Hawkins and Hull, 1990)</td>
</tr>
<tr>
<td>Gluteus maximus</td>
<td>7</td>
<td></td>
<td></td>
<td>(Hawkins and Hull, 1990)</td>
</tr>
<tr>
<td>Hamstrings</td>
<td>6</td>
<td>–6</td>
<td></td>
<td>(Visser et al., 1990)</td>
</tr>
<tr>
<td>Rectus femoris</td>
<td>–4</td>
<td>6</td>
<td></td>
<td>(Visser et al., 1990)</td>
</tr>
<tr>
<td>Vasti</td>
<td>6</td>
<td></td>
<td></td>
<td>(Visser et al., 1990)</td>
</tr>
<tr>
<td>Soleus</td>
<td></td>
<td>5</td>
<td></td>
<td>(Spoor et al., 1990)</td>
</tr>
<tr>
<td>Gastrocnemius</td>
<td>–2.5</td>
<td>5</td>
<td></td>
<td>(Spoor et al., 1990)</td>
</tr>
</tbody>
</table>

2. Work

When a single muscle is active and when the end of the kinematic chain moves with a displacement $\Delta s$, an amount of work is done equal to

$$W = F \Delta s$$

Assuming no friction, this external work must be equal to the work done by the muscle $W_m = F_m \Delta l_m$, so

$$W_m = F_m \Delta l_m = F \Delta s \cos \alpha,$$

in which $\alpha$ is the angle between the directions of $F$ (the principal direction) and $\Delta s$.

One of the roles of the principal direction is revealed here. When a combination of several muscles is active, the total action force, being the vector sum of the separate muscle action forces, can have any specified direction. For every separate muscle, on the other hand, the work done depends on the angle between its own principal direction and the direction of the displacement $\Delta s$: it is greatest when both are in the same direction, zero when the two are perpendicular, and negative (eccentric) when $\Delta s$ is in the half-plane opposite to the principal direction. Fig. 2 gives the relative magnitude of $W_m$ as a function of the direction of $\Delta s$ for the hip extensor muscles; this is a figure-of-eight, consisting of two circles, one for positive and one for negative work.

By combining Eq. (4) with Eqs. (1)–(3), the ‘gear ratio’, the length change $\Delta l_m$ of the muscle per unit displacement $\Delta s$, can be found, e.g. for the hip:

$$\Delta l_m = \frac{d_{hm}}{r_H} \cos \alpha \Delta s.$$
3. Discussion

The main conclusion of this paper has already been stated: while the lines of action of the monoarticular muscles are all directed more or less lengthwise, those of the biarticular muscles can have a strong transverse component. This is especially evident when the leg is almost straight. In that case the monoarticulars generate only very little force and do little work in the transverse direction, which the biarticulars can do.

The effects of gastrocnemius action deserve special mention. When the action force is calculated for this muscle alone, a rather unrealistic result is obtained (Fig. 1f). The calculations in the Appendix give this muscle alone, a rather unrealistic result is obtained. When the action force is calculated for

\[
\frac{p_1}{(p_1 + p_2)} \quad \text{and} \quad \frac{p_2}{(p_1 + p_2)}
\]

are both equal to 0.5. The most recent anatomical data (Spoor et al., 1990) indicate a ratio of knee- and ankle moment arm of also about 0.5 with the knee extended. The result is that according to (A.3) gastrocnemius does not generate a lengthwise force at all. Only a transverse force is generated (A.2) and its main action is that the centre of pressure is shifted forward (A.4). From this it may be predicted that gastrocnemius will hardly ever be active without simultaneous activity from other extensor muscles, soleus or the vasti. As yet we have never observed from EMG that one of the gastrocnemius heads was active without the activity of soleus.

The arguments presented here may help to interpret the findings of (Jacobs et al., 1996) and (Doorenbosch and Ingen Schenau, 1995) in which subjects were instructed to apply isometric forces in various directions. Forces directed forward/ backward with respect to the line HA involved high EMG activity of biarticular rectus femoris and hamstrings, respectively. EMGs of these two muscles were much lower when the force was directed downward, in line with hip and ankle.

Cycling is another instructive example. In general the precise direction of the force exerted on the pedal is quite immaterial, it is in no way necessary to direct the pedal force in the direction tangential to the crank (Doorenbosch et al., 1997). This is only necessary at the upper (TDC) and lower dead centre (BDC). In order for the crank rotation to continue smoothly, at those two instants, a forward (TDC) or backward (BDC) force needs to be applied, which is provided by the activity of rectus femoris (TDC) and hamstrings and gastrocnemius (BDC), respectively.

The theory presented here is rather elementary in the sense that it is restricted to incremental movements only and does not take the segment masses and accelerations into account. A more complete treatment should include a full forward dynamics linked segment model. Some results have been given by Zajac and Gordon (1989).

Their results on gastrocnemius action suggest similar effects as described here.

Acknowledgements

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Appendix A

In this section, an analytical model of the above will be presented. It may serve for calculations and to show that the above simple reasoning is indeed valid. A simpler version has been presented in (Hof, 2000).

In Fig. 3, again a leg is shown with a ground action force \( F \). This force has its centre of pressure a distance \( a \) in front of the line HA. \( F \) is decomposed into a component \( F_p \), parallel to HA, and a transverse component \( F_t \) perpendicular to it. The distance from K to HA is indicated as the ‘knee eccentricity’ \( q \). The perpendicular from K on HA divides HA in sections with lengths \( p_1 \) and \( p_2 \). The centre of pressure is a distance \( p_1 \) below (i.e. in the direction HA) and \( a \) to the right of the ankle A. These and other length measures are indicated in Fig. 3. For the three moments the following equations holds:

\[
M_H = aF_p + (p_1 + p_2 + p_3)F_t,
\]

\[
M_K = (q - a)F_p - (p_2 + p_3)F_t,
\]

\[
M_A = aF_p + p_3F_t.
\]

\[ (A.1) \]

Fig. 3. Diagram used for the analytical calculation of action forces (see Appendix). Action force \( F \) is decomposed in a component in the direction of HA, \( F_p \), and a component perpendicular to HA, \( F_t \). The perpendicular from K on HA has length \( q \), and divides HA into sections with lengths \( p_1 \) and \( p_2 \), respectively. \( p_1 \) is the distance from A to the projection of the centre of pressure on HA. Knee flexion angle is \( \psi \).
This set of equations can be solved to give the ‘output’ \( F_t, F_p \) and \( a \) as a function of the ‘inputs’ \( M_H, M_K \) and \( M_A \):

\[
F_t = \frac{M_H - M_A}{p_1 + p_2}, \quad \text{(A.2)}
\]

\[
F_p = \frac{M_H(p_2/(p_1 + p_2)) + M_K + M_A(p_1/(p_1 + p_2))}{q}, \quad \text{(A.3)}
\]

\[
a = \frac{M_A(p_1 + p_2 + p_3) - M_HP_3}{M_A p_1 + M_K(p_1 + p_2) + M_HP_2} q \approx \frac{M_A}{F_P}. \quad \text{(A.4)}
\]

By inserting \( M_A = d_A F_m \), etc. it can be verified that magnitude and direction of \( F \) agree with the predictions in the first section. It is also seen that the resultant \( F \) is indeed a linear combination of the contributions from the three moments.

When necessary, \( q \) can be computed from geometry as

\[
q = \frac{2 \cdot \text{area(\Delta HKA)}}{\text{length(HA)}} = \frac{l_1 l_2 \sin \psi}{\sqrt{l_1^2 + l_2^2 + 2l_1 l_2 \cos \psi}}, \quad \text{(A.5)}
\]

\( p_1 \) and \( p_2 \) can then be found from \( p_1 = \sqrt{l_1^2 - q^2} \) and \( p_2 = \sqrt{l_2^2 - q^2} \).

References


