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Static and dynamic aspects of the demagnetizing factor in magnetic thin films with random rough surfaces

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We investigate static and dynamic aspects of the demagnetizing factor $N$ in magnetic thin films with random rough surfaces which are described by the rms amplitude $\Delta$, the correlation length $\xi$, and the roughness exponent $H$ ($0 \leq H \leq 1$). The demagnetizing factor decreases as the surface smoothens (increasing $H$ and/or decreasing ratio $\Delta/\xi$), with the exponent $H$ yielding a comparable contribution to $N$ as the roughness ratio $\Delta/\xi$. Moreover, for growing films with self-affine surfaces, $N$ decreases with film thickness, closely as a power law for large roughness exponents ($H \sim 1$). Finally, estimates of the demagnetizing factor based on sinusoidal models are shown to be inadequate since they neglect fine roughness details at short wavelengths ($<\xi$) as depicted by the roughness exponent $H$. © 1999 American Institute of Physics. [S0021-8979(99)03915-8]

I. INTRODUCTION

The magnetic properties on thin films are influenced by a variety of parameters such as film thickness, crystalline structure, composition, preparation conditions, and surface/interface roughness. More precisely, surface/interface roughness influences magnetic properties such as magnetic anisotropy, coercivity, magnetic domain structure, and magnetoresistance. For example, the coercivity of chemically etched NiFeCo films was found to increase with increasing film surface roughness, and with increasing substrate roughness in Cu/Cu(100) films. Other coercivity studies in Ni-Co-alloy films and CoPt/Si$_3$N$_4$ multilayers revealed a more complex dependence on surface roughness. In the diffuse limit in magnetic multilayers, interface roughness enhances the giant magnetoresistance (GMR) when bulk and interface electron roughness scattering is stronger for electrons with spins of the same kind (up or down), while in the opposite case it diminishes GMR.

Magnetic anisotropy and magnetic domain structure of materials used in read/write heads are critical in optimizing the head performance especially at high frequencies. Studies of Co films deposited on plasma etched Co/Si(100) substrates revealed that by increasing surface roughness the uniaxial anisotropy decreased and disappeared for the roughest films. Moreover, with increasing surface roughness the magnetization reversal changed gradually from magnetization rotation which was dominant for smooth films to domain-wall motion that was dominant for the roughest films. Finally, in the same study a decrease of the hysteresis loop squareness was speculated to be related to the increase of the in-plane roughness induced demagnetizing factor.

In thin magnetic films with a relatively high saturation magnetization, even a small surface roughness can produce significant demagnetizing fields, which tend to demagnetize the film in the vicinity of the rough surface unless a strong magnetic field is applied. A measure of the roughness induced demagnetizing effect is represented by the demagnetizing factor $N$. For the etched Co/Si(100) films the in-plane demagnetizing factor was estimated by the formula $N = \pi \Delta^2/4 \xi^2$ with $d$ the film thickness, $\Delta$ the rms roughness amplitude, and $\xi$ the in-plane roughness correlation length. It was shown to increase with etching time or equivalently increasing surface roughness (especially at early etching stages) closely by two orders of magnitude.

Nevertheless, such a formula for the demagnetizing factor was based on a sinusoidal grooved model, and was translated for random rough surfaces by substitution of the groove amplitude and wavelength, respectively, by $\Delta$ and $\xi$. On the other hand, random rough surfaces are in many cases characterized (besides the parameters $\Delta$ and $\xi$) by an additional component, the roughness exponent $H$ ($0 \leq H \leq 1$). The latter quantifies the degree of surface irregularity at short length scales ($r<\xi$), and is associated with a local fractal dimension $D = 3 - H$. For the etched Co/Si(100) films the roughness exponent was measured by atomic force microscopy to be $H \sim 0.93$. Other past x-ray reflectivity and scanning probe microscopy roughness studies of magnetic thin films (including, e.g., eroded Fe films, Fe/Au, NiFe/Au, and Co/Au multilayers) were shown the existence of surface/interface roughness exponents $H$ that span almost the whole range of values $0<H<1$. Therefore, in the present article we will focus on a more precise calculation of the roughness induced demagnetizing factor as a function of all the roughness parameters $\Delta$, $\xi$, and $H$, as well as we will investigate its dynamic evolution with increasing film thickness. Moreover, direct comparisons of the demagnetizing factor with predictions of sinusoidal roughness models (which were used in the past) will be performed.

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II. DEMAGNETIZING FACTOR THEORY FOR RANDOM ROUGH SURFACES

We assume a film of thickness $d$ with homogeneous magnetization $\mathbf{J}_0$. The local magnetization at every point in the film is given by $\mathbf{J}(r,z) = \mathbf{J}_0 [\theta(h(r,z) - \theta(-d - z)]$ with $\theta(x)$ the step function, $\mathbf{r}$ the in-plane two-dimensional vector, and $h(\mathbf{r})$ the film surface fluctuations from flatness ($\langle h(\mathbf{r}) \rangle = 0$). The film/substrate interface is assumed for simplicity to be flat. If $W$ is the magnetostatic free energy, the tensorial demagnetizing factor $N$ is defined by the relation $W = (2\pi a d)\langle \mathbf{J}_0 \cdot \mathbf{N} \cdot \mathbf{J}_0 \rangle$. The diagonal elements of $N$ in the limit of weak roughness ($|\nabla h| \ll 1$) are given to first order by

$$N_{xx} = (4\pi da)^{-1} \times \int \left[ \partial_{x_1} h(\mathbf{r}) \partial_{x_2} h(\mathbf{r})' \right] \frac{d^2r_2}{|\mathbf{r} - \mathbf{r}'|},$$

$$N_{zz} = 1 - (N_{xx} + N_{yy}),$$

with $A$ the average flat film surface, and $\partial_{x_2} h(\mathbf{r})$ the partial derivative with respect to $x$ ($y$). The nondiagonal elements $N_{xy}$ vanish with suitable choice of the $xy$-coordinates system, while the elements $N_{xz}$ will vanish to first order upon ensemble average over possible roughness configurations assuming $h(\mathbf{r})$ to be a Gaussian random variable.

We define the Fourier transform of $h(\mathbf{r})$ by $h(\mathbf{q}) = \int h(\mathbf{r}) e^{-i\mathbf{q} \cdot \mathbf{r}} d^2\mathbf{r}$ which yields the equation $\partial_q h(\mathbf{r}) = \int (-iq_{x_2} h(\mathbf{q}) e^{-i\mathbf{q} \cdot \mathbf{r}} d^2\mathbf{q})$ and assumes translation invariant surfaces or $\langle h(\mathbf{q}) h(\mathbf{q}') \rangle = [(2\pi)^4/A] \langle |h(\mathbf{q})|^2 \rangle \delta^2(\mathbf{q} + \mathbf{q}')$. By considering the identity $\int |\mathbf{r}'|^{-1} e^{-i\mathbf{q} \cdot \mathbf{r}' \cdot d^2\mathbf{r}' = 2\pi q \delta^2(\mathbf{q})}$ and substituting in Eq. (1), we obtain after ensemble average over possible roughness configurations

$$N_{xx} = \frac{1}{2d} \left[ \frac{\Delta^2}{(2\pi)^4/A} \right] \int \langle |h(\mathbf{q})|^2 \rangle d^2(q),$$

with $Q_c = \pi a_0$ an upper roughness cutoff and $a_0$ to the order of the atomic spacing. As Eq. (2) indicates, further calculation of the demagnetizing factor components requires knowledge of the surface roughness spectrum $\langle |h(\mathbf{q})|^2 \rangle$.

III. SURFACE ROUGHNESS MODEL

A wide variety of surfaces/interfaces occurring in nature are well described by a kind of roughness associated with self-affine fractal scaling which was defined by Mandelbrodt in terms of fractional Brownian motion. Examples include the nanometer scale topology of vapor deposited metal films, eroded and fractured surfaces, liquid-gas interface fluctuations, kilometer scale structures of mountain terrain, etc. Physical processes which produce such surfaces/interfaces include molecular-beam epitaxy, erosion, fracture, fluid invasion in porous media, etc.

For self-affine fractals the roughness spectrum $\langle |h(\mathbf{q})|^2 \rangle$ scales as

$$\langle |h(\mathbf{q})|^2 \rangle \propto q^{-2 - 2H} \quad \text{if} \quad q\xi \gg 1$$

$$\text{const} \quad \text{if} \quad q\xi \ll 1$$

with the roughness exponent $H$ being a measure of the degree of surface irregularity, such that small values of $H$ characterize more jagged or irregular surfaces at short roughness wavelengths ($<\xi$). The scaling behavior of Eq. (3) is satisfied by the simple Lorentzian model, $\langle |h(\mathbf{q})|^2 \rangle = [\pi/(2\pi)^2] \langle |\xi|^{2H}/(1 + a^2 \xi^2)^{1/2} \rangle$ with $a = (1/2H)[1 - (1 + aQ_c^2\xi^2)^{-H}]$ if $0 < H < 1$, and $a = (1/2)\ln[1 + aQ_c^2\xi^2]$ if $H = 0$ (logarithmic roughness). Other roughness models which satisfy the scaling relation depicted by Eq. (3) can be found in Refs. 12 and 14.

IV. RESULTS AND DISCUSSION

Equation (2) can be simplified further if we take into account the isotropic nature of the roughness spectrum; $\langle |h(\mathbf{q})|^2 \rangle = \langle |h(q)|^2 \rangle$. Thus, we obtain

$$N_{xx} = N_{yy} = \frac{\Delta^2}{4d \sqrt{a}} \int_{0 < \xi < \sqrt{a}Q_c} x^2(1 + x^2)^{-1-H} dx$$

with $N_{xx} = 1 - 2N_{xx}$. Analytic calculations of the demagnetizing factor can be performed for three characteristic values of the roughness exponent $H$; namely $H = 0$ (logarithmic roughness), $H = 0.5$ (simple ‘Brownian’ roughness), and $H = 1$ (smooth hill—valley roughness). From Eq. (4) we obtain

$$N_{xx} = N_{yy} = \frac{\Delta^2}{4d \sqrt{a}} \left[ \sqrt{a}Q_c \tan^{-1}(\sqrt{a}Q_c) \right] \quad (H = 0),$$

$$N_{xx} = N_{yy} = \frac{\Delta^2}{4d \sqrt{a}} \left[ -\sqrt{a}Q_c \ln(\sqrt{a}Q_c) + \sqrt{a}Q_c \right] \quad (H = 1/2),$$

$$N_{xx} = N_{yy} = \frac{\Delta^2}{4d \sqrt{a}} \left[ (3/2)\tan^{-1}(\sqrt{a}Q_c) - (1/4)\sin[\tan^{-1}(\sqrt{a}Q_c)] \right] \quad (H = 1).$$

For sufficiently large correlation lengths ($\xi Q_c \gg 1$) we obtain from Eqs. (5)–(7) $N_{xx} = N_{yy} \approx (\Delta^2Q_c/4d)$ for $H = 0$, $N_{xx} = N_{yy} \approx (\Delta^2/4d\xi^{3/2}) \ln(2\sqrt{a}Q_c)$ for $H = 1/2$, and $N_{xx} = N_{yy} \approx (3\pi\Delta^2/16\sqrt{a})$ for $H = 1$. Clearly for roughness exponents $H > 0$, the demagnetizing factor depends strongly on the ratio $\Delta^2/\xi$.

As was explained by Schömann, the contribution of higher order terms on the demagnetizing factors can be limited up to 10% with respect to that of the first order term as long as the maximum surface local slope is less than 1. For random rough surfaces the rms local surface slope is given by $\rho = (\langle |\nabla h|^2 \rangle)^{1/2}$, and it has been shown to scale as $\rho \propto \Delta^{1-H}$ for self-affine fractal surfaces. Moreover, as can be seen in Fig. 1, the local surface slope is strongly influenced...
by the roughness exponent $H$ in such a way that it decreases by more than an order of magnitude as $H$ increases from 0 to 1 (especially for small roughness ratios $D/j!).

The in-plane demagnetizing factors $N_{xx(yy)}$ have a trivial dependence on the rms roughness amplitude $\Delta$: namely $N_{xx(yy)} \propto \Delta^2$, while any complex dependence will arise solely from the roughness exponent $H$ and the in-plane correlation length $\xi$. Figures 2 and 3 depict $N_{xx}$ vs $\xi$ and $H$, respectively. The demagnetizing factor is a monotonic decreasing function of the roughness exponent $H$ and the correlation length $\xi$. In other words, roughness induced demagnetizing decreases as the surface smoothens at short and/or long wavelengths ($H \sim 1$ and/or $\Delta/\xi \approx 1$). Moreover, since $N_{xx}$ decreases with increasing $H$, Eqs. (5) and (7) define, respectively, its upper and lower limits; $N_{xx(H=1)} \leq N_{xx(H)} \leq N_{xx(H=0)}$. Finally, comparing Figs. 2 and 3 we can infer that the roughness exponent $H$ has a contribution on the in-plane demagnetization comparable to that of the correlation length $\xi$ or equivalently the ratio $\Delta/\xi$ (for $\Delta$ fixed).

Figure 4 depicts the ratio of $N_{xx}$ and the demagnetizing factor $N^s = \pi \Delta^2 / d\xi$ obtained in the past for a sinusoidal roughness model. Estimation of the demagnetizing factor for random rough surfaces from the formula $N_s$ appears to be closer to the actual predictions for moderate roughness exponents ($H \sim 0.4–0.5$). However, for small ($H \sim 0$) and large ($H \sim 1$) roughness exponents such an estimation can be different by more than an order of magnitude. In addition, as the correlation length $\xi$ increases (or $\Delta/\xi$ decreases, leading to surface smoothing at long wavelengths), the ratio $N_{xx}/N_s$ increases for small roughness exponents ($H < 0.5$), while it decreases for large roughness exponents ($H > 0.5$). Therefore, the effect of the roughness exponent $H$ on the demagnetizing factor should be taken properly into account in order that a more precise estimation of roughness effects on measurable magnetic properties be achieved.

FIG. 1. The rms local surface slope $\rho = (|\nabla h|^2)^{1/2}$ vs the roughness exponent $H$ for $a_0=0.3$ nm, $\Delta=1$ nm, and ratios $\Delta/\xi$ as indicated.

FIG. 2. Demagnetizing factor $N_{xx}$ vs correlation length $\xi$ for roughness exponents $H$ as indicated, $a_0=0.3$ nm, and $\Delta=1$ nm.

FIG. 3. Demagnetizing factor $N_{xx}$ vs roughness exponent $H$ for correlation length $\xi$ as indicated, $a_0=0.3$ nm, and $\Delta=1$ nm.

FIG. 4. Demagnetizing factor $N_{xx}/N_s$ vs correlation length $\xi$ for roughness exponents $H$ as indicated, $a_0=0.3$ nm, and $\Delta=1$ nm. The actual value of the demagnetizing factor for random rough surfaces can differ significantly from that estimated for sinusoidal model predictions.
Indeed, in nonequilibrium film growth it was predicted\(^1\) and verified experimentally\(^9,17\) that the demagnetizing factor decays as a power law \(N_{xx} \propto d^{-(H+\beta-2\beta H)/H}\). For growing self-affine surfaces, \(N\) decreases significantly with film thickness, closely as a power law for large roughness exponents \(H \gg 0.8\) reflecting the dominant dependence on the ratio \(\Delta^2 / \xi\). Finally, analytic calculations of the demagnetizing factor were performed for the characteristic roughness exponents \(H = 0, 0.5, 1\), which can be useful in future studies of roughness induced demagnetizing effects.

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V. CONCLUSIONS

In conclusion, we investigated static and dynamic aspects of the demagnetizing factor for magnetic thin films with rough self-affine surfaces. It was shown that the roughness exponent \(H\) has comparable contribution on the demagnetizing factor \(N\) with that of the long wavelength ratio \(\Delta / \xi\). Indeed, \(N\) can decrease closely by two orders of magnitude if they applied in random surfaces and neglecting the effect of the roughness exponent \(H\).